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Crack Layer Theory

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CRACK LAYER THEORY

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I. INTRODUCTION

Studies of crack propagation and stability have developed in two main directions. One, related to materials science, concerns studies of the hierarchy of microdefects, their nucleation, interaction and development in association with propagation of a main crack. The microdefects are very different by nature depending on material structure and method of observation, specifically the level of magnification chosen. Using a progressively finer scale of observation, a hierarchy of defects can be visualized. For instance, the various elements of damage in the vicinity of a fatigue crack in ASI 301 stainless steel are shown in Fig. 1 [1]. A zone of a large plastic deformation surrounding the crack tip appears under 25X (Fig. 1a). The randomly oriented, "turbulent" field of lines representing the localization of deformation can be distinctly observed under 500X (Fig. 1b). Elements of discontinuity constituting an essential part of overall deformation can be identified under 20,000X (Fig. 1c). Obviously, other details could be seen on intervening magnifications. Individual dislocations, the atomic structure etc can be observed under larger magnification. Which elements of this hierarchy of defects should be parametrized in order to be included into a quantitative model of a crack surrounded by damage? Apparently this question should not be addressed to the materials science only. The continuum mechanics, constituting the foundatiaon of the second main direction in studies of fracture propagation, is addressed as well.

Conventionally in continuum mechanics a crack is considered as an ideal cut in an elastic, elasto-plastic or visco-elasto-plastic medium. The concept of crack-cut with associated surface energy was the first and a very important step in studies of brittle failure. It reflects some essen-

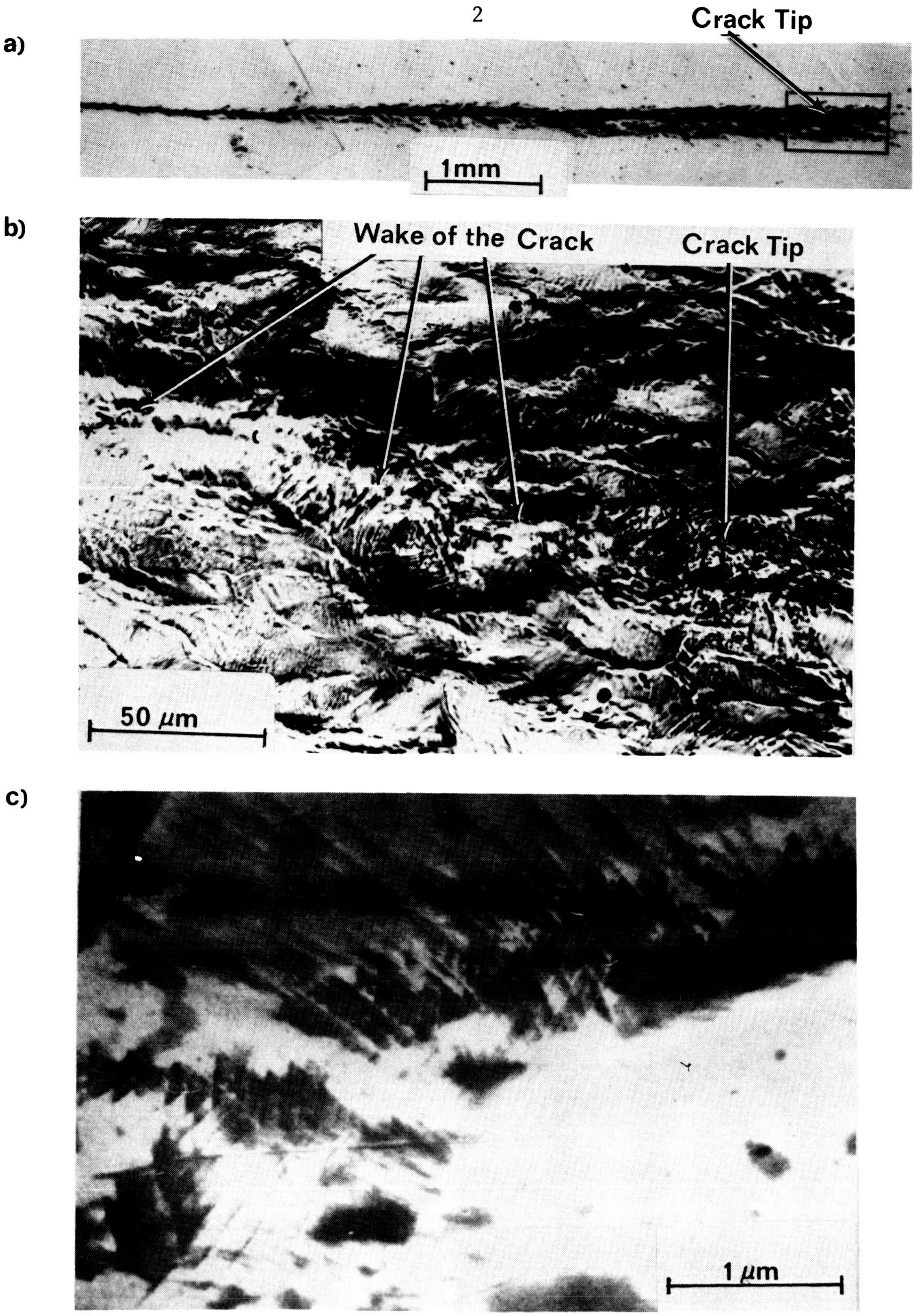

Fig. 1

Morphology of the crack layer on different magnifications.

a - General view of the fatigue crack layer at low magnification.

b - The crack tip region. Extensive damage is seen around and in front of the crack tip.

c - SEM picture of an element of damage from the area in b taken at 20,000x magnification.

tial features of fracture processes and has served as a solid foundation for many engineering and scientific applications. Following this approach, microdefects surrounding the crack are modeled as a plastic zone in a very general sense. Such modeling describes the macroscopic deformation and stress state reasonably well for plastic metals. It does not, however, describe the microstructure of plastic deformation within the "plastic zone" (see for example Fig. 1b, c). For brittle materials such as ceramics and rocks the models of plasticity seem inadequate.

Recent achievements in materials science challenge the continuum approach to model the fracture processes. Obviously the complexity of a hierarchy of interacting defects briefly mentioned above is the main obstacle.

In order to simplify the picture we may examine a crack surrounded by damage under relatively low magnification (Figs. 2,3,4) [1-3]. These micrographs have been obtained from various materials: polystyrene (an amorphous polymer), polypropylene (a semicrystalline polymer) and stainless steel (a polycrystalline metal). The observations [1-3] indicate that under similar loading conditions the global geometry and the evolution of an array of microdefects surrounding a main crack have many similar features for various materials in spite of all the differences in molecular structure and morphology.

Fracture propagation is usually an irreversable process. Hence the general framework of the thermodynamics of irreversable processes can be employed for modeling the phenomenon.

A system of a crack and its surrounding damage is referred to as a crack layer (CL). The theory of crack layer propagation based on irreversible thermodynamics has been proposed in [4,5, 6]. Supporting experimental

FATIGUE CRACK — POLYSTYRENE

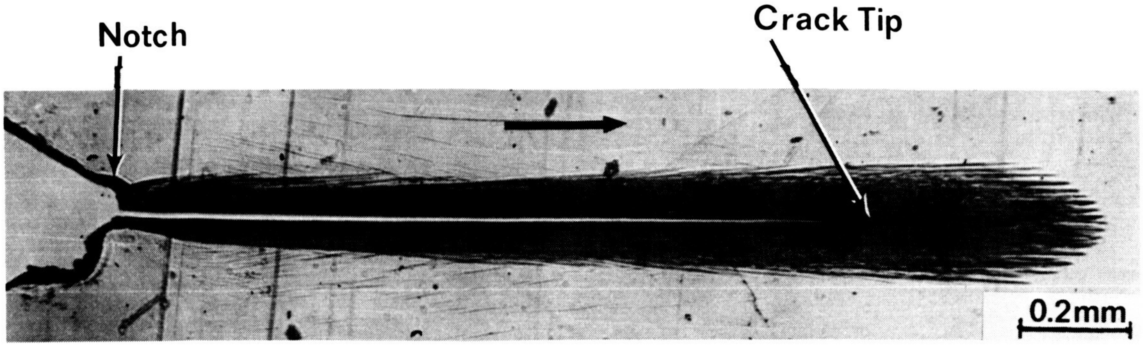

Fig. 2

FATIGUE CRACK — POLYPROPYLENE

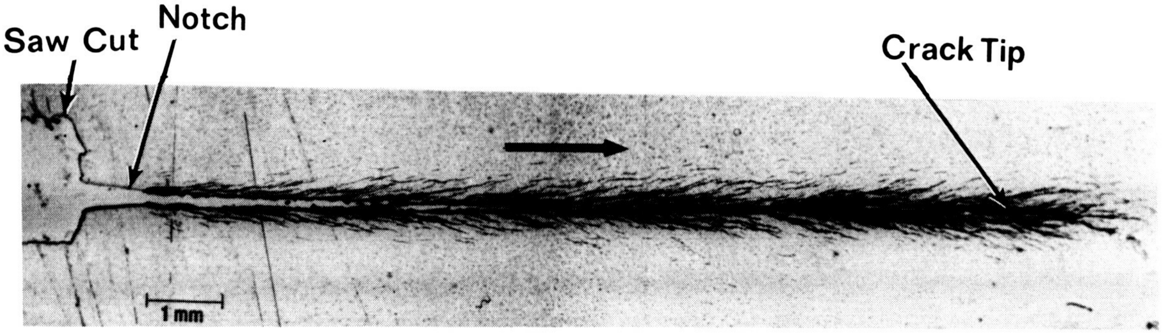

Fig. 3

FATIGUE CRACK — STAINLESS STEEL

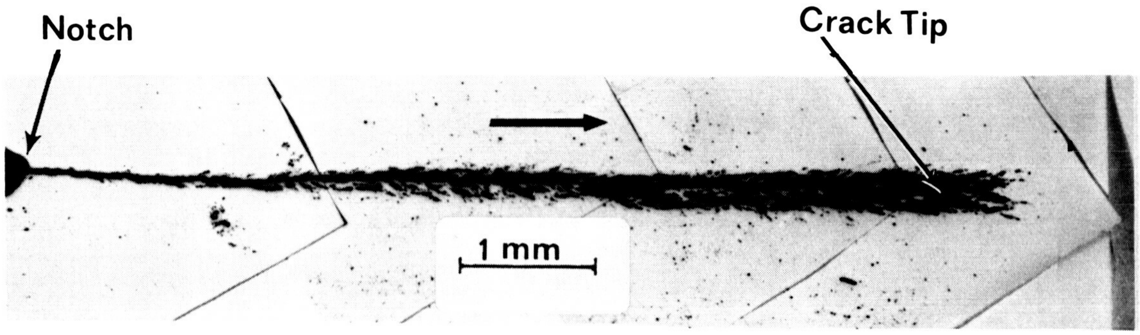

Fig. 4

evidences were reported recently [1-3,7,8].

