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PAIR PRODUCTION RATES IN MILDLY RELATIVISTIC, MAGNETIZED PLASMAS

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ABSTRACT

Electron-positron pairs may be produced by either one or two photons in the presence of a strong magnetic field. In magnetized plasmas with temperatures $kT \sim mc^2$, both of these processes may be important and could be competitive. We calculate the rates of one-photon and two-photon pair production by photons with Maxwellian, thermal bremsstrahlung, thermal synchrotron and power law spectra, as a function of temperature or power law index and field strength. This allows a comparison of the two rates and a determination of the conditions under which each process will be a significant source of pairs in astrophysical plasmas. We find that for photon densities $n_\gamma \leq 10^{25}$ cm^{-3} and magnetic field strengths $B \geq 10^{12}$ G, one-photon pair production dominates at $kT \gtrsim mc^2$ for a Maxwellian, at $kT \gtrsim 2\ mc^2$ for a thermal bremsstrahlung spectrum, at all temperatures for a thermal synchrotron spectrum, and for power law spectra with indices $s \leq 4$.

I. INTRODUCTION

Relativistic and trans-relativistic plasmas, where temperatures approach and exceed the electron rest mass energy, have been of increasing interest in astrophysics. The existence of such plasmas has been inferred from the study of sources such as gamma-ray bursts, where observed spectra require temperatures of order mc^2, and active galaxies, whose gamma-ray emission implies temperatures in excess of mc^2. These plasmas may have significant concentrations of electron-positron pairs, whose presence could strongly affect the spectrum, energetics and dynamics of the source regions. Study of the microphysics of pair production and annihilation is therefore of central importance to understanding the properties of hot plasmas. There has recently been a substantial amount of work on the processes governing the behavior of non-magnetized and weakly magnetized relativistic thermal plasmas. Rates of photon-photon pair production at relativistic temperatures have been calculated by Weaver (1976) and Stoeger (1977), and rates for particle pair production processes have been studied in the relativistic limit by Lightman (1982). Pair annihilation in hot thermal plasmas has been investigated by Ramaty and Mészáros (1981), Zdziarski (1981), and Svensson (1982a). Using results of these studies, Lightman (1982), Svensson (1982b), Stepney (1983), Takahara and Kusunose (1983) and McKinley and Ramaty (1983) have determined the steady state equilibria of a thermal plasma, where pair production would balance annihilation.

The properties of trans-relativistic plasmas in strong magnetic fields are virtually unexplored, but have potentially interesting applications to gamma-ray burst source models. In comparison, the characteristics of strongly magnetized plasmas at lower temperatures (10 - 100 keV) have been extensively

studied in connection with models for X-ray pulsars. The coincidence of fields around 10^{12} G with plasma temperatures around mc^2 introduces first order processes, one-photon (photon-magnetic field) pair production and annihilation, which may be as important as the two-photon processes which operate in both magnetized and non-magnetized plasmas. In addition, synchrotron emission will be far more important as a source of pair-producing photons than bremsstrahlung or annihilation radiation. As a result, the behavior of strongly magnetized pair plasmas could be quite different from what has thus far been determined for non-magnetized pair plasmas. In view of the evidence for gamma-ray burst source regions being near the surfaces of strongly magnetized neutron stars (Katz 1982, Lamb 1981), it seems worthwhile to evaluate quantitatively the relative importance of the one-photon processes in a trans-relativistic plasma.

This paper calculates and compares the one-photon and two-photon pair production rates for various thermal and power law photon distributions in the presence of a strong (> 10^{12} G) magnetic field. We consider pair production by photons with Maxwellian, thermal bremsstrahlung, thermal synchrotron, and power law energy spectra, with preliminary results having been given for the Maxwellian case in Burns and Harding (1983). With the exception of the two-photon rates for Maxwellian and bremsstrahlung photon distributions, none of these have previously been calculated. Guilbert (1983) has considered one-photon pair production by a power law photon spectrum, but did not give a calculation of the rate. In Section II, we present an outline of the calculations and a review of the less familiar one-photon pair production process. Sections III and IV give, respectively, the results for the one-photon and two-photon rates as well as the energy distributions of the created electron-positron pairs. We also to provide approximate analytic expressions

for the one-photon rates. In Section V, we look at the ratios of the two processes as a function of magnetic field strength and either temperature or power law index of the photon distribution. Section VI discusses the implications of the results for self-consistent models of strongly magnetized pair plasmas and for models of gamma-ray bursts.

II. OUTLINE OF CALCULATION

The total rate of pair production by photon processes in a plasma is an integration of the cross section for the various processes over the photon distribution function. If the photons involved in the pair production are themselves produced by the particles of the plasma, the photon distribution in a self-consistent model would depend on the radiation mechanisms, the energy distribution of the particles, and the optical depth of the source region. The present calculation does not attempt self-consistency, but starts from assumed photon distributions which could plausibly be produced by mildly relativistic plasmas. This will give reasonable estimates for both the total rate of e+e- pair production and the relative importance of the one- and two-photon processes as a function of parameters which can be related to the properties of the plasma.

In a strong magnetic field, the energies of the electron and positron perpendicular to B are quantized and the pair can therefore only be created in discrete states. When the photon center-of-mass energies are not large compared to the spacing of these states, the pair production rates are significantly influenced, primarily near threshold, by quantum effects. While an expression exists for the one-photon pair production rate near threshold (Daugherty and Harding 1983), none exists for the two-photon rate in

a strong magnetic field. For the sake of consistency in comparing the two rates, we will neglect the effect of the discreteness of the pair states, although we will present additional calculations of the one-photon rates with quantum effects included.

Throughout the rest of the paper, we will use the dimensionless variables: E for photon energy in units of the electron rest mass mc^2, E_+ for electron/positron energy in mc^2, $B' = B/B_{cr}$ for the magnetic field strength in units of the critical field strength, $B_{cr} = m^2c^3/e\hbar = 4.414 \times 10^{13}$ G, and temperature $T_* = kT/mc^2$.

a) One-photon Pair Production

The creation of e+e- pairs by single photons is possible in the presence of a magnetic field, because the field is able to participate in the momentum transfer, allowing momentum to be conserved (for a complete discussion of the kinematics, see Daugherty and Harding 1983). The probability of one-photon pair production, however, is negligibly small unless the quantity, $\chi = 1/2$ E B'sinθ > 0.1, where E is the photon energy, and θ is the angle between the photon direction and B. The photon must also be able to supply at least the rest mass of the pair in the frame where θ = π/2, and must therefore have an energy in excess of the threshold, $2mc^2/\sin\theta$.

The polarization-averaged pair production attenuation coefficient for a photon of energy E propagating at an angle θ to a constant, homogeneous field of strength B' is (Toll 1952, Erber 1966)

$$\alpha_{\gamma B} = \frac{1}{2} \frac{\alpha}{\pi} B' \sin\theta \, T(\chi) \qquad (1)$$

$$T(\chi) \sim 4.74 \, \chi^{-1/3} \, Ai^2(\chi^{-2/3}) \qquad (1a)$$

$$\sim 0.46 \exp [-4/3\chi], \; \chi \ll 1 \tag{1b}$$

$$\sim 0.60 \; \chi^{-1/3}, \; \chi \gg 1 \tag{1c}$$

where Ai is the Airy function, $\lambdabar$ is the electron Compton wavelength and α is the fine structure constant. Eqn (1) is the asymptotic expression for the attenuation coefficient in the limit where the number of kinematically allowed states for the e+e- pair is large. Daugherty and Harding (1983) have found that the change in the attenuation coefficient due to quantum effects in the case where the number of allowed pair states is not large can be approximated by making the substitution:

$$\chi \rightarrow \chi/F,$$
$$F = 1 + 0.42 \; (E \sin\theta)^{-2.7}$$

in Eqn (1). The largest values of F, and thus the most significant changes in $\alpha_{\gamma B}$, occur near threshold, where $E \sin\theta = 2$. The pair production rate "per photon" for a photon distribution,

$$n_\gamma \; (E, \; \Omega) \; dEd\Omega = \frac{n_0}{4\pi\kappa} \; f(E, \; \Omega) \; dEd\Omega \tag{2}$$

will be

$$R_{\gamma B} = \frac{1}{n_0} \int d\Omega \int_{E_{min}}^{\infty} n_\gamma \; (E, \; \Omega) \; r_{\gamma B} \; (B', \; E, \; \Omega) \; dE \tag{3}$$

