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Optimal Cooperative Control Synthesis Applied to a Control-Configured Aircraft

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I. INTRODUCTION

Quantitative handling qualities specifications are almost non existent for flight vehicles exhibiting non conventional dynamic characteristics. Examples include vehicles in completely foreign operating environments, or radically new aerodynamic and flight control designs such as Control Configured Vehicles (CCV's). Furthermore, higher order-system dynamics or even augmentation itself, have been found to significantly alter pilot opinion ratings so that the existing handling qualities specifications, based on conventional modes, are not appropriate for use with such systems.

A methodology that would effectively take into account both augmentation system design and the human evaluation in a single analytical framework was proposed in Reference [1]. Optimal control theory was used to synthesize the augmentation control law as well as to model the human pilot control input. More recently, the methodology was extended to include a more complete pilot model and the restriction that the augmentation is a linear combination of selected system measurements [2].

The aim of the present paper is to apply the extended approach to the synthesis of an augmentation system, consisting of a control law of simple structure, for a control-configured flight vehicle similar to the AFTI/F-16 [3], and to compare the resulting controller with two alternate control designs - a rate command system and a simplified, linearized version of this AFTI/F-16 air-to-air combat mode control law.

II. METHODOLOGY REVIEW

At this point, a brief review of the methodology is appropriate - the complete derivation can be found in Reference [4].

The aircraft dynamics, linearized about a steady state level flight condition is expressed by the following linear time invariant system

$$\dot{\bar{x}} = A\bar{x} + B_p\bar{u}_p + B_A\bar{u}_A + D\bar{w} \tag{1}$$

with $\bar{x}\varepsilon R^n$, $\bar{u}_p$ and $\bar{u}_A$ εR^m. The vector $\bar{w}$ is a zero-mean Gaussian white-noise process with intensity W. In addition to (1), we assume that measurements or outputs available to the two "controllers" $\bar{u}_p$ and $\bar{u}_A$ are

$$\begin{cases} \bar{y}_p = C_p\bar{x} + \bar{v}_p \\ \bar{y}_A = C_x\bar{x} + C_u\bar{u}_p \end{cases} \tag{2}$$

respectively. The vector $\bar{v}_p$ is also a zero-mean Gaussian white-noise process (with intensity V_p) representing the error in the pilot's observation, and $\bar{y}_A$ are the measurements for feedback augmentation. A schematic diagram of the aircraft plus control dynamics is shown in Figure 1.

The input $\bar{u}_p$ represents the human operator component of the total control vector $\bar{u}$, or for example, control surface deflections associated with the pilot's stick input. The mathematical model for the pilot has been chosen to be similar to the optimal control model of Kleinman and others [5]. The input $\bar{u}_A$, associated with the augmentation system, is constrained here to be the direct feedback of

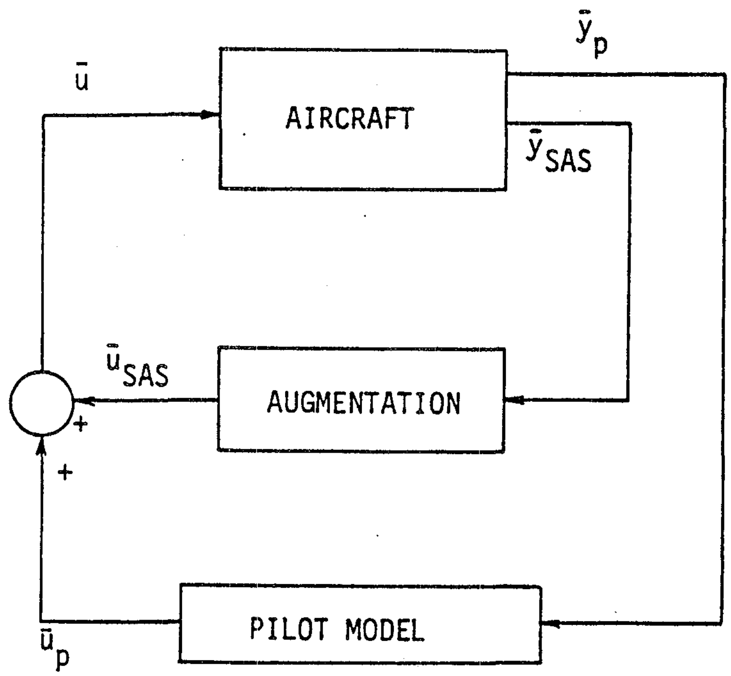


Figure 1 LOCR Block Diagram

measured outputs, or

$$\bar{u}_A = G\bar{y}_A \tag{3}$$

(Note that this is consistent with the desire for simple, easy to implement control laws).

Solution for $\bar{u}_p$

The optimal controller $\bar{u}_p^*$ is chosen to minimize the performance index J_p, the pilot's objective in the task. Now J_p is taken to be

$$J_p = E\{\lim_{T\to\infty} \frac{1}{T} \int_0^T (\bar{x}^T Q\bar{x} + \bar{u}_p^T R_1 \bar{u}_p + \dot{\bar{u}}_p^T R_2 \dot{\bar{u}}_p)\, dt\} \tag{4}$$

where $E\{\cdot\}$ indicates the expected value operator, and the weighting matrices are $Q \geq 0$, $R_1 \geq 0$, $R_2 > 0$. Since the pilot controls the augmented aircraft, the minimizing control policy for $\bar{u}_p$ must be found subject to the dynamic constraint

$$\dot{\bar{x}} = (A + B_A G C_x)\,\bar{x} + (B_p + B_A G C_u)\bar{u}_p + D\bar{w} \tag{5}$$

where G is the matrix of augmentation gains yet to be found. Now defining $\bar{\chi}^T = [\bar{x}^T \,\vdots\, u_p]$ the pilot's optimal control input is given by

$$\begin{aligned} \dot{\bar{u}}_p &= K\hat{\bar{\chi}} \\ K &= -R_2^{-1}\,[0 \,\vdots\, I]P \end{aligned} \tag{6}$$

where $\hat{\bar{\chi}}$ is the best estimate of $\bar{\chi}$ obtained from the Kalman filter

$$\dot{\hat{\bar{\chi}}} = \left[\begin{array}{c|c} A+B_A GC_x & B_p + B_A GC_u \\ \hline 0 & 0 \end{array}\right] \hat{\bar{\chi}} + \left[\begin{array}{c} 0 \\ \hline I \end{array}\right] \dot{\bar{u}}_p + M[\bar{y}_p - \{C_p \,\vdots\, 0\}\,\hat{\bar{\chi}}]$$

$$M = \Sigma C_p^T V_p^{-1} \tag{7}$$

Finally, P and Σ are obtained from their respective Ricatti equations

$$\left[\begin{array}{c|c} A+B_A GC_x & B_p+B_A GC_u \\ \hline 0 & 0 \end{array}\right]^T P + P \left[\begin{array}{c|c} A+B_A GC_x & B_p+B_A GC_u \\ \hline 0 & 0 \end{array}\right]$$

$$+ \left[\begin{array}{c|c} Q & 0 \\ \hline 0 & R_1 \end{array}\right] - P \left[\begin{array}{c} 0 \\ \hline I \end{array}\right] R_2^{-1}\,[0 \,\vdots\, I]\,P = 0 \tag{8}$$

and

$$\dot{\Sigma} = \left[\begin{array}{c|c} A+B_AGC_x & B_p+B_AGC_u \\ \hline 0 & 0 \end{array}\right] \Sigma + \Sigma \left[\begin{array}{c|c} A+B_AGC_x & B_p+B_AGC_u \\ \hline 0 & 0 \end{array}\right]^T$$

$$+ \left[\begin{array}{c|c} DWD^T & 0 \\ \hline 0 & V_m \end{array}\right] - \Sigma \left[\begin{array}{c} C_p^T \\ \hline 0 \end{array}\right] V_y^{-1} [C_p \mid 0] \, \Sigma = 0$$

Now, consistent with the human operator model [5] the control input is modified to the (sub-optimal) relation

$$\dot{\bar{u}}_p^* = K\hat{\chi} + \bar{v}_m$$

or

$$\dot{\bar{u}}_p^* = K_x\hat{x} + K_u \bar{u}_p + \bar{v}_m \tag{9}$$

or for scalar u_p

$$\tau_n \dot{u}_p^* = - \bar{g}\bar{x} - u_p + v_m'$$

where τ_n is the human's neuromuscular lag time constant and $\bar{v}_m$ is a zero-mean Gaussian white-noise process with intensity V_m that represents the error contaminating the pilot's commanded control.