This work gives a comprehensive presentation of crack-layer (CL) as a model encompassing essential features of fracture propagation in various materials. Relation of the CL model to applied fracture mechanics problems is discussed.

II. Thermodynamics of an Elastic Medium With Damage

Fracture processes consist of nucleation and growth of microdefects. For thermodynamic description of fracture one needs to introduce a list of parameters of state by incorporating a damage parameter. To my knowledge, L.M. Kachanov was the first one who did it explicitly [9]. Since then, numerous papers suggesting various damage parameters and constitutive equations for them have been published. Summaries of some of the proposed damage models can be found in [10,11]. Specific interpretation of a damage is vitally important for establishing a correspondence between experimental studies and a damage model. It is not so important for general thermodynamic analysis, which is presented below. However, it is always useful to have in mind a particular damage model. Therefore, in this section we introduce a damage parameter (P) following [12]. Surfaces supporting the discontinuities within an initially continuous solid are considered as the elements of damage (microcracks, crazes, shear bands, martensite transformation). A damage parameter P is defined as a pairing of scalar damage density ρ (i.e., an area of discontinuity surfaces per unit volume $[\rho] = \frac{m^2}{m^3}$) and damage orientation parameter O: $P = \{\rho, O\}$. Thus, the following system of thermodynamic parameters of state is considered.

$$\{\underset{\sim}{\sigma}, T, P\} \tag{1}$$

Here stress tensor $\underset{\sim}{\sigma}$ and the absolute temperature T constitute a conventional for an elastic medium list of parameters of state. This list is extended by adding the damage parameter P in order to describe the fracture processes. To deduce 'thermodynamic causes' of damage we derive the entropy production associated with damage.

The local energy balance can be written as:

$$\dot{u} = \underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}} - \underset{\sim}{\nabla} \cdot \underset{\sim}{j}^{Q} \qquad (2)$$

Here $\dot{u}$ stands for the rate of internal energy density, $\underset{\sim}{\varepsilon}$ is a strain rate tensor, the product $\underset{\sim}{\sigma}:\underset{\sim}{\varepsilon}$ represents the rate of work density, $\underset{\sim}{\nabla}\cdot\underset{\sim}{j}^{Q}$ gives the rate of internal energy density due to heat transfer, (j^{Q} stands for heat flux).

Considering small deformations, we decompose the total strain tensor into perfectly elastic $\underset{\sim}{\varepsilon}^{(e)}$ (thermodynamically reversible) and nonelastic $\underset{\sim}{\varepsilon}^{(i)}$ (irreversible) parts, i.e.,

$$\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^{(e)} + \underset{\sim}{\varepsilon}^{(i)} \qquad (3)$$

The work done on nonelastic deformation is spent partially on damage and partially by being converted into heat. It can be expressed as follows: $\alpha\, \underset{\sim}{\sigma}:\dot{\underset{\sim}{\varepsilon}}^{(i)}$ is a part of the irreversible work associated with damage nucleation and growth and $(1-\alpha)\underset{\sim}{\sigma}:\dot{\underset{\sim}{\varepsilon}}^{(i)}$ is converted into heat, (α is a phenomenological coefficient).

Let us consider the left part of equation (2). Conventionally, the internal energy density u consists of Helmholtz free energy density f and the entropic part Ts:

$$u = f + T\,s \tag{4}$$

consequently:

$$\dot{u} = \dot{f} + T\dot{s} + \dot{T}s \tag{5}$$

According to the basic concept of irreversible thermodynamics, the time rate of changes of the entropy density $\dot{s}$ can be decomposed into two terms:

$$\dot{s} = \dot{s}_i + \dot{s}_e \quad , \tag{6}$$

where $\dot{s}_i$ stands for the <u>entropy production</u> due to irreversible processes, and $\dot{s}_e$ is the entropy density rate due to exchanges with the surrounding by heat and other kinds of energy. In the equilibrial thermodynamics the entropy increment Δs is defined as the ratio: heat/temperature. In non-equilibrial thermodynamics the heat exchange with the surrounding is usually assumed to be <u>equilibrial</u>. Using this assumption we introduce the equilibrial entropy rate $\dot{s}_e$ in the following form:

$$\dot{s}_e = -\,\underset{\sim}{\nabla}\cdot\left(\frac{1}{T}\,\underset{\sim}{j}^{Q}\right) + \frac{1-\alpha}{T}\,\underset{\sim}{\sigma}:\underset{\sim}{\dot{\varepsilon}}^{(i)} + \Delta s\dot{\rho} \tag{7}$$

Here the first term represents the exchange entropy rate due to the entropy flux $\underset{\sim}{j}^{s} = \frac{1}{T}\,\underset{\sim}{j}^{Q}$, the second term reflects the entropy increases due to the heat generated by the irreversible work $\underset{\sim}{\sigma}:\underset{\sim}{\varepsilon}^{(i)}$ and the third term reflects the rate of the entropy changes due to the localized transformation of the termodynamic state such as cracking, crazing, shear banding, etc. $\Delta s(\underset{\sim}{\sigma},T,o)$ stands for the difference between the entropies of damaged and undamaged matter.

Since the stress tensor and the absolute temperature constitute the conventional part of the list of parameters of state, it is convenient to use Gibbs free energy density g. We define g as the difference between Helmholtz free energy density and the density of the work done on elastic deformation:

$$g = f - \underset{\sim}{\sigma}:\underset{\sim}{\varepsilon}^{(e)} \tag{8}$$

Obviously,

$$\dot{f} - \underset{\sim}{\sigma}:\dot{\underset{\sim}{\varepsilon}}^{e} = \dot{g} + \dot{\underset{\sim}{\sigma}}:\underset{\sim}{\varepsilon}^{(e)} \tag{9}$$

Substituting (3)-(7) into (2), solving energy balance equation (2) with respect to the entropy production $\dot{s}_i$, and taking into account (9) one an find:

$$T\dot{s}_i = \alpha\underset{\sim}{\sigma}:\dot{\underset{\sim}{\varepsilon}}^{(i)} - \dot{g} - \dot{\underset{\sim}{\sigma}}:\underset{\sim}{\varepsilon}^{(e)} - s\dot{T} - \Delta S\dot{\rho} - \frac{1}{T}\underset{\sim}{j}^{Q}\underset{\sim}{\nabla}T \tag{10}$$

For further transformations, we decompose the rate g into two terms:

$$g(\underset{\sim}{\sigma},T,P) = \Delta g(\underset{\sim}{\sigma},T,0)\dot{\rho} + \dot{\pi}(\underset{\sim}{\sigma},T,P) \tag{11}$$

where Δg is the difference between the Gibbs free energy densities of damaged and undamaged matter and π is the elastic potential energy density.

The assumption of local equilibrium yields the following conventional consitutive equations:

$$\left.\frac{\partial\pi}{\partial T}\right|_{\substack{\underset{\sim}{\sigma}\\ P}} = - S \tag{12}$$

$$\left.\frac{\partial\pi}{\partial\underset{\sim}{\sigma}}\right|_{\substack{P\\ T}} = - \underset{\sim}{\varepsilon}^{(e)} \tag{13}$$

We also make use of conventional relationship between the densities of the enthalpy h, Gibbs free energy g and entropy s in the following way:

$$\Delta g + T\Delta s = \Delta h(\underset{\sim}{\sigma},T,0) \tag{14}$$

Then substituting (11)-(14) into (10) and having defined[1] $\frac{\partial h}{\partial P}\dot{P} \overset{\text{def}}{=} \Delta h \dot{\rho}$ the following expression for the entropy production can be obtained:

$$Ts_i = \alpha\underset{\sim}{\sigma}:\dot{\underset{\sim}{\varepsilon}}^{(i)} - \frac{\partial(h+\pi)}{\partial P}\dot{P} - \frac{1}{T}\underset{\sim}{j}^{Q}\cdot\underset{\sim}{\nabla}T \tag{15}$$

The last term in (15) describes the entropy production due to heat transfer which is out of the scope of this paper. In order to concentrate attention on the damage process ($\dot{P}$), we assume an isothermal condition and homogeneity of temperature field ($\underset{\sim}{\nabla}\dot{T} = 0$). Under these conditions the entropy production (15) reduces to two terms, the first $\alpha\underset{\sim}{\sigma}:\dot{\underset{\sim}{\varepsilon}}^{(i)}$ is indirectly associated with damage and therefore is nonzero only within a region where $\dot{P}\neq 0$. The second term represents the entropy production due to damage growth ($\dot{P}$) directly. Therefore,

$$T\dot{s}_i = \alpha\underset{\sim}{\sigma}:\dot{\underset{\sim}{\varepsilon}}^{(i)} - \frac{\partial(h+\pi)}{\partial P}\dot{P} \tag{16}$$

The rate of damage growth $\dot{P}$ may be considered as a thermodynamic flux. Then $-\frac{\partial(h+\pi)}{\partial P}$ is the reciprocal force. The physical interpretation of this force can be done using a simple example. If one visualizes the damage as a field of microcracks with microcrack density ρ then the rate of the potential energy density $\pi(\underset{\sim}{\sigma},T,P)$ with respect to ρ is always negative [12], i.e.

$$-\frac{\partial\pi}{\partial P}\Big|_{\underset{\sim}{\sigma},T} > 0 \tag{17}$$

It means that the contribution of the potential energy density change into the thermodynamic forces is always <u>positive</u>. It can be shown for the same

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[1] See also equations (18) and (44).

conditions that the enthalpy increment Δh as well as the entropy increment Δs due to cracking is always positive, i.e., $\Delta h \geq 0$ [12]. Thus, the thermodynamic force $- \frac{\partial (h + \pi)}{\partial P}$ reciprocal to the rate of damage $\dot{P}$ results from the competition between the driving $(- \frac{\partial \pi}{\partial P} > 0)$ and the resisting $(- \frac{\partial h}{\partial P} < 0)$ parts.