where $r_{\gamma B} = c \; \alpha_{\gamma B}$ is the rate for an individual photon and $E_{min} = 2/\sin\theta$ is threshold. The photon distribution is normalized to the total number density,

n_0, so that κ is determined by

$$\iint n_\gamma (E, \Omega) \, dE \, d\Omega = n_0 \qquad (4)$$

The energy distribution of the created pairs can be found from the differential single photon rate (Daugherty and Harding 1983):

$$\frac{dr_{\gamma B}(\epsilon)}{d\epsilon} = \sin\theta \, \frac{\alpha c B' \sqrt{3}}{\chi} \frac{[2+\epsilon \, (1-\epsilon)]}{\epsilon(1-\epsilon)} \, K_{2/3} \, [\frac{1}{3\chi\epsilon(1-\epsilon)}] \qquad (5)$$

where $\epsilon = E_+/E$ is the fractional energy of one member of the pair (note that the distribution is symmetric about $\epsilon = 0.5$ because the electron and positron are identical). The differential positron (electron) production rate for a distribution of photons is then,

$$\frac{dR_{\gamma B}(E_+)}{dE_+} = \frac{1}{2n_0} \int d\Omega \int_{E_{min}}^{\infty} n_\gamma (E, \Omega) \, \frac{dr_{\gamma B}(\epsilon)}{d\epsilon} \, \frac{d\epsilon}{dE_+} \, dE \qquad (6)$$

where $E_{min} = E_+ + 1/\sin\theta$.

b) Two-photon Pair Production

Although two-photon pair production is a process which occurs in free space, it will be modified in the presence of a strong magnetic field to the extent that the quantization of pair states discussed above is important (Wunner, Herold and Ruder 1983). Daugherty and Bussard (1980) have derived the cross section for the inverse process, two-photon annihilation of pairs in a magnetic field, and find that it does not depart significantly from the free-space cross section until $B > 10^{13}$ G. However, their calculation assumes that the pairs are in the ground state when they annihilate. Since this

initial state is a small part of the phase space which would be kinematically available to the pairs in the final state of pair production, we are not able from their results to estimate the general quantum behavior of the two-photon pair production rate. Since the cross section for photon-photon pair production in a strong magnetic field has not yet been derived we use the invariant free-space cross section (Berestetskii, Lifschitz and Pitaevskii 1971):

$$\sigma\,(E,\,E',\,\psi) = \frac{3}{2}\,\frac{\sigma_T}{\tau^3}\,\{(\tau^2 + 4\tau - 8)\,\ln\,[\frac{\sqrt{\tau} + \sqrt{\tau-4}}{\sqrt{\tau} - \sqrt{\tau-4}}] - (\tau+4)\,\sqrt{\tau(\tau-4)}\,\}$$

(7)

where $\tau\,(E,\,E',\,\psi) = 2EE'\,(1-\cos\psi)$, σ_T is the Thomson cross section, and ψ is the angle between the propagation vectors of the two photons. The pair production rate for a distribution of photons requires an integration of the cross section over both photon energies, E and E', and the solid angles for their propagation directions:

$$\frac{R_{\gamma\gamma}}{n_0} = \frac{c}{2}\,\int d\Omega \int d\Omega' \int_0^\infty dE \int_{E'_{min}}^\infty dE'\,\frac{n_\gamma(E,\,\Omega)}{n_0}\,\frac{n_\gamma\,(E',\,\Omega')}{n_0}\,(1-\cos\psi)\,\sigma\,(E,E',\,\psi)$$

(8)

where $E'_{min}\,(E,\,\psi) = 2/[E(1-\cos\psi)]$ (9)

is threshold energy for the primed photon. In the case of an isotropic photon distribution, the above angular integrations can be reduced to an integration over just one angle, which is usually chosen to be ψ, the angle between the photons (see Weaver 1976).

c) Photon Distributions

We calculate the rate of pair production for both processes using four different photon distributions in energy and angle: Maxwell-Boltzmann (or Wien spectrum) (MB), thermal bremsstrahlung (with gaunt factors ignored) (TB), thermal synchrotron (TS), and power law (P). In terms of the dimensionless variables T_*, E and B', defined above, we take the following expressions for the function $f(E,\Omega)$ in Eqn (2):

$$f^{MB}(E, \Omega) = E^2 \exp(-E/T_*) \tag{10a}$$

$$f^{TB}(E, \Omega) = \frac{1}{E} \exp(-E/T_*) \tag{10b}$$

$$f^{TS}(E, \Omega) = \exp\left[-\left(\frac{4.5 E}{T_*^2 B' \sin\theta}\right)^{1/3}\right] \tag{10c}$$

$$f^{P}(E, \Omega) = E^{-s}, \quad s > 2 \tag{10d}$$

The expression for TS is the optically thin synchrotron spectrum from mildly relativistic electrons with a Maxwellian distribution at temperature $T_* \lesssim 1$ derived by Petrosian (1981) in the limit $E \gg T_*^2 B'$. The dependence on θ, the angle between the emitted photon and the magnetic field, makes the TS distribution anisotropic, with a higher density of photons propagating perpendicular to the field. The other three distributions are assumed to be isotropic, even though thermal bremsstrahlung in a magnetic field should be anisotropic to a certain degree (see Nagel and Ventura 1983). From Eqn (4), the normalization parameter κ for the distributions in Eqn (10) are,

$$\kappa^{MB} = 2T_*^3 \tag{11a}$$

$$\kappa^{TB} = E_1 \ (E_L/T_*) \tag{11b}$$

$$\kappa^{TS} = \frac{\pi}{3} \ T_*^2 \ B' \tag{11c}$$

$$\kappa^{P} = \frac{E_L^{(1-s)}}{(s-1)} \tag{11d}$$

where E_L is an arbitrary lower energy cutoff in the the bremsstrahlung and power law spectra to make the integration finite, and E_1 is the exponential integral. The lower limit of E=0 has been used to normalize the TS spectrum even though emission is expected to fall off below the first harmonic, E=B'. Thermal broadening at $T_* \sim 1$, however, will cause the effective lower energy cutoff to be E << B', and in any case, the cutoff is always below pair production thresholds.

III. ONE-PHOTON RATE

a) Analytic Expressions

In the limit, χ << 1 and E >> 2, it is possible to perform the integration over the cross section to obtain approximate expressions for the one-photon pair production rate for the four photon distribution functions. Using the limiting expression (1b) for the magnetic single photon pair production rate, we can rewrite Eqn (3) as,

$$R_{\gamma B} \simeq \frac{0.23}{2\kappa} \frac{\alpha c}{\pi} B' \int_o^\pi d\theta \ \sin^2\theta \int_{\frac{2}{\sin\theta}}^\infty dE \ f(E, \theta) \ \exp \left[- \frac{8}{3EB'\sin\theta} \right] \tag{12}$$

Since all of the photon distributions are decreasing functions of energy and the pair production rate is sharply increasing with energy, the E integration

can be performed by the method of steepest descents. The integrand has saddle
points at the energies:

$$E_0^{MB} = (\frac{8T_*}{3B'\sin\theta})^{1/2} \tag{13a}$$

$$E_0^{TB} = (\frac{8T_*}{3B'\sin\theta})^{1/2} \tag{13b}$$

$$E_0^{TS} = 2(\frac{8T_*}{3\ B'\ \sin\theta})^{1/2} \tag{13c}$$

$$E_0^{P} = \frac{8}{3\ s\ B'\ \sin\theta}, \ s > 2 \tag{13d}$$

for the different photon distributions. These are the energies at which most
of the photons pair produce. The major contribution to the integral lies in a
relatively small region about the saddle point and there is negligible
contribution elsewhere. Since the use of the steepest descents method
requires extension of the lower limit of the integral to 0, the result will be
most accurate in cases where $E_0 \gg E_{min}$ (ie., when the saddle point lies well
above threshold). The θ integration can then be performed by noting that the
integrand also peaks sharply about $\sin\theta = 1$, and by making an expansion about
this point, retaining only the linear terms. The resulting pair production
rates for the four photon distributions are:

$$R_{\gamma B}^{MB} = 1.11 \frac{\alpha c}{\lambda} \frac{1}{T_*} \exp [-2 (\frac{8}{3B'T_*})^{1/2}] \tag{14a}$$

$$R_{\gamma B}^{TB} = .313 \frac{\alpha c}{\lambda} [E_1(\frac{E_L}{T_*})]^{-1} B'^{3/2} \exp [-2(\frac{8}{3B'T_*})^{1/2}] \tag{14b}$$

$$R_{\gamma B}^{TS} = 1.65 \frac{\alpha c}{\lambda} \frac{1}{T_*} \exp [-2(\frac{8}{3B'T_*})^{1/2}] \tag{14c}$$

$$R_{\gamma B}^P = .511 \frac{\alpha c}{\lambda} E_L^{s-1} \left(\frac{3}{8}\right)^{s-1} \frac{\Gamma(\frac{s}{2}+1)}{\Gamma(\frac{s}{2}+\frac{3}{2})} s^{s-\frac{3}{2}} (s-1)e^{-s}B'^s \qquad (14d)$$