Solution for $\bar{u}_A$

For the input $\bar{u}_A$, we wish to find the controller $\bar{u}_A$ (or gain G) as in (3) that minimizes the index of performance that includes J_p or

$$J_A = J_p + E\{\lim_{T\to\infty} \frac{1}{T} \int_0^T \bar{u}_A^T F \bar{u}_A \, dt\} \tag{10}$$

subject to the constraints of Equations (5) and (7). Thus , the augmentation is chosen to be "pilot-optimal" in the sense that its index of performance J_A incorporates J_p , which, as in [6], [7], and [8], is taken to be correlated with the pilot rating.

Now in solving for $\bar{u}_A$, we must include the dynamics of the (pilot's) state estimator, Eqn. 7, in addition to the plant dynamics, Eqn. 5. Substituting Eqn. 9 into the above two relations, and defining the augmented state vector $\bar{q}^T = [\bar{x}^T \,\vdots\, \hat{x}^T]$, we may write the system dynamics in the form

$$\dot{\bar{q}} = \bar{A}\,\bar{q} + \bar{B}\,\bar{u}_A + \bar{D}\,\tilde{\bar{w}} \tag{11}$$

with

$$\tilde{\bar{w}}^T = [\bar{w}^T\ \bar{v}_m^T\ \bar{v}_p^T]\ ; \quad E[\tilde{\bar{w}}(t)\tilde{\bar{w}}^T(\sigma)] = \bar{W}\delta(t-\sigma)$$

$$\bar{A} = \left[\begin{array}{cc|cc} A & B_p & 0 & 0 \\ 0 & 0 & K_x & K_u \\ \hline & 0 & A+B_AGC_x-MC_p & B_p+B_AGC_u \\ MC_p & & & \\ & 0 & K_x & K_u \end{array}\right]$$

$$\bar{B}^T = [B_A^T \,\vdots\, 0 \,\vdots\, \cdots \,\vdots\, 0]$$

$$\bar{D} = \left[\begin{array}{cc|c} D & 0 & 0 \\ 0 & I & 0 \\ \hline 0 & 0 & \\ & & M \\ 0 & 0 & \end{array}\right]$$

We may now express the objective function J_A as

$$J_A = E\{\lim_{T\to\infty} \frac{1}{T}\int_0^T (\bar{q}^T\bar{Q}\bar{q} + \bar{u}_A^T F\bar{u}_A)dt\} \tag{12}$$

where

$$\bar{Q} = \begin{bmatrix} Q & 0 & 0 \\ 0 & R_1 & 0 \\ 0 & 0 & K^T R_2 K \end{bmatrix}$$

With the control law taken as

$$\bar{u}_A = G\,\bar{y}_A = G[C_A \mid 0]\bar{q}$$

the set of gains G that minimizes Eqn. 12, subject to Eqn. 11 may be shown to be [2]

$$G = -F^{-1}\{\bar{B}^T HL\bar{C}^T + \underline{B}^T HL\underline{C}^T\}\,[\bar{C}L\bar{C}^T]^{-1} \tag{13}$$

where $\bar{B}^T = [B_A^T \vdots \cdots \vdots\; 0]$ $\qquad$ $\bar{C} = [C_A \;\vdots\; 0]$

$\underline{B}^T = [0 \;\vdots\; 0 \;\vdots\; B_A^T \;\vdots\; 0]$ $\qquad$ $\underline{C} = [0 \;\vdots\; C_A]$

with $L = E\{\bar{q}\;\bar{q}^T\}$ satisfying the relation

$$\begin{aligned} &[\bar{A} + \bar{B}G(C_A \;\; 0)]L + L[\bar{A}+\bar{B}G(C_A \;\; 0)]^T \\ &\qquad + \bar{D}\bar{W}\bar{D}^T = 0 \end{aligned} \tag{14}$$

and H satisfying

$$\begin{aligned} &[\bar{A}+\bar{B}G(C_A \mid 0)]^T H + H[\bar{A}+\bar{B}G(C_A \mid 0)] \\ &\qquad + \bar{Q} + \left[\begin{array}{c|c} C_A{}^T G^T F G C_A & 0 \\ \hline 0 & 0 \end{array}\right] = 0 \end{aligned}$$

III. APPLICATION

In the following, we will apply the technique to the augmentation synthesis of a CCV vehicle similar to the AFTI/F-16 aircraft [3]. We will sepcifically have as the design objective that of optimizing pitch tracking performance. The vehicle state vector is taken as $\bar{x}_A^T = [u, \alpha, \dot{\theta}, \theta]$ the perturbation forward velocity, angle of attack, pitch rate, and pitch attitude angle. The control vector is $\bar{u}^T = [\delta_E, \delta_F]$, where δ_E is the elevator and δ_F the direct-lift flap deflection. (Note that in the following, the forward velocity u is nondimensionalized with the reference velocity U_0 and all the angular displacements and angular rates will have units of degrees and degrees per second respectively).

The motion of the aircraft is referenced to the steady-state level flight condition given in Table 1.

Table 1 Flight Condition Specifications

MACH	$M = 0.8$
Altitude	$h = 23000$ ft
Trim Velocity	$U_0 = 819.57$ ft/sec
Trim angle of attack	$\alpha_0 = 2.345$ deg
Load factor	1.0 g

The attitude "command" signal to be tracked θ_c is chosen consistent with previous pitch tracking studies [9] and is generated by a white-noise process ω_c with zero mean and intensity $\sigma^2_{\omega_c}$, passed through a second order filter having a break frequency .5 rad/sec and a damping ratio of 0.5. In state variable form

$$\dot{\bar{x}}_c = \begin{bmatrix} \dot{\theta}_c \\ \ddot{\theta}_c \end{bmatrix} = \begin{bmatrix} 0 & 1. \\ -.25 & -.5 \end{bmatrix} \bar{x}_c + \begin{bmatrix} 0. \\ 1. \end{bmatrix} \omega_c = A_c\bar{x}_c + D_c\omega_c \qquad (15)$$

The covariance $\sigma^2_{\omega_c}$ is chosen in this case to yield $\sigma^2_{\theta_c} = 10.\text{deg}^2$.

The information perceived by the pilot, or his observation vector $\bar{y}_p$ is chosen to be

$$\bar{y}_p = \begin{bmatrix} \varepsilon \\ \dot{\varepsilon} \\ \dot{\theta} \\ \theta \end{bmatrix} = C_p\,\bar{x} + v_p\,, \qquad \varepsilon = \theta - \theta_c \qquad (16)$$

representative of a pursuit tracking task. Also the pilot's objective function for the pitch tracking task is taken as

$$J_p = E\{\lim_{T\to\infty} \frac{1}{T} \int_0^T [16(\theta_c-\theta)^2 + (\dot{\theta}_c-\dot{\theta}) + r\,\dot{u}_p^2]\,dt\} \qquad (17)$$

This selection of weightings has been shown [9], [10] to be consistent with experiment data on the modeled task for a wide variety of system dynamics. Likewise, the augmentation system objective function is

$$J_A = J_p + E\{\lim_{T\to\infty} \frac{1}{T} \int_0^T \bar{u}_A{}^T F \bar{u}_A\,dt\} \qquad (18)$$

with $F = \rho I > 0$ the augmentation weighting matrix (I = identity).

Now in Eqn. 17, as with the complete pilot model [5] the parameter r is adjusted to produce a pilot's neuromuscular lag time constant (Eqn. 9) $\tau_n = 0.1$ seconds.

The measurements selected in this analysis for augmentation feedback were simply $\bar{y}_A{}^T = [\alpha, \dot{\theta}, \theta]$, or we are performing the optimization assuming the control law

$$\begin{bmatrix} \delta_E \\ \delta_A \end{bmatrix}_{ang} = G_\alpha \, \alpha + G_{\dot{\theta}} \, \dot{\theta} + G_\theta \, \theta$$

(It should be noted that the augmentation measurements in this case do not include pilot control input, although this is admissible in the formulation. Finally, in this exploratory investigation a constant set of stick gains were selected, and the pilot control input was taken as

$$u_p = \delta_{stick} \quad ; \quad \begin{bmatrix} \delta_E \\ \delta_F \end{bmatrix}_{pilot} = \begin{bmatrix} K_{E_{st}} \\ K_{F_{st}} \end{bmatrix} \delta_{stick}$$

with $K_{E_{st}} = 1.0$ and $K_{F_{st}} = 0.25$. Further studies will address the possibilities of letting the stick gains be free to be selected in the optimization, and whether to include pilot control input in the augmentation measurements).