To emphasize the importance of the enthalpy increment for the thermodynamics of failure we introduce <u>the specific enthalphy of damage γ^*</u> with respect to the unit of damage chosen $(\rho \frac{m^2}{m^3})$:

$$\gamma^*(\underset{\sim}{\sigma}, T, 0) \overset{\text{def}}{=} \Delta h (J m^{-2}) \tag{18}$$

The establishment of a constitutive equation for damage growth $\dot{P}$ would be the most desirable goal. This requires an effort to specify the damage parameter P and a wide program of microscopical observation of damage evolution under various loading conditions. However, at the present time we do not have sufficient experimental data to construct a constitutive equation for damage growth on a sound foundation. An attempt to do it, although useful, would be just a speculation with many adjustable parameters involved. In addition, for crack propagation studies, integral characteristics of damage surrounding the crack rather than details of the damage distribution, are important. Therefore, following the spirit of thermodynamics we introduce <u>average characteristics of the entire damage zone</u> including the crack (CRACK LAYER) and constitutive equations for the crack layer without reference to a constitutive law for local damage (P). Obviously, the constitutive equations for the crack layer could be deduced from a constitutive law of local damage. The absence of this law prohibits the stated deduction. The crack layer approach presents a valuable solution to this problem.

III. The Concept of CRACK LAYER

We consider a system of a crack and the surrounding array of microdefects as one macroscopic entity, crack layer (CL). As an illustrative example the trace of fatigue CL propagation in polystyrene [2] is shown in Fig. 5.

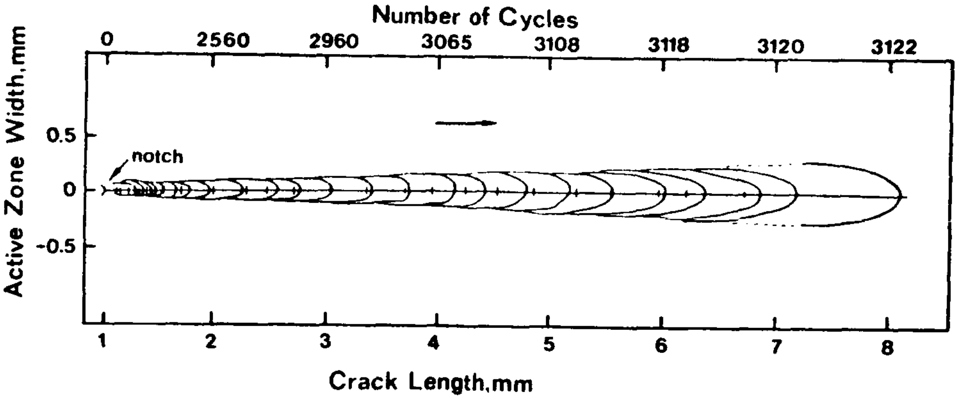

Fig. 5

The vertical markers indicate the positions of the crack tip corresponding to various numbers of cycles. The damaged zone expands in a self similar fashion. The observation of damage (crazes) in the vicinity of the crack tip [2] suggests that the damage distribution appears as a manifestation of actual stress field. Consequently, the similarity criteria for damage distribution are the same as that for the stress field.

Formally, a crack layer is described as zone V_L within which the damage density ρ is positive, i.e.,

$$V_L = \{(x_1, x_2):\rho(x_1, x_2) > 0 \} \tag{19}$$

It is worth noting that a certain level of damage could exist independently of crack propagation. In such a case, a level of reference damage density ρ_o should be determined. The CL is then defined as a zone V_L within which the damage density is above the reference level.

To describe CL propagation, we distinguish active and inert zones (see Fig. 6) within the CL. In a zone adjacent to the crack tip the damage density keeps growing under the influence of stress concentration. The zone V_A, within which the damage density is above the reference level ρ_o and the rate of damage density is positive, is called the active zone (see Fig. 6).

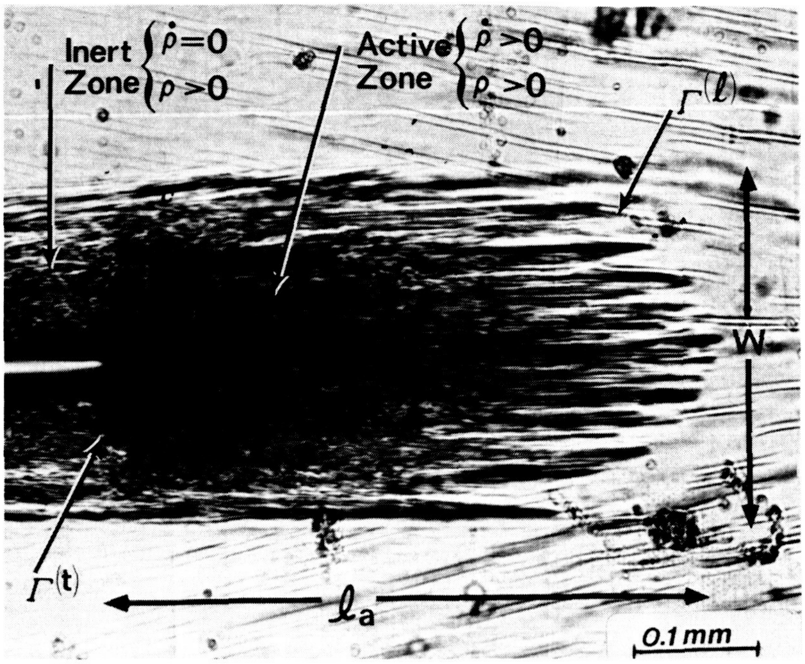

Fig.6

When a crack propagates through the active zone the stresses are released and consequently the process of damage growth practically stops. Thus, the inert zone appears as a trace of the active zone propagation. The inert zone V_I is the part of V_L complementary to the active zone V_A. The active zone boundary ∂V_A can be represented as consisting of the leading $\Gamma^{(\ell)}$ and trailing $\Gamma^{(t)}$ edges (see Fig. 6). The trailing edge $\Gamma^{(t)}$ is defined as the border between the active and inert zones. The leading edge $\Gamma^{(\ell)}$ is part of ∂V_A complementary to $\Gamma^{(t)}$.

A characteristic width w_a and length ℓ_a of the active zone are shown in Fig. 6. Further, for simplicity, we consider the case when both w_a and ℓ_a are small in comparison with the main crack length ℓ, i.e.,

$$\frac{W_a}{\ell} \sim \frac{\ell_a}{\ell} << 1 . \qquad (20)$$

IV. Kinematics of CL Propagation

The mechanism of crack layer propagation can generally be described as follows. At first, damage nucleates and grows to a critical level at which the local instability conditions are met [12]. Then a crack appears within the damage zone. The crack creates a stress concentration which intensifies the processes of damage growth. The damage density in front of the crack reaches a critical level which leads to a crack extension and so on.

The critical level of damage relates to the stresses through the conditions of instability [12]. Since the stress distribution in the vicinity of a crack has an invarient shape with respect to crack length (the crack length just scales the stresses by a stress intensity factor K), the critical level of damage is maintained constant during the crack propagation.

The crack layer propagation can be visualized as an active zone (damage distribution) movement. The latter can be decomposed into translation and rotation as a rigid body and an active zone deformation (i.e. damage dissemination). Considering homogeneous deformation only, one can express the rate of the damage parameter $\dot{P}$ resulting from the active zone movement without explicit time dependency in the following form:

$$\dot{P} = \underset{\sim}{v}\ \underset{\sim}{\delta}^{tr} P + \underset{\sim}{\omega}\ \underset{\sim}{\delta}^{rot} P + \dot{e}\ \delta^{exp} P + \underset{\sim}{\dot{d}} : \underset{\sim}{\delta}^{dev} P \qquad (21)$$

Here v, ω, $\dot{e}$, $\dot{d}$ stand for the rates of translation, rotation, isotropic expansion and deviatoric deformation, correspondingly, and δ^{tr}, δ^{rot}, δ^{exp}, δ^{dev} are the operators of translational, rotational, expansional and deviatoric transformations.

V. Global Entropy Production Due to Crack Layer Propagation

The global entropy production is the integral over the entire volume of a solid V from the entropy production:

$$\dot{S}_i^{global} = \int_V \dot{s}_i \, dV \tag{22}$$

Since the entropy production due to damage growth (16) is nonzero only within the active zone, the integral in (22) is reduced into the integral over V_A. Substituting (21) into (16) and integrating we obtain,

$$T\dot{S}_i^{global} = \dot{D} + \underset{\sim}{v}\cdot\underset{\sim}{X}^{tr} + \underset{\sim}{\omega}\cdot\underset{\sim}{X}^{rot} + \dot{e}\cdot X^{exp} + \underset{\sim}{\dot{d}}:\underset{\sim}{X}^{dev} \tag{23}$$

Here $\dot{D}$ is the part of irreversible work within the active zone spent on damage, i.e.,

$$D = \alpha \int_{V_A} \underset{\sim}{\sigma} : \underset{\sim}{\varepsilon}^{(i)} \, dV = \dot{W}^{(i)} + \dot{Q} \tag{24}$$

where $\dot{W}^{(i)} = \int_{V_A} \underset{\sim}{\sigma}:\underset{\sim}{\varepsilon}^{(i)} \, dV$ is the rate of the total work dissipated within V_A and the rate of heat radiation $\dot{Q} = \int_{\partial V} \underset{\sim}{j}^Q \cdot \underset{\sim}{n} dA$. The thermodynamic forces translational $\underset{\sim}{X}^{tr}$, rotational $\underset{\sim}{X}^{rot}$, expentional X^{exp} and deviatoric $\underset{\sim}{X}^{dev}$ can be presented as follows:

$$\underset{\sim}{X}^{tr} = \int_{V_A} \frac{\partial(h + \pi)}{\partial P} \underset{\sim}{\delta}^{tr} P \, dV \tag{25}$$

$$\underset{\sim}{X}^{rot} = \int_{V_A} \frac{\partial(h + \pi)}{\partial P} \underset{\sim}{\delta}^{rot} P \, dV \tag{26}$$

$$X^{exp} = \int_{V_A} \frac{\partial(h + \pi)}{\partial P} \delta^{exp} P \, dV \tag{27}$$

$$\underset{\sim}{X}^{dev} = \int_{V_A} \frac{\partial(h + \pi)}{\partial P} \underset{\sim}{\delta}^{dev} P \, dV \tag{28}$$

VI. Thermodynamic Forces for the CL

In order to calculate the forces we consider the enthalpic (resisting) and potential energy release (driving) parts separately. Let us start with the driving parts.