It is an interesting coincidence that this calculation gives the same exponential dependence in the TS case as in the MB and TB cases. Zheleznyakov (1982) has performed a similar computation of the one-photon pair production rate for a thermal bremsstrahlung photon distribution, except that his expression was not integrated over angle. The approximations we have made here, $\chi \ll 1$ and $E \gg 2$, with the expressions for E_0 from Eqn (13) substituted for E, give the following regions of validity for the above expressions:

$$\text{MB,TB: } (T_* B')^{1/2} \ll 1, \quad T_* \gtrsim \frac{3}{2} B' \qquad (15a)$$

$$\text{TS: } \left(\frac{8}{3} T_* B'\right)^{1/2} \ll 1, \quad T_* \gtrsim \frac{3}{8} B' \qquad (15b)$$

$$\text{P: } s > \frac{4}{3}, \quad sB' \lesssim \frac{4}{3} \qquad (15c)$$

The above expressions for $R_{\gamma B}$ can be modified to take quantum effects into account by considering the second term introduced in the exponential in Eqn (1b) by the substitution $\chi \rightarrow \chi/F$ as small. This term then becomes a slowly varying multiplicative factor in the steepest descents analysis which is taken outside the integral and evaluated at $E=E_0$ and $\sin\theta = 1$. The result is a mulitplication of the expressions in Eqn (14) by the factor,

$$\text{MB, TB: } \exp\left[-1.186\, B'^{.85}/T_*^{1.85}\right] \qquad (16a)$$

$$\text{TS: } \exp\left[-.09\, B'^{.85}/T_*^{1.85}\right] \qquad (16b)$$

P: $\exp\left[-.193\ B'^{2.7}\ s^{3.7}\right]$ (16c)

These factors give a good approximation to quantum effects in the pair production rate within the regions of validity estimated by Eqn (15). However, the most significant changes in the rate will occur for combinations of B' and T_* or s outside the region where the analytic expressions are accurate.

b) Numerical Results

According to Eqn (15), at low temperatures or in magnetic fields approaching the critical value, where most of the pair-producing photons have energies near threshold, the analytic expressions and approximation to quantum effects derived above are not accurate. We have therefore also numerically integrated Eqn (3), explicitly including the threshold photon energy and the full expression [Eqns (1),(1a)] for the asymptotic rate, which does not require χ to be small. The values for the saddle points [Eqn (13)] and the second derivatives from the steepest descents analysis were used as an estimate of the peak and width of the integrand to determine the numerical limits of integration. Figures 1a-1d present the numerical results for the pair production rate "per photon", both with and without quantum effects included, as a function of T_* (or power law index s) and B' along with the dashed analytic curves from Eqn (14) for the same parameters. Since expressing the rate "per photon" requires normalizing to the total number of photons in the spectrum, the TB and P rates are dependent on the additional free parameter E_L which cuts off the spectrum at some low value of the energy. We will thus plot $\bar{R}_{\gamma B} = R_{\gamma B}\ \kappa$ for these cases. However, if $E_L \ll T_*$

in the TB case or $E_L \sim 2$ in the P case, the TB and P one-photon rates can be compared directly with the MB and TS rates which are exactly normalizable. It is most convenient to discuss the thermal and non-thermal distributions separately, since the parameterizations are different.

The pair production rates for the thermal photon distributions all have a similar behavior, indicated by the same exponential dependence in the analytic expressions. This exponential dependence requires the product T_*B' to be reasonably large (> .01) before pair production becomes important. It also means that for temperatures below a few times mc^2, small changes in the parameters can cause order-of-magnitude changes in the rate. The rates for the MB distribution tend to be larger than those for TB because there are more photons above 1 MeV at a given temperature in the case of MB. The TS rates are larger than either of the others because there are more photons in the TS distribution which propagate perpendicular to the field, where the probability for pair production is highest. The TS distribution therefore picks out the most favorable phase space for pair production. The analytic curves match the numerical ones reasonably well over the parameter space where they were estimated to be accurate [Eqn (15)], but they overestimate the rates at low temperatures and high field strengths, where energies are near threshold, and at very high temperatures, where the single-photon rate is no longer increasing exponentially with energy. The quantum effect of discrete pair states is strongest for low temperatures, where the pair producing photons are near threshold, and tends to decrease the rate of pair production (by limiting the phase space allowed in the final state). The effect is not as strong in the TS case, because the value of E_0 in Eqn (13c) indicates that the pair producing photons have energies higher above threshold.

In the case of the P distribution, the temperature is replaced by the

power law index as the parameter determining the number of photons above pair production threshold. The rates for the P distribution are generally power laws in B' with the same index s (only negative) as the photon distribution. Quantum effects are important here only at the highest field strengths and for s $\gtrsim$ 3, where pair producing photons have lower energies.

c) Pair Energy Distributions

The differential one-photon rate which gives the energy distribution of the created pairs is too complicated to evaluate analytically, even in the limit $\chi \ll 1$, so we evaluate Eqn (6) for several cases by numerical integration. Figures 2 and 3 show the results for the MB and TS distributions at two temperatures, 0.1 mc^2 and mc^2. Figure 4 shows the differential rates for the P distribution with two values of power law index. For nearly all values of T_* and B', each member of the pair will receive half of the photon energy (ie., the differential rate in Eqn [5] for a single photon is sharply peaked at $\varepsilon = 0.5$). Therefore, the pair energy distribution has its maximum at around half of E_0, the most probable pair-producing photon energy. The width of the distribution is the dispersion in photon energy convolved with the dispersion of positron (electron) energy about $\varepsilon = 0.5$. The TB distribution gives results which are very similar to MB, since they share the same values for E_0 and for the dispersion of pair-producing photon energies. The TS spectrum produces pairs at higher energies for the same values of T_* and B', since the value of E_0 is larger than for the MB or TB cases. At low temperatures and higher field strengths, most of the pairs are produced "cold" (ie., with only their rest mass). Photons in a power law spectrum produce pairs with a much wider range of energies, the width being inversely proportional to power law index, but there is still a well defined maximum in

the distribution.

It is interesting to note that in a number of cases, most of the pairs are produced with energies considerably greater than the average energy of the particles in the plasma. The average energy in a Maxwellian distribution of electrons either in equilibrium with photons in the MB distribution at kT or which are radiating photons in the TS spectrum is 3kT. The curves shown in Figure 2 indicate that pairs are being produced with energies as high as 20-30kT in the case of the low field strengths. The rate of producing these high energy pairs, however, is very low.

IV. TWO-PHOTON RATES

Deriving analytic expressions for the rate of photon-photon pair production by means of the steepest descents method is not accurate enough to give useful results. The approximation which must be made for the cross section depends on $E^{1/2}$, a much slower dependence than the one-photon cross section which is an exponential. This produces an overlap with the distribution function which is neither sharply peaked nor sufficiently symmetric about the saddle point for the Gaussian approximation made by the steepest descents method. We are therefore limited to a numerical evaluation of the two-photon rate.

In the case of the isotropic photon distributions, we need only perform integrals over the photon energies and the angle between their propagation directions. Results for the MB and TB spectra are plotted together in Figure 5a, although $R_{\gamma\gamma}^{TB}$ could shift up or down relative to $R_{\gamma\gamma}^{MB}$ depending on the low energy cutoff in the spectrum which determines the normalization. However, in the limit $E_L \ll T_*$, the dependence should be weak. The curve

for $R_{\gamma\gamma}^{MB}$ reproduces Weaver's (1976) result, except that he computes the quantity $\langle\sigma v\rangle = 2 R_{\gamma\gamma} / n_0$, which differs by a factor of two. In order to directly compare the one-photon and two-photon rates we must compute the rate of pair production events per photon, which is 1/2 of the total rate of events in the case of the two-photon process.

