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COLLEGE OF ENGINEERING

Department of Mechanical Engineering and Applied Mechanics
Heat Transfer Laboratory

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Interim Report

Modeling of Zero Gravity Venting

Herman Merte, Jr.

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Studies of Two-Phase Heat Transfer
Under Reduced Gravity
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MODELING OF ZERO GRAVITY VENTING

The University of Michigan
Ann Arbor, MI

ABSTRACT

An experimental investigation of the venting of cylindrical containers partially filled with initially saturated liquids was previously conducted under zero-gravity conditions at the NASA Lewis Research Center 5-second zero gravity facility, and compared with an analytical model which determined the effect of interfacial mass transfer on the ullage pressure response during venting. A new model is proposed here to improve the estimation of the interfacial mass transfer. Duhammel's superposition integral is incorporated in this analysis to approximate the transient temperature response of the interface, treating the liquid as a semi-infinite solid with conduction heat transfer. The results show that this approach to estimating interfacial mass transfer gives improved response when compared to previous models. However, the present model still predicts a pressure decrease greater than those in the experiments reported.

INTRODUCTION

The use of high-energy liquid propellants in the space program has led to a need for information concerning the thermodynamic behavior of cryogenic fluids in tanks which are vented or depressurized to space. Low vapor vent rates are used as a method of tank pressure control. The task of venting in low gravity has been successfully accomplished during a number of past missions with venting systems that rely exclusively on auxiliary thrusters to actively position the liquid propellant away from the tank vent. This method of pressure control was adequate for short term missions and deemed economically more feasible than the weight penalty of additional insulation. (Ref. 7) The objective of the present study is to predict the pressure response of a saturated liquid-vapor system when undergoing a venting or depressurization process in zero gravity at low vent rates.

Fig. 1 is a schematic of a typical test container, with the liquid vapor interface assuming a hemispherical shape in zero-gravity. Fig. 2 is a schematic of the proposed venting model. The l-v interface is assumed to be planar, but with the surface area of the hemispherical interface, and the contents of the container are assumed to be at saturation conditions corresponding to Pv prior to venting, $t<0$. Upon initiation of venting, $t>0$, all properties are considered spacially uniform but time dependent, except for the liquid, whose temperature varies spacially one-dimensionally as well. The interfacial temperature is the saturation temperature corresponding to the system pressure Pv. The analysis consists of applying the appropriate governing equations to three control volumes; the vapor, the liquid-vapor interface, and the liquid. Figures 3-5 are schematics of these three control volumes. The vapor is treated as a lumped or uniform property control volume, and the conservation of mass and energy are applied. The interfacial mass transfer is found by applying the conservaton of energy to the liquid-vapor interface. The liquid is treated as a semi-infinite planar solid in order to calculate the temperature gradient of the liquid at the interface.

For purposes of comparison, an adiabatic model, which assumes no interfacial mass transfer, is constructed. The analysis, presented in Appendix D, is otherwise identical to that developed below. This model, when compared with the interfacial mass transfer model, will aid in evaluating the impact of interfacial mass transfer on the pressure response of the system.

The pressure responses determined with the interfacial mass transfer and adiabatic models are compared with the results from previous models and with the experimental results obtained from the short duration drop tower tests conducted at the Lewis zero-gravity facility.

NOMENCLATURE

a	thermal diffusivity, m2/sec
A	area, m2
Cd	discharge coefficient
Cv	specific heat at constant volume, J/kg-K
δ	penetration depth, m
h	specific enthalpy, J/kg
hfg	heat of vaporization, J/kg
k	thermal conductivity, W/m-K
m	mass, kg
n	unit normal vector
P	pressure, N/m2
q	heat flux, W/m2
R	gas constant, m-N/kg-K
T	temperature, K
t	time, sec
U	internal energy, J
u	specific internal energy, J/kg
v	velocity, m/sec

Subscripts:

e	vented vapor

i	liquid-vapor interface
l	liquid
o	initial
sat	saturated conditions
v	ullage vapor

ANALYSIS

The integral form of the continuity and energy equations for a control volume are used.

The continuity equation is

$$\int_{\forall} (\partial\rho/\partial t)\, d\forall + \int_{A} \rho \vec{V}\cdot\vec{n}\, dA = 0 \qquad (1)$$

The volume V may be assumed constant, since the actual volume changes due evaporation are small. Then, Eq. (1) becomes

$$\frac{d}{dt}\int_{\forall} \rho\, d\forall = -\int_{A} \rho \vec{V}\cdot\vec{n}\, dA \qquad (2)$$

For the vapor region, Eq. (2) becomes

$$dmv/dt=\dot{m}i-\dot{m}e \qquad (3)$$

where mi is the rate of generation of vapor at the liquid-vapor interface, and me is the mass flow rate of the vapor vented. For the liquid region

$$dml/dt=-\dot{m}l \qquad (4)$$

The energy equation is

$$\frac{d}{dt}\int_{\forall} \rho u\, d\forall + \int_{A} \rho h \vec{V}\cdot\vec{n}\, dA + \int_{A} \vec{q}\cdot\vec{n}\, dA = 0 \qquad (5)$$

For purposes of the present analysis, it will be assumed that:

1. Heat transfer from the walls is negligibly small.
2. No heat transfer takes place between the vapor and the l-v interface.
3. The internal energy in the vapor is spacially uniform, varying only with time.
4. The vapor volume is constant (volume increases due to evaporation are neglected).
5. The interface surface area remains constant.
6. The liquid mass is large compared to the amount evaporated.
7. All vapor properties are uniform at the state defined by Tv and Pv.
8. The interface temperature Ti=Tsat @ Pv.
9. The liquid-vapor mixture is initially saturated at Tv=Ti=Tsat @ Pv.

For the relatively short test times being modeled, along with the low venting rates assumed, these assumptions are reasonable. For longer test times, conduction from the walls must be taken into consideration. For the vapor then, Eq. (5) reduces to

$$\frac{d}{dt}(m_v u_v) + \dot{m}_e h_v - \dot{m}_i h_i = 0 \tag{6}$$

Expanding Eq. (6):

$$u_v \frac{dm_v}{dt} + m_v \frac{du_v}{dt} + \dot{m}_e h_v - \dot{m}_i h_i = 0 \tag{7}$$

Now, assuming Cv=constant over a small temperature range, and substituting Eq. (3) into Eq. (7):

$$m_v C_v \frac{dT_v}{dt} + \dot{m}_i (u_v - h_i) + \dot{m}_e (h_v - u_v) = 0 \tag{8}$$

Expressions for $\dot{m}i$ and $\dot{m}e$ will now be developed.

The mass flow rate through the vent, $\dot{m}e$, is determined by using a classical choked flow analysis (Ref. 9). Since the gas is vented directly to a vacuum, the choked flow assumption is valid and the exiting mass flow rate is a function of upstream vapor properties only, given by:

$$\dot{m}_e = \frac{P_v C_D A_T K_D}{(R T_v)^{1/2}} \tag{9}$$

where Cd is an experimentally determined discharge coefficient and:

$$k_b = \frac{\left(\frac{c_p}{c_v}\right)^{1/2}\left[\frac{2}{\left(\frac{c_p}{c_v}+1\right)^{\frac{c_p}{c_v}+1}}\right]}{2\left(\frac{c_p}{c_v}+1\right)} \tag{10}$$

The rate of vapor generation, mi, is determined from the conservation of energy equation (Eq. (5)) applied to the liquid-vapor interface. Assuming no heat transfer to the vapor, all energy transfered to the interface by conduction in the liquid results in vaporizaion of liquid at the interface. Eq. (5) reduces to:

$$q_l = \dot{m}_i h_{fg} \tag{11}$$

For relatively short periods, where the temperature boundary layer is small compared to any radii of curvature present at the interface, the liquid may be treated as a semi-infinite planar solid. The surface area term, Ai, will be the surface area of the hemisphere, the shape the interface takes in zero gravity. Referring to Fig. 5, the one dimensional conduction equation is:

$$q_l = -k_l A_i \left(\frac{dT}{dx}\right)\Big|_{x=0} \tag{12}$$

Combining equations (11) and (12) gives

$$\dot{m}_i = \frac{-k_l A_i \left(\frac{dT}{dx}\right)\Big|_{x=0}}{h_{fg}} \tag{13}$$

Thus, the problem of determining to interfacial mass transfer is reduced to determining the temperature gradient of the liquid at the interface, which requires that the transient temperature distribution in the liquid near the l-v interface be determined. If the liquid near the l-v interface can be considered to approximate a one-dimensional semi-infinite solid in it's thermal behavior the analytic solution for a step change in surface temperature, in connection with the finite form of Duhammel's superposition integral, can be used to determine the transient temperature distribution in the liquid. The time varying interface temperature is taken as the saturation temperature corresponding to the instantaneous system pressure, which must be determined appropriately from the system of governing equations.

Accordingly, the differential form of the governing equation and the initial and boundary conditions for the one-dimensional semi-infinite solid, initially at uniform temperature To and with a step change in surface temperature to Ti are:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \tag{14}$$

$$T(x,0) = T_o \tag{15}$$

$$T(0,t) = T_i \tag{16}$$

$$T(\infty,t) = T_o \tag{17}$$

The solution is (Ref. 2):

$$\frac{T(x,t) - T_i}{T_o - T_i} = erf\left(\frac{x}{2(at)^{1/2}}\right) \tag{18}$$

The interface temperature, being the saturation temperature corresponding to the ullage pressure, will be time varying in the present case since the pressure will change as the tank is vented. This time varying boundary condition Ti(t) is incorporated into the solution using Duhammel's superpostion intregral (Ref. 2) in the form:

$$\theta(x,t) = \theta_i(0) \cdot \psi(x,t) + \int_0^t \psi(x,t-s) \frac{d\theta_i(s)}{ds} ds \tag{19}$$

Here,

$$\begin{aligned} \theta(x,t) &= T(x,t) - T_o \\ \theta_i(t) &= T_i(t) - T_o \end{aligned} \tag{20}$$

and we let

$$\phi(x,t) \equiv \frac{\theta(x,t)}{\theta_i(t)} \tag{21}$$

$\psi(x,t)$ is the unsteady temperature resulting from a stepwise unit increase in surface temperature, relative to a uniform initial temperature. If the increase is kept at zero until a certain time t=s, and at that instant raised to unity and maintained constant, the new temperature $\phi(x,t)$ may be expressed in terms of $\psi(x,t)$ as

$$\phi(x,t) = \begin{cases} 0, & t < s \\ \psi(x,t-s), & t > s \end{cases} \tag{22}$$

The solution for $\psi(x,t)$ is given by Eq.(18), transformed to the form of Eq. (21) as

$$\psi(x,t) = \frac{\theta(x,t)}{\theta_i} = erf\,c\left(\frac{x}{2(at)^{1/2}}\right) \tag{23}$$

Solution of the system of equations for the venting problem will be performed in discrete time steps, and the discrete form of Eq. (19) is given by:

$$\theta(x,t) = \theta_i(0) \cdot \psi(x,t) + \sum_{m=1}^{n} \Delta\theta_{i_m} \cdot \psi(x, t-s_m) \tag{24}$$

where

$$\Delta\theta_{i_m} = \theta_i(s_m) - \theta_i(s_{m-1}) \tag{25}$$

Here, n is the total number of time steps into which the process has been divided, m is a running time index, 1<m<n, and $\Delta\theta_{i_m}$ is the incremental change in surface temperature, related to the system vapor pressure.

It is difficult to obtain a temperature gradient in the liquid at the interface to the desired degree of precision from the solution in the form of Eq. (24). Rather, the procedure followed here is to compute the instantaneous temperatures at a finite number of points in the liquid near the interface, using Eq. (24), and fit these points to a third order polynomial using a least squares fit. The polynomial used is of the form:

$$T = A + Bx + Cx^2 + Dx^3 \tag{26}$$

The temperature gradient of the liquid at the l-v interface, x=0, is then:

$$\left.\frac{dT}{dx}\right|_{x=0} = B \tag{27}$$

The number and spacing of the nodes at which the temperatures of the liquid are to be calculated, and with which the coefficients A, B, C, and D in Eq. (26) will be determined, must next be specified. Six nodes were taken arbitrarily as being sufficient to obtain the four coefficients in Eq. (26). Intuitively, nodes nearest to the l-v interface will give the most accurate value of the liquid temperature gradient at the l-v interface. The method used was to estimate a temperature penetration depth, , taken here to be the depth at which the dimensionless temperature change computed by equation (18) is 95%. This is determined as:

$$0.95 = \operatorname{erf}\left(\frac{S}{2(at)^{1/2}}\right) \tag{28}$$

or

$$S = 1.39 \times 2(at)^{1/2} \tag{29}$$

The actual penetration depth will be somewhat less than this value, since the actual system does not undergo a single step change in surface temperature, but rather a transient change in surface temperature. The six equally spaced nodes are taken to be within the 10% of this penetration depth nearest the l-v interface, shown schematically in Fig. 6.

Now that the temperature of the liquid at each of the six nodes near the l-v interface is known, the constants A,B,C, and D of Eq. (26) may be determined. A least squares algorithm was used (Ref. 4), which determines the polynomial coefficients which minimize the error between the data points and the polynomial.

Appendix A describes a test program devised to evaluate the effect of the fraction of penetration depth used when fitting a polynomial by computing the accuracy of the polynomial in predicting the temperature gradient at the l-v interface. The temperature gradient obtained with the above procedure is compared with the analytical value for a single step change in surface temperature, being the most severe test possible. This is done for different fractions of the penetration depth. Figures A1 and A2 show that with the nodes spaced in a region of 10% of the penetration depth from the surface and using a third order order polynomial, an error of less than 0.5% in temperature gradient at the surface is obtained.

For the adiabatic model, the mass transfer at the interface is taken as zero, and the above analysis for the interfacial mass transfer is not used.

When combined with the proper initial conditions, equations (3), (8), (9), and (13), along with the liquid temperature distribution, provide a complete description of the vapor space. These equations were numerically solved by computer. A program listing and description is included in appendix B . A comparison of these results with the experimental data available to date is presented below.

