Calculation of Transonic Flow in a Linear Cascade

Leo F. Donovan
Lewis Research Center
Cleveland, Ohio

Prepared for the
Twentieth Joint Propulsion Conference
cosponsored by the AIAA, SAE, and ASME
Cincinnati, Ohio June 11-13, 1984
CALCULATION OF TRANSONIC FLOW IN A LINEAR CASCADE

Leo F. Donovan
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

Turbomachinery blade designs are becoming more aggressive in order to achieve higher loading and greater range. New analysis tools are required to cope with these heavily loaded blades that may operate with a thin separated region near the trailing edge on the suction surface. An existing, viscous airfoil code was adapted to cascade conditions in an attempt to provide this capability. Comparisons with recently obtained data show that calculated and experimental surface Mach numbers were in good agreement but loss coefficients and outlet air angles were not.

INTRODUCTION

The new, highly loaded turbomachinery blades are straining current design methods to their limit and designers are being forced to seek more powerful analysis techniques before beginning fabrication and test. Two-dimensional Navier-Stokes codes have been successfully used for several years to calculate flows around isolated airfoils. Such codes should be adaptable to cascade flows and may provide blade designers with a valuable analysis tool. A recent example illustrating this is given in the paper by Schmidt et al. (ref. 1) describing the redesign of a supercritical, controlled diffusion compressor stator blade. The Navier-Stokes calculations performed for that study were for near design conditions. This paper will discuss the code and boundary conditions, and compare calculations and data both near design conditions and at off-design conditions for that blade. The author wishes to thank D. R. Boldman for making available the experimental data.

NOMENCLATURE

AVDR  axial velocity density ratio, $\frac{P_2 V_2 X_2}{P_1 V_1 X_1}$
C    chord
M    mach number
p    pressure
V    velocity
X    axial position
$\beta$  air angle, deg
$\Delta \beta$  difference between actual and design air inlet angles, degrees
DESCRIPTION OF CALCULATIONS

The calculations reported in this work were performed using a modification of Steger's two-dimensional, isolated airfoil code (refs. 2 and 3). Although the code can be used for either inviscid or viscous flows, only viscous results are reported here. Other modifications of the code have been used successfully to calculate, for example, transonic aileron buzz (ref. 4) and three-dimensional flows over simple bodies (ref. 5). The code is quite robust and converges reasonably well for external flow calculations.

A general coordinate transformation is applied to the two-dimensional, unsteady Navier-Stokes equations. The thin-layer approximations to the resulting equations are solved using an implicit finite difference algorithm developed by Beam and Warming (ref. 6). The Baldwin-Lomax (ref. 7) turbulence model is used without modification.

The major differences between the present work and other applications are the nondimensionalization with respect to inlet total conditions and the boundary conditions, which are chosen to be appropriate for turbomachinery calculations. Since the solution technique has been adequately described elsewhere, only the boundary conditions will be discussed here.

Inlet. - Since the flows of interest are subsonic at the inlet, three boundary conditions must be specified and one obtained from the flow. We have chosen to maintain constant total inlet conditions and either constant inlet flow angle or constant inlet tangential velocity. Only the constant inlet flow angle boundary condition was used for the calculations reported in this paper. The fourth condition is obtained by extrapolation of pressure along a characteristic, as suggested by Gopalakrishnan and Bozzola (ref. 8). Note that with these inlet conditions mass flow is not constant but develops as the calculations proceed.

Periodic boundary. - The flow is assumed to be periodic from blade passage to blade passage. This is imposed numerically by averaging the solution on the "upper" and "lower" periodic grid lines after each iteration.
Outlet. - Subsonic outflow requires that one condition be specified and three obtained from the flow. To be consistent with experiments, static pressure has been held constant. Density and the two velocity components are extrapolated along a characteristic.

Initial conditions. - Uniform initial conditions are used, with the no slip blade boundary conditions ramped in over a small number of time steps.

Cascade flow calculations converge much more slowly than external flow calculations. This may be because the conditions on the periodic boundary are not fixed at constant free stream conditions as they are for external flows but vary with time. The initial conditions were chosen such that the outlet static pressure, which was held constant throughout the calculation, would result in approximately the design inlet Mach number. Convergence was established when inlet Mach number was no longer changing. By that time the pressure distribution on the blade and the outlet air angle were constant.