The rates for the TS spectrum are plotted in Figure 5b for three field strengths. Even though we have used the free-space cross section, the two-photon rates are now dependent on B' through the dependence on B'$\sin\theta$ in the TS distribution function. It is therefore necessary to integrate over three angles in Eqn (8), which we have chosen to be θ, θ', and $\bar{\phi} = \phi' - \phi$, in the coordinate frame where **B** lies along the z axis. The angle ψ is then related to θ, θ', and $\bar{\phi}$ through the law of cosines for spherical triangles: $\cos\psi = \cos\theta \cos\theta' + \sin\theta \sin\theta' \cos\bar{\phi}$. The rate of two-photon pair production from a TS spectrum is significantly less than from the MB or TB spectra, because of the anisotropy of the distribution function. The departure of the TS rate from the MB rate is largest for low temperatures and low fields, where the beaming of photons in the TS spectrum is most pronounced.

The rate for the P case is plotted in Figure 5c as a function of power law index. Like the TB case, the two-photon rate for the power law distribution is dependent on the low energy cutoff in the spectrum. Except for some dependence on s in κ, the two-photon rate decreases with increasing power law index because the steeper spectra have fewer photons at energies around the maximum in the cross section.

V. RATIO OF ONE-PHOTON TO TWO-PHOTON PAIR PRODUCTION RATES

The ratio of rates for one-photon and two-photon pair production "per photon" can be computed for each value of field strength and either temperature or power law index. However, the total photon density, n_0, is introduced as an additional parameter, since the relative importance of the two processes will depend on the relative densities of real photons and virtual photons of the magnetic field. Figures 6a-6d show the ratio of the one-photon to two-photon rates as a function of kT and B' (or s and B' in the P case). On the left-hand vertical scale is the quantity $n_0 \, R_{\gamma B}/R_{\gamma\gamma}$, which shows the photon density at which the two rates are equal. On the right-hand side is the actual ratio of the rates for a fixed photon density of $n_0 = 10^{25}$ cm^{-3}. For the thermal distributions, we also plot the blackbody or thermal equilibrium photon density, n_{BB}, which is an upper limit to the photon density at kT. Therefore, in the region of Figures 6a-6c above n_{BB}, the one-photon rate always exceeds the two-photon rate.

For the thermal photon distributions, temperatures below 1 MeV are of most interest to models of gamma-ray bursts. In this region, small changes in the magnetic field strength can produce order-of-magnitude changes in the ratio of the two rates. For example, for the MB distribution and a photon density of $n_0 = 10^{25}$ cm^{-3}, the two-photon process dominates up to $T_* \sim .6$ for $B_{12} = 2$, whereas for $B_{12} = 4.4$ the one-photon process dominates at all temperatures. The ratio of one- to two-photon rates is significantly larger at all field strengths and temperatures for the thermal synchrotron spectrum. In this case, one-photon pair production is the dominant mechanism at all temperatures for $B > 10^{12}$ G, and $n_0 \leq 10^{25}$ cm^{-3}.

When the pairs are in thermal equilibrium with the photons at temperature

$T_* \sim B'$, most of the pairs will be in the lowest Landau state and we can
compare our results for the ratio of one-to two-photon rates with those of
Daugherty and Bussard (1980) for the inverse processes. They find that the
one-photon and two-photon annihilation rates for pairs in the ground state are
equal at $B = 10^{13}$ G. Using Fig. 6a for a Maxwellian photon distribution, we
find that at the thermal equilibrium photon density, n_{BB}, one-photon and two-
photon pair production rates are also equal around $B = 10^{13}$ G under the
constraint that $T_* = B'$.

VI. DISCUSSION

It is of interest to examine the implications of the above calculations
of pair production in mildly relativistic plasmas for models of gamma-ray
burst source regions and spectra. The observed spectra of many bursts show
line features at energies around 400 keV which could be red-shifted .511 MeV
photons from positron annihilation. Fits to the continuum yield temperatures
of 100 keV to 1 MeV for thermal bremsstrahlung spectra (Mazets et. al. 1981)
and temperatures of 100 - 500 keV and $B \sim 10^{12}$ G for thermal synchrotron
spectra (Liang 1982). These properties suggest that creation of electron-
positron pairs must be taking place in the source regions of these objects.
Two-photon pair production has been considered as the source of pairs in most
models for bursts (Cavallo and Rees 1978; Ramaty, Lingenfelter and Bussard
1981), but if strong magnetic fields are present our calculations indicate
that one-photon pair production could be at least as important. The observed
ratio of energy in the annihilation line to energy in the continuum (about
0.1) requires at least this fraction of energy of the plasma to be in pairs.
The contribution of pairs from magnetic pair production of bremsstrahlung

photons in gamma-ray bursts was estimated by Zheleznyakov (1982), who found that this process gives the observed line-to-continuum ratio provided that the source region is optically thick to pair production, that all the pairs annihilate, and that continuum radiation from the pairs is not important. However, in neutron star magnetic fields, the synchrotron loss rates are much higher than bremsstrahlung loss rates, so that in estimating the rate of pair production, a thermal synchrotron spectrum should be used. Since the corresponding rate of pair production from synchrotron radiation is much higher at a given temperature, an optically thick source will give too large a line-to-continuum ratio. One is led to conclude that gamma-ray burst source regions, if they have strong magnetic fields, are not optically thick to pair production.

Clearly, a more sophisticated approach is needed in order to model gamma-ray burst spectra. To self-consistently determine the concentration of pairs in the plasma and the rate of pair annihilation, it is necessary to include all important radiation and pair production processes, and optical depth effects, both from pair production attenuation and scattering. In a thermal pair plasma, ie. one where the pairs have a Maxwellian distribution, the pair density may be found from a steady state solution which balances pair production and annihilation. A simplified model of steady state pair equilibrium in a strongly magnetized plasma, where the only processes included are one-photon pair production by synchrotron photons and two-photon annihilation, is considered by Harding (1984). This calculation gives relatively high pair densities, ie, near or above thermal equilibrium densities, for values of field strength and temperature estimated for gamma-ray bursts, unless the source size is much smaller than a neutron star polar cap radius. Not included, however, were Compton scattering of photons by the

pairs and optical depth of the photons from pair production.

The optical depth of the sources to pair production is also an observational consideration. Photons above threshold would be severely attenuated in sources having high pair production optical depths. The observed spectra do not show strong attenuation above 1 MeV; in fact, a number of burst spectra have significant flux up to ~ 10 MeV and are best fitted by thermal synchrotron or power law models (Nolan et. al. 1984). These photons in the high energy part of the spectrum must be produced in regions of low optical depth. If thermal synchrotron models are to explain the continuum photon production then parameters derived from fits to observed spectra must give a low optical depth for pair production. The results presented in this paper can be used to obtain an average optical depth for photons in the spectrum. The optical depth for one-photon pair production is $\langle \tau_{\gamma B} \rangle \sim R_{\gamma B} \, \ell/c$ where ℓ is the path length of the photon through the strong field region. From Figures 1c and 1d, $\langle \tau_{\gamma B} \rangle \sim (.3 - 10^9) \, \ell_6$, where ℓ_6 is in units of 10^6 cm, a typical size for the strong field region near a neutron star. It appears that the observed photons with energies above 1 MeV cannot be produced within fields $\gg 10^{12}$ G. Even if the source of observed photons is in a low field region, optical depths for two-photon pair production could attenuate photons above ~ .511 MeV. The two-photon optical depth $\langle \tau_{\gamma\gamma} \rangle = (R_{\gamma\gamma}/n_0) \, Rn_0/c$, will depend on both source size R and photon density n_0. Using a typical burst luminosity, $L \approx 10^{37} \, (d/100 \, pc)^2$ erg s^{-1} where d is the distance to the source, we can estimate the photon density in the source as,

$$n_0 \approx \frac{L}{\langle E \rangle Ac} \approx \frac{10^{23} \; cm^3 \; (d/100 \; pc)^2}{T_* \, A_{10}}$$

where A_{10} is the area of the emitting region in units of 10^{10} cm^2. This gives

for the pair production optical depth at $T_* = 1$ of a bremsstrahlung spectrum (using Fig. 5a):

$$\langle\tau_{\gamma\gamma}\rangle^{TB} \simeq 10^{-4} \frac{R}{A_{10}} (d/100 \text{ pc})^2.$$

So, unless $R/A_{10} < 10^4$ cm. or the sources are very nearby, two-photon pair production will also attenuate high energy photons. This argument has in fact been used to place distance limits on gamma-ray burst sources (Schmidt 1978). However Figure 5b suggests one possible solution to the problem. At temperatures below $\sim .3$ mc^2, the two-photon rate for a thermal synchrotron spectrum decreases with decreasing field strength significantly below the values for an isotropic distribution. The resulting pair production optical depths are

$$\langle\tau_{\gamma\gamma}\rangle^{TS} \simeq (10^{-9} - 10^{-5}) \frac{R}{A_{10}} (d/100 \text{ pc})^2.$$

If the photons in burst sources are produced by thermal synchrotron radiation and the fields are not high enough to cause one-photon pair production, the photons in the > 1 MeV range may be able to escape without strict requirements on either source geometry or distance.