1. Calculation of the potential energy release rates. The following transformation can be applied to the elastic potential energy density $\pi(\underset{\sim}{\sigma},T,P)$ using arbitrary operator "δ" and assuming homogeneous temperature field.

$$\frac{\partial\pi}{\partial P} \delta P = \delta\pi - \frac{\partial\pi}{\partial\underset{\sim}{\sigma}}:\delta\underset{\sim}{\sigma} \tag{29}$$

Therefore

$$\int_{V_A} \frac{\partial\pi}{\partial P} \partial P \, dV = \int_{V_A} [\delta\pi - \frac{\partial\pi}{\partial\underset{\sim}{\sigma}}:\delta\underset{\sim}{\sigma}]dV \tag{30}$$

Now one can use (30) for various operators.

(a) The operator of an infinitisimal translation in k-th direction (opposit to the active zone advance) is

$$\delta_K^{tr} = - \frac{\partial}{\partial x_K} \overset{def}{=} - \partial_K \tag{31}$$

Substituting (31) into (30) we obtain

$$- \int_{V_A} \frac{\partial\pi}{\partial P} \delta_K^{tr} P \, dV = \int [\partial_K \pi - \frac{\partial\pi}{\partial\underset{\sim}{\sigma}}:\partial_K\sigma]dV \tag{32}$$

Using the constitutive equation (13) $(\frac{\partial\pi}{\partial\sigma_{ij}} = -\varepsilon_{ij})$, expression (8) written in the following form $\pi = f(\varepsilon^e, T, P) - \sigma_{ij}\varepsilon_{ij}^e$ (f is the elastic strain energy), equations of equilibrium in the absence of mass forces:

$$\partial_j \sigma_{ij} = 0 \tag{33}$$

and the definition of a small deformation

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad , \tag{34}$$

the expression within the brackets in (32) can be simplified:

$$\begin{aligned}\partial_K \pi - \frac{\partial\pi}{\partial\sigma_{ij}} \partial_K \sigma_{ij} &= \partial_K(f - \sigma_{ij}\varepsilon_{ij}^{(e)}) + \varepsilon_{ij}^{(e)} \partial_K \sigma_{ij} \\ &= \partial_K f - \partial_K(\sigma_{ij}\varepsilon_{ij}^{(e)}) + \partial_K(\sigma_{ij}\varepsilon_{ij}^{(e)}) - \sigma_{ij} u_{i,jK} \\ &= \partial_K f - \partial_j(\sigma_{ij} u_{i,K})\end{aligned} \tag{35}$$

Substituting (35) into (32) and using Green's theorem we find

$$-\int_{V_A} \frac{\partial\pi}{\partial P} \delta_K^{tr} P \, dV = J_K \quad , K = 1,2 \tag{36}$$

Where J_1 stands for the conventionally used energy release rate with respect to crack extension in the tangent direction:

$$J_1 = \int_{\partial V_A} [f n_1 - \sigma_{ij} n_j u_{i,1}] \, dT \tag{37}$$

If one chooses a closed contour Γ outside of the active zone V_A, then the integral (37) is identically equal to zero since the left part of equation (36) vanishes (see Fig. 7). The path independency of (37) is direct consequence of this statement. Therefore, instead of $\partial V_A = \Gamma^{(t)} \cup \Gamma^{(\ell)}$ in (37), one can choose a partially arbitrary contour $\Gamma = \Gamma^{(t)} \cup \Gamma^{*}$ (Fig. 7). The integral in (37) is invariant with respect to an arbitrary path Γ^{*}.

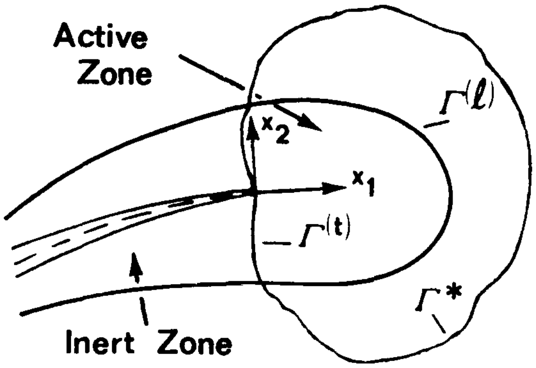

Fig. 7

b). The operator of an infinitesimal rotation is

$$\delta_m^{rot} = \varepsilon_{k\ell m} x_k \partial_\ell + \left(\begin{array}{l} \text{correction for tensor components} \\ \text{associated with rotation of coordinate} \\ \text{system spinorial terms.} \end{array} \right) \qquad (38)$$

where $\varepsilon_{k\ell m}$ is the alternating symbol with components equal to + 1 or − 1 for even and odd permutations of 1, 2, 3 respectively and zero for all other combinations.

Substituting (38) into (30) and providing transformations similar to that in (35), one can obtain:

$$\int_{V_A} \frac{\partial \pi}{\partial P} \delta_m^{rot} P dV = L_m \qquad (39)$$

Here L is a pseudovector (m = 3 for 2-D problems)

$$L_m = \epsilon_{K\ell m} \int_{\partial V_A} [n_K x_\ell f - x_\ell \sigma_{ij} n_j u_{i,K} - \sigma_{\ell i} n_i u_K] d\Gamma \tag{40}$$

For an isotropic medium the path independency of L_m remains similar to that of J_k for homogeneous medium. Consequently, the partially arbitrary contour $\Gamma = \Gamma^{(t)} \cup \Gamma^*$ (see Fig. 7) can be used instead of ∂V_A in (40).

c). The operator of an infinitesimal isotropic expansion is

$$\delta^{exp} = - x_K \partial_K \tag{41}$$

Substituting (41) into (30) one can obtain,

$$- \int_{V_A} \frac{\partial \pi}{\partial \rho} \delta^{exp} \underset{\sim}{p} dV = M \tag{42}$$

where $$M = \int_{\partial V_A} [x_K n_K f_\varepsilon - \sigma_{ij} n_j x_K u_{i,K}] d\Gamma \tag{43}$$

The M-integral (43) possesses the path-invariancy for linear medium only.

The J_1, L_3 and M integrals represent the potential energy release rates with respect to the translation, rotation and isotropic expansion of the active zone. In a similar fashion, a second rank tensor $N_{K\ell}$ can be introduced as the potential energy release due to deviatoric deformation of the active zone [6]. $N_{K\ell}$ is particularly sensitive to deviations in the stress and strain fields near the crack tip. In contrast to $\underset{\sim}{J}$, $\underset{\sim}{L}$ and M, $\underset{\sim}{N}$ does not possess path independency. (See some historical remarks about J, L and M in the appendix.)

Notably, the path independency of $\underset{\sim}{J}$, $\underset{\sim}{L}$ and M renders their evaluation more convenient, it has no physical significance. For instance, for nonhomogeneous, anisotropic and nonlinear medium the equations (37), (40) and (43) express the same potential energy rates as discussed above, but the integrals are not path independent.

2. Calculation of the resistance parts of the thermodynamic forces. In order to calculate the enthalpic part of the thermodynamic forces one needs to specify the damage parameter. We make use of the damage parameter discussed in Section II, i.e. $P = \{\rho , 0\}$, where ρ is the scalar damage density and 0 stands for an average damage orientation. If the orientation of an element of the damage (a plane microcrack, for instance) is described by a vector attached to the center of the element, then the translation and isotropic expansion transform the damage density ρ only and do not affect the orientation 0 . The rotation and the deviatoric deformation affect both the damage density and the orientation.

The change of the enthalpy h due to damage according to (7), (11), (14) and (18) is given by:

$$\frac{\partial h}{\partial P}\,\delta P = \gamma^{*}(\sigma,T,0)\,\delta\rho + \rho\frac{\partial\gamma^{*}}{\partial 0}\,\delta 0 \qquad (44)$$

Here γ^{*} is the specific enthalpy of damage introduced in Section II. The general expression (44) can be used to derive the resistance parts of the thermodynamic forces.

(a) Substituting (44), the expression (31) for the operator of translations into the first term of (25) and keeping in mind that a translation affects the damage density only, one can find:

$$\begin{aligned} - \int_{V_A} \frac{\partial h}{\partial P}\,\delta_K^{tr} P\, dV &= \int_{V_A} \gamma^{*}\, \partial_K \rho\, dV = \\ \gamma^{*}\int_{\partial V} P n_K d\Gamma &= - \gamma^{*} R_K \end{aligned} \qquad (45)$$

where $R_K = -\int_{\partial V_A} \rho\, n_K d\Gamma$ is defined as the translational resistance moment.