Finally, the knowledge of the (observation and motor) noise intensities is required (or V_y and V_m). The complete pilot model [5] is developed on the basis of nearly constant noise to signal ratios, rather than constant noise intensities. Therefore, an iterative procedure was used, beginning with the augmentation synthesis with assumed V_m and V_y, then the augmented system dynamics were evaluated with the complete pilot model (with time delays and attention sharing) to verify that the covariances (V_m and V_y) utilized were consistent with a properly calibrated pilot model.

The synthesis procedure was performed in a parametric fashion by varying the scalar ρ in (18), or control energy weighting. In this way, effects of different levels of augmentation authority on system performance can be determined. Table 2 reveals the parametric optimization results, obtained from evaluations of the augmented aircraft with the complete pilot model. The augmentation control gains are listed in Table 3. Finally the eigenvalue locus with increasing augmentation level (ρ) is shown in Figure 2. (Time responses are shown in the next section of the paper.)

Table 2

Optimization Results

ρ	r	J_p(cost)	τ_N	σ_ε (deg)	$\sigma_{\dot{u}_p}$ ($\frac{deg}{sec}$)
5.0	.025	11.6	0.08	0.68	8.26
1.0	.025	11.9	0.10	0.68	9.08
0.5	.02	12.0	0.10	0.68	10.48
0.1	.01	13.3	0.10	0.70	16.00

A significant result lies in the fact that there appears to be a minimum value of J_p for the control authority level associated with the control energy weighting of $\rho = 1 \rightarrow 5$, rather than a monotonic reduction with increasing augmentation level (decreasing ρ). This would in fact be the case if the cost weighting r and the covariance matrix $\bar{W}$ remained constant for each case. However, to maintain a pilot control loop ($\dot{\bar{u}}_p$) consistent with the optimal control pilot model for each solution, r varies to obtain a $\tau_N \cong .1$ sec, and $\bar{W}$ is adjusted to maintain the appropriate noise to signal ratios. Finally, adding the motor noise $\bar{v}_M$ to

TABLE 3 Augmentation Gains with ρ

ρ		α(deg)	θ(deg/sec)	θ(deg)
5.0	δ_E	-.148	.429	.087
	δ_F	-.032	.103	.020
1.0	δ_E	-.208	.777	.211
	δ_F	-.042	.185	.050
.5	δ_E	-.130	.939	.245
	δ_F	-.190	.209	.057
.1	δ_E	.236	1.218	.424
	δ_F	.043	.267	.101

EIGENVALUE LOCUS

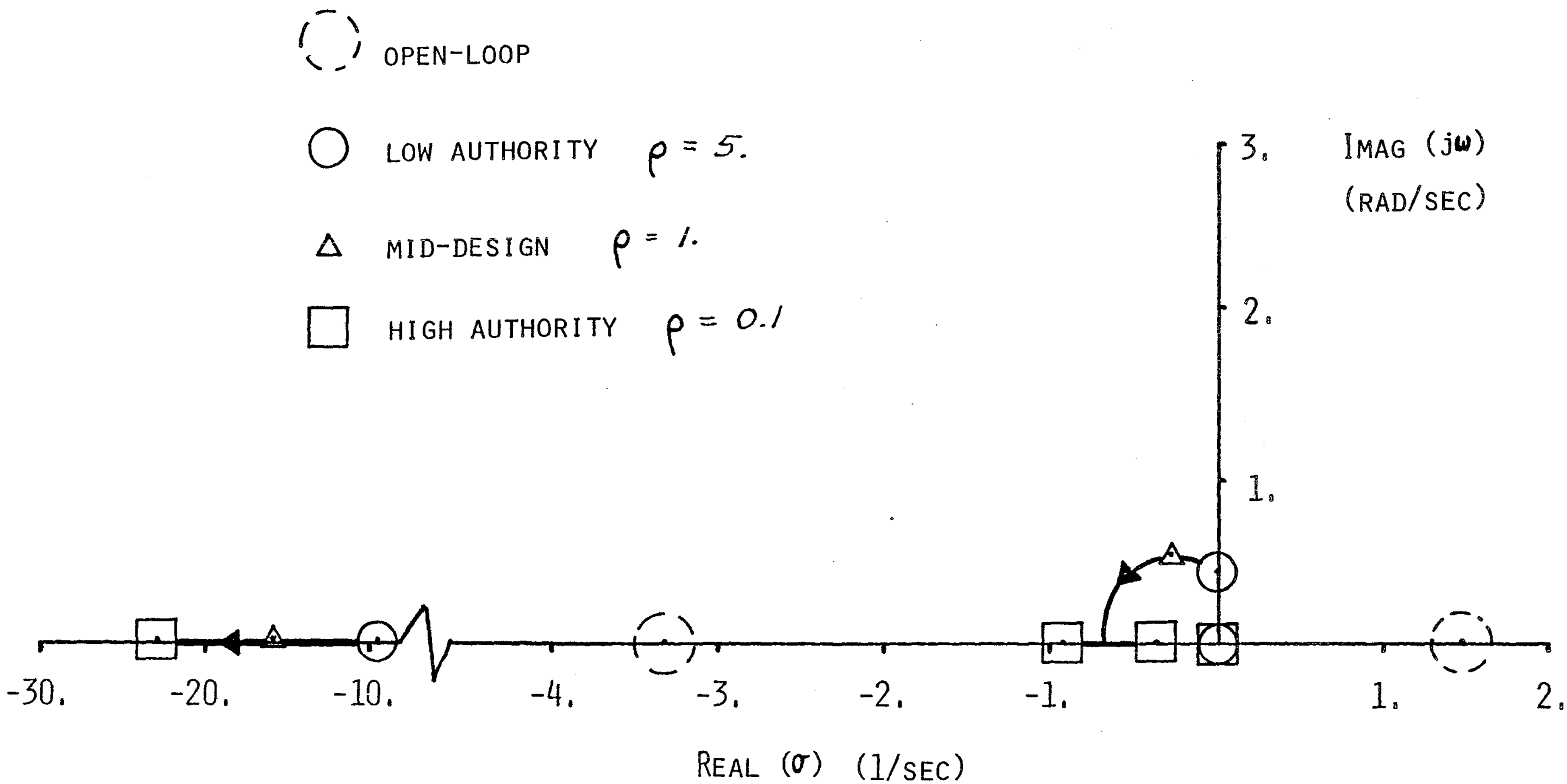


Figure 2

the pilot's input (in Eqn. 9) results in a sub-optimal pilot-model solution for $\dot{\bar{u}}_p$. Further, these effects appear to become significant at higher augmentation levels of authority.

As a result, for this chosen control law the optimization suggests a candidate design corresponding to a value of ρ near 1.0. We will evaluate this system further.

IV. CONTROL SYSTEMS EVALUATION

In an attempt to evaluate results from the proposed methodology, two alternate augmentation synthesis methods are chosen for comparison with our simple output feedback control law. They are an optimal pitch-rate command augmentation system, and a linearized air-to-air combat mode augmentation, similar to the standard air-to-air mode on the AFTI/F-16 aircraft [3].

A rate command control system is chosen because it is considered to be very effective in attitude tracking. Briefly, it consists of an augmentation system that is designed to minimize the error between the aircraft pitch rate and the pilot's stick input, which is taken as the commanded pitch rate from the pilot. The synthesis of the controller used here is summarized in the Appendix.

In addition, an augmentation system similar to the air-to-air standard normal mode present in the AFTI/F-16 aircraft is simplified and linearized about the steady state level flight condition of Table 1. This mode is also intended to provide precise tracking capabilities. The characteristics of these two controllers are listed in Tables 4 and 5.

Table 4

Rate Command Control Law

Veh. Dynamics $\dot{\bar{x}} = A\bar{x} + B\,\bar{u}_A$

Control Law: $\bar{u}_A = \begin{bmatrix} \delta_E \\ \delta_F \end{bmatrix}_A = G_x\bar{x} + G_u\,\delta_{stick}$

Gains*

	u	α	$\dot{\theta}$	θ	δ_{stick}
δ_E	~ 0	.330	.942	.028	-.746
δ_F	~ 0	.073	.232	.007	-.187

*All angles in degrees, u non-dimensionalized with U_o

Table 6 Performance Comparison

CONFIGURATION	RMS ERROR (deg) ε	RMS STICK RATE (deg/sec) $\dot{u}_p$
Cand. Design	.68	9.08
Rate Command	.69	15.69
Air-to-Air Mode	.69	24.02 (lb/sec)*

*This system developed for pilot input in force.