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FIGURE CAPTIONS

Figure 1a - One-photon (magnetic) pair production rate per photon at temperature kT/mc^2 for a Maxwellian photon distribution. Curves are labeled with different magnetic field strengths in units of 10^{12} Gauss. Solid lines are numerical calculations, dashed lines are the analytic results, both without quantum effects; dot-dashed lines are numerical calculations including quantum effects.

Figure 1b - Same as Fig. 1a but for a thermal bremsstrahlung spectrum. $\bar{R}_{\gamma B} = R_{\gamma B} \kappa$.

Figure 1c - Same as Fig. 1a but for a thermal synchrotron spectrum.

Figure 1d - One-photon pair production rate per photon versus magnetic field in units of the critical field for a power law photon spectrum. Curves are labeled with power law index, s.

Figure 2 - Spectrum of pairs produced by the one-photon process from a Maxwellian distribution for two different temperatures and magnetic field strengths. The vertical scale is the differential rate of pair production normalized to the total rate, and the horizontal scale is pair energy in mc^2. Curves are labeled with values of magnetic field strength in units of 10^{12} Gauss.

Figure 3 - Same as Fig. 2 but for a thermal synchrotron spectrum.

Figure 4 - Same as Fig. 2 but for a power law spectrum, for two values of power law index, s.

Figure 5a - Two-photon pair production rate versus kT/mc^2 for Maxwellian, $R_{\gamma\gamma}^{MB}$ and thermal bremsstrahlung, $R_{\gamma\gamma}^{TB}$ photon distributions. $\bar{R}_{\gamma\gamma} = R_{\gamma\gamma} \, \kappa^2$.

Figure 5b - Two-photon pair production rate versus kT/mc^2 for a thermal synchrotron spectrum and different magnetic field strengths.

Figure 5c - Two-photon pair production rate versus power law index, s, for a power law photon spectrum.

Figure 6a - Ratio of one-photon to two-photon pair production rates at temperature kT for a Maxwellian photon distribution. Curves are labeled with different values of magnetic field. The dashed line is the thermal equilibrium photon density, n_{BB}.

Figure 6b - Same as Fig. 6a but for a thermal bremsstrahlung spectrum.

Figure 6c - Same as Fig. 6a but for a thermal synchrotron spectrum.