RESULTS

The model described above differs primarily in two respects from previous models used to predict the pressure response of an initially saturated liquid vapor mixture vented to a vacuum in zero gravity. The most significant difference is the procedure used to approximate the interfacial mass transfer. The present model assumes the liquid to be a semi-infinite solid with a planar surface and a transient surface temperature determined from the coupling between the liquid conduction process and the vapor behavior. Duhammel's superpostion integral is used to incorporate the effect of a transient surface temperature in computing the liquid temperature profile. The interfacial mass flux is then determined from the temperature gradient at the liquid-vapor interface.

The second difference from past models is that the vapor temperature is *not* assumed to be at the saturation temperature corresponding to the vapor pressure. This now couples the energy and continuity equations for the vapor system and makes for a more difficult numerical solution. The effect of this change in assumption can be seen in figures 7 and 8, where both the mean vapor temperature and the instantaneous saturation temperatures are plotted for two test runs. The difference between the vapor temperature and the saturation temperature can be as much as 30 degrees K. The vapor temperature is higher than the saturation temperature and is thus superheated. Since $\dot{m}_e$ is inversely proportional to vapor temperature, higher vapor temperatures result in slightly lower vent rates, and thus slower ullage pressure drop.

Comparison between the pressure response predicted by the present model, the present adiabatic model, and previous models (Ref. 1) are given in Table 1, together with measurements obtained previously (Ref. 1). The data in Table 1 shows that the proposed model gives pressure responses closer to the experimental data than does any previous model. The data in Table 1 also shows that both the present model and previous models incorporating interfacial mass transfer yield better results than does the adiabatic model, which assumes no interfacial mass transfer. It is evident that interfacial mass transfer must be considered when using low vent rates such as the ones used in this study. Hence,

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It may be concluded that the proposed model better approximates interfacial mass transfer than previous models, but the sizeable error when compared to the experimental data indicates that certain elements are still lacking in the description of the process. It is also possible that the experiments themselves should reexamined.

Additional detailed transient behavior of Runs 2 and 4 in Table 1 are plotted in figures 9-14, with system pressures in Figs. 9 and 10, vent rates in Figs. 11 and 12, and evaporation rates in Figs. 13 and 14. Run 4 has a discharge area 2.2 times that for Run 2, approximately the same initial volume, and an initial pressure approximately 10% higher. This is consistent with the higher pressure drop rate, higher vent rate, and higher evaporation rate that occurs wiith Run 4.

Evaluation of this model assumes that the experimental data accurately describes the system being modeled. The small test vessels used would tend to make the geometry of the system important. The flow coefficients, Cd, were experimentally determined, and there is no way of evaluating their accuracy. Future experiments should be conducted before making a final evaluation of the model proposed here.

CONCLUSION

An analytical model was constructed to predict the pressure response of cylindrical containers initially filled with a saturated liquid-vapor mixture vented to a vacuum under zero gravity conditions. The response predicted by this model was compared to that of previous models and to the experimental data obtained at the NASA Lewis Researach Center.

Previous models predicted too large a pressure drop. The model proposed here gives a pressure response closer to the experimental data than other models, but still predicts too large a pressure drop. This means that the present model still underestimates the amount of interfacial mass transfer. Higher rates of evaporation will yield a lower pressure drop in the system. An additonal source of vapor formation not considered in the present model is the thin liquid layer existing at the liquid-vapor-solid triple interline formed by a hemispherical liquid-vapor interface. It can be expected that rapid evaporation would take place in this region, involving conduction effects from the container walls (neglected in the present analysis). This would reduce the pressure drop predicted by the model, with perhaps better agreement with experiments conducted to date.

Future experiments might be considered for comparison with the present model in which the presence of the triple interline would be minimized by using larger size vessels and by conducting the experiments at standard earth gravity.

TABLE 1. SUMMARY OF PARAMETERS AND RESULTS

Test Run No	Initial vapor volume	Nozzle diameter	Discharge coeffic.	Initial ullage pressure	Initial ullage temp.	Final exp. pressure	Final analy. press.	Final past analy. press.	Final adiabatic press.	Dimen-sionless anal. press drop	Dimen-sionless exp. press dr
	m3	m	Cd	kPa	K	kPa	kPa	kPa	kPa		
1	1.93E-4	0.406E-3	0.64	89.6	294.3	86.2	85.2	81.6	83.2	0.07	0.06
2	2.01	0.889	0.69	87.9	294.7	70.3	64.4	56.3	56.1	0.31	0.25
3	1.90	1.07	0.86	91.0	293.7	60.7	46.8	40.7	33.6	0.48	0.33
4	1.93	1.32	0.875	97.2	296.5	53.8	37.9	29.4	21.8	0.62	0.46
5	1.93	1.93	0.77	101.0	295.4	41.4	21.4	13.1	5.3	0.78	0.57

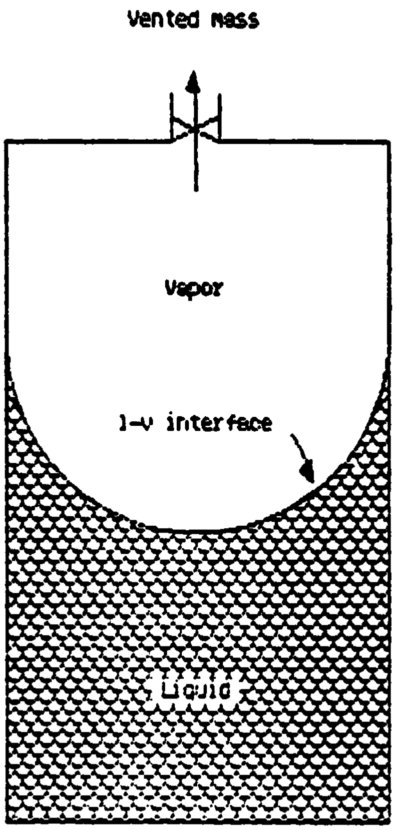

Figure 1. - Schematic of typical test container.

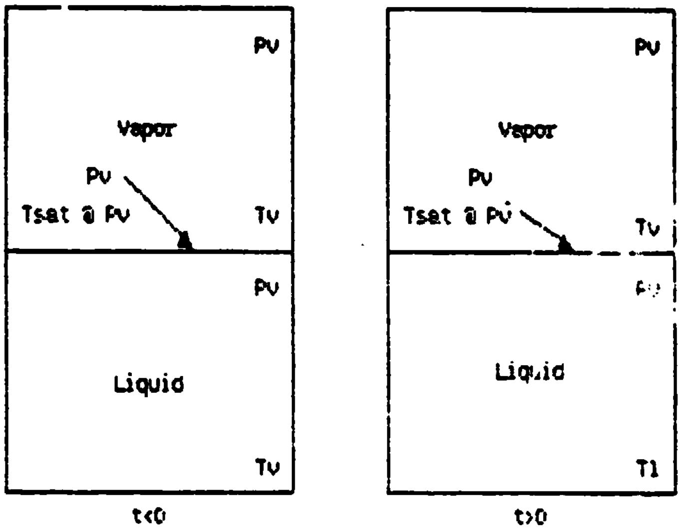

Figure 2. - Schematic drawing of venting model.

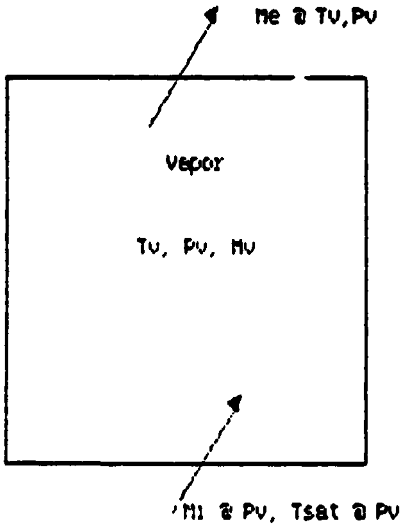

Figure 3. - Vapor region control volume.

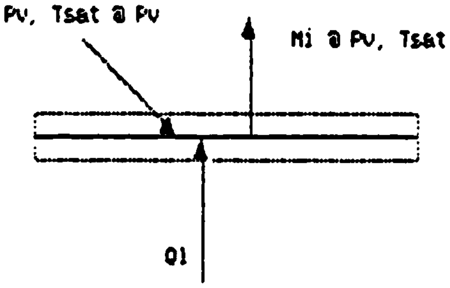

Figure 4. - Interface region control volume.

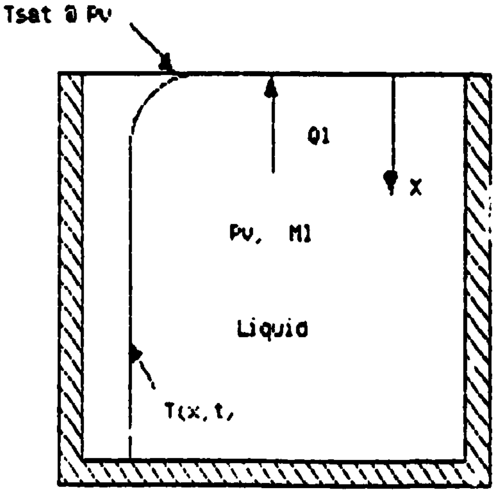

Figure 5. - Liquid region control volume.

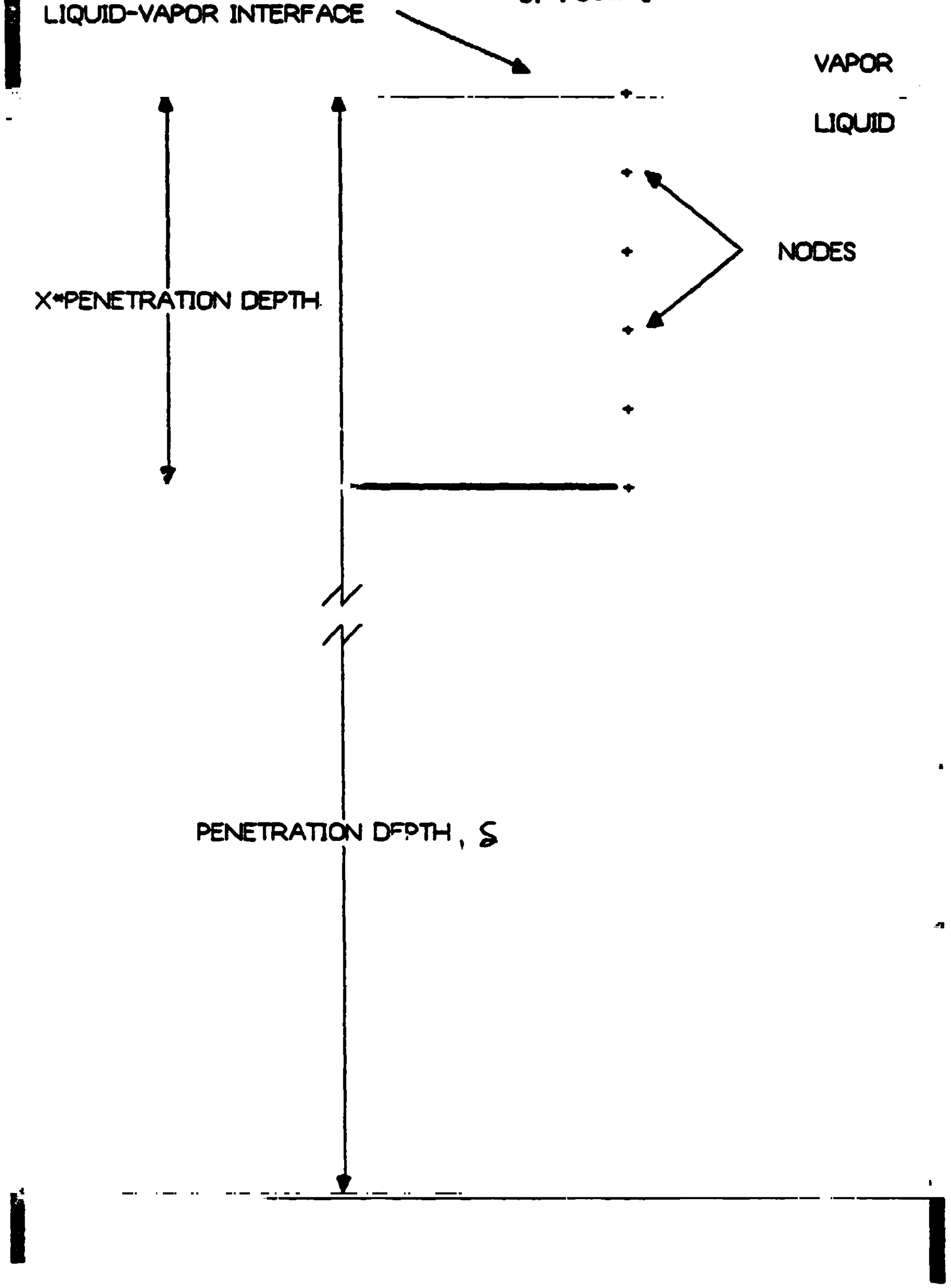

Figure 6. - Location of nodes in liquid.