Table 5

Air-to-Air Control Law

Veh. Dynamics: $\dot{\bar{x}} = A\bar{x} + B\bar{u}_A$

Control Law: $\bar{u}_A = \begin{bmatrix} \delta_E \\ \delta_F \end{bmatrix} = L\bar{\xi} + Mx + N\delta_{st}$

$\dot{\bar{\xi}} = F\bar{\xi} + G\bar{x} + H\delta_{st}$

$$L = \begin{bmatrix} 1. & 0. & .1056 \\ 0. & 0. & -1. \end{bmatrix}, \quad M = \begin{bmatrix} .0007 & .5076 & 2.336 & 0. \\ 0. & 0. & 0. & 0. \end{bmatrix}$$

$$N = \begin{bmatrix} -.9308 \\ .24 \end{bmatrix}, \quad F = \begin{bmatrix} -1. & 1. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & -1. \end{bmatrix}$$

(Note: All angles in degrees, stick input in pounds).

$$G = \begin{bmatrix} 0. & 0. & 7.66 & 0. \\ 0. & 0. & 7.67 & 0. \\ 0. & 0. & 0. & 0. \end{bmatrix}, \quad H = \begin{bmatrix} -2.685 \\ -3.066 \\ .24 \end{bmatrix}$$

The aircraft dynamics will now be compared in terms of tracking errors and stick rates, eigenvalues and eigenvectors, time responses, and predicted pilot rating.

Model based predictions of "mission performance" are shown in Table 6.

Although the higher stick rates of the two comparison systems may be reduced with higher stick gains, it might be at the expense of higher tracking errors. It would then seem fair to state that the candidate design exhibits equivalent predicted tracking performance scores.

The eigenvalues of the three systems are compared in Figure 3, along with the "description" of the mode shape from the eigenvectors of the systems. All the systems exhibit a relatively fast pitch rate ($\dot{\theta}$) pole, with the candidate system's eigenvalue near -16. (1/sec) while the others are at -18. and -35. (1/sec), respectively.

Both the rate command and air-to-air systems have a real mode dominating angle of attack near -1. (1/sec), and the traditional phugoid pair near the origin dominating pitch and speed (or θ and u). (The air-to-air mode also has three control system roots, one at -3.5 (1/sec) and two at -1. (1/sec.))

In definite contrast to these two systems, the candidate system has a <u>single</u> root at the origin associated with velocity perturbations u. Then a complex coupled mode is present near $-.3 \pm .4j$, that includes a significant amount of angle of attack α as well as attitude and velocity θ and u. (The participation of this mode in the angle of attack will be clearly evident in the time histories shown later). Clearly with a higher frequency and a significant α participation, this mode is <u>not</u> a conventional phugoid mode.

Now, consider the time responses to a step of one stick input unit, shown in Figures 4-8. The similarity between the rate command and air-to-air systems is evident, both clearly showing pitch rate command characteristics in the $\dot{\theta}$ and θ responses, and nearly first order (α mode) response in angle of attack.