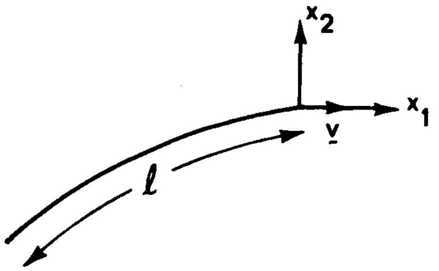

Fig. 8

Let us consider a smooth crack trajectory (Fig. 8) and a local cartesian coordinate system with the origin at the crack tip and the axes ox_1 directed along the tangent to the crack trajectory. Then, the crack speed vector v has only one nonzero component $v_1 (v_2 = 0)$. Accordingly, only one component of the translational resistance moment R_1 is of interest. That is

$$\begin{aligned} R_1 &= -\int_{\partial V_A} \rho n_1 d\Gamma = -\int_{\Gamma(\ell)} \rho n_1 d\Gamma - \int_{\Gamma^{(t)}} \rho n_1 d\Gamma = \\ &- \int_{\Gamma^{\ell}} \rho_0 n_1^{(+)} d\Gamma + \int_{\Gamma} \rho n_1^{(-)} d\Gamma = \underset{\sim}{\tau} \int_{\Gamma^{(t)}} (\rho - \rho_0) dx_2 \end{aligned} \qquad (46)$$

where $n^{(\pm)}$ stands for a unit vector normal to ∂V which has the positive or negative projection on the unit tangent $\tau((n.\tau) \lessgtr 0)$, respectively (Fig. 9).

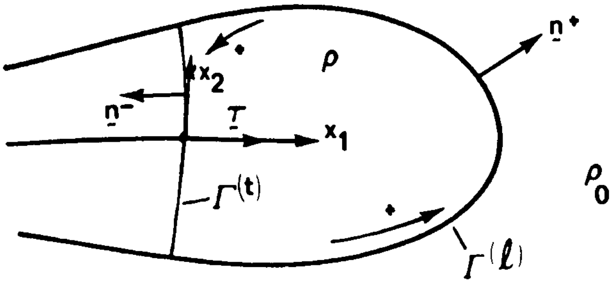

Fig. 9

The path of integration in (46) (the trailing edge) contains a singular point (the crack tip) where the damage density may be singular. Thus, we describe the damage density $\rho(x_2)$ along $\Gamma^{(t)}$ as the sum of a singular $R_1^c\, \delta(x_2)$ and regular $\rho^r(x_2)$ densities, i.e., $\rho(x_2) = R_1^c\, \delta(x_2) + \rho^r(x_2)$, respectively. R_1^c is defined as the core of damage. Substituting $\rho(x_2)$ into (46) and integrating we find:

$$R_1 = [R_1^c + R_1^r] \tag{46a}$$

(b) In a similar fashion substituting (44) and the expression (41) for the isotropic expansion operator into the first term of (25) we obtain,

$$\int_{V_A} \frac{\partial h}{\partial P} \delta^{exp} P \, dV = - \gamma^* \int_{V_A} x_K \partial_K \rho \, dV = - \gamma^* R_o \tag{47}$$

Here the expansion resistance moment R_o is defined as:

$$R_o = - \int_{V_A} x_K \partial_K \rho \, dV = - \int_{V_A} [\partial_K (x_K \rho) - \rho] \, dV =$$

$$- \int_{\partial V_A} x_K \rho n_K \, d\Gamma + \int \rho \, dV \tag{48}$$

In a case when the reference damage density $\rho_o = 0$ the first term on the right hand side of (48) can be neglected. Indeed,

$$\int_{\partial V_A} x_K \rho h_K \, d\Gamma = \int_{\Gamma^{(\ell)}} x_K \rho_o h_K d\Gamma + \int_{\Gamma^{(t)}} x_K \rho h_K \, d\Gamma \sim 0$$

Since $x_1 \sim 0$ and $n_2 \sim 0$ along the trailing edge $\Gamma^{(t)}$. Therefore,

$$R_o = \int_{V_A} \rho \, dV = \langle \rho \rangle \cdot A \tag{48a}$$

where $\langle \rho \rangle$ stands for an average over V_A damage density and A is the area of V_A.

(c) Substituting (44) and the expression (38) for the operator of rotation into the first term of (26) we obtain

$$- \int \frac{\partial h}{\partial P} \delta_m^{rot} P \, dV = - \gamma^* R_m - \frac{\partial \gamma^*}{\partial O} R_m^o \tag{49}$$

Where R_m and R_m^o stand for rotational resistance moments.

Assuming that the enthalpic barrier does not depend on the orientation of damage (a reasonable assumption for an isotropic medium) i.e., $\frac{\partial \gamma^*}{\partial O} = 0$, then (49) can be reduced into

$$- \int_{V_A} \frac{\partial h}{\partial P} \delta_m^{rot} P \, dV = - \gamma^* R_m \qquad (50)$$

For deviatoric deformation of the active zone a similar expression can be obtained:

$$- \int_{V_A} \frac{\partial h}{\partial P} \delta_{k\ell}^{dev} P \, dV = - \gamma^* R_{K\ell} \qquad (51)$$

where $$R_{k\ell} = (1 - \delta_{k\ell})1/2 \int_{V_A} (x_k \partial_\ell + x_\ell \partial_k) \rho dV \qquad (52)$$

and $\delta_{K\ell}$ stands for Kroneker's symbol.

Finally, summarizing the results of this section one can express the CL thermodynamic forces in an index form, which is convenient for practical application.

$$X_1^{tr} = (J_1 + \gamma^* R_1) \, , \qquad (53)$$
$$X_3^{rot} = (L_3 + \gamma^* R^{rot}), \qquad (54)$$
$$X^{exp} = (M + \gamma^* R_o), \qquad (55)$$
$$X_k^{dev} = (N_k - \gamma^* R_{k\ell}), \ k,\ell = 1,2 \qquad (56)$$

It should be noted that the singular damage density $R_1^c \delta(x_2)$ contributes for translational resistance R_1 only, it produces no effects for R_o, R_m^{rot} or $R_{K\ell}$.

VII. Rectilinear Crack Layer Propagation. Single Parameter Model

To analyze the constitutive laws of crack layer propagation we start from the simplest case when all degrees of freedom are frozen except one. We assume that crack layer propagation appears as a translation of the active zone along a rectilinear path with neither deformation nor rotation. Therefore, $\dot{\omega} = 0$; $\dot{e} = 0$; $\dot{\underset{\sim}{d}} = 0$ and $v_2 = 0$ (in terms of the coordinates of Fig. 8). Since the crack trajectory is a rectilinear $v_1 = \dot{\ell}$ where ℓ is the crack length. Using (53) the global entropy production (23) can be rewritten as follows:

$$T\dot{S}_i^{global} = \dot{D} + \dot{\ell}(J_1 - \gamma^* R_1) \tag{57}$$

According to the second law of thermodynamics, the entropy production (global as well as local) is nonnegative and equals zero for reversible processes. This does not mean that the processes causing negative entropy production $\dot{S}_i$ not take place at all. Such processes are well known in chemical thermodynamics, [13,14]. The negative entropy producing processes may occur if some other dissipative processes produce a sufficient amount of entropy to make the total entropy production nonnegative. In this case the rates of entropy consuming processes are controlled by other sources of entropy production. The constitutive law can therefore be obtained from first principles. This is the case for the crack layer propagation.

In order to derive the law of rectilinear CL propagation, we need to analyze the stability of the CL. Assuming that there are no other sources of dissipation except of CL growth, i.e., $\dot{D} = 0$, the global entropy production takes the form

$$T\dot{S}_i^{global} = \dot{\ell}(J_1 - \gamma^* R_1) \tag{58}$$

In classical thermodynamics, the instability condition is the prerogative of the second law. There are no disagreements about the criteria of instability of an equilibrial state. However, it is not so clear what instability criteria should be accepted for an irreversible process. In this study we make use of the "universal criterion of evolution" which has been recently proposed and successfully applied to various irreversible processes [14]. Since the entropy production is a bilinear form of thermodynamic fluxes j_k and reciprocal forces X_k:

$$\dot{s}_i = \sum_{(k)} j_k \cdot X_k \quad , \tag{59}$$

the time rate of the entropy production can be naturally decomposed into two terms:

$$\frac{d\dot{S}_i}{dt} = \frac{d_j\dot{S}_i}{dt} + \frac{d_x\dot{S}_i}{dt} \tag{60}$$

where
$$\frac{d_j\dot{S}_i}{dt} = \sum_{(k)} X_k \frac{dj_k}{dt} \tag{61}$$

and
$$\frac{d_x\dot{S}_i}{dt} = \sum_{(k)} j_k \frac{dX_k}{dt} \tag{62}$$

The criterion of evolution [14] states that the following inequality

$$\frac{d_x\dot{S}_i}{dt} \leq 0 \tag{63}$$

always holds true for stable processes. The equality is met for either a stationary process or a critical situation when a sudden (uncontrolled) transition becomes possible. Analysis of the second variation $\delta^2\dot{s}_i$ is necessary to distinguish these cases.

Let us apply this criterion to the entropy production due to crack layer extension (58):

$$T\frac{d_x \dot{S}_1^{global}}{dt} = \dot{\ell}^2 \cdot \frac{d}{d\ell}\left(J_1 - \gamma^* R_1\right) = \dot{\ell}^2 \frac{dJ_1}{d\ell} \leq 0 \qquad (64)$$

Indeed, the crack length ℓ is the only variable, since $\underset{\sim}{\sigma}$ is constant. R_1 is constant according to our assumption (there is no deformation of the active zone).

From (64) one can conclude that $\frac{dJ}{d\ell}$ should be < 0 for a stable crack growth.

Two types of crack configuration can be distinguished from the stability view point: stable ($\frac{dJ}{d\ell}$ < 0) and unstable ($\frac{dJ}{d\ell}$ >0). Illustrative examples of these two distinct types are shown in Fig. 10.

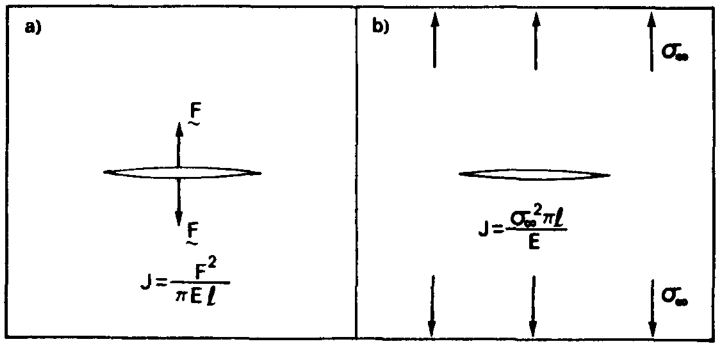

Fig. 10

Crack layer exhibits different behavior for stable and unstable configurations.