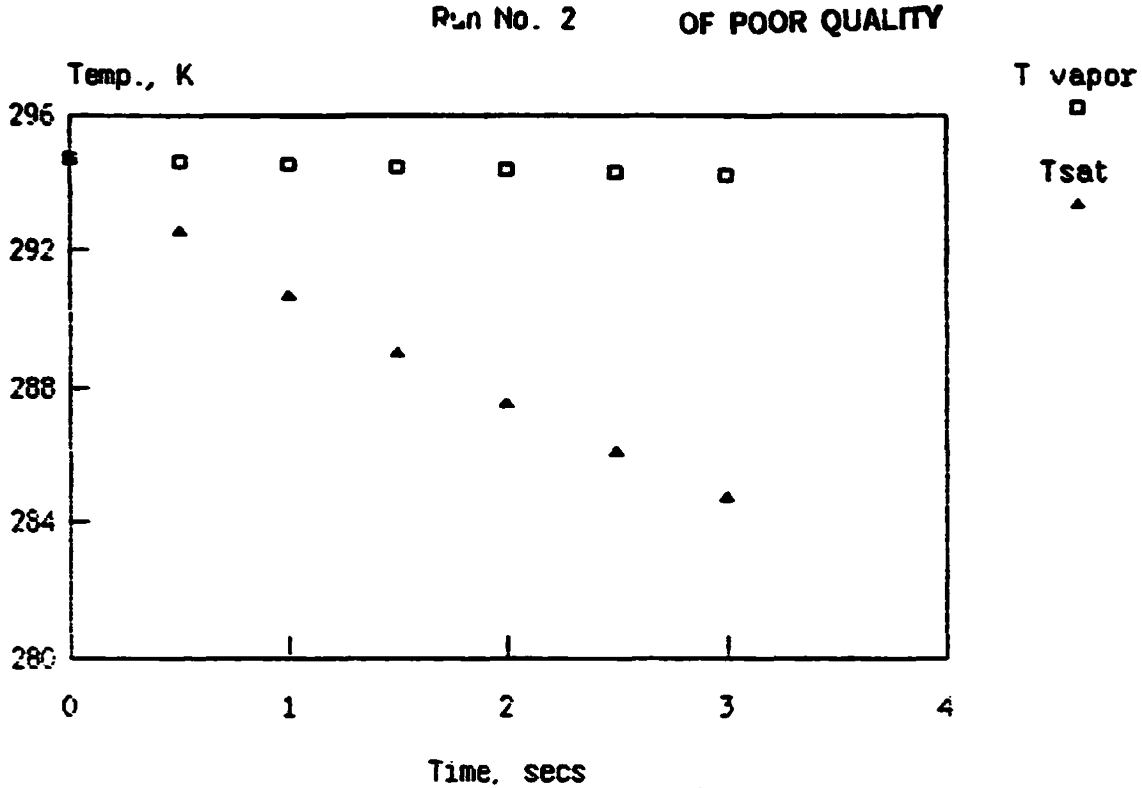

Figure 7. - Transient Vapor and Saturation Temperatures.

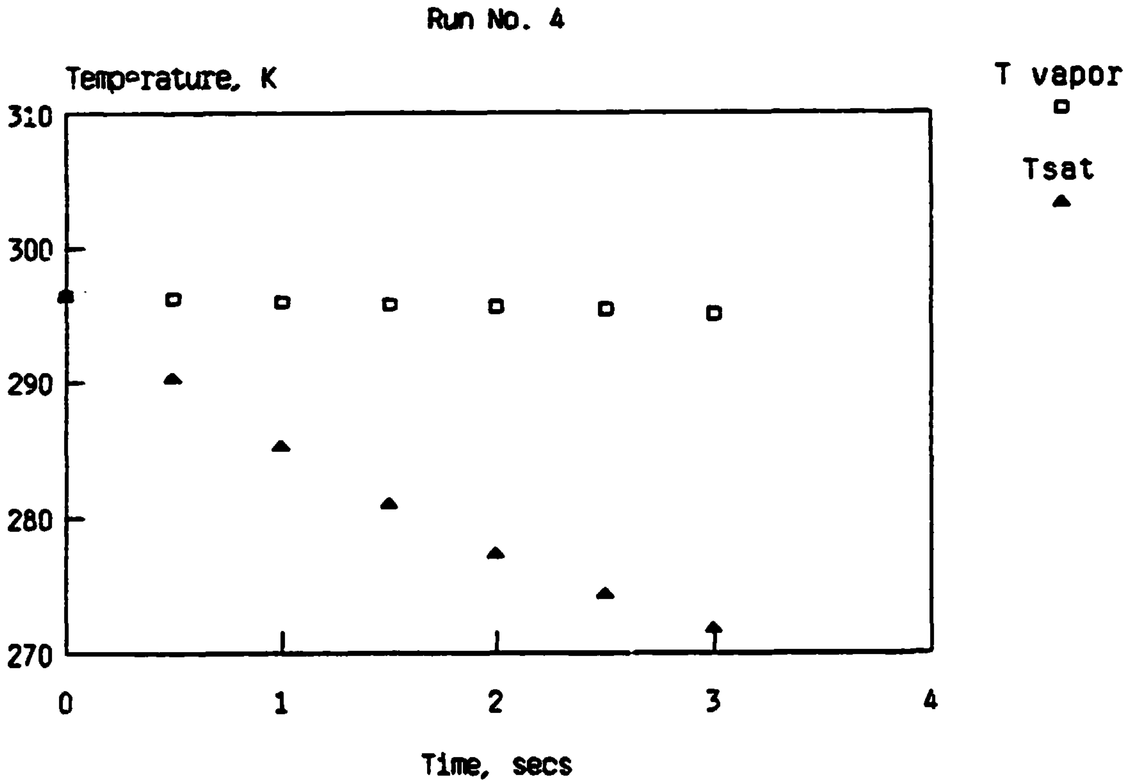

Figure 8. - Transient Vapor and Saturation Temperatures

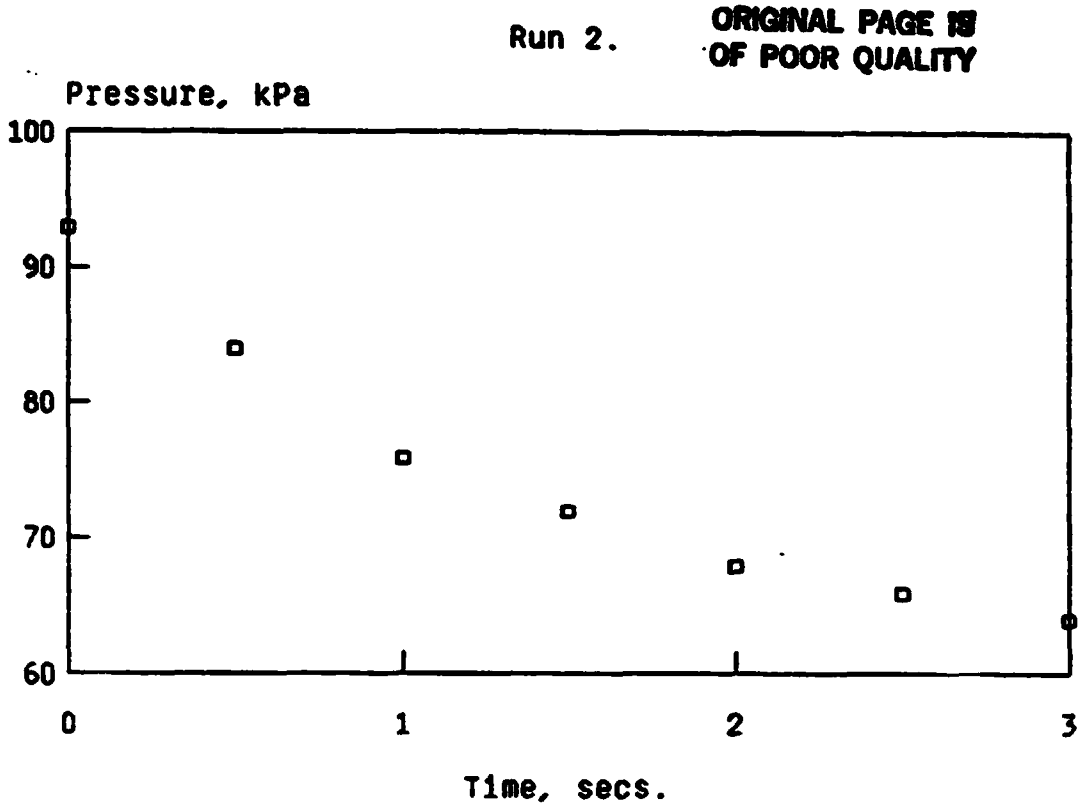

Figure 9. - Transient System Pressure Response

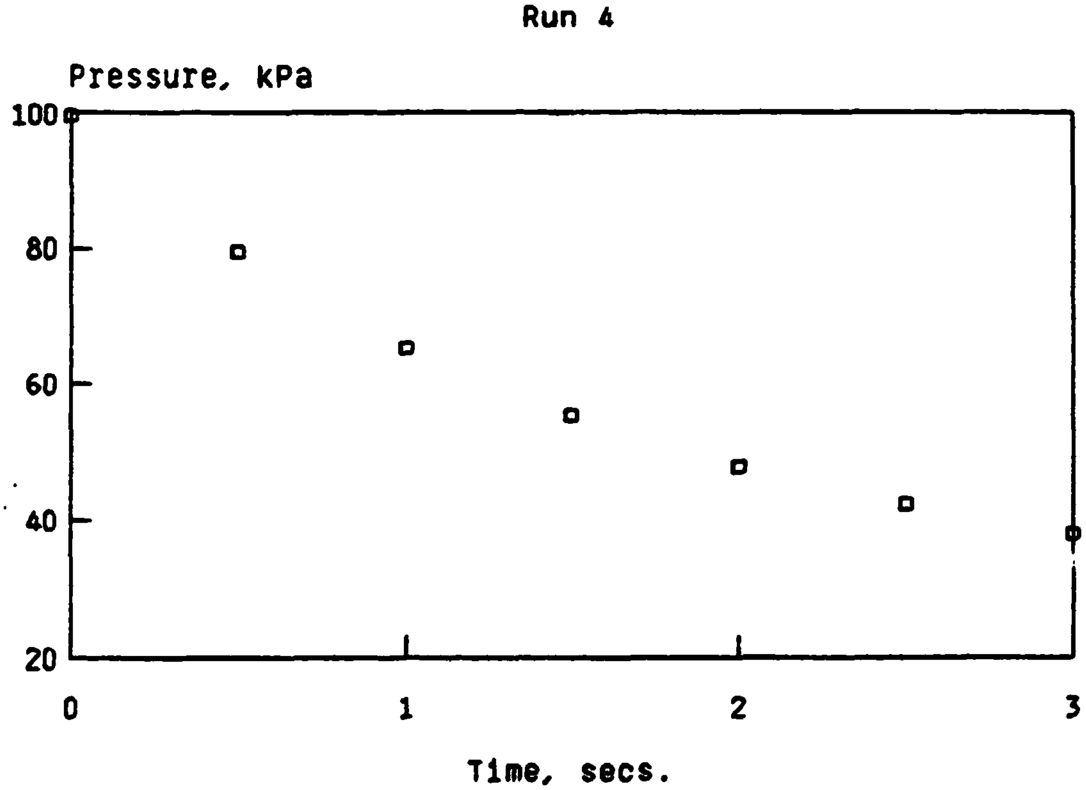

Figure 10.- Transient System Pressure Response.

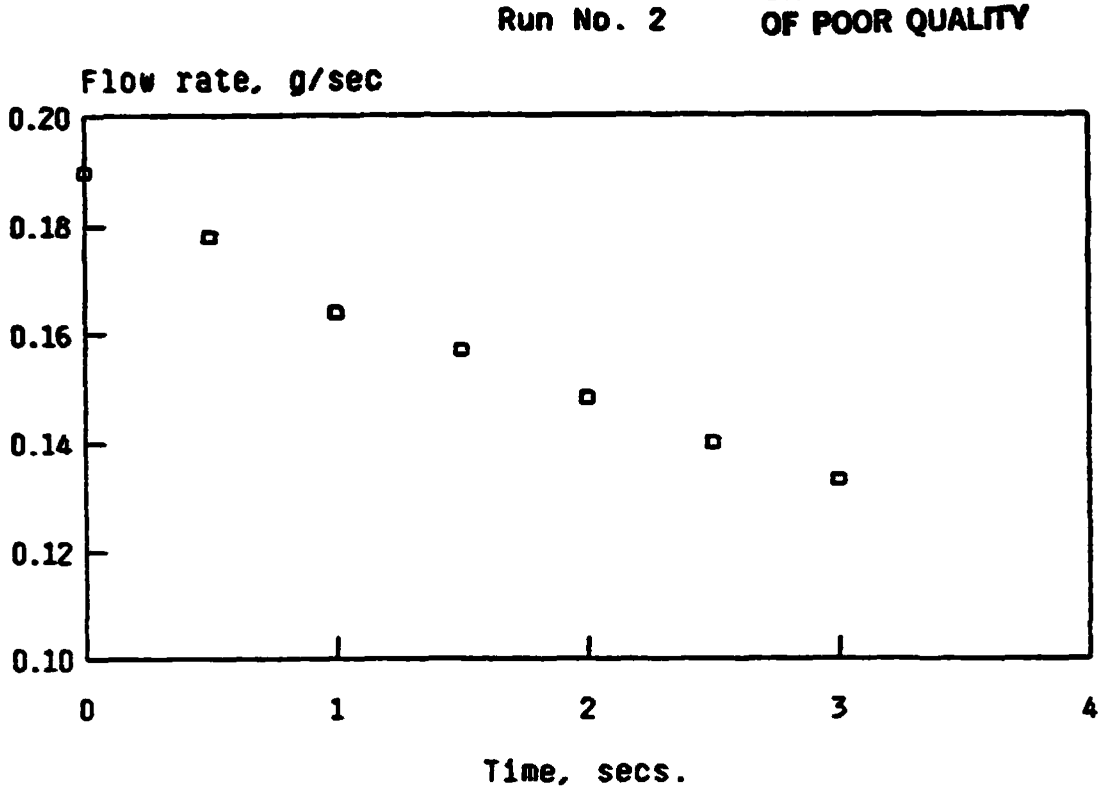

Figure 11. - Transient Vent Rate

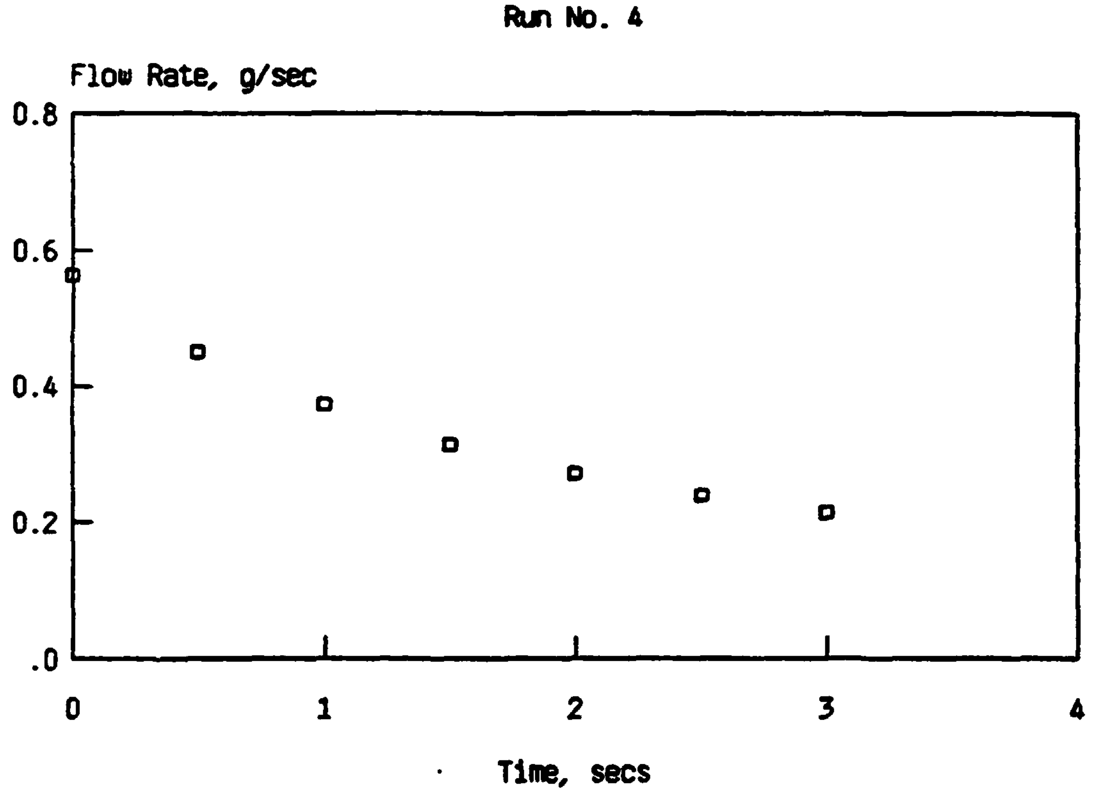

Figure 12. - Transient Vent Rate.