Figure 6d - Ratio of one-photon to two-photon pair production rates at different magnetic field strengths for a power law spectrum. Curves are labeled with power law index.
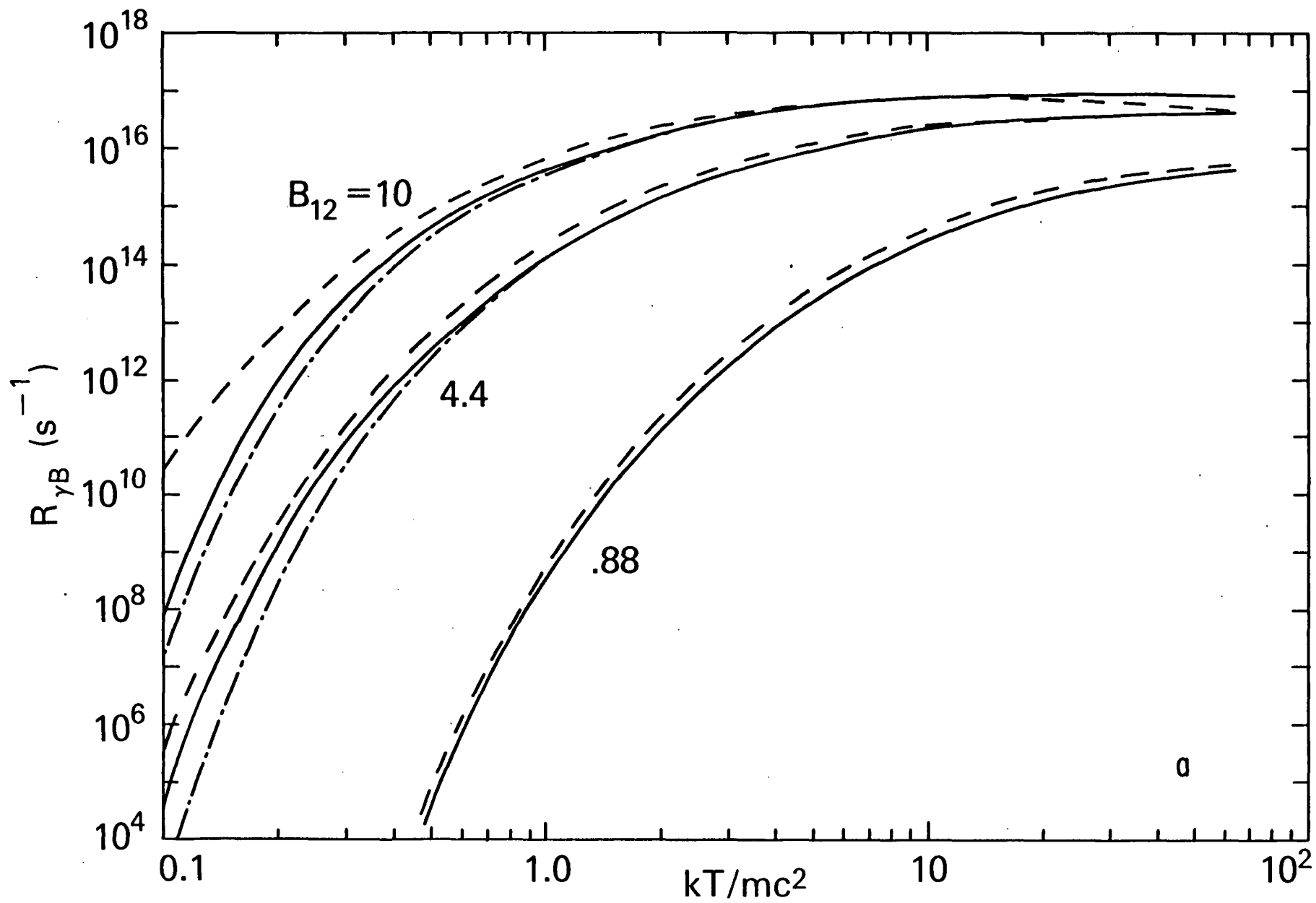