1) CL propagation in a stable configuration

For the configuration shown in Fig. 10a the energy release rate J is given by $\frac{F^2}{E\pi\ell}$. Consequently $\frac{dJ}{d\ell}$ is always negative, i.e., the configuration is always stable. Then, the requirements of the second law (58) is the only "controller" of CL propagation. For $J \geq \gamma^* R_1$ the requirement (58) is met and a crack layer is "allowed" to grow with undefined speed $\dot{\ell} > 0$. To specify the speed, we consider stationary CL growth. The condition of stationarity, i.e., the equality in (63) yields

$$J_1 = \gamma^* R_1 \qquad (65)$$

Then, the applied load F controls the crack propagation velocity. Indeed, from (65) and constancy of $\gamma^* R_1$ one can deduce:

$$\frac{d}{dt}\,(J_1 = \gamma^* R_1) \rightarrow \frac{\partial J}{\partial F}\dot{F} + \frac{\partial J}{\partial \ell}\dot{\ell} = 0 \tag{66}$$

Substituting $J = \frac{F^2}{E\pi\ell}$ into (66) we obtain

$$\dot{\ell} = \frac{2\ell}{F}\dot{F} \tag{67}$$

The condition (65) is obviously in agreement with the principle of minimum entropy production [21].

If $J < \gamma^* R_1$ the CL growth (i.e. $\dot{\ell} > 0$), consumes entropy (negative entropy production). According to the second law, it is possible if the dissipation $\dot{D}$ is sufficient to compensate the negative term $\dot{\ell}(J_1 - \gamma^* R_1)$. Applying the principle of minimum entropy production (the minimal value of $\dot{S}_i^{\text{global}}$ is zero), we find:

$$\dot{D} + \dot{\ell}(J_1 - \gamma^* R_1) = 0 \tag{68}$$

from which

$$\dot{\ell} = \frac{\dot{D}}{\gamma^* R_1 - J}$$

Summarizing the cases discussed, one can write a general expression for a stationary CL propagation in a stable configuration:

$$\dot{\ell} = \begin{cases} \dfrac{\dot{D}}{\gamma^* R_1 - J_1} \quad \text{if } J_1 < \gamma^* R_1 & \text{Dissipation controlled process.} \\[2ex] -\dfrac{\frac{\partial J}{\partial F}}{\frac{\partial J}{\partial \ell}}\dot{F} \quad \text{if } J_1 = \gamma^* R_1 & \text{Equilibrial crack layer growth.} \end{cases} \tag{69}$$

2) CL propagation in an unstable configuration

For the configuration shown in Fig. 10b the energy release rate $J = \frac{\sigma^2 \pi \ell}{E}$. Consequently $\frac{\partial J}{\partial \ell} = \frac{\sigma^2 \pi}{E}$ is always positive, i.e. the configuration is always unstable. The limitation introduced by the second law prevents the crack from avalanche-like propagation, i.e., the crack cannot grow if

$$T\dot{S}_1^{global} = \dot{\ell}(J_1 - \gamma^* R_1) < 0$$

which yields (for $\dot{\ell} > 0$) the following requirements for stable CL growth:

$$J_1 - \gamma^* R < 0 \tag{70}$$

Therefore, a slow CL propagation, which is compatible with (70) can occur due to the dissipative term $\dot{D}$ only. Therefore, if one accepts the principle of minimum entropy production, then equation (68) holds true for an unstable configuration as well as for a stable one.

When J_1 approaches $\gamma^* R_1$ the requirement of the second law is met. Then for an unstable configuration the crack propagates avalanche-like if $J_1 = \gamma^* R_1$.

Summarizing the results, one can write:

$$\dot{\ell} = \begin{cases} \dfrac{\dot{D}}{\gamma^* R_1 - J_1} \text{ if } J_1 < \gamma^* R_1 & \text{Dissipation controlled process.} \\ \text{Undefined if } J_1 = \gamma^* R_1 & \text{Transition to the dynamic process.} \end{cases} \tag{71}$$

Although both characteristics $\dot{D}$ and $\gamma^* R_1$ can be studied using either stable or unstable configuration, the first is more convenient for analysis of the dissipation $\dot{D}$ while unstable configuration is preferable for evaluation of $\gamma^* R_1$. The later can be obtained from the conventional fracture toughness test and additional microscopic studies of damage distribution along $\Gamma^{(t)}$.

The rate of dissipation within the active zone $\dot{D}$ can be experimentally measured as the difference between the rate of total dissipated work $\dot{W}^{(i)}$ and the heat $\dot{Q}$ radiated (see 24).

3) Example

Slow CL propagation in unstable configuration (equation 71) is considered as an illustrative example. Since, we do not have experimental data on heat radiation, we assume that the rate of dissipation $\dot{D}$ is proportional to the dissipated work $W^{(i)}$. The coefficient of proportionality apparently depends on mechanisms of dissipation. One would expect the coefficient to be dependent upon strain rate, temperature and a characteristic time of the process. An evaluation of the irreversible work $W^{(i)}$ due to an array of crazes constituting the CL active zone has been done in [8]. The evaluation is based on a new CL stress analysis [15] and an experimental CL characterization [2]. These results suggests that $W^{(i)}$ is proportional to the product $J\langle d\rangle$, where $\langle d\rangle$ stands for a characteristic size of V_A. Thus, the rate of dissipation $\dot{D}$ can be written as follows:

$$\dot{D} = \beta\langle d\rangle J \qquad (72)$$

Here β is a phenomenological coefficient with dimension $[\beta] = \sec^{-1}$. At fixed temperature β depends on the strain rate. The latter can be expressed in terms of the rate of applied load and a dimensionless crack propagation rate $\dot{\ell}/\langle d\rangle$ [8].

Substituting (72) into (71) we obtain

$$\dot{\ell} = \begin{cases} \dfrac{\beta\cdot\langle d\rangle\cdot J}{\gamma^* R_1 - J} & \text{if } J < \gamma^* R_1 \\ \text{Undefined} & \text{if } J = \gamma^* R_1 \end{cases} \qquad (73)$$

The relationship (73) is schematically represented in Fig. 11. One can distinguish three stages of slow crack propagation in unstable configuration. The first stage corresponds to crack propagation through previously damaged material. A new damage is not being developed at this stage. It means that the resistance moment R_1 mainly consists of the core of damage R_1^c which is small in comparison with a developed CL resistance R_1. Therefore, the rate of crack acceleration with respect to the energy release rate J_1 is relatively high. This is indicated by the slope of the stage I portion of the curve in Fig. 11.

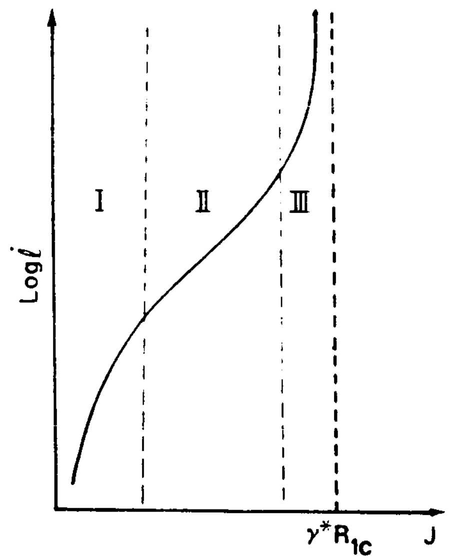

Fig. 11

The second, intermediate, state is characterized by monotonic growth of damage accompanied by crack propagation. It is reflected in a monotonic (approximately linear) increase of the translational resistance moment R_1 and the characteristic length <d> with increasing of J_1. In this case the translational driving force $\underset{\sim}{X}^{tr}$ is maintained approximately constant and the rate of crack propagation (73) can be approximated by a power type equation $\dot{\ell} \sim J^2$(i.e., K^4).

The third, subcritical stage of CL propagation is characterized by deceleration of the resistance moment R_1 with respect to J. This results in the translational force $X_1^{tr} = \gamma^* R_1 - J$ approaching zero, which corresponds to the critical state (the asymptote in Fig. 11). When $\gamma^* R_1 - J_1 = 0$ the requirement of the second law (58) is met and instability becomes permissible. Thus, at the end of the third stage a slow crack growth transforms into uncontrolled (avalanche-like) crack propagation.

Obviously, the value of J_1 at which the translational force $\gamma^* R_1 - J_1$ equals zero corresponds to the critical energy release rate J_c (or G_c) in conventional fracture mechanics, i.e.,

$$J_{1c} = \gamma^* R_{1c} \tag{74}$$

J_{1c} is a parameter which can be experimentally evaluated using fracture mechanics. γ^* and R_{1c} can be measured by materials science methods. Thus, (74) suggests a link between the two approaches. It is discussed in more details in section IX.

The simple model described above generally predicts the shape of $\dot{\ell}$ vs. J_1 curve. However, it does not describe crack deceleration phenomena, history dependency of J_c, etc.

VIII. Rectilinear CL Propagation. Two Parameter Model

Limitations of the previous model appear due to the employment of a single parameter only (the crack length). A natural way to overcome the limitations is to use an additional degree of freedom offered by the CL model. Following this idea, we consider crack layer propagation by translation along the rectilinear path and isotropic expansion of the active zone. Similar to the previous case, one can write: $v_1 = \dot{\ell}$; $v_2 = 0$ (rectilinearity); $\omega = 0$ (no rotation), $\dot{e} = 1/2\left(\frac{\dot{W}_a}{W_a} + \frac{\dot{\ell}_a}{\ell_a}\right)$ (isotropic expansion from the crack tip as an origin), $\underset{\sim}{d} = 0$ (no deviatoric deformation).