The code converges slowly, thousands of iterations being required for steady state. This requires several hours of run time on a Cray-1S. No special attempts have been made yet to speed up the code. It is expected that spatially varying time steps would prove helpful, as it has for external flows (ref. 9). Also, since most of the time is consumed in the solution of block tridiagonal systems of equations, removing constructs that inhibit vectorization on the Cray from the tridiagonal solution subroutine will reduce run time.

DESCRIPTION OF EXPERIMENTS

Sanz (ref. 10) has recently published a technique for the design of supercritical cascades. A stator blade section with a "flat-roof" type Mach number distribution on the suction surface was designed using this method. The blade was tested in the Lewis Research Center linear cascade by Boldman et al. (ref. 11).

After the cascade tests showed a large, laminar separation bubble the forward end of the suction surface of the blade was reshaped. The redesigned blade has now been tested in the same cascade. Using the nomenclature of reference 11, the cascade consisted of 5 blade passages with chord, \( C = 10.7 \text{ cm} \), gap, \( \tau = 11.7 \text{ cm} \), and stagger angle, \( \gamma = 14.1^\circ \). A sketch of the cascade geometry is shown in figure 1.

Three independent suction systems were employed for boundary layer control. AVDR, the axial velocity density ratio, which indicates flow blockage, was controlled by optimizing end wall suction at the design Mach number and performing the off-design tests without altering the suction valve setting. The optimum suction condition was established on the basis of blade-to-blade wake consistency combined with wake minimum pressure loss. For the results reported in this paper AVDR was between 1.00 and 1.04, indicating only slight flow blockage.

Mach number and air angle were measured 0.13 to 0.15 chord length upstream of the cascade. Since the flow field is highly nonuniform in this region because of the close proximity to the blades, Mach number and air angle at the inlet were determined by an indirect method involving these experimental pressure measurement in combination with the calculated potential flow.
field (ref. 12). Inlet static pressure was taken to be the mean inlet side-wall pressure. A combination probe, located 0.5 chord length downstream of the cascade, was used to measure total and static pressures and air angles as it was traversed in the tangential direction. These measurements were used to calculate mass averaged Mach number, total pressure, and air angle. Also, static pressures were measured at 10 positions on the pressure surface of one blade and 16 positions on the suction surface of another blade. These blades were arranged so that the pressure corresponding to the flow in the central passage was measured.

The design conditions specified were that inlet Mach number, \( M_1 = 0.754 \), inlet air angle, \( \beta_1 = 35.7^\circ \), and Reynolds number based on chord of 1 400 000. Experiments were conducted over a range of Mach numbers and differences between actual and design air angles, \( \Delta \beta_1 \). Results are reported at \( \Delta \beta_1 \) of +1°, -0.4°, -2°, and -6° at inlet Mach numbers close to the design value.

RESULTS

The grid generation procedure described in reference 2 has been adapted to cascades. All calculations reported in this paper were performed using a C-grid with 34 points in the crossflow direction and 99 points in the wrap around direction. Of these 99 points, 79 were distributed around the blade. The inlet was located a chord length upstream of the blade leading edge and the outlet was located a chord length downstream of the trailing edge. The overall grid is shown in figure 2 and expanded views near the leading and trailing edges are shown in figures 3 and 4.

Near design conditions. - Experimental results are presented as surface Mach numbers, calculated from measured static pressures on the blade and assuming that total pressure remains constant. The surface Mach number distribution for \( \Delta \beta_1 = -0.4^\circ \) is shown in figure 5. Since the inlet Mach number cannot be specified in the calculation, the comparison shows the results of two experimental runs that bracket the calculated inlet Mach number. The spurious pressure spike, caused by acceleration around the trailing edge, and typical of thick trailing edge blades, appears on the pressure surface. The grid spacing may not be fine enough to resolve this turning. Agreement between experiment and calculation is good except at the spike and just before the peak Mach number on the suction surface. One would expect from such good agreement that overall cascade performance would be well predicted. However, as shown in table 1, the calculated loss coefficient, \( \omega \), was about one and one half times the experimental \( \omega \) and the air outlet angle, \( \beta_2 \), was about 4° larger than the experimental \( \beta_2 \).

A velocity vector plot of the resulting flow field showed a very thin separated region on the downstream end of the suction surface. This is consistent with the limited flow visualization studies that were conducted.