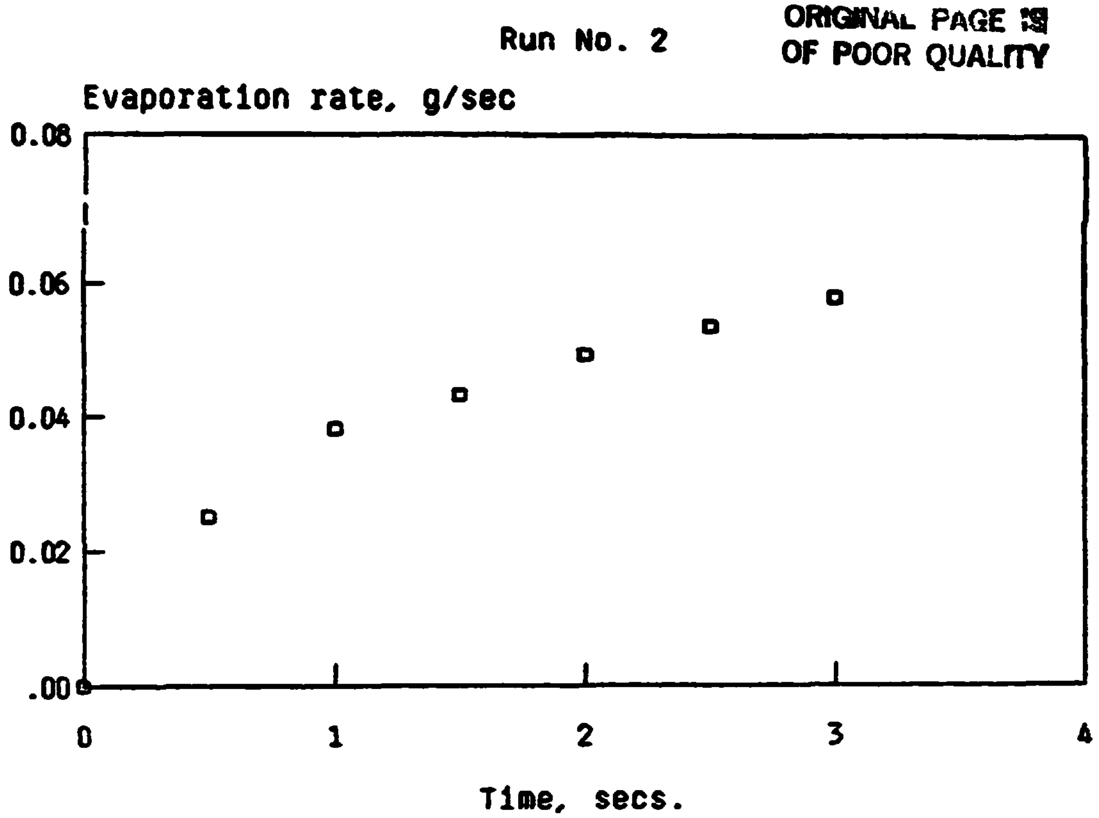

Figure 13. - Transient Interface Evaporation Rate

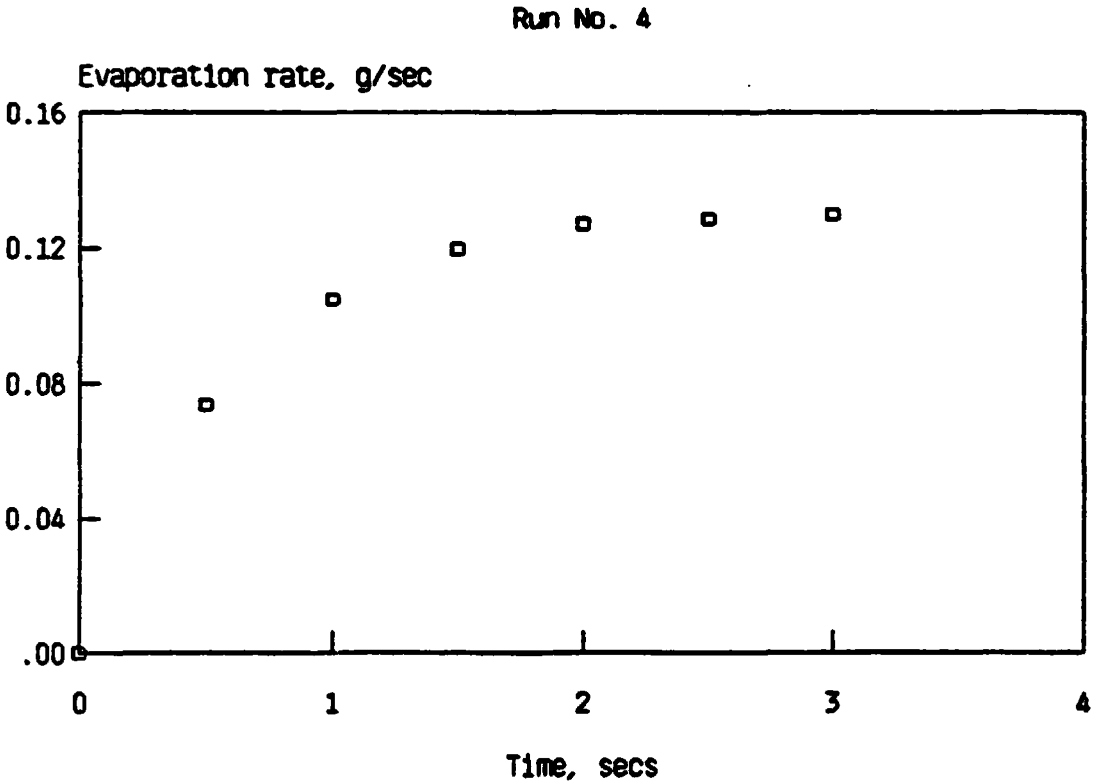

Figure 14. - Transient Interface Evaporation Rate.

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APPENDIX A

EXAMINATION OF THIRD ORDER CURVE FIT ACCURACY

A test was run to determine the accuracy of the third order least squares curve fit used in calculating the interfacial mass transfer. The test also determined the spacing of the nodes in the liquid which would give the best curve fit. The temperature distribution in a semi-infinite solid with constant surface temperature is derived in the ANALYSIS and given by Eq. (18):

$$\frac{T(x,t) - T_i}{T_o - T_i} = erf\left(\frac{x}{2(a\,t)^{1/2}}\right) \tag{A1}$$

From this the temperature gradient at x=0 is:

$$\left.\frac{dT}{dx}\right|_{x=0} = \frac{T_o - T_i}{(\pi a t)^{1/2}} \tag{A2}$$

Equation (A2) represents an exact solution for the gradient. An approximate solution is obtained via the method described in ANALYSIS. The penetration depth is calculated. A percentage of this depth near the surface is then divided into six equally spaced nodes at which the temperature is calculated. First, second, and third order curves are fit to the data obtained using different percentages of the penetration depth. As can be expected, the calculated gradient and intercepts were most accurate when the nodes were space closest to the interface, i.e. a small percentage of the penetration depth. Figures A1 and A2 show that using a third order polynomial with nodes very close to the interface give the best gradient and intercept results.

Note that equation (A2) gives the exact gradient for a semi-infinite solid with a step change in surface temperature. This can not be used in determining interfacial mass transfer in the proposed model of this report since the surface temperature in reality is a function of time.

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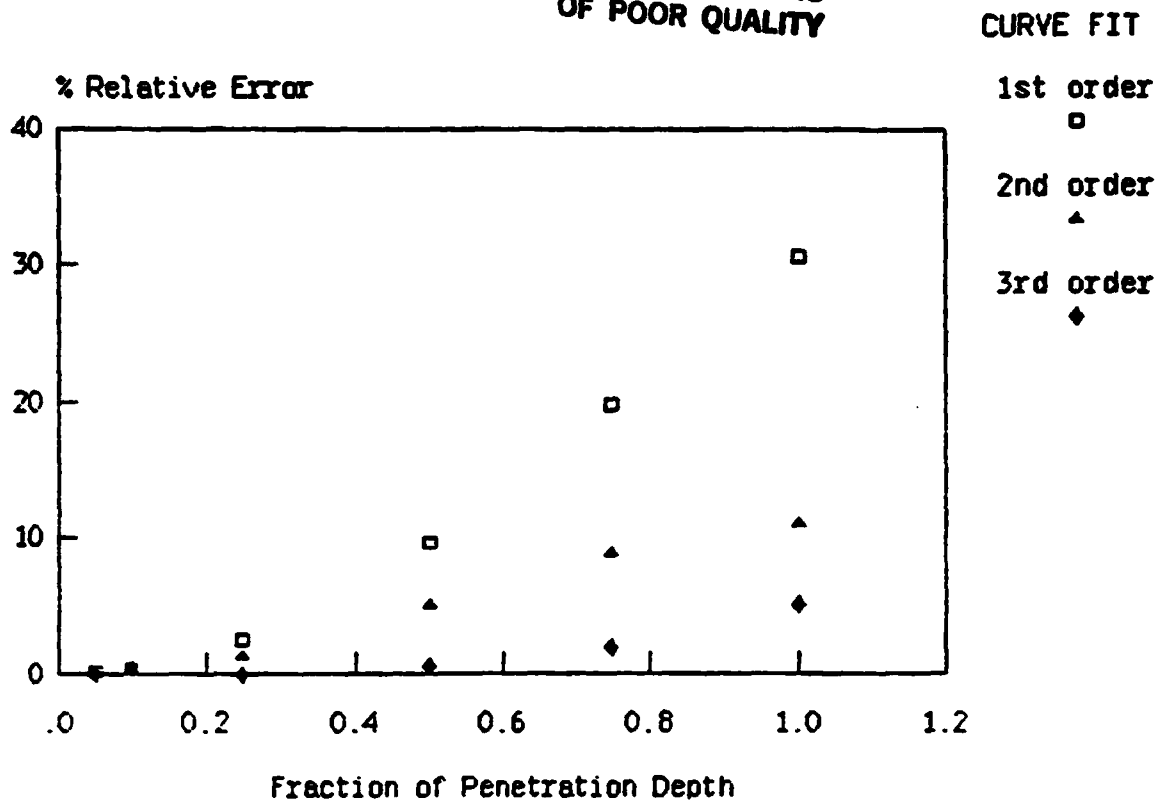

Fig. A1.-Error in Surface Temperature Gradient vs Fraction of Penet. Depth

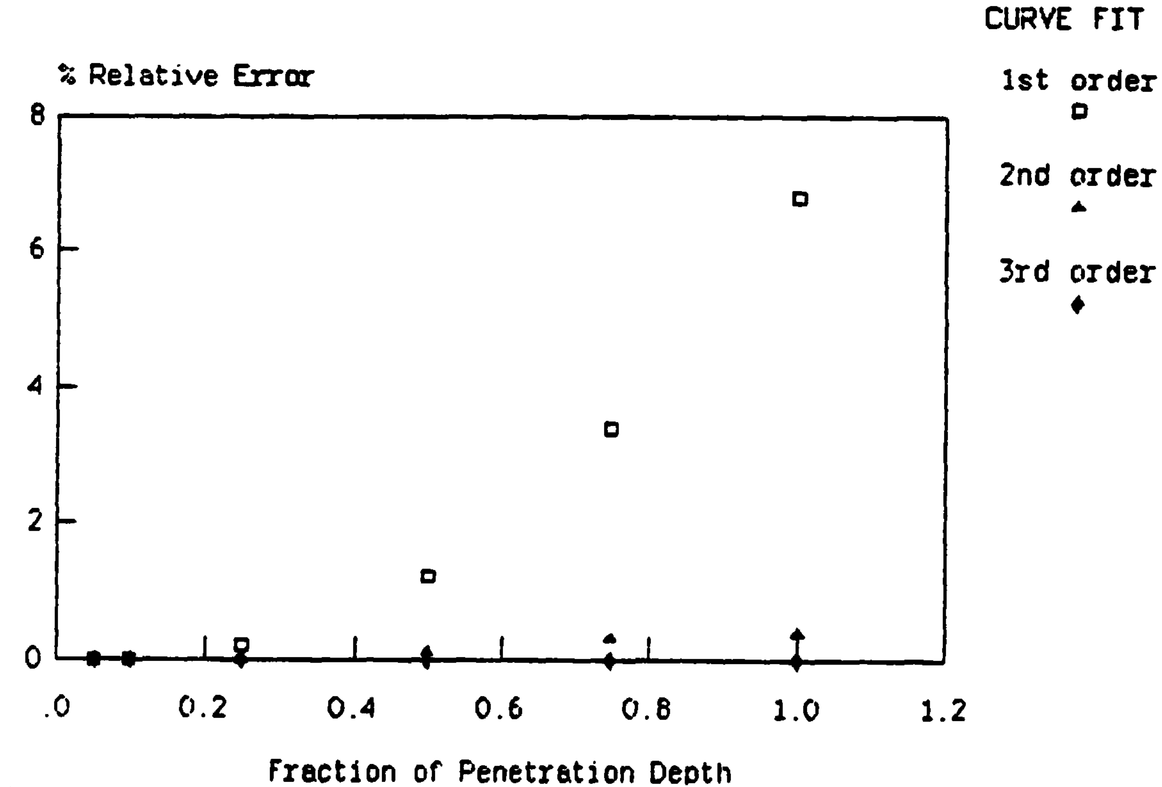

Fig. A2.-Error in Surface Temperature vs Fraction of Penetration Depth

APPENDIX B

COMPUTER ALGORITHM

The computer algorithm used to numerically solve the governing equations consists of a main program and eight subroutines. The basic outline of the numerical solution is as follows. At time= t, the vapor temperature and mass(Y(1), Y(2)) are known, along with the ullage volume, which is constant. Thus the state of the vapor and the interface are defined and all thermodynamic properties can be determined. With the state at time=t completely defined, values of Y(1) and Y(2) at t=t + 0.05 are found by solving the governing differential equations by a fourth-order Runge-Kutta method. With the values of Y(1) and Y(2) now determined at time=t + 0.05, this state is now completely defined, and the algorithm can be incremented by one time step and repeated. The following is a brief description of the function of each subroutine.