The global entropy production (23) now can be rewritten as follows:

$$T\dot{S}_i^{global} = \dot{D} + \dot{\ell}(J_1 - \gamma^* R_1) + \dot{e}\,(M - \gamma^* R_0) \tag{75}$$

The criterion of stability (63) for the crack layer only (ignoring $\dot{D}$ in (75) as we did in the previous example) can be written as follows:

$$T \frac{d_x \dot{S}_i^{global}}{dt} = (\dot{\ell})^2 \frac{\partial (J_1 - \gamma^* R_1)}{\partial \ell} + \dot{\ell}\dot{e} \left[\frac{\partial (J_1 - \gamma^* R_1)}{\partial e} + \frac{\partial (M - \gamma^* R_0)}{\partial \ell} \right] + (\dot{e})^2 \frac{\partial (M - \gamma^* R_0)}{\partial e} \le 0$$

Since $\dot{\ell}$ and $\dot{e}$ are independent variables the above expression yields:

$$\frac{\partial (J_1 - \gamma^* R_1)}{\partial \ell} \le 0$$

$$\frac{\partial (M - \gamma^* R_0)}{\partial e} \le 0$$

$$\begin{vmatrix} \dfrac{\partial (J_1 - \gamma^* R_1)}{\partial \ell} & \frac{1}{2}\left[\dfrac{\partial (J - \gamma^* R_1)}{\partial e} + \dfrac{\partial (M - \gamma^* R_0)}{\partial \ell} \right] \\ \frac{1}{2}\left[\dfrac{\partial (J - \gamma^* R_1)}{\partial e} + \dfrac{\partial (M - \gamma^* R_0)}{\partial \ell} \right] & \dfrac{\partial (M - \gamma^* R_0)}{\partial e} \end{vmatrix} \le 0 \qquad (76)$$

Using the same argument as in the single parameter model, one can conclude that both the expansional $(M - \gamma^* R_0)$ and translational $(J_1 - \gamma^* R_1)$ forces are always nonpositive for a stable crack layer propagation. Therefore, two types of crack layer configurations (stable and unstable) can be distinguished. Incorporating the of principle of minimum entropy production one can analyze stationary crack layer propagation following the formalism used in the single parameter model.

The stationary crack layer propagation controlled by dissipation is described by the same equation (68) for both stable and unstable configuration (compare (69) and (71). For this reason, below we consider the dissipation controlled propagation only.

The principle of minimum entropy production for slow CL propagation can be expressed in the form:

$$TS_1^{global} = \dot{D} - \dot{\ell}\,(\gamma^* R_1 - J_1) - \dot{e}(\gamma^* R_o - M) = 0 \tag{77}$$

It implies, that the dissipation $\dot{D}$ is distributed between two entropy sinks associated with two independent degrees of freedom: $\dot{\ell}$ and $\dot{e}$. As in the previous case we assumed the rate of dissipation $\dot{D}$ to be expressed by (72). In this case two parameters β_1 and β_2 substitute β to describe the distribution of the dissipation between two degrees of freedom. Then (77) yields

$$\dot{\ell} = \frac{\beta_1 <d> J}{\gamma^* R_1 - J_1} \tag{78}$$

$$\dot{e} = \frac{\beta_2 <d> J}{\gamma^* R_o - M} \tag{79}$$

Since the isotropic expansion is the only source for the resistance moment changes (it affects only the regular part R_1^r) one can write

$$\dot{R}_1^r = R_1^r(0)\dot{e} \tag{80}$$

Accordingly, equation (79) can be converted into an equation for R_1^r evolution using (80):

$$\dot{R}_1^r = \frac{\beta_2 R_1^r(0) <d> J_1}{\gamma^* R_o - M} \tag{81}$$

To analyze the changes of R_1^r with crack length one can calculate $\frac{dR_1^r}{d\ell}$ by taking the ratio of (81) and (78):

$$\frac{dR_1^r}{d\ell} = k\, R_1{}^r(0)\, \frac{\gamma^* R_1 - J}{\gamma^* R_o - M} \tag{82}$$

where $k = \frac{\beta_2}{\beta_1}$, is a phenomenological coefficient. It can be shown that J approaches $\gamma^* R_1$ faster than M approaches $\gamma^* R_o$ for unstable configuration. Therefore $\frac{dR_1^r}{d\ell}$ vanishes when $J_1 = \gamma^* R_1$ and $(\gamma^* R_o - M > 0)$. Equation (82) suggests a law of R_1^r evolution. The total translational resistance moment $R_1 = R_1^c + R_1^r$ (see (46a)) displays a behavior similar to R_1^r due to the constancy of R_1^c.

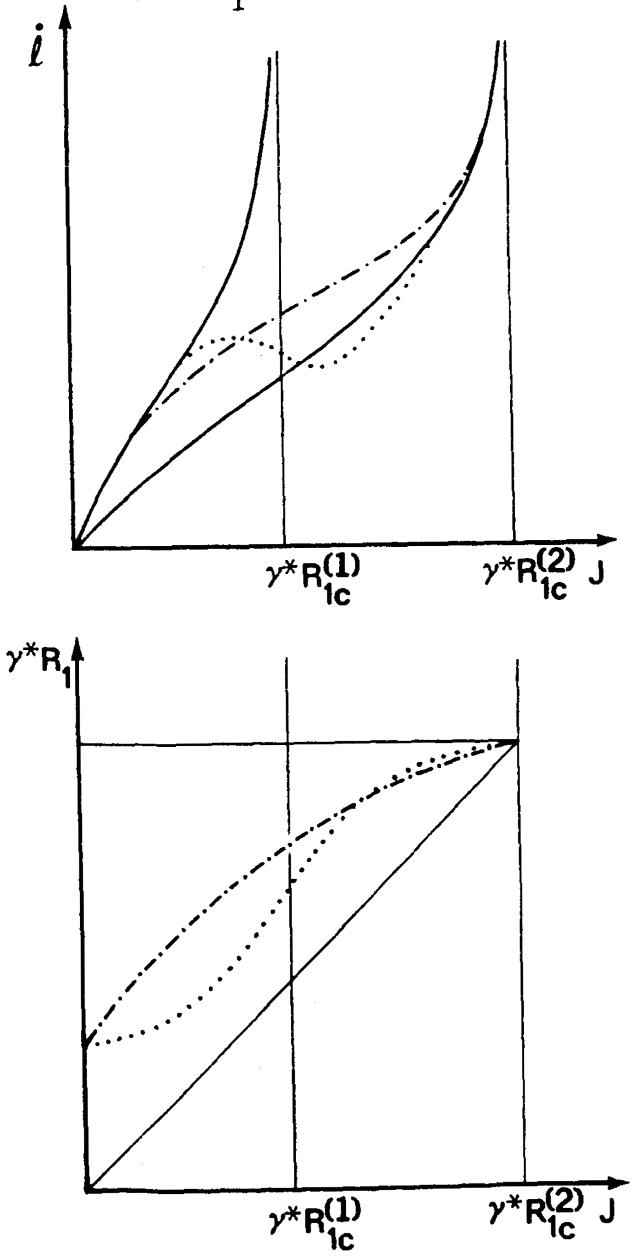

Fig. 12

Since $\gamma^* R_1$ is not a constant in the two parameter model, the crack growth rate vs. J_1 (equation (76)) essentially differs from that of the single parameter model. The solid lines I and II in Fig. 12a represent the crack growth rate ($\dot{\ell}$ vs. J) according to the single parameter model with two different values of R_1, i.e., $R_1^{(1)} < R_1^{(2)}$, respectively. Evolution of R_1 (Fig. 12b) yields a transition from the curve I to the curve II (Fig. 12a). Various ways of the transition depending on R_1 evolution are shown by dotted lines for monotonic transition and dashed line for a transition with crack disceleration (Fig. 12).

Applications of the two parameter model to slow crack layer propagation in unstable configuration of PS sheet under fatigue loading was recently achieved [8]. Fig. 13 taken from [8] shows a reasonable agreement between the theory and the experimental data for more than four order of magnitudes in crack growth rate. The two fatigue tests presented in Fig. 13 were identical except of the values of σ_{mean}. The higher stress yields faster crack propagation and shorter fatigue time which was expected. Most important is the fact that the critical value of J_1(at which avalanche-like crack propagation was observed) is much smaller for higher stress. The micrographs of the cross-section along the trailing edges of the two active zones shown in Fig. 13 explain this phenomenon. The lower stress produces more dense damage, i.e., larger translational resistance moment which consequently leads to larger J_{1c}.

IX. Material Toughness Characterization in Two Parameter Model

The necessary condition of crack layer instability (see sections VII and VIII) is

$$J_1 = \gamma^* R_{1c} \tag{83}$$

where $R_1 = R_1^c + R_1^r(t)$ according to (46a). When the sufficient condition of instability $(\frac{\partial J}{\partial \ell} > 0)$ is met, as it happens for various loading conditions, the condition (83) expresses the only requirement for the critical state.