Off-design conditions. - As with the near design calculations, the off-design calculations showed a pressure spike on the pressure surface near the trailing edge and a thin separated region on the downstream end of the suction surface.

4
A straightforward comparison of calculated and measured surface Mach numbers at off-design air inlet angles was not fruitful. It was apparent that the calculations and experiments were describing different flows. This problem has arisen before; Carta (ref. 13) compared steady-state experiments at incidences of 2° and 6° with calculations at -0.27° and 2.23°. Stephens (ref. 14) reported comparisons of data and calculation for supercritical airfoils that were best at air inlet angles that differ by 2° and experimental AVDR of 1.15. Since AVDR was not systematically varied in the work reported here, a comparison was sought at different air inlet angles. The best results were obtained with the calculations at more positive \( \Delta \beta_1 \) than the experiments. Figures 6 to 8 show surface Mach number distributions at experimental \( \Delta \beta_1 \) of -2°, -6°, and +1° compared to calculations at \( \Delta \beta_1 \) of -1°, -2°, and +2°. In general the agreement is quite good although there is some variation due partly to differences in inlet Mach number.

As at near design conditions, calculated and measured overall performance do not agree well. Table 1 gives calculated and experimental loss coefficients and air outlet angles for comparable inlet Mach numbers. It can be seen that for negative \( \beta_1 \) the calculated loss is again about one and one half times the experimental loss and the calculated air outlet angle is about 3° larger than the experimental \( \beta_2 \). Air outlet angle was not measured for positive \( \Delta \beta_1 \) but since the calculated loss coefficient is about the same as the measured one, it is expected that the air outlet angle is approximately the same also.

**DISCUSSION**

The original calculations for this blade were performed near the design conditions to support an experimental program. The good agreement between calculated and measured surface Mach number distributions encouraged us to continue the calculations at off-design conditions at a later date in spite of the difference between calculated and experimental performance.

Two questions arise from the comparison of calculated and experimental values. First, what is the cause of the discrepancy in air inlet angles at off-design conditions? The solution code has not in any sense been "tuned" for the near design condition. It is possible that the potential flow code used to determine the air inlet angle does not include some physics of importance. But, if this is so, then why is there no difference between air inlet angles at near design conditions

Second, why is the surface Mach number agreement good but the overall performance poor? The grid may not be fine enough to resolve all important effects. Or, the turbulence model, developed for isolated airfoils, may be inadequate in the blade wake. However, while one would expect the agreement to be worse for positive \( \Delta \beta_1 \), where the wake is largest, it is precisely here that the agreement between calculation and experiment is best. The answers to these questions may have to evolve in a stepwise manner starting from comparisons with data for less ambitious blade designs.
REFERENCES


<table>
<thead>
<tr>
<th>Experimental</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \beta_1$</td>
<td>$M_1$</td>
</tr>
<tr>
<td>$+1$</td>
<td>0.780</td>
</tr>
<tr>
<td>$+1$</td>
<td>0.754</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>0.755</td>
</tr>
<tr>
<td>$-0.4$</td>
<td>0.732</td>
</tr>
<tr>
<td>$-2$</td>
<td>0.734</td>
</tr>
<tr>
<td>$-2$</td>
<td>0.757</td>
</tr>
<tr>
<td>$-6$</td>
<td>0.732</td>
</tr>
<tr>
<td>$-6$</td>
<td>0.754</td>
</tr>
</tbody>
</table>
Figure 1. - Cascade geometry.

Figure 2. - Overall view of grid.
Figure 3. - Expanded view of grid near leading edge.

Figure 4. - Expanded view of grid near trailing edge.
Figure 5. - Surface Mach number distribution for near design conditions.

Figure 6. - Surface Mach number distribution.
Figure 7. - Surface Mach number distribution.

Figure 8. - Surface Mach number distribution.
Abstract

Turbomachinery blade designs are becoming more aggressive in order to achieve higher loading and greater range. New analysis tools are required to cope with these heavily loaded blades that may operate with a thin separated region near the trailing edge on the suction surface. An existing, viscous airfoil code was adapted to cascade conditions in an attempt to provide this capability. Comparisons with recently obtained data show that calculated and experimental surface Mach numbers were in good agreement but loss coefficients and outlet air angles were not.

Key Words (Suggested by Author(s))

2D Navier-Stokes equations
Linear cascade