RUNGE - A fourth order Runge-Kutta algorithm to solve first order differential equations with non-constant coefficients. This routine uses a fixed time step, with the time step being the independent variable.

DERY - Calculates the derivatives of Y(1) and Y(2) with respect to time for use in the RUNGE algorithm.

PROPS - Determines the necessaary thermodynamic properties of the working fluid, given vapor temperature, mass, and volume. The four basic equations used to calculate the properties are; vapor-pressure equation, equation of state, density of saturated liquid, and heat capacity of vapor(Ref.3). All properties can be determined from these equations(App.C).

NEWTTS - The vapor-pressure equation is of the form P=f(Tsat). This routine uses the Newton-Rapson method(Ref.4) to solve this equation for Tsat, given P.

NEWTV - The equation of state is of the form P=f(v,Tv). This routine uses the Newton-Rapson method to solve the equation of state for the specific volume v, given P and Tv. These values of v are needed in PROPS to calculate internal energy and enthalpy.

MASS - Determines the rate of mass transfer across the liquid-vapor interface. As discussed in ANALYSIS, the liquid temperature gradient at the interface is needed to compute the interfacial mass transfer. Duhammel's superposition integral and the one dimensional conduction equation for a semi-infinite solid with a step change in surface temperature are used to compute the temperature of the liquid at various depths near the interface. A third order least squares curve fit(Ref.4) is used to find the best curve through these points and thus the surface temperature gradient.

SLUD - Along with SLIR, solves the system of equations describing the third order least squares curve fit. This routine computes the LU decomposition of the coefficient matrix.

SLIR - Computes the solution to the system of linear equations AX=B using iterative refinement. SLUD and SLIR are called from the MTS Numerical Analysis Library(Ref.5). Similar routines are readily available for users not on the MTS network(Ref.4).

FORTRAN Symbol	Engineering Symbol	Description	Units
AS	As	Interface surface area	ft2
AT	At	Nozzle cross sectional area	ft2
CD	Cd	Discharge coefficient	-
CVTVP	cv	Specific heat of vapor @ TV,P	ft-lbf/slug-R
HFGTS	hfg	Enthalpy of evaporation @ TS	ft-lbf/slug
HVTSP	hg	Enthalpy of vapor @ TS,P	ft-lbf/slug
HVTVP	h	Enthalpy of vapor @ TV,P	ft-lbf/slug
KLTS	k	Thermal conductivity of liquid @ TS	lbf/sec-R
ME	me	Mass flow rate of vapor vented	slug/sec
MI	mi	Mass flux across l-v interface	slug/sec
P	P	Ullage pressure	lbf/in2
PR	Pref	Peference pressure	lbf/in2
R	R	Ideal gas constant	psi-ft3/lbm-R
T	t	Time	seconds
TC	Tc	Critical temperature	R
TR	Tref	Reference pressure	R
TS	Tsat	Saturation temperature @ P	R
UVTRPR	uref	Reference internal energy @ TR,PR	ft-lbf/slug
UVTSP	ug	Internal energy of vapor @ TS,P	ft-lbf/slug
UVTVP	u	Internal energy of vapor @ TV,P	ft-lbf/slug
VU	Vu	Ullage volume	ft3
VLTSP	vf	Specific volume of liquid @ TS,P	ft3/lbm
VVTSP	vg	Specific volume of vapor @ TS,P	ft3/lbm
VVTVP	v	Specific volume of vapor @ TV,P	ft3/lbm
VVTVPR	v	Specific volume of vapor @ TV,PR	ft3/lbm
Y(1)	Tv	Temperature of ullage vapor	R
Y(2)	mv	Mass of ullage vapor	lbm
YP(1)	dTv/dt	Time rate of change of vapor temp.	R/sec
YP(2)	dmv/dt	Time rate of change of vapor mass	lbm/sec