EIGENVALUE COMPARISON

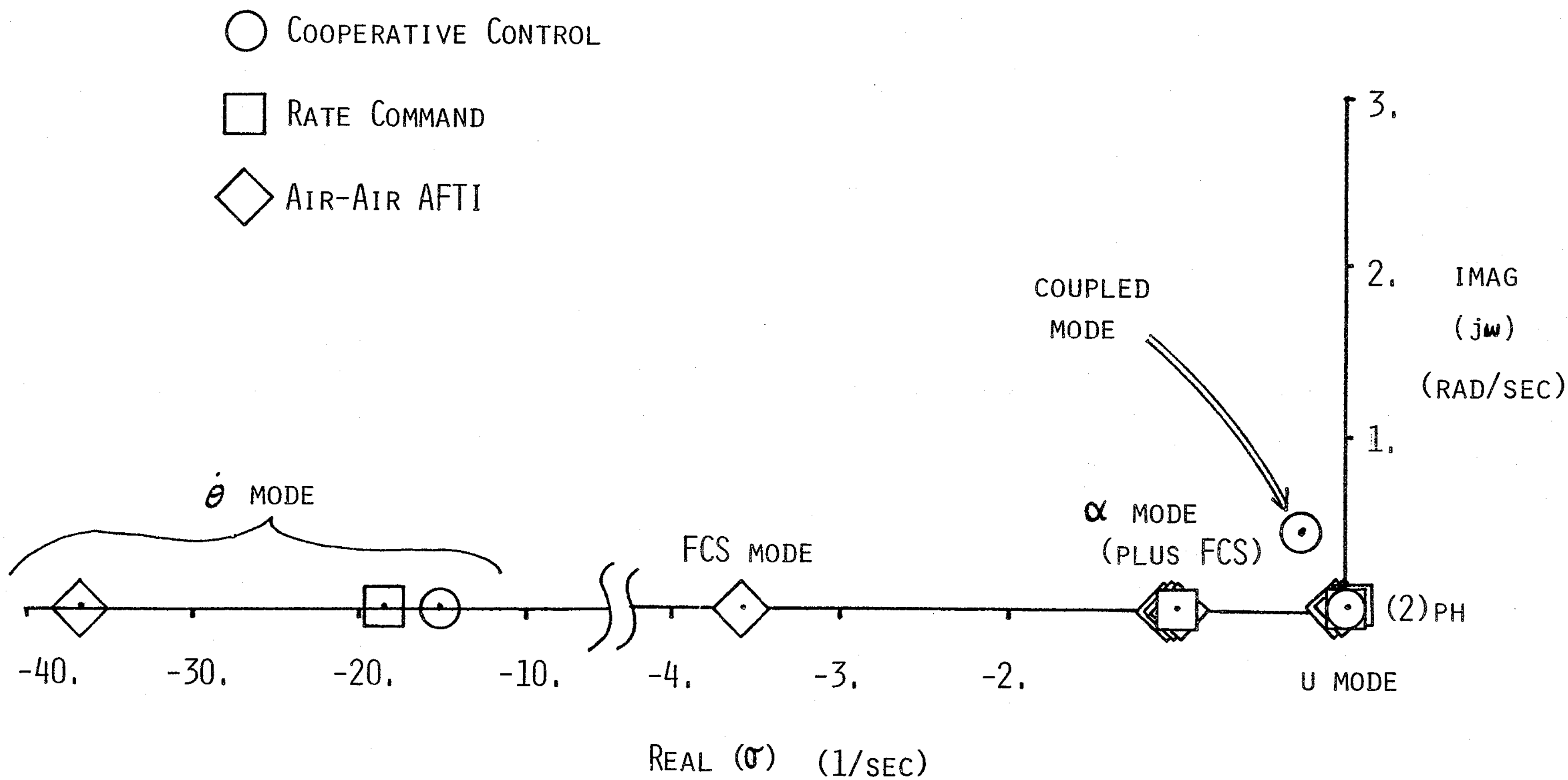


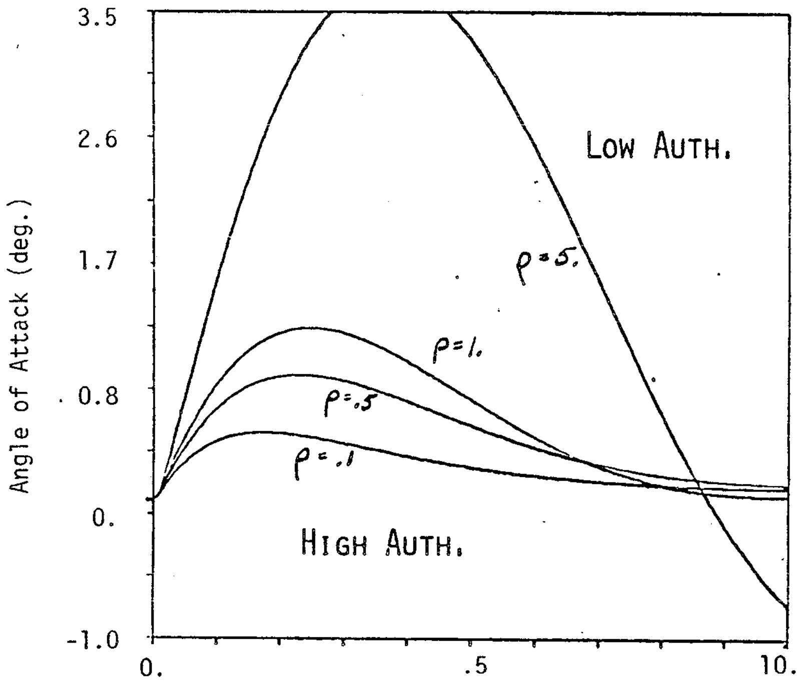
Figure 3

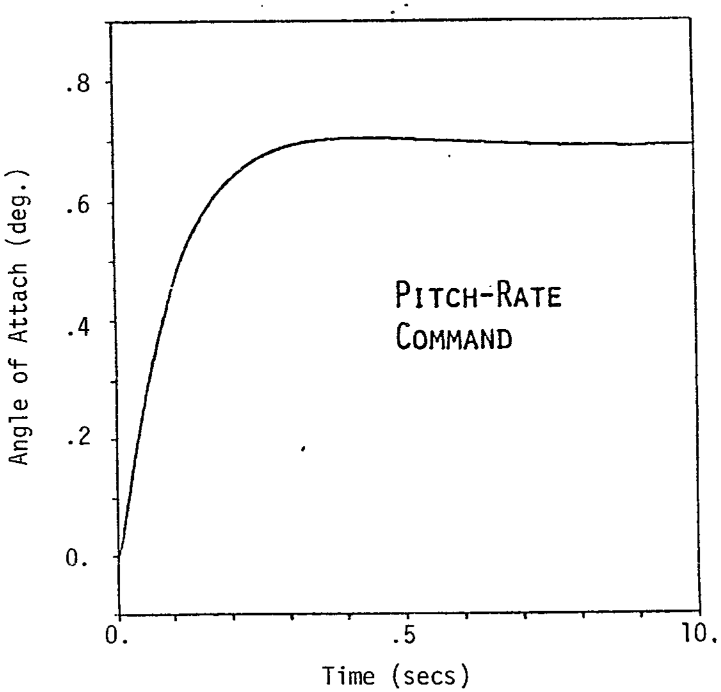
COMPARISON OF

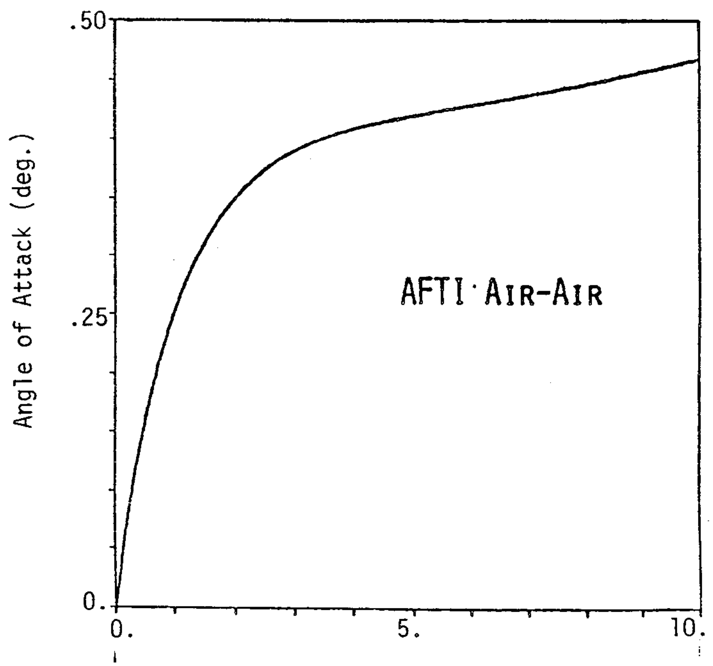
TIME RESPONSES - AOA

STEP INPUT

Figure 4





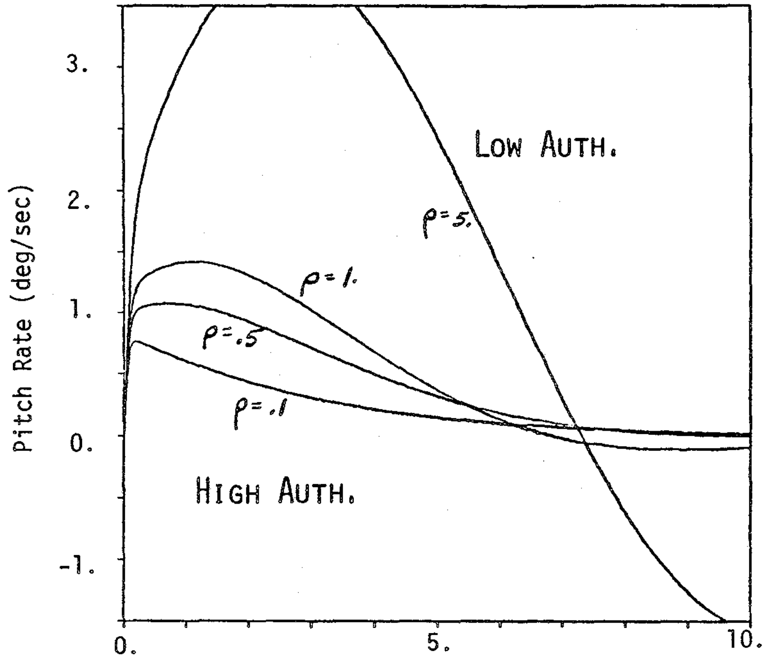


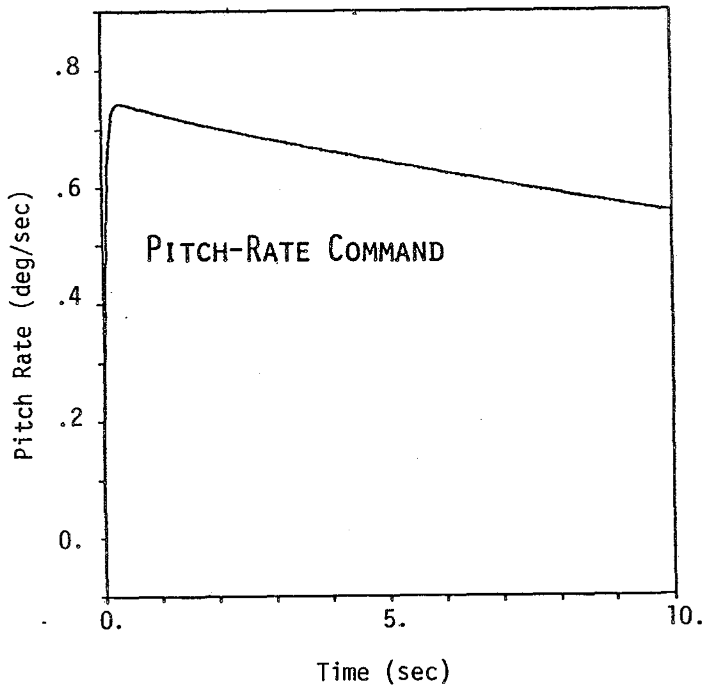
COMPARISON OF

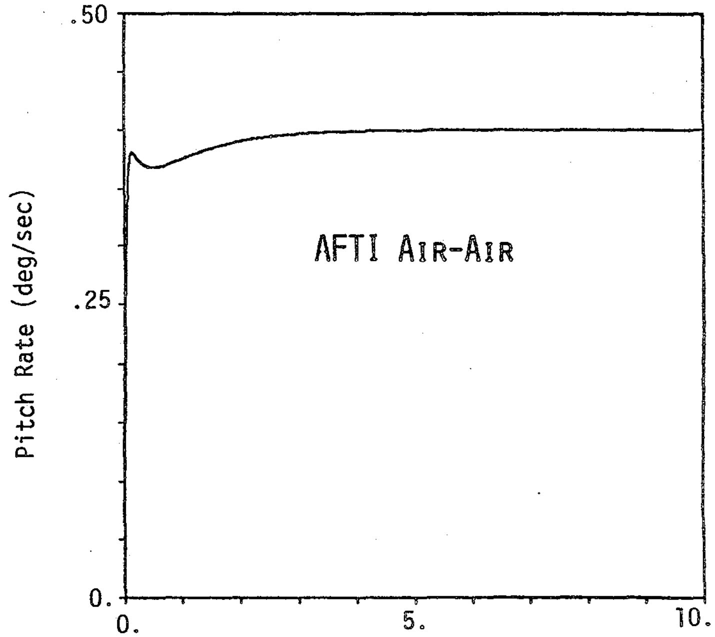
TIME RESPONSES -- PITCH RATE

STEP INPUT

Figure 5





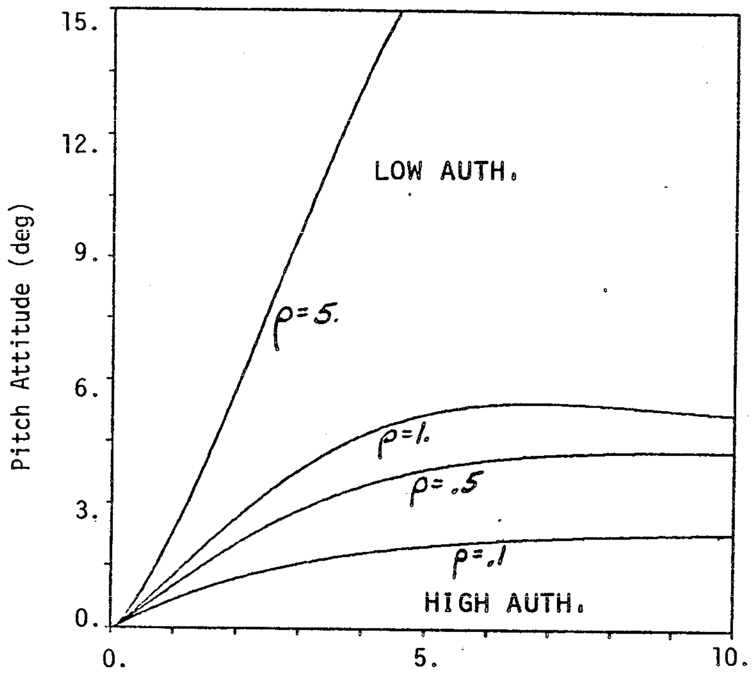


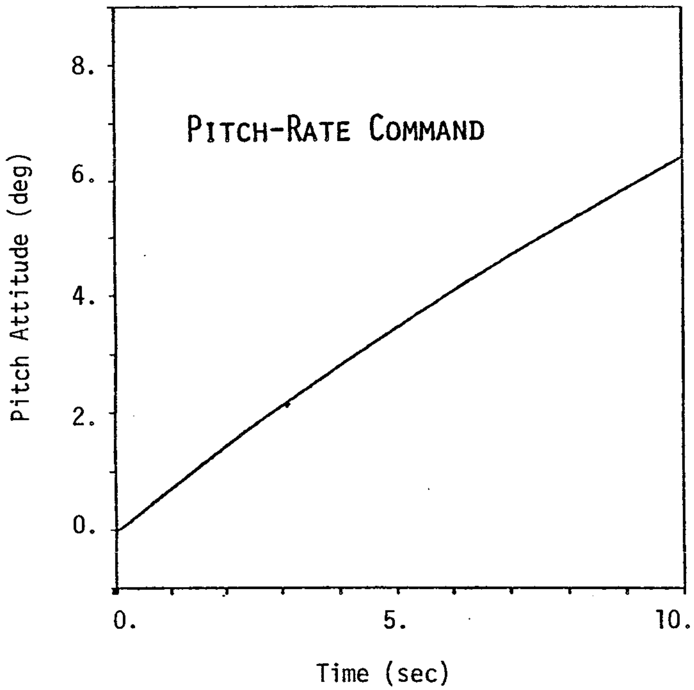
COMPARISON OF

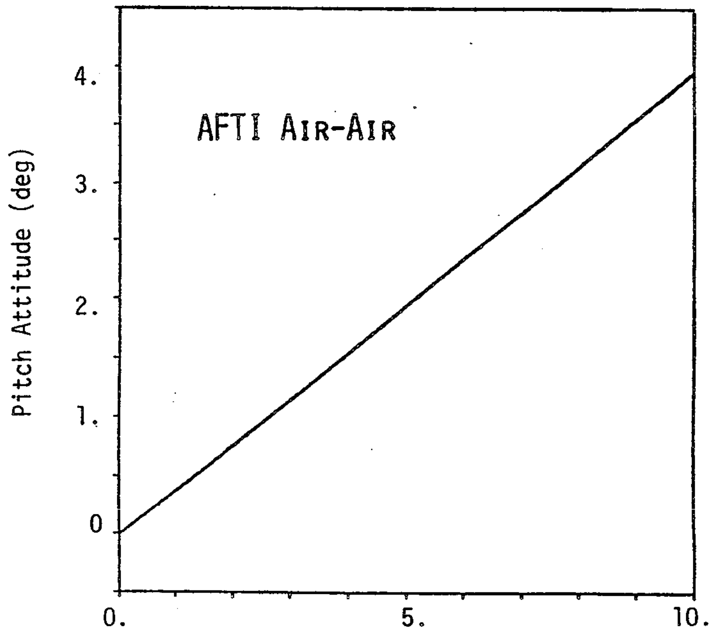
TIME RESPONSES -- ATTITUDE

STEP INPUT

Figure 6



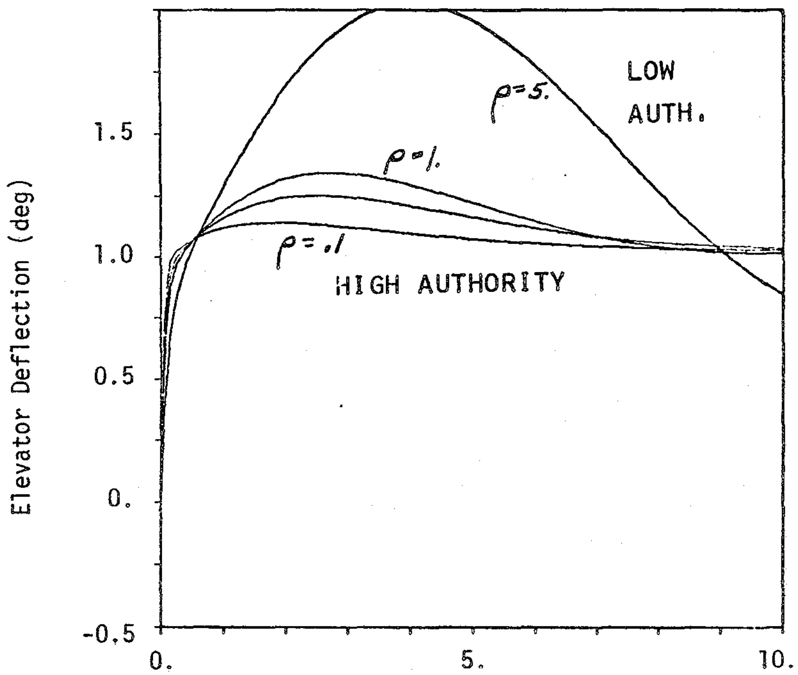


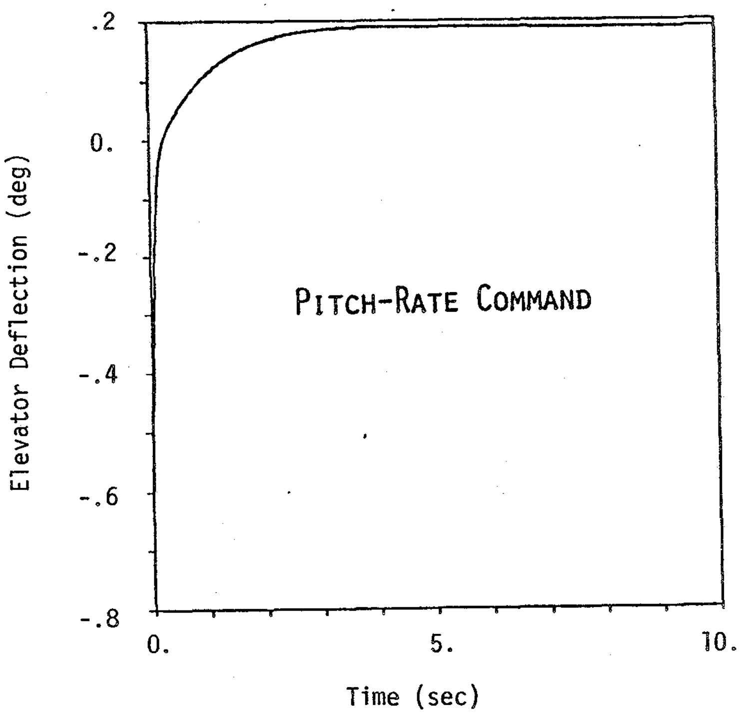


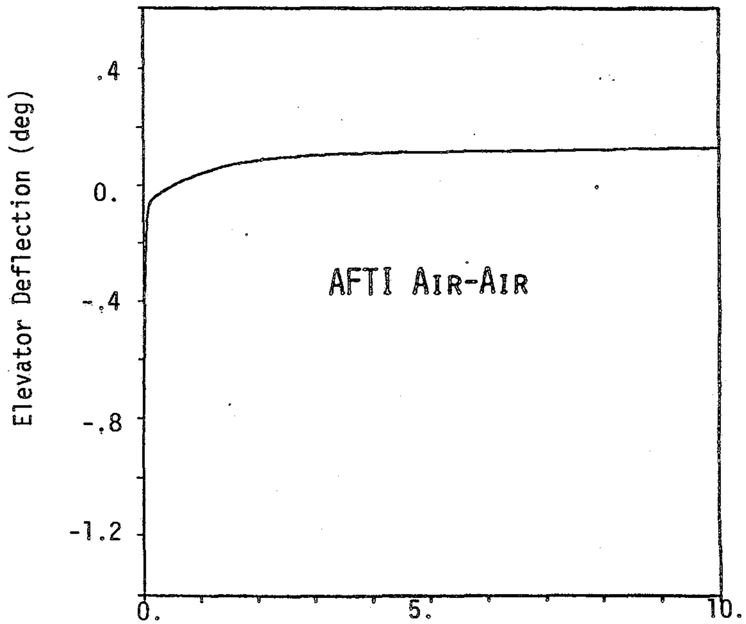
COMPARISON OF TIME RESPONSES -- ELEVATOR

STEP INPUT

Figure 7



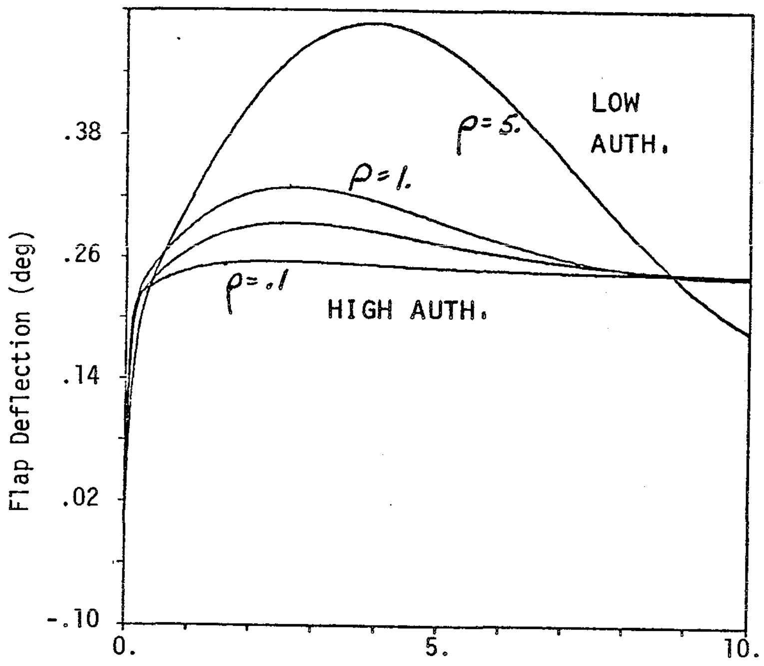


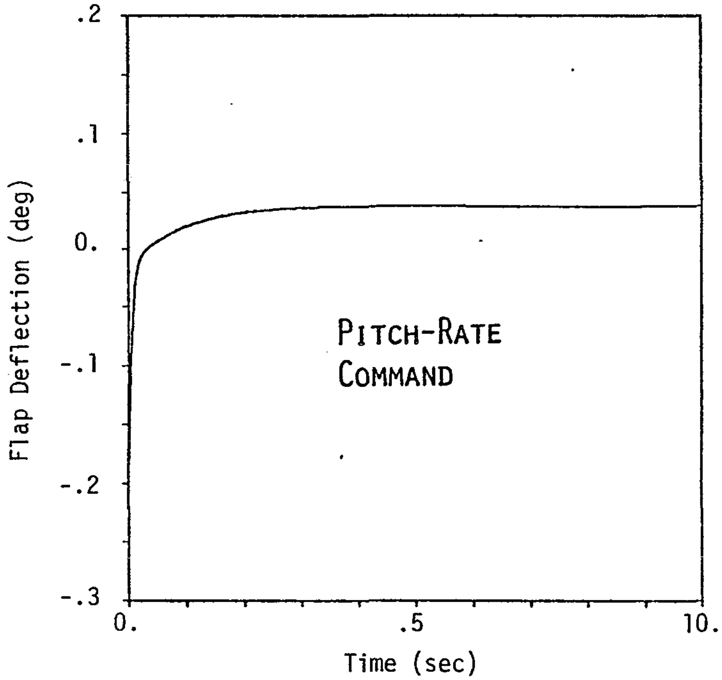


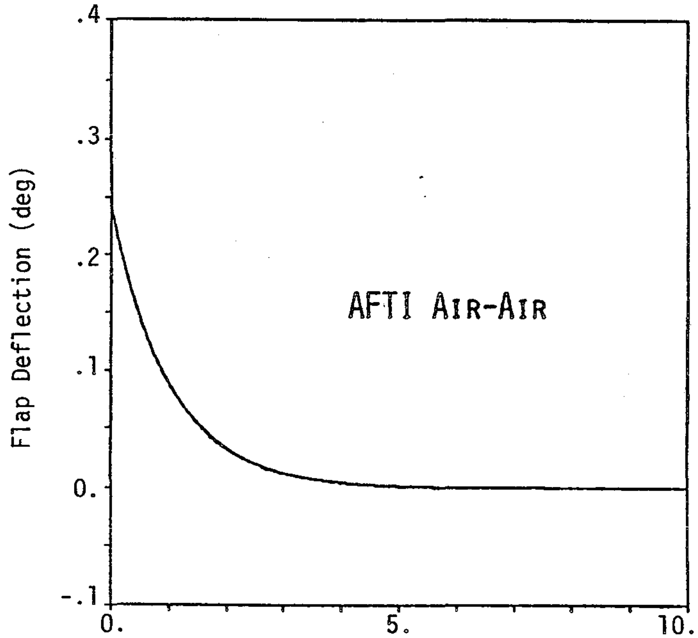
COMPARISON OF TIME RESPONSES -- FLAP

STEP INPUT

Figure 8







In constrast, we see that the candidate design exhibits characteristics similar to a lightly damped attitude command system in that the attitude response tends to a steady state value. Also note the oscillatory response in angle of attack, as mentioned in discussing the presence of a coupled (u, θ, α) mode.