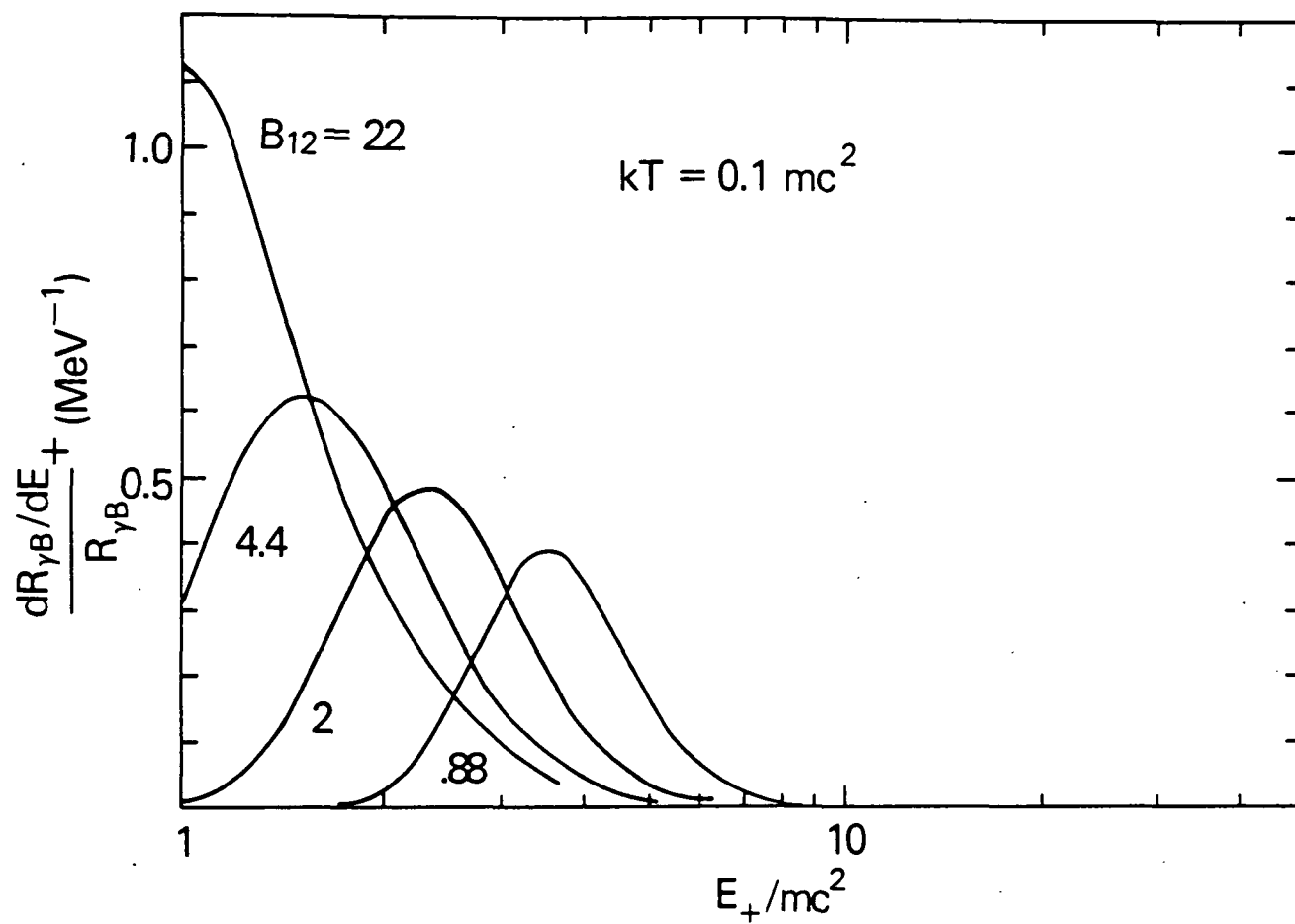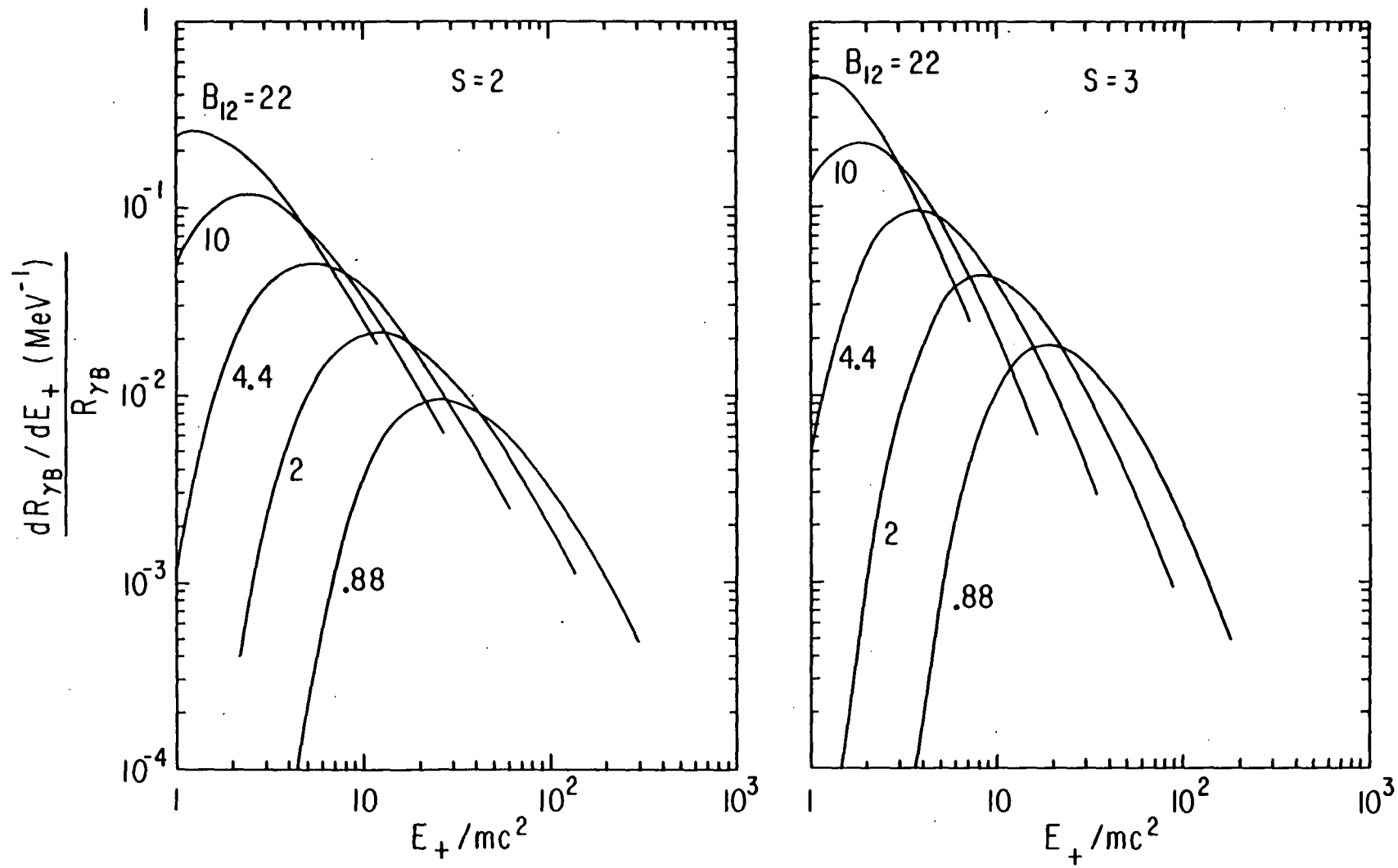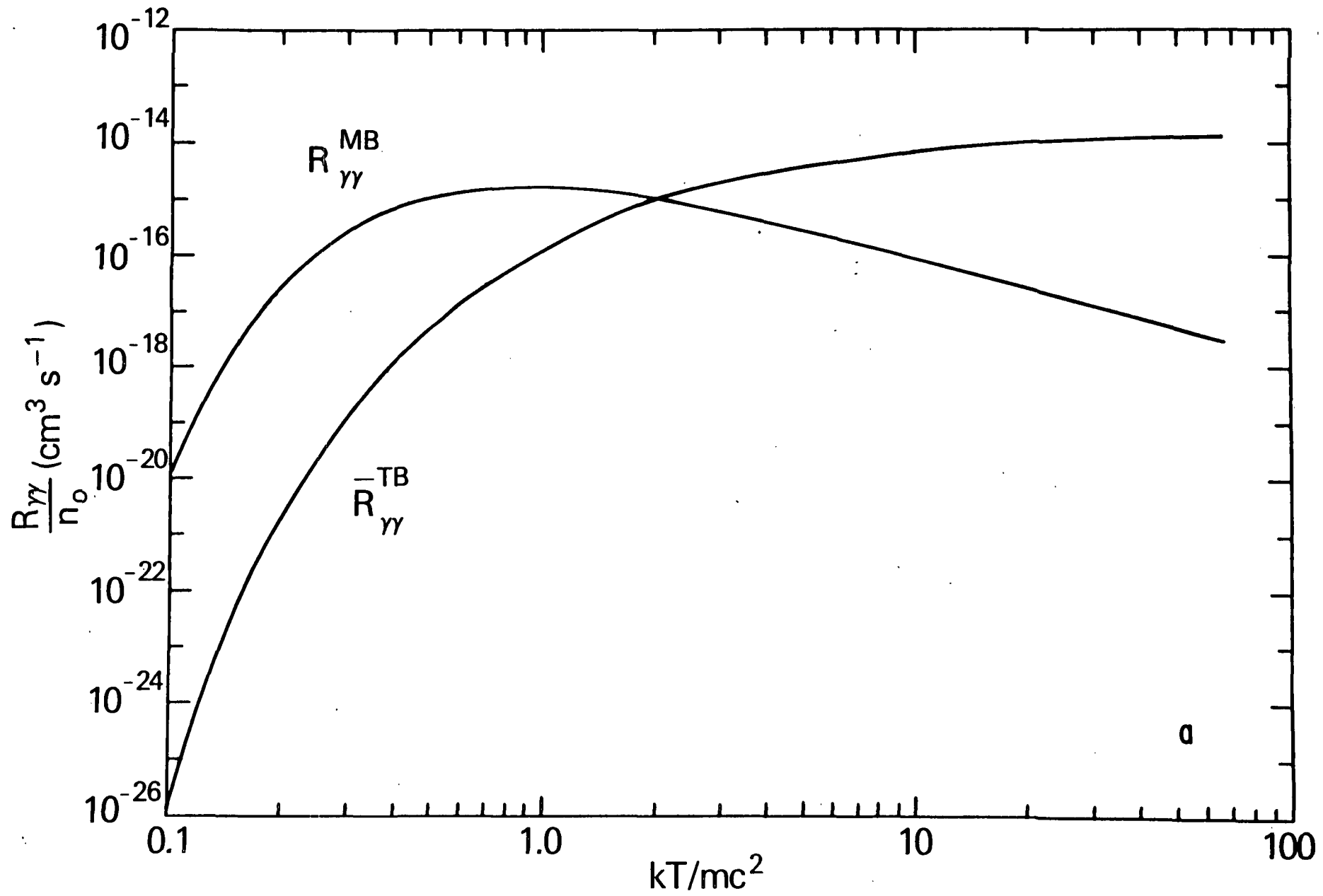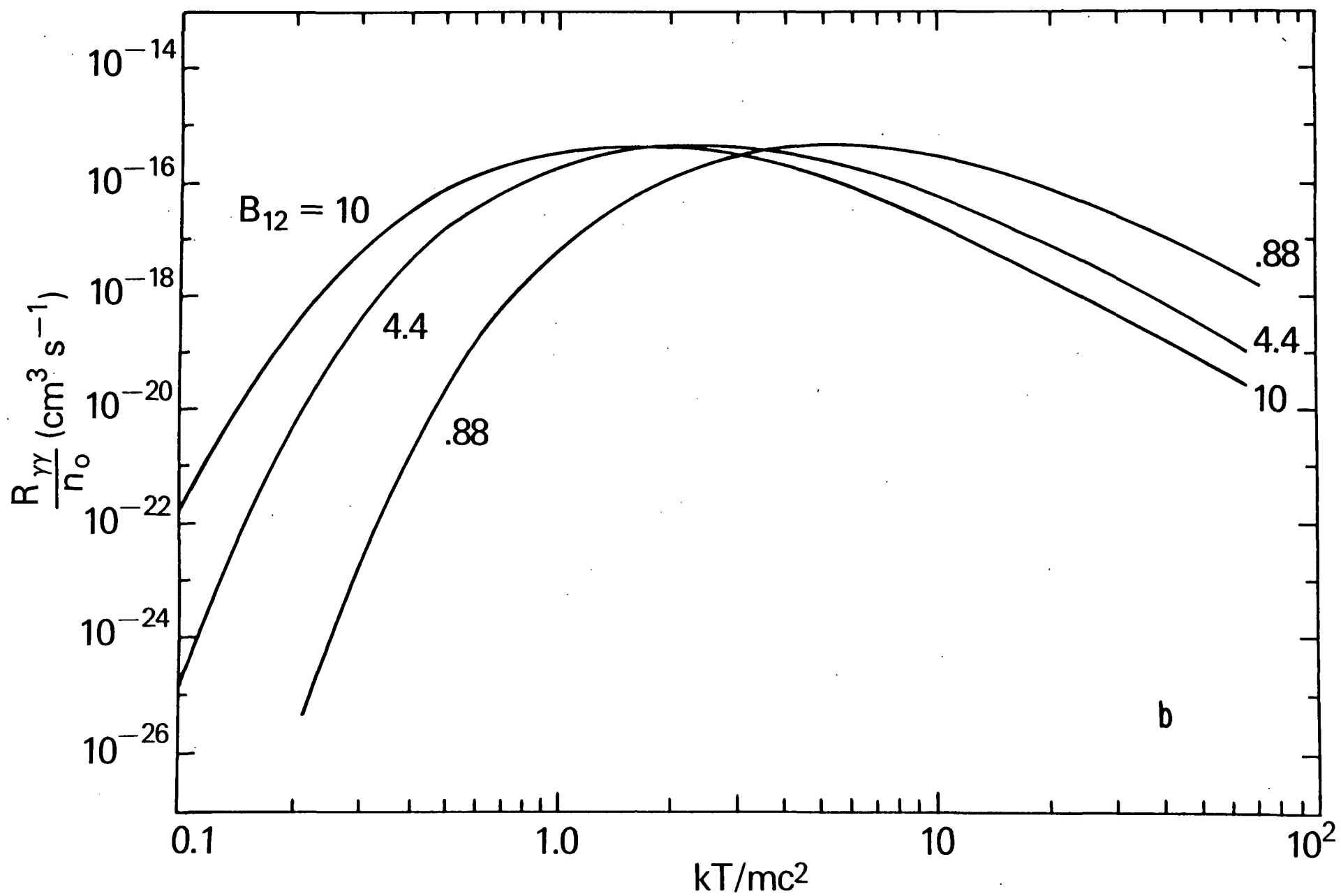
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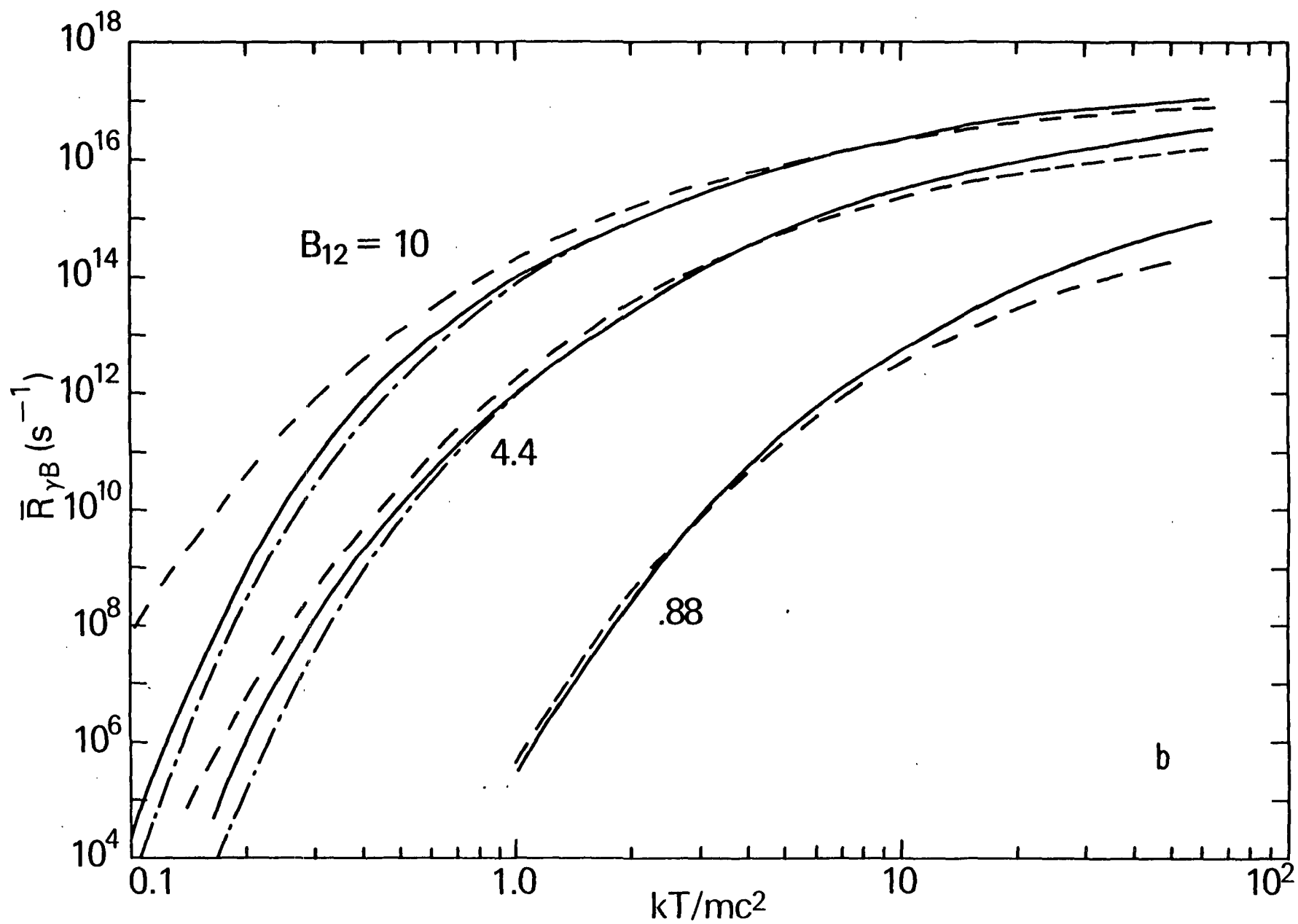
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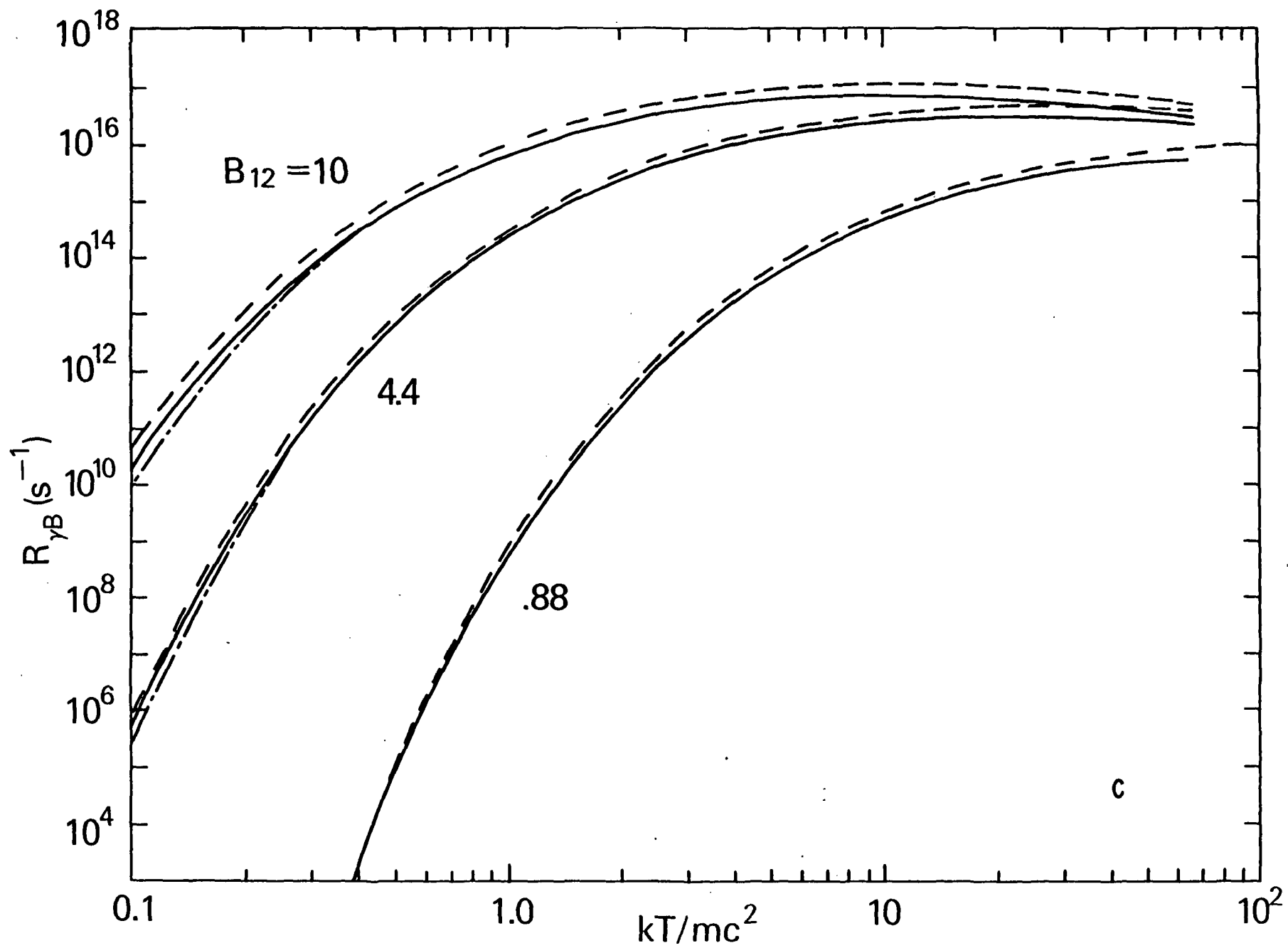
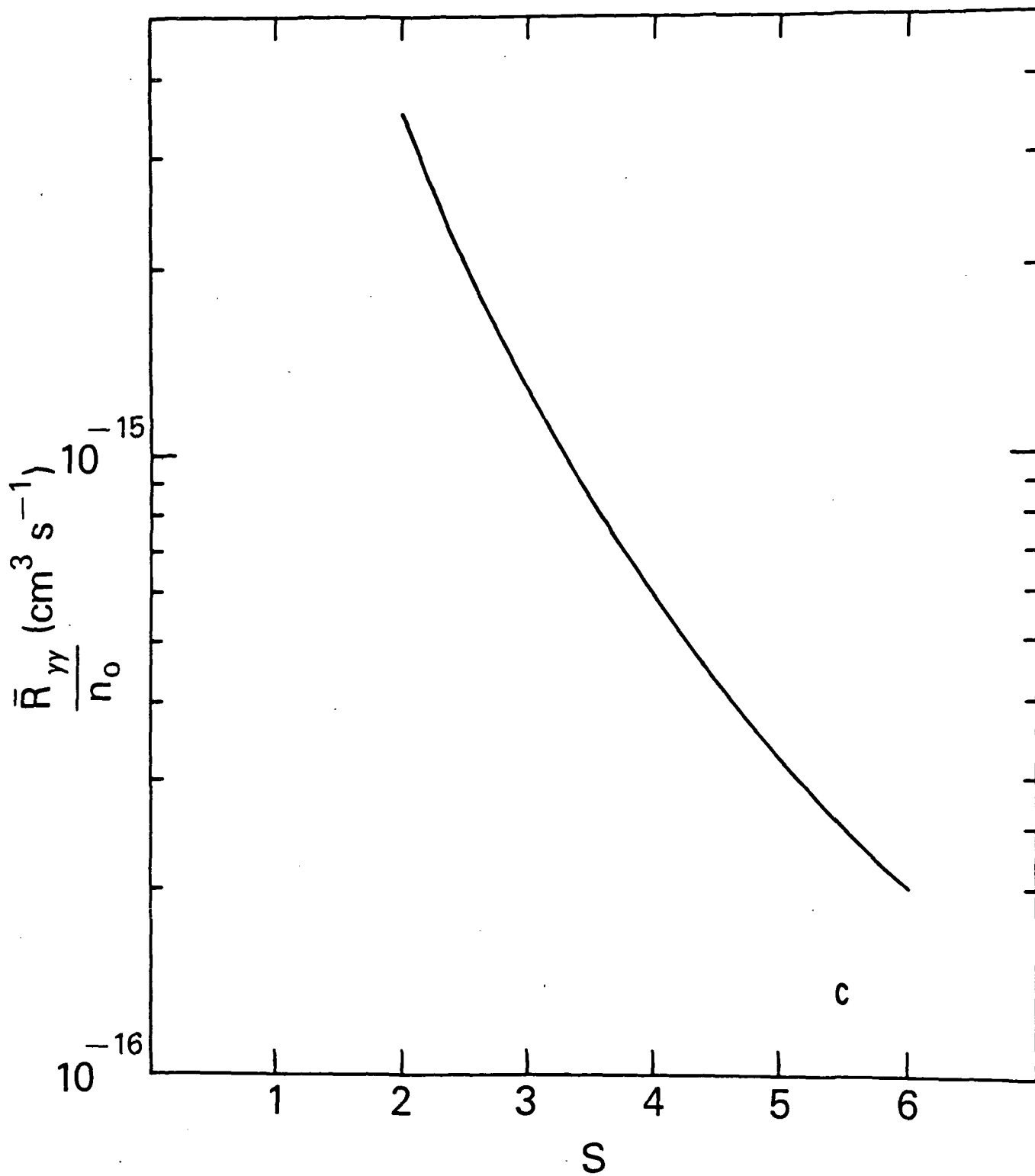Figure 1c