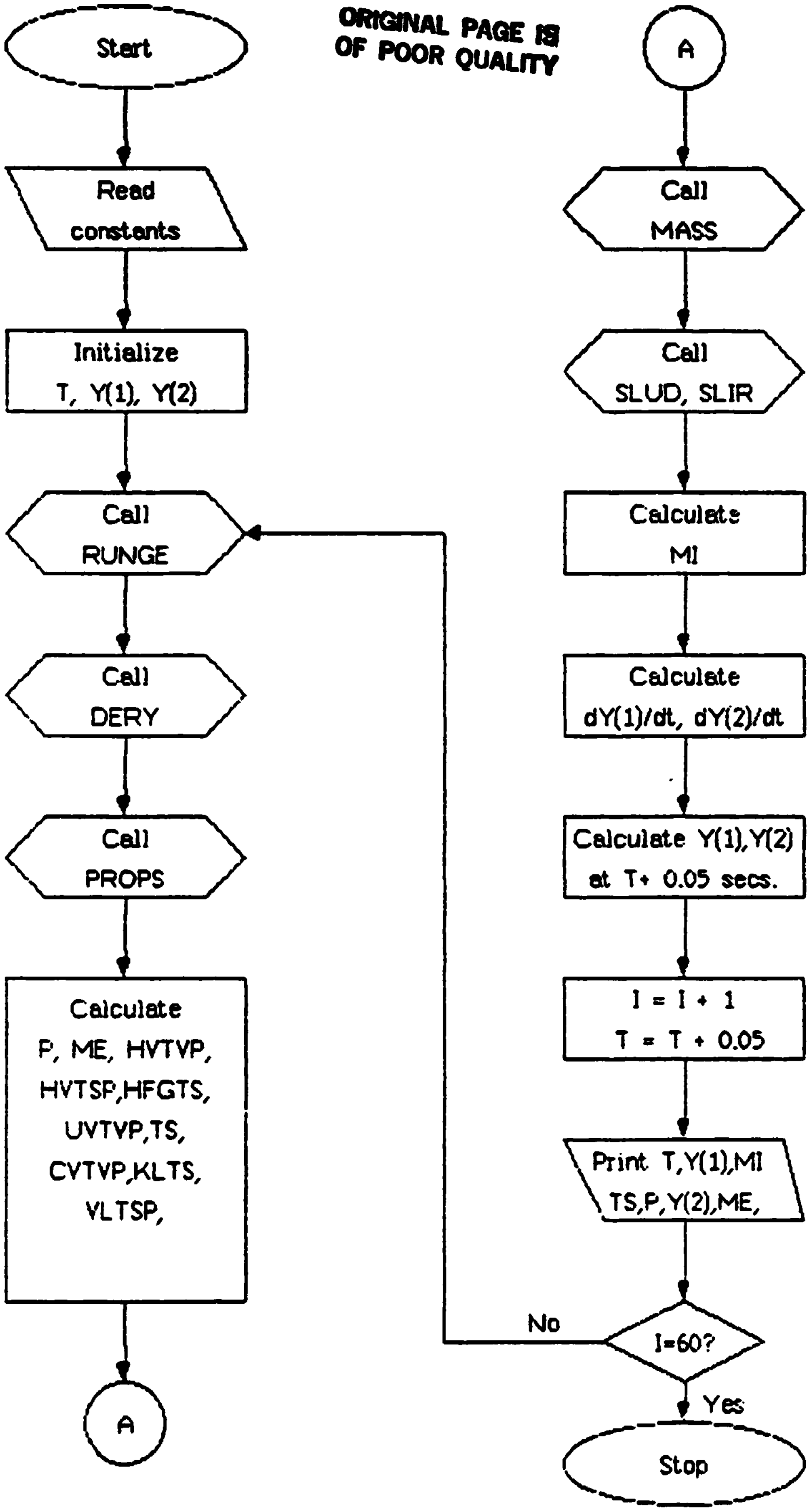

Figure 6. - Algorithm Flow Chart

B1

Listing of MAIN+... at 10:07:26 on APR 6, 1984 for CCid=SS3X

```
C
C  READ IN CONSTANTS FOR FREON-11 VAPOR PRESSURE CURVE, EQUATION
C  OF STATE, HEAT CAPACITY OF THE VAPOR, AND DENSITY OF SATURATED
C LIQUID.
C
        BLOCK DATA
        REAL A(6),B(6),C(6),D(6),E(6),F(6),R,CK,
     1  SB,P,MS,TCRIT,TR,PR,CD,CC,AT,UV,AS,KLTS
C
        COMMON/ALPHA/A,B,C,D,E,F,R,CC,SB,CD,AT,UV,TCRIT,TR,PR,
     1     UVTRPR,AS
C
       DATA A/0.0,-3.126759,-0.025341,0.001687277,-2.35893E-5,
     1  1.057504E8/
      DATA B/0.0,0.001318523,4.875121E-5,-1.805062E-6,2.448303E-8,
     1  -9.472103E4/
C
       DATA C/0.0,-35.76999,1.220367,0.0,-1.478379E-4,0.0/
C
       DATA D/42.14702865,-4344.343807,-12.84596753,
     1  0.004008372507,0.0313605356,862.07/
C
       DATA E/34.57,57.63811,43.63220,-42.82356,36.70663,0.0/
C
       DATA F/0.023815,-336.80703,2.798823E-4,-2.123734E-7,
     1  5.999018E-11,0.0/
C
       DATA R,TCRIT,SB,CC/0.078117,848.07,0.0019,-4.5/
        DATA TR,PR,UVTRPR/419.67,0.74137,2032163.0/
         END
C
C  BEGIN MAIN PROGRAM.
C
C
        REAL Y(2),YP(2),A(6),B(6),C(6),D(6),E(6),F(6),R,CK,
     1  SB,P,MS,ERF(150,2),TCRIT,TR,PR,CD,CC,AT,UV,AS,KLTS
         REAL Z(4)
        REAL MI,ME,WME,WMI,WTS,TS,WMASS
C
        COMMON/ALPHA/A,B,C,D,E,F,R,CC,SB,CD,AT,UV,TCRIT,TR,PR,
     1     UVTRPR,AS
C
C  READ IN ERROR FUNCTION VALUES FOR USE IN SUBROUTINE MASS.  DATA
C  LOCATED IN FILE 'ERF'.
C
         DO 22 K=1,102
         READ(7,34) ERF(K,1),ERF(K,2)
 34      FORMAT(2F20.9)
 22      CONTINUE
C
C
C
C  TR,PR,UVTRPR INITIALIZED IN SUBROUTINE PROPS
C
C    INITIALIZE T,Y(1),Y(2), AT T=0.0 SECONDS.
C
C    SET AT AND CD, THE VARIABLES WHICH CONTROL VENT FLOW RATE.
C    SET UV, THE ULLAGE VOLUME
```

```
C
C  UNITS: T IN SECS.,Y(1) IN RANKINE, Y(2) IN LBM, AT IN FT2,
C        UV IN FT3.
C  INITIALIZE VARIABLES AT T=0.0
C
        T=0.0
          Y(1)=531.74
          Y(2)=0.00237026
          AT=0.0000314902
          UV=0.006815371
          CD=0.77
C
C  WRITE OUT INPUT VALUES
        WRITE(6,38) Y(1),Y(2),AT,UV,CD
 38     FORMAT('INITIAL VAPOR TEMPERATURE IS',2X,F8.3,'RANKINE'/
     1   'INITIAL VAPOR MASS IS',2X,E11.5,2X,'LBM'/
     1   'NOZZLE AREA IS',2X,E11.5,2X,'SQUARE FEET'/
     1   'ULLAGE VOLUME IS',2X,E11.5,2X,'CUBIC FEET'/
     1   'DISCHARGE COEFFICIENT IS',2X,F6.2)
C
C   WRITE OUT HEADINGS
C
        WRITE(6,71)
         WRITE(6,72)
         WRITE(6,73)
 73      FORMAT(' ')
 71      FORMAT('TIME',2X,'T VAPOR',4X,'TSAT',5X,'P VAPOR',3X,
     1    'VAPOR MASS',2X,'VENT RATE',3X,'EVAP RATE')
 72      FORMAT('SECS',3X,'KELVIN',4X,'KELVIN',3X,'PASCALS',6X,'KG',
     1   8X,'KG/SEC',8X,'KG/SEC')

C
C  USING A FOURTH ORDER RUNGE KUTTA METHOD TO EVALUATE THE INTEGRALS
C  THE FOLLOWING LOOP WILL BE RUN THROUGH 60 TIMES WITH A TIME STEP
C  OF 0.05 SECONDS.  TOTAL TEST TIME BEING 3.0 SECONDS.
C
          DO 23 KL=1,60
          CALL RUNGE(Y,T,YP,P,MI,ME,ERF,TS)
C
C  CONVERT UNITS FROM ENGLISH TO MKS AND WRITE OUT RESULTS
C
         WTEMP= (Y(1)-459.67)*5/9-17.77778+ 273.14
         WTS=(TS-459.67)*5/9 -17.77778 + 273.14
        WMASS=Y(2)/2.205
           WP=P*6895.0
          WMI=MI*14.59
          WME=ME*14.59
C
C     INCREMENT TIME
C
         T=T+0.05
         WRITE(6,24) T,WTEMP,WTS,WP,WMASS,WME,WMI
 24    FORMAT(F4.2,2X,F8.4,2X,F8.4,2X,F8.2,2X,F10.8,1X,E10.4,1X,E11.
 23      CONTINUE
        STOP
        END
C
C  SUBROUTINE RUNGE RUNS A 4TH ORDER RUNGE-KUTTA METHOD TO NUMERI-
```

```
C  CALLY SOLVE THE DIFFERENTIAL EQUATIONS GOVERNING THE SYSTEM.
C
       SUBROUTINE RUNGE(Y,T,YP,P,MI,ME,ERF,TS)
        REAL A(6),B(6),C(6),D(6),E(6),F(6),R,CK,
     1  SB,P,MS,ERF(150,2),TCRIT,TR,PR,CD,CC,AT,UV,AS,KLTS
        REAL Z(4)
C
        COMMON/ALPHA/A,B,C,D,E,F,R,CC,SB,CD,AT,UV,TCRIT,TR,PR,
     1     UVTRPR,AS
           REAL ENDRKS,MI,ME
       REAL K0(2),K1(2),K2(2),K3(2),Y(2),YP(2),NEWY(2)
       H=0.05
C  TIME STEP, H, SET AT 0.05 SEC
C
C  COMPUTE FIRST APPROX OF SLOPE
C
         ENDRKS=1.0
              CALL DERY(Y,T,YP,P,MI,ME,ENDRKS,ERF,TS)
         DO 90 J=1,2
              K0(J)=H*YP(J)
  90          CONTINUE
C
C  SECOND APPROX OF SLOPE
C
            ENDRKS=0.0
       Z(2)=Y(2)+K0(2)/2.
            Z(1)=Y(1)+K0(1)/2.0
            V=T+H/2.0
            CALL DERY(Z,V,YP,P,MI,ME,ENDRKS,ERF,TS)
            K1(1)=H*YP(1)
                    K1(2)=H*YP(2)
C
C THIRD APPROX OF SLOPE
C
         Z(1)=Y(1)+K1(1)/2.0
         Z(2)=Y(2)+K1(2)/2.0
         CALL DERY(Z,V,YP,P,MI,ME,ENDRKS,ERF,TS)
         K2(1)=YP(1)*H
         K2(2)=YP(2)*H
C
C  FOURTH APPROX OF SLOPE
C
        Z(1)=Y(1)+K2(1)
        Z(2)=Y(2)+K2(2)
                 V=T+H
                 CALL DERY(Z,V,YP,P,MI,ME,ENDRKS,   F,TS)
        K3(1)=H*YP(1)
        K3(2)=H*YP(2)
C
C PREDICT FUTURE Y BASED ON AN AVERAGE SLOPE
C
           DO 93 M=1,2
                 Y(M)=Y(M)+(K0(M)+2*K1(M)+2*K2(M)+K3(M))/6.0
                 NEWY(M)=Y(M)

  93             CONTINUE
C
C
```

```
                RETURN
              END
      C
      C  SUBROUTINE EVALUATES THE VALUES dY(1)/dt AND dY(2)/dt FOR EACH
      C  CALL FROM THE SUBROUTINE RUNGE.
      C
               SUBROUTINE DERY(Y,T,YP,P,MI,ME,ENDRKS,ERF,TS)
               REAL ENDRKS
               REAL A(6),B(6),C(6),D(6),E(6),F(6),R,CK,
           1   SB,P,MS,ERF(150,2),TCRIT,TR,PR,CD,CC,AT,UV,AS,KLTS
      C
               COMMON/ALPHA/A,B,C,D,E,F,R,CC,SB,CD,AT,UV,TCRIT,TR,PR,
           1      UVTRPR,AS
      C
               REAL Y(3),T,YP(3),MI,ME
      C
      C  CALL SUBROUTINE PROPS TO FIND THERMODYNAMIC PROPERTIES OF THE
      C  LIQUID AND VAPOR, GIVEN Y(1),AND Y(2); THE TEMPERATURE AND MASS
      C  THE VAPOR.
      C
                CALL PROPS(Y,T,YP,P,ME,HVTVP,UVTVP,HVTSP,HFGTS,TS,CVTVP,KL
      C
      C  CALL SUBROUTINE MASS TO COMPUTE THE MASS FLOW RATE ACROSS THE
      C  LIQUID-VAPOR INTERFACE
      C
                CALL MASS(Y,T,TS,HFGTS,KLTS,VLTSP,MI,ENDRKS,ERF)
      C
      C  COMPUTE DY(1)/DT AND DY(2)/DT, THE DERIVATIVES OF VAPOR TEMPERAT
      C  AND VAPOR MASS WITH RESPECT TO TIME.
      C
              YP(1)=(HVTSP-UVTVP)*MI/(Y(2)*CVTVP)+(UVTVP-HVTVP)*ME/(Y(2)*C
              YP(2)=(MI-ME)*32.174
      C
      C

               RETURN
               END
      C
      C  SUBROUTINE PROPS COMPUTES THE THERMODYNAMIC PROPERTIES OF THE
      C  WORKING FLUID, GIVEN THE VAPOR TEMPERATURE AND MASS.
               SUBROUTINE PROPS(Y,T,YP,P,ME,HVTVP,UVTVP,HVTSP,HFGTS,
           1   TS,CVTVP,KLTS,VLTSP)
      C
               COMMON/ALPHA/A,B,C,D,E,F,R,CC,SB,CD,AT,UV,TCRIT,TR,PR,
           1      UVTRPR,AS
               REAL Y(2),YP(2),A(6),B(6),C(6),D(6),E(6),F(6),ERF(150,2)
               REAL XV(4),XT(4),WV(4),WT(4)
                REAL KLTS,ME,XCV(4)
      C
      C  CRITICAL TEMPERATURE AND RELATIVE TEMPERATURE AND PESSURE OF FRE
      C
                T     8.07
                P    .74317
                TR-427.0
      C
      C  COMPUTE SPECIFIC VOLUME OF ULLAGE VAPOR, FT3/LBM.
      C
               VVTVP=UV/Y(2)
```

```
C
C  COMPUTE ULLAGE PRESSURE FROM EQUATION OF STATE, KNOWING TEMPERAT
C  OF ULLAGE AND SPECIFIC VOLUME OF ULLAGE VAPOR , UNITS OF P ARE P
C

        P=R*Y(1)/(VVTVP-SB) +(A(2)+B(2)*Y(1)+C(2)*EXP(CC*Y(1)/TCRIT
     1  /((VVTVP-SB)**2)    +(A(3)+B(3)*Y(1)+C(3)*EXP(CC*Y(1)/TCRIT
     1  /((VVTVP-SB)**3) +(A(4)+B(4)*Y(1))/((VVTVP-SB)**4)
     1  + (A(5)+B(5)*Y(1)+C(5)*EXP(CC*Y(1)/TCRIT))/((VVTVP-SB)**5)
C
 3       FORMAT(3F15.9)
C  CALL THE NEWTONS METHOD SUBROUTINES TO FIND TS,VVTSP,VVTVPR, AND
C  VVTSPR.  THESE SPECIFIC VOLUMES ARE NEEDED FOR THE CALCULATION
C  OF ENTHALPY AND INTERNAL ENERGY.
C   NEWTTS USES NEWTONS METHOD TO SOLVE THE VAPOR PRESSURE EQUATION
C  FOR TSAT, GIVEN PSAT.  NEWTV SOLVES THE EQUATION OF STATE FOR
C  SPECIFIC VOLUME, GIVEN TEMPERATURE AND PRESSURE OF THE VAPOR.
C    UNITS ARE: TS IN DEGREES RANKINE, SPEC. VOL. IN FT3/LBM
C
        VTOL=0.005
        TSTOL=0.5
         CALL NEWTTS(TSTOL,P,TS)
        CALL NEWTV(VTOL,P,TS,VVTSP)
        CALL NEWTV(VTOL,PR,Y(1),VVTVPR)
        CALL NEWTV(VTOL,PR,TS,VVTSPR)

C
C ASSIGN TEMPORARY VALUES TO SPECIFIC VOLUMES AND TEMPERATURES TO C
C INTERNAL ENERGY AND EHTHALPY
C
       XT(1)=TR
        XT(2)=TS
       XT(3)=Y(1)
       XT(4)=TS
       XV(1)=VVTSPR
       XV(2)=VVTVP
       XV(3)=VVTSP
       XV(4)=VVTVPR
C
C  EVALUATE THE INTERNAL ENERGY INTEGRALS IN  THIS LOOP
C  UNITS OF WV AND WX ARE FT-LBF/SLUG OR FT2/SEC2
C
        DO 6 I=1,4
        IF(I.LE.2) TEMP=TS
        IF(I.GT.2) TEMP=Y(1)
        BETA=TEMP*CC/TCRIT
        WV(I)=((A(2)-C(2)*(BETA-1.)*EXP(BETA))/(XV(I)-SB)
     1      +(A(3)-C(3)*(BETA-1.)*EXP(BETA))/(2.*(XV(I)-SB)**2.)
     1      +A(4)/(3.*(XV(I)-SB)**3.) +(A(5)-C(5)*(BETA-1.)*EXP(BET
     1  )/(4.*(XV(I)-SB)**4.))*144.0*32.174

C
C
          WT(I)=(F(1)*XT(I)-F(2)/XT(I [illegible] 3)*XT(I)**2.)/2.
     1    +(F(4)*XT(I)**3.)/3. +(F( [illegible] .)/4.)*778.17*32.174
C
C
 6         CONTINUE
```

```
C
C  COMPUTE INTERNAL ENERGY AND ENTHALPY USING THE VALUES OF WV AND
C  UNITS ARE FT2/SEC2 OR FT-LBF/SLUG
C
          UVTVP=WV(2)-WV(4)+WT(3)-WT(1)+UVTRPR
          UVTSP=WV(3)-WV(1)+WT(2)-WT(1)+UVTRPR
          HVTVP=UVTVP + P*144.0*32.174*VVTVP
          HVTSP=UVTSP + P*144.0*32.174*VVTSP
C
C  COMPUTE SPECIFIC VOLUME OF LIQUID IN FT3/SLUG
C
       CON=1-(TS/TCRIT)
        RHOL=E(1) +E(2)*CON**(1./3.) +E(3)*CON**(2./3.) +
     1   E(4)*CON +E(5)*CON**(4./3.)
        VLTSP=32.174/RHOL
C
C COMPUTE DP/DT
C
C
        DPDT=(-D(2)*ALOG(10.0)/(TS**2.) +D(4)*ALOG(10.) +D(3)/TS
     1  -D(5)*D(6)*ALOG(D(6)-TS)/(TS**2.) +D(5)/TS)*EXP(ALOG(10.)
     1  *(D(1)+D(2)/TS +D(4)*TS) +D(3)*ALOG(TS) +D(5)*(D(6)-TS)*
     1  ALOG(D(6)-TS)/TS)
C
C  COMPUTE ENTHALPY OF FORMATION
C
C IN FT2/SEC2
C
        HFGTS=TS*(VVTSP-(VLTSP/32.174))*DPDT*144.0*32.174
C
C COMPUTE K, THERMAL CONDUCTIVITY OF THE LIQUID
C   UNITS ARE LBF/SEC-R.  A LINEAR CURVE FIT IS USED.
C
        KLTS=(0.111562-TS*0.000115)*0.216158
C
C  COMPUTE CV, THE SPECIFIC HEAT, AT TEMPERATURE OF THE VAPOR
C
        CV0=F(1)+F(2)/(Y(1)**2.) +F(3)*Y(1) +F(4)*Y(1)**2.+F(5)*(Y(
C
C   THE DO LOOP EVALUATES AN INTEGRAL TO FIND SPECIFIC HEAT AT TV
C RELATIVE TO THE SPECIFIC HEAT AT T-RELATIVE
C
         DO 357 L=2,4,2
         XCV(L)=Y(1)*(-CC/TC)**2.*EXP(CC*Y(1)/TC)*(-C(2)/(XV(L)-SB)
     1   -C(3)/(2.*(XV(L)-SB)**2.) -C(5)/(4*(XV(L)-SB)**4.))*144.*3
  357   CONTINUE
C
C  XCV IN UNITS OF FT2/SEC2-R, CONVERT CV0,R TO THOSE UNITS
C
         CV0=CV0*778.16*32.174
         R=R*144.*32.174
C