Following the conventional symbolism we introduce J_{1c} as a critical value of J_1. Then using (83) and (46a) one can write

$$J_c = \gamma_o^* + \gamma^* R_1^r(t) \tag{84}$$

where the first term $\gamma_o^* = \gamma^* R_1^c$ is a Griffith's type energy associated with either the crack surfaces or, in a more general sense, a near surface layer of intensive damage (a core of damage). During crack layer growth γ_o^* assumingly remains constant. The second term describes the loading history

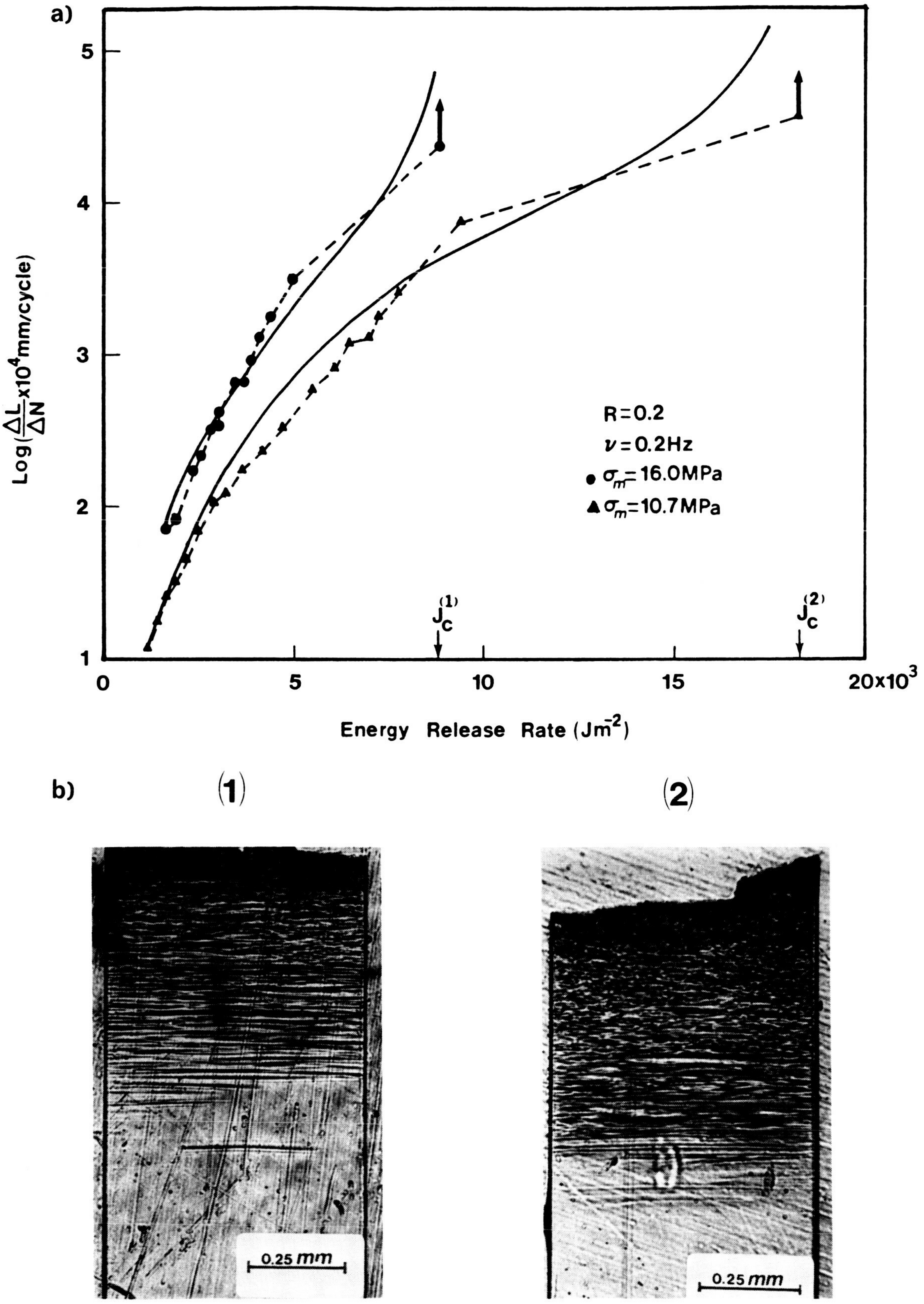

Fig.13

a) Crack growth rates.

b) Transverse sections along the trailing edge at critical CL configuration. ($J_c^{(1)}/J_c^{(2)} = R_{1c}^{(1)}/R_{1c}^{(2)}$)

dependency of J_{1c} and is associated with a damage dissimination accompanying the crack growth.

Equation (84) presents J_{1c} as a linear function of R_1^r (Fig. 14). The slope and the intersect in Fig. 14 gives rise to γ^* and γ_0^*, respectively. As shown in Figs. 15, 16, 17 the linear $J_{1c} - R_1^r$ relationship has been demonstrated by the results on stainless steel [1], polycarbonate and polystyrene, respectively [7].

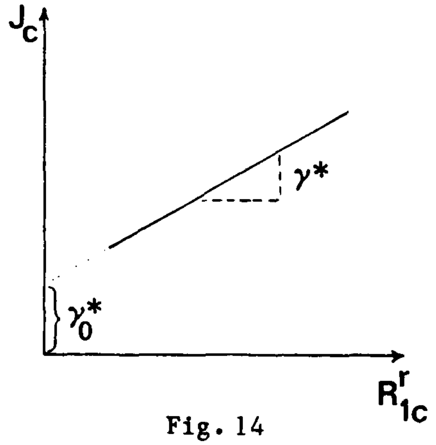

Fig. 14

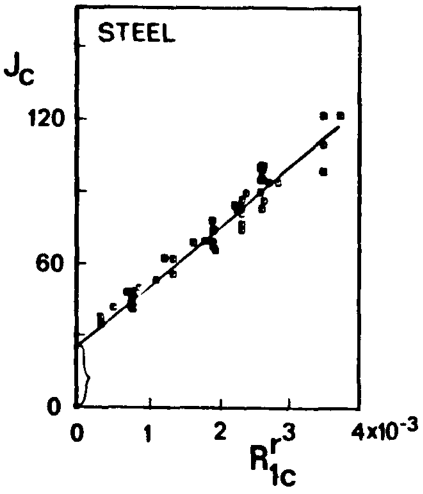

Fig. 15

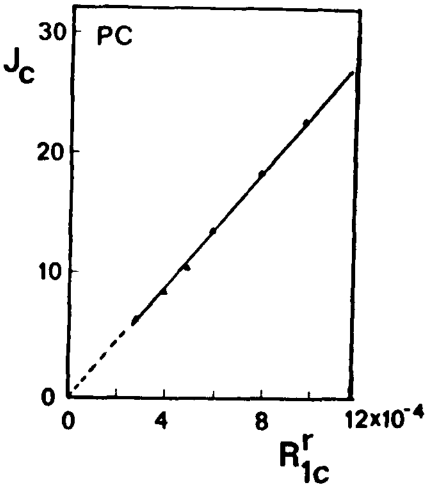

Fig. 16

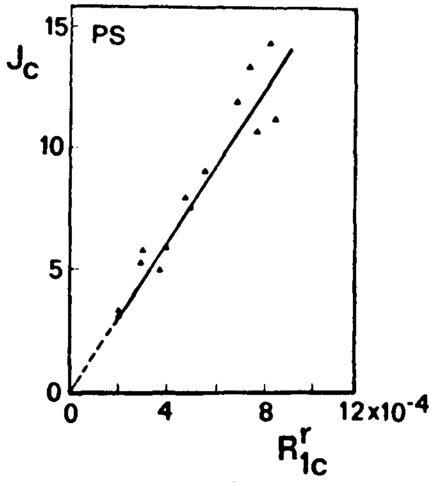

Fig. 17

As it was suggested [12], γ^* appears to be a constant quantity of the same order of magnitude as the latent energy of a phase transition for the material considered [1,7].

X. Conclusion

The model described above suggests three independent parameters γ_0^*, γ^* and R_1 to characterize material toughness. Two of them γ_0^* and γ^* are material constants, reflecting the mode of damage (microstructural features). The third is a history dependent parameter. Therefore, a complete toughness characterization requires establishment of the constitutive equation for R_1^r.

The model described is in a good agreement with available experimental data. At the same time the limitations of the model are obvious. It does not predict the crack trajectory (we assumed a rectilinear path), it does not describe the active zone shape changes, observed recently [3], etc. Therefore the necessity of employing the rest of the crack layer degrees of freedom is clear.

XI. Acknowledgements

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Appendix

Historical Remarks

J-integral was first introduced by J. Eshelby in 1951 [16] to express the force acting on a singularity within an elastic solid. Later, J-integral was rederived independently by J. Sanders in 1960 [17], G. Cherepanov in 1967 [18] and J. Rice in 1968 [19]. The most clear and popular interpretation of J as the energy release rate with respect to crack length has been done by J. Rice. W. Gunter [20] (1962) and later J. Knowles and E. Sternbert [21] (1972) applied Noether's theorem to elastostatics and obtained three path-independent for a linear, homogeneous and isotropic medium integrals J, L and M associated with translational, rotational and expansional invariances. These integrals express the general conservation laws of elastostatics. Shortly after J. Knowles and E. Sternberg' publication, the physical interpretation of L and M integrals was discussed by B. Budiansky and J. Rice [22]. The same J, L and M integrals (and additional one N) appeared in the crack layer theory as active parts of thermodynamic forces reciprocal to crack layer extension rotation, expansion and distortion ([5], [6] (1978)). In recent publication of S. Aoki, K. Kishimoto and M. Sakata [23] a generalization of J, L and M integrals is proposed for cases in which plastic deformation, body forces, thermal strains may exist. The formalism used in [23] is very similar to that in [5,6]. Path independent integrals for inelastic materials are in details discussed by Stonesifer and Atluri [24].

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16. Abstract
Studies of crack propagation and stability have developed in two main directions: One related to materials science, which concerns studies of the hierarchy of microdefects, their nucleation, interaction and development in association with propagation of a main crack. Another, based on continuum mechanics, considers crack as an ideal cut in an elastic, elastoplastic or viscoelastic medium. To bridge these approaches we introduce a damage parameter in addition to conventional parameters of continuum mechanics and consider a crack surrounded by an array of microdefects within the continuum mechanics framework. A system consisting of the main crack and surrounding damage is called "crack layer." Crack layer propagation is an irreversible process. Hence, the general framework of the thermodynamics of irreversible processes have been employed to identify the driving forces (causes) and to derive the constitutive equation of CL propagation, that is, the relationship between the rates of the crack growth and damage dissimenation from one side and the conjugated themodynamic forces from another. The proposed law of CL propagation is in good agreement with the experimental data on fatigue CL propagation in various materials. The theory also elaborates material toughness characterization. It proposes the following relationship between the critical energy release rate J_{1C} (widely used as a material toughness parameter) and the damage dissemination characteristics R_1 (which is called translational resistance moment):

$$J_{1C} = \gamma_0^* + \gamma_1^* R_1 \tag{1}$$

Here γ_0^* is a Griffith's type energy associated with either the crack surfaces or, in a more general sense, a near surface layer of intensive damage (a core of damage); γ^* is the specific enthalpy of damage. J_{1C} is measured on the basis of the fracture mechanics methods, γ_0^*, γ^* and R_1 results from the materials science technique. Thus, equation (1) suggests a practical link between the two approaches to crack stability analysis.

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