In this regard they are neither like the AFTI air-to-air mode discussed here, nor the decoupled pitch pointing mode [3] that decouples attitude response from flight path response - but rather somewhere between these modes.

It is significant that in further application of this methodology, by allowing only pitch-rate and angle of attack feedback (instead of α, $\dot{\theta}$, and θ as in the above cases), the resulting pilot-optimal control laws were similar to the rate-command systems presented here. In this case (α and $\dot{\theta}$ feedback only) the $\dot{\theta}$ mode, referring to Figure 3, remained near -16. (1/sec), but the angle of attack was dominated by a single pole at -0.6 (1/sec), and two phugoid roots appeared near the origin. Therefore, these eigenvalue locations are very near those in the rate command and AFTI air-to-air systems (still referring to Fig. 3). Likewise the time responses were similar to these two "rate-command" systems. Additionally, even with feedback of attitude angle <u>not</u> included, the tracking error only increased to 0.69 degrees while the stick rate increased slightly also to 9.4 deg./sec. (referring to Table 6). Therefore, it would appear that in the absence of attitude-angle for feedback, the optimum system dynamics for attitude tracking are like K/s, agreeing with well known results. However, if attitude feedback is allowed, significantly different dynamics are optimum for this pitch tracking task, but with only slightly improved tracking errors, however.

Another closed-loop analysis method, the Neal-Smith criterion, has been proposed to obtain pilot-rating predictions from frequency response characteristics of the pilot-aircraft system in pitch tracking tasks [11]. In their work, Neal and Smith hypothesized that "pilot rating is correlated with the pilot's compensation required to achieve good low frequency performance (good tracking) and the pilot/vehicle oscillation that resulted". In recent studies by Bacon and Schmidt [10], the same approach was considered but with the use of the optimal control pilot model instead of a describing function modeling approach. Using this technique, a relationship was established between predicted pilot rating, pilot phase (lead or lag) compensation, and the resonance peak of the closed-loop system transfer function, or $\left|\frac{\theta}{\theta_c}\right|_{max}$.

Based on the above, we compare the three configurations of interest in our research in terms of the Neal-Smith parameters, and results are given in Figure 9. Here, the levels of handling qualities are defined by

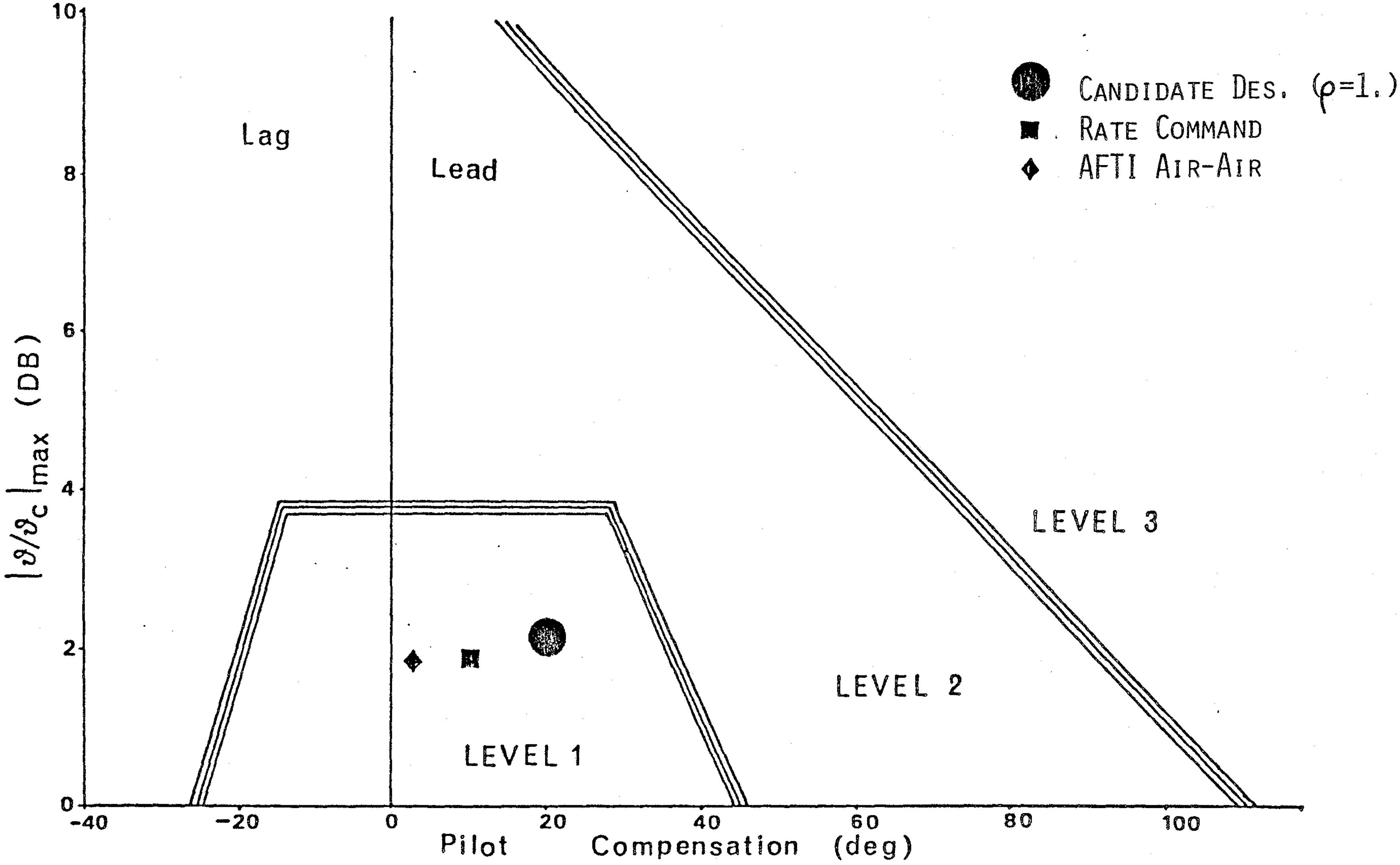


Figure 9 Neal-Smith Analysis Results

Level 1 = 1.0 - 3.5 Cooper-Harper Scale, good

Level 2 = 3.5 - 6.5 Cooper-Harper Scale, fair

Level 3 = 6.5 - 10.0 Cooper-Harper Scale, poor

From Figure 9, all the configurations fall within the bounds of Level 1 and therefore they are predicted to attain good handling qualities characteristics according to the Neal-Smith criterion.

V. SUMMARY

Analytical evaluations have shown a favorable comparison between the simple augmentation system obtained and systems obtained using two alternate methods. The same level of tracking performance is predicted for all the controllers, which are also predicted to be acceptable to the pilot in the task considered. It was noted that the augmented vehicle dynamics are significantly different, showing the presence of nonconventional modes. This leads one to conjecture about the potential of significantly different dynamics in future vehicle designs.

Finally, we note that the procedure resulted in a system that, at least for low frequencies, behaved like a pure gain plant (or $\theta(s)/\delta_{st}(s) \approx K$) while the other design approaches (and the proposed method without θ feedback) tended to lead to plant characteristics more like K/S. Which are ultimately optimum in the variety of tasks over the flight envelope for future vehicles are yet to be determined.

APPENDIX

The equations of motion of the aircraft in state variable form were given for the flight condition of interest as

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} \tag{A.1}$$

with

$$\bar{x}^T = [u, \alpha, \dot{\theta}, \theta] \; ,$$

$$\bar{u}^T = [\delta_E, \delta_F]$$

Assuming the stick input to be proportional to commanded pitch rate, yields

$$\delta_{st} \triangleq \dot{\theta}_c \; .$$

The commanded signal is modeled then as a first order Markov process

$$\dot{u}_p = \dot{\delta}_{st} = -\frac{1}{\tau_s}\delta_{st} + \xi \tag{A.2}$$

with:

τ_s = .2 sec time constant related to the pilot dynamics

ξ = Gaussian random variable with zero mean and intensity σ_ξ^2

Augmenting (A.1) with (A.2), yields

$$\begin{bmatrix} \dot{\bar{x}} \\ \hline \dot{u}_p \end{bmatrix} = \left[\begin{array}{c|c} A & 0 \\ \hline 0 & -1/\tau_s \end{array}\right] \begin{bmatrix} \bar{x} \\ \hline u_p \end{bmatrix} + \begin{bmatrix} B \\ \hline 0 \end{bmatrix} \bar{u} + \begin{bmatrix} 0 \\ \hline 1 \end{bmatrix} \xi$$

The optimal $\bar{u}^*$ is chosen such that the following index of performance is minimized

$$J = E\{\lim_{T\to\infty} \frac{1}{T} \int_0^T [(\dot{\theta}-\dot{\theta}_c)^2 + \bar{u}^T F \bar{u} \; dt\}$$

$$F = fI \;, \quad f \text{ a scalar}$$

According to linear optimal control theory we have

$$\bar{u}^* = -\frac{1}{f}[B^T \;\; 0]\, P \begin{bmatrix} \bar{x} \\ \hline u_p \end{bmatrix} = [K_1 \;\; K_2] \begin{bmatrix} \bar{x} \\ \hline u_p \end{bmatrix} \tag{A.4}$$

where P is the solution of the algebraic Riccati equation

$$A^T P + PA + CC^T - P \begin{bmatrix} B \\ \hline 0 \end{bmatrix} F^{-1} [B^T \;\; 0]\, P = 0$$

where

$$[\theta - \theta_c] = C \begin{bmatrix} \bar{x} \\ \hline \theta_c \end{bmatrix} \quad .$$

Using (A.4) the augmented dynamics have the form

$$\dot{\bar{x}} = (A + BK_1)\,\bar{x} + BK_2 u_p$$

or

$$\dot{\bar{x}} = A_{aug}\,\bar{x} + B_{st}\,\delta_{stick}$$

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15. Supplementary Notes

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16. Abstract

A multivariable control augmentation synthesis method is presented that is intended to enable the designer to directly optimize pilot opinion rating of the augmented system. The approach involves the simultaneous solution for the augmentation and predicted pilot's compensation via optimal control techniques. In this paper, the methodology is applied to the control law synthesis for a vehicle similar to the AFTI-F-16 control-configured aircraft. The resulting dynamics, expressed in terms of eigen-structure and time/frequency responses, are presented with analytical predictions of closed-loop tracking performance, pilot compensation, and other predictors of pilot acceptance.

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