C  COMPUTE CV, THE SPECIFIC HEAT CONSTANT
C
         CV=CV0 + XCV(2) - XCV(4)
        CVTVP=CV
C
C  COMPUTE THE MASS FLOW RATE THROUGH THE NOZZLE BASED ON THE BULK
```

```
C  PROPERTIES OF THE VAPOR
C   IN UNITS OF SLUGS/SEC
C
          CP=R + CV
          ALPHA=(CP/CV)**0.5 *(2/((CP/CV)+1.))**(((CP/CV)+1.)/(2.
     1     *((CP/CV)-1.)))
          ME=CD*ALPHA*P*AT*144./((R*Y(1))**0.5)
C
C
        R=R/(144.*32.174)
      RETURN
      END
C
C
C
        SUBROUTINE NEWTV(ERROR,PRESS,TEMP,X)
        REAL A(6),B(6),C(6),D(6),E(6),F(6),R,CK,
     1  SB,P,MS,ERF(150,2),TCRIT,TR,PR,CD,CC,AT,VU,AS,KLTS
C
C
        COMMON/ALPHA/A,B,C,D,E,F,R,CC,SB,CD,AT,VU,TC,TR,PR,
     1     UVTRPR,AS
C
C
C
C
C    THIS ROUTINE USES NEWTONS METHOD TO FIND THE ROOTS OF THE
C  EQUATION OF STATE EQUATION, THE SPECIFIC VOLUME.
C
C INITIAL GUESS FOR SPECIFIC VOLUME
C
        X=R*TEMP/PRESS
C
C  PERFORM NEWTONS METHOD UNTIL ERROR IS LESS THAN VTOL
C
        DO 40  J=1,7
       Z=R*TEMP/(X-SB) +(A(2)+B(2)*TEMP+C(2)*EXP(CC*TEMP/TC))
     1 /((X-SB)**2)    +(A(3)+B(3)*TEMP+C(3)*EXP(CC*TEMP/TC))
     1 /((X-SB)**3)    +(A(4)+B(4)*TEMP)/((X-SB)**4) +
     1 (A(5)+B(5)*TEMP+C(5)*EXP(TEMP*CC/TC))/((X-SB)**5)-PRESS
        CON=CC*TEMP/TC
        DZDV=-(R*TEMP)/(X-SB)**2. -2.*(A(2)+B(2)*TEMP+C(2)*EXP(CON))
     1  /(X-SB)**3. -3.*(A(3)+B(3)*TEMP+C(3)*EXP(CON))/(X-SB)**4.
     1  -4.0*(A(4)+B(4)*TEMP)/(X-SB)**5. -5.*(A(5)+B(5)*TEMP
     1  +C(5)*EXP(CON))/(X-SB)**6.
C
C  COMPUTE NEW SPECIFIC VOLUME
C
          X=X-Z/DZDV
          IF(Z/DZDV.LT.ERROR) GO TO 40
 40        CONTINUE
           RETURN
           END
       SUBROUTINE NEWTTS(ERROR,PRESS,X)
        REAL A(6),B(6),C(6),D(6),E(6),F(6),R,CK,
     1  SB,P,MS,ERF(150,2),TCRIT,TR,PR,CD,CC,AT,VU,AS,KLTS
C
```

```
C
      COMMON/ALPHA/A,B,C,D,E,F,R,CC,SB,CD,AT,VU,TCRIT,TR,PR,
     1   UVTRPR,AS
C
C  THIS ROUTINE USES NEWTONS METHOD TO FIND THE ROOTS OF THE
C VAPOR-PRESSURE EQUATION; THE SATURATED TEMP CORRESPONDING TO
C THE GIVEN P
C
C  AN INITIAL GUESS FOR X
C
      X=560.0
C
C  USE NEWTONS METHOD UNTIL ERROR IS LESS THAN TSTOL
C
      DO 75 K=1,7
      DZDT=(-D(2)*ALOG(10.0)/(X**2.) +D(4)*ALOG(10.) +D(3)/X
     1 -D(5)*D(6)*ALOG(D(6)-X)/(X**2.) +D(5)/X)*EXP(ALOG(10.)
     1 *(D(1)+D(2)/X +D(4)*X) +D(3)*ALOG(X) +D(5)*(D(6)-X)*
     1 ALOG(D(6)-X)/X)
      Z=EXP((D(1)+D(2)/X +D(4)*X)*ALOG(10.) +D(3)*ALOG(X) +
     1 D(5)*(D(6)-X)*ALOG(D(6)-X)/X) -PRESS
C
C   COMPUTE NEW VALUE FOR TEMP SATURATED
C
      X=X-Z/DZDT
      IF(Z/DZDT .LT. ERROR) GO TO 75
 75    CONTINUE
       RETURN
       END
C
C
C  THIS ROUTINE COMPUTES THE MASS FLUX ACROSS THE LIQUID VAOR INTER-
C  FACE, THE EVAPORATION RATE.  DUHAMMELS SUPERPOOSITION INTEGRAL IS
C  USED IN APLYING THE SEMI-INFINITE SOLID WITH TRANSIENT SURFACE
C  TEMPERATURE.
C   IMPROVED MASS USING NEW INDICIES TO GIVE PHI(1)=TS
C
        SUBROUTINE MASS(Y,T,TS,HFGTS,KLTS,VLTSP,MI,ENDRKS,ERF)
       COMMON/ALPHA/A,B,C,D,E,F,R,CC,SB,CD,AT,UV,TCRIT,TR,PR,
     1   UVTRPR,AS
        REAL ENDRKS
        REAL  PHI(6),KLTS,MI,ERF(150,2),THETA(100)
       REAL U(10,10),MT(10,10),MX(10),MB(10),MR(10)
         INTEGER N,NN,IV(10)
C
       NN=(T+0.01)/0.05 + 1
       IF(NN.LT.2) SAVED=0.0
C
C  NN IS THE NUMBER OF TIME STEPS WHICH HAVE TAKEN PLACE UP TILL NOW
C
C
C COMPUTE MASS FLOW RATE 1 TIME PER RUNGE-KUTTA STEP
        IF(ENDRKS.EQ. 0.0) GO TO 123
C
C
        THETA(NN)=TS
         IF(NN.LT.2) GO TO 123
C
```

```
C  AS IS THE LIQUID SURFACE AREA IN FT2
C
           AS=0.0599332
C CALCULATE SPECIFIC HEAT OF LIQUID BY LINEAR CURVE FIT
C UNITS FT-LBF/SLUG-R
C
           CLTS=(TS*0.000031666+0.190144)*778.16*32.174
C
C  CALCULATE AALPHA, THERMAL DIFFUSIVITY
C
            AALPHA=KLTS*VLTSP/CLTS
C
C  COMPUTE THE DEPTH  AT WHICH THE TEMPERATURES IN THE FLUID
C  WILL BE APPROXIMATED.  THE PENETRATION DEPTH IS FOUND, AND THEN
C  OF THIS VALUE IS USED AS THE REGION IN WHICH THE TEMPERATURES WI
C  BE DETERMINED. THIS DEPTH IS THEN DIVIDED INTO 6 LOCATIONS.
C
         DEPTH=0.10*1.39*2.*((AALPHA*T)**0.5)/6.
C
C COMPUTE TEMP AT SIX LOCATIONS, STARTING AT THE LIGUID-VAPOR IN-
C TERFACE USING DUHAMMEL'S SUPERPOSITION APPLIED TO A SEMI-INFINITE
C SOLID WITH TRANSIENT SURFACE TEMPERATURE
          DO 88 I=1,6
          DELX=(I-1)*   DEPTH
C
C    IF TIME=0.0, LIQUID IS UNIFORM TEMP AT TSAT
C
          IF(T.EQ.0.0) PHI(I)=TS
          IF(T.EQ.0.0) GO TO 88
C
       PHI(I)=THETA(1)
        DO 90 K=2,NN
C
        DELT=T-(K-2)*0.05
C
              VAL=DELX/(2.*(DELT*AALPHA)**0.5)
C
C  FIND ERF(VAL)
C
                      DO 77 J=1,102
                      IF(VAL.LT.ERF(J,1)) GO TO 83
  77                  CONTINUE
          WRITE(6,5) J
  5       FORMAT(I3)
  83      ERFVAL=ERF(J-1,2)+(ERF(J,2)-ERF(J-1,2))*(VAL-ERF(J-1,1))/
     1       (ERF(J,1)-ERF(J-1,1))
  19         ERFC=1-ERFVAL
C
              PHI(I)=(THETA(K)-THETA(K-1))*ERFC+PHI(I)
              IF(K.GT.70) GO TO 90
 90     CONTINUE
  88    CONTINUE
C
C  SET UP THE COEFFICIENT MATRIX FOR A LEAST SQUARES THIRD
C  ORDER CURVE FIT.
C
        N=4
        U(1,1)=6.
```

```
       U(1,2)=15.*DEPTH
       U(2,1)=15.*DEPTH
       U(1,3)=55.*DEPTH**2.
       U(2,2)=55.*DEPTH**2.
       U(3,1)=55.*DEPTH**2.
       U(1,4)=225.*DEPTH**3.
       U(2,3)=225.*DEPTH**3.
       U(3,2)=225.*DEPTH**3.
       U(4,1)=225.*DEPTH**3.
       U(2,4)=979.*DEPTH**4.
       U(3,3)=979.*DEPTH**4.
       U(4,2)=979.*DEPTH**4.
       U(3,4)=4425.*DEPTH**5.
       U(4,3)=4425.*DEPTH**5.
       U(4,4)=20515.*DEPTH**6.
C
C   IN ORDER FOR MORE ACCURATE MARTRIX ARITHMETIC, THE VALUES OF PHI
C      WILL BE SCALED DOWN TO THE SAME ORDER OF MAGNITUDE AS THAT OF
C      DELTA X.
C
C       WRITE(6,65) THETA(1)
C      WRITE(6,65) TS
       DO 66  II=1,6
        PHI(II)=PHI(II)-THETA(1)
C      WRITE(6,65) PHI(II)
 65    FORMAT(F15.6)
 66    CONTINUE
C
C
       MB(1)=PHI(1)+PHI(2)+PHI(3)+PHI(4)+PHI(5)+PHI(6)
       MB(2)=DEPTH*(PHI(2)+2*PHI(3)+3*PHI(4)+4*PHI(5)
     1  +5*PHI(6))
      MB(3)=DEPTH**2*(PHI(2)+4*PHI(3)+9*PHI(4)+16*PHI(5)
     1  +25*PHI(6))
      MB(4)=DEPTH**3*(PHI(2)+8*PHI(3)+27*PHI(4) +64*PHI(5)
     1  +125*PHI(6))
C
C   CALL THE SUBROUTINES SLUD AND SLIR. SLUD COMPUTES THE LU-DECOMP-
C   OSITION OF THE MATRIX U. SLIR COMPUTES A SOLUTION TO THE SYSTEM
C   OF LINEAR EQUATIONS U*MX=MB.
C
       CALL SLUD(N,10,U,10,MT,IV)
       CALL SIR(N,10,U,10,MT,IV,MX,MB,MR,IER)
C
C
C       WRITE(6,122) MX(1),MX(2),MX(3),MX(4)
 122    FORMAT(4E15.8)
C
C
         DTDX0=MX(2)
         SAVED=DTDX0
         GO TO 124
 123     DTDX0=SAVED
C
C  COMUTE MASS FLOW RATE BASED ON THE SLOPE AT THE INTERFACE, REPRE-
C  SENTED BY MX(2).
C
 124      MI=AS*KLTS*DTDX0/HFGTS
```

```
      RETURN
      END
```

```
Listing of -ONE at 10:18:28 on APR  6, 1984 for CCid=SS3X

INITIAL VAPOR TEMPERATURE IS   531.740RANKINE
INITIAL VAPOR MASS IS  0.23703E-02  LBM
NOZZLE AREA IS  0.31490E-04  SQUARE FEET
ULLAGE VOLUME IS  0.68154E-02  CUBIC FEET
DISCHARGE COEFFICIENT IS    0.77
TIME   T VAPOR     TSAT      P VAPOR    VAPOR MASS  VENT RATE    EVAP RAT
SECS   KELVIN     KELVIN     PASCALS        KG       KG/SEC         KG/SE

0.05  295.3511  294.1243  91584.75  0.00102505 0.9752E-03  0.0
0.10  295.3027  292.8953  87597.38  0.00097898 0.9328E-03  0.3253E-0
0.15  295.2554  291.7068  83870.31  0.00093605 0.8932E-03  0.5444E-0
0.20  295.2085  290.5537  80373.00  0.00089588 0.8561E-03  0.7133E-0
0.25  295.1621  289.4314  77084.81  0.00085822 0.8211E-03  0.8537E-0
0.30  295.1165  288.3411  73989.19  0.00082285 0.7882E-03  0.9737E-0
0.35  295.0713  287.2834  71085.00  0.00078975 0.7574E-03  0.1108E-0
0.40  295.0264  286.2517  68333.63  0.00075845 0.7281E-03  0.1170E-0
0.45  294.9817  285.2500  65745.81  0.00072908 0.7006E-03  0.1270E-0
0.50  294.9373  284.2720  63295.86  0.00070133 0.6746E-03  0.1327E-0
0.55  294.8931  283.3237  60991.94  0.00067529 0.6501E-03  0.1415E-0
0.60  294.8489  282.4033  58814.86  0.00065071 0.6269E-03  0.1472E-0
0.65  294.8049  281.5078  56759.19  0.00062755 0.6051E-03  0.1528E-0
0.70  294.7610  280.6326  54806.49  0.00060558 0.5843E-03  0.1554E-0
0.75  294.7170  279.7825  52959.86  0.00058484 0.5647E-03  0.1597E-0
0.80  294.6731  278.9617  51223.87  0.00056536 0.5462E-03  0.1661E-0
0.85  294.6292  278.1584  49571.90  0.00054686 0.5286E-03  0.1674E-0
0.90  294.5852  277.3801  48010.46  0.00052938 0.5120E-03  0.1710E-0
0.95  294.5415  276.6360  46552.53  0.00051309 0.4965E-03  0.1785E-0
1.00  294.4976  275.9126  45170.95  0.00049767 0.4818E-03  0.1808E-0
1.05  294.4534  275.2104  43861.89  0.00048307 0.4679E-03  0.1830E-0
1.10  294.4092  274.5315  42624.61  0.00046929 0.4548E-03  0.1858E-0
1.15  294.3645  273.8606  41428.80  0.00045598 0.4420E-03  0.1823E-0
1.20  294.3198  273.2078  40293.94  0.00044336 0.4300E-03  0.1837E-0
1.25  294.2751  272.5894  39239.15  0.00043164 0.4187E-03  0.1901E-0
1.30  294.2300  271.9839  38228.84  0.00042042 0.4080E-03  0.1892E-0
1.35  294.1846  271.3838  37248.34  0.00040955 0.3976E-03  0.1854E-0
1.40  294.1389  270.8091  36328.45  0.00039935 0.3878E-03  0.1889E-0
1.45  294.0933  270.2588  35464.63  0.00038979 0.3786E-03  0.1920E-0
1.50  294.0471  269.7190  34633.59  0.00038059 0.3698E-03  0.1903E-0
1.55  294.0010  269.2026  33853.73  0.00037196 0.3615E-03  0.1932E-0
1.60  293.9543  268.7019  33110.49  0.00036375 0.3536E-03  0.1933E-0
1.65  293.9075  268.2014  32380.82  0.00035569 0.3458E-03  0.1886E-0
1.70  293.8604  267.7234  31696.46  0.00034814 0.3385E-03  0.1912E-0
1.75  293.8127  267.2688  31055.70  0.00034107 0.3317E-03  0.1938E-0
1.80  293.7651  266.8276  30445.05  0.00033434 0.3252E-03  0.1939E-0
1.85  293.7170  266.3921  29850.50  0.00032779 0.3189E-03  0.1911E-0
1.90  293.6687  265.9607  29271.58  0.00032141 0.3127E-03  0.1884E-0
1.95  293.6201  265.5596  28741.83  0.00031559 0.3071E-03  0.1934E-0
2.00  293.5713  265.1802  28247.04  0.00031014 0.3018E-03  0.1957E-0
2.05  293.5220  264.7917  27748.74  0.00030467 0.2965E-03  0.1897E-0
2.10  293.4724  264.4243  27283.37  0.00029955 0.2916E-03  0.1919E-0
2.15  293.4226  264.0605  26828.18  0.00029455 0.2868E-03  0.1892E-0
2.20  293.3726  263.7139  26401.30  0.00028987 0.2822E-03  0.1909E-0
2.25  293.3223  263.3708  25983.02  0.00028528 0.2778E-03  0.1883E-0
2.30  293.2715  263.0532  25601.21  0.00028110 0.2737E-03  0.1921E-0
2.35  293.2205  262.7253  25212.00  0.00027683 0.2696E-03  0.1864E-0
2.40  293.1692  262.4270  24861.89  0.00027300 0.2659E-03  0.1911E-0
2.45  293.1177  262.1365  24524.09  0.00026930 0.2623E-03  0.1902E-0
2.50  293.0659  261.8228  24164.07  0.00026536 0.2584E-03  0.1815E-0
```

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Listing of -ONE at 10:18:28 on APR  6, 1984 for CCid=SS3X
    59     2.55  293.0137  261.5383  23841.94  0.00026184 0.2550E-03  0.1863E-
    60     2.60  292.9614  261.2417  23509.52  0.00025820 0.2515E-03  0.1806E-
    61     2.65  292.9087  260.9753  23214.31  0.00025497 0.2484E-03  0.1855E-
    62     2.70  292.8560  260.7061  22918.13  0.00025174 0.2452E-03  0.1821E-
    63     2.75  292.8027  260.4424  22631.80  0.00024861 0.2422E-03  0.1812E-
    64     2.80  292.7493  260.1946  22365.78  0.00024571 0.2394E-03  0.1828E-
    65     2.85  292.6956  259.9565  22111.23  0.00024293 0.2367E-03  0.1825E-
    66     2.90  292.6418  259.7234  21864.62  0.00024025 0.2340E-03  0.1816E-
    67     2.95  292.5876  259.5034  21635.49  0.00023775 0.2316E-03  0.1830E-
    68     3.00  292.5334  259.2656  21388.68  0.00023506 0.2290E-03  0.1765E-
```

```
[illegible]

   1      0.0,0.0,
   2      0.02,0.02256,
   3      0.04,0.04511,
   4      0.06,0.06762,
   5      0.08,0.09003,
   6      0.10,0.11246,
   7      0.12,0.13476,
   8      0.14,0.15695,
   9      0.16,0.17901,
  10      0.18,0.20094,
  11      0.20,0.22270,
  12      0.22,0.24430,
  13      0.24,0.26570,
  14      0.26,0.28690,
  15      0.28,0.30788,
  16      0.30,0.32863,
  17      0.32,0.34913,
  18      0.34,0.36936,
  19      0.36,0.38933,
  20      0.38,0.40901,
  21      0.40,0.42839,
  22      0.42,0.44749,
  23      0.44,0.46622,
  24      0.46,0.48466,
  25      0.48,0.50275,
  26      0.50,0.52050,
  27      0.52,0.53790,
  28      0.54,0.55494,
  29      0.56,0.57162,
  30      0.58,0.58792,
  31      0.60,0.60386,
  32      0.62,0.61941,
  33      0.64,0.63459,
  34      0.66,0.64938,
  35      0.68,0.66278,
  36      0.70,0.67780,
  37      0.72,0.69143,
  38      0.74,0.70468,
  39      0.76,0.71754,
  40      0.78,0.73001,
  41      0.80,0.74210,
  42      0.82,0.75381,
  43      0.84,0.76514,
  44      0.86,0.77610,
  45      0.88,0.78669,
  46      0.90,0.79691,
  47      0.92,0.80677,
  48      0.94,0.81627,
  49      0.96,0.82542,
  50      0.98,0.83423,
  51      1.00,0.84270,
  52      1.02,0.85084,
  53      1.04,0.85865,
  54      1.06,0.86614,
  55      1.08,0.87333,
  56      1.10,0.88020,
  57      1.12,0.88079,
  58      1.14,0.89308,
  59      1.16,0.89910,
  60      1.18,0.90484,
  61      1.20,0.91031,
  62      1.22,0.91553,
  63      1.24,0.92050,
  64      1.26,0.92524,
  65      1.28,0.92973,
  66      1.30,0.93401,
  67      1.32,0.93806,
  68      1.34,0.94191,
  69      1.36,0.94556,
  70      1.38,0.94902,
  71      1.40,0.95228,
  72      1.42,0.95538,
  73      1.44,0.95830,
  74      1.46,0.96105,
  75      1.48,0.96365,
  76      1.50,0.96610,
  77      1.52,0.96841,
  78      1.56,0.97263,
  79      1.60,0.97635,
  80      1.64,0.97962,
  81      1.68,0.98249,
  82      1.72,0.98500,
  83      1.76,0.98719,
  84      1.80,0.98909
```

```
[illegible]
 88        1.94,0.99443,
 89        2.00,0.995322,
 90        2.10,0.997020,
 91        2.20,0.998137,
 92        2.30,0.998857,
 93        2.40,0.999311,
 94        2.50,0.999593,
 95        2.60,0.999764,
 96        2.70,0.999866,
 97        2.80,0.999925,
 98        2.90,0.999959,
 99        3.00,0.999978,
100        3.20,0.999994,
101        3.40,0.999998,
102        3.60,1.000000,
```

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APPENDIX C

ALGORITHM FOR DERIVING THERMODYNAMIC PROPERTIES

Four governing equations were obtained from DuPont, (Ref. 3) for R-11; the equation of state, the vapor pressure curve, density of the saturated liquid, and the heat capacity of the vapor.

$P=P(v,T)$; equation of state.

$P_{sat}=P_{sat}(T_{sat})$; vapor-pressure curve.

$\rho_\ell = \rho_\ell(T_{sat})$; density of saturated liquid.

$C_{v_0} = C_{v_0}(T)$; heat capacity of vapor.

From these four equations, and given Tv and mv, the thermodynamic properties of the liquid and vapor may be determined as follows. Refer to Fig. C1, a T-S diagram, to identify the states being determined.

1) $v_v(T_v, P) = \dfrac{V_v}{m_v}$

2) $P=P(v,T_v)$; determine system pressure from equation of state.

3) $T_{sat}=T_{sat}(P)$; determine Tsat from vapor-pressure curve.

4) Find $v_v(T_r,P_r)$, $v_v(T_{sat},P)$, $v_v(T_{sat},P_r)$, $v_v(T_v,P_r)$, from equation of state.

5) $C_v = C_v(T_v)$; find heat capacity of vapor.

6) $$u(T_v, P) = \int_{v_v(T_v,P_r)}^{v_v(T_v,P)} \left[T\frac{\partial P}{\partial T} - P\right] dv + \int_{T_r}^{T_v} C_{v_0}\, dT_P + u(T_r, P_r)$$

7) $$u(T_{SAT}, P) = \int_{v_v(T_{SAT},P_r)}^{v_v(T_{SAT},P)} \left[T\frac{\partial P}{\partial T} - P\right] dv + \int_{T_r}^{T_{SAT}} C_{v_0}\, dT_P + u(T_r, P_r)$$

8) $v_\ell = v_\ell(T_{SAT})$ find density of saturated liquid.

9) $$h_{fg} = \left(\frac{dP}{dT}\right)_{SAT} \cdot T \cdot \left(v_v(T_{SAT}, P) - v_\ell(T_{SAT}, P) \right)$$

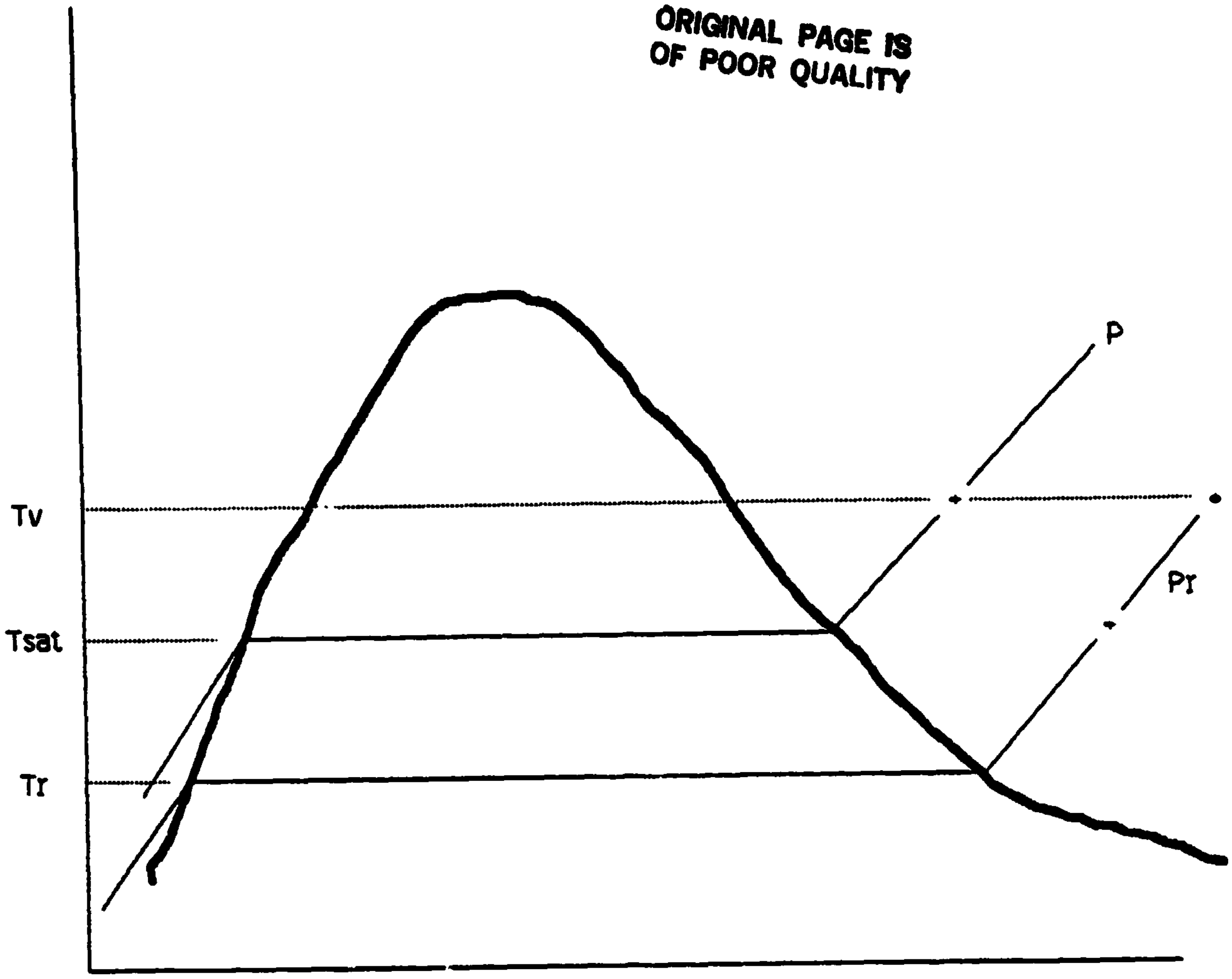

Figure C1.- T-S diagram for thermodynamic property calculations.

APPENDIX D

ADIABATIC DECOMPRESSION MODEL

In order to evaluate the effect of interfacial mass transfer on the ullage pressure response, an adiabatic model was constructed. Derivation is identical to that of the interfacial mass transfer model discussed in the ANALYSIS section of this report, except that the evaporation rate, $\dot{m}1$, is assumed to be zero. The continuity equation, Eq. (3), then becomes

$$\frac{dm_v}{dt} = -\dot{m}_e \qquad \text{(D1)}$$

The energy equation, Eq.(8), becomes

$$m_v c_v \frac{dT_v}{dt} + \dot{m}_e (h_v - u_v) = 0 \qquad \text{(D2)}$$

These two equations, combined with Eq.(9) now define the vapor space behavior. The computer algorithm in App. B is easily modified to solve these governing equations. The MASS subroutine, which calculates $\dot{m}i$, is removed and in place is put $\dot{m}1$=0.0. The remainder of the program is unchanged.

APPENDIX E

ANALYSIS OF PAST VENTING MODELS

As discussed earlier, the critical element in modeling the pressure response of a cylinder initially filled with a suturated mixture and slowly vented is the method used to evaluate interfacial mass transfer. Labus, et al (Ref. 1) used the equation

$$\dot{m}_i \simeq \frac{A_i \rho C_v (T_o - T_i)}{(\pi a t)^{1/2} h_{fg}} \qquad \text{(E1)}$$

This equation was obtained by simplifying an analytical expression for the interfacial mass transfer during depressurization for an infinitely planar interface obtained by Thomas and Morse (Ref. 8). Now, assuming that there is no heat transfer across the interface, equations (11) and (12) again apply

$$q = \dot{m}_i h_{fg} = k A_i \left.\frac{dT}{dx}\right|_{x=0} \qquad \text{(E2)}$$

With the definition a=k/pc, equation (E1) becomes

$$\dot{m}_i = \frac{A_i k (T_o - T_i)}{h_{fg} (\pi a t)^{1/2}} \qquad \text{(E3)}$$

Now, solving for $dT/dx|_{x=0}$ from equation (E2):

$$\left.\frac{dT}{dx}\right|_{x=0} = \frac{(T_o - T_i)}{(\pi a t)^{1/2}} \qquad \text{(E4)}$$

Equation (E4) is the temperature gradient of the liquid at the interface, and is precisely the temperature gradient at the surface of a semi-infinite planar solid undergoing a step change in surface temperature(ref.2). But, the system being modeled undergoes a transient change in surface temperature. Hence, some method of incorporating this transient effect, such as Dunammel's superposition integral must be employed for proper application of equation (E4).

In deriving equation (E1), Labus, et al made a number of assumptions which greatly reduced the complexity of the equation derived by Thomas and Morse. It was assumed that Tv=Tsat @ Pv. The effect of this assumption was discusssed earlier. Also, a term in the original expression of Thomas and Morse was dropped, assuming the effect of that term to be negligible. The validity of this assumption was not evaluated. The equation derived by Thomas and Morse was not used in the present work. Future models may wish to evaluate the behavior of this equation in it's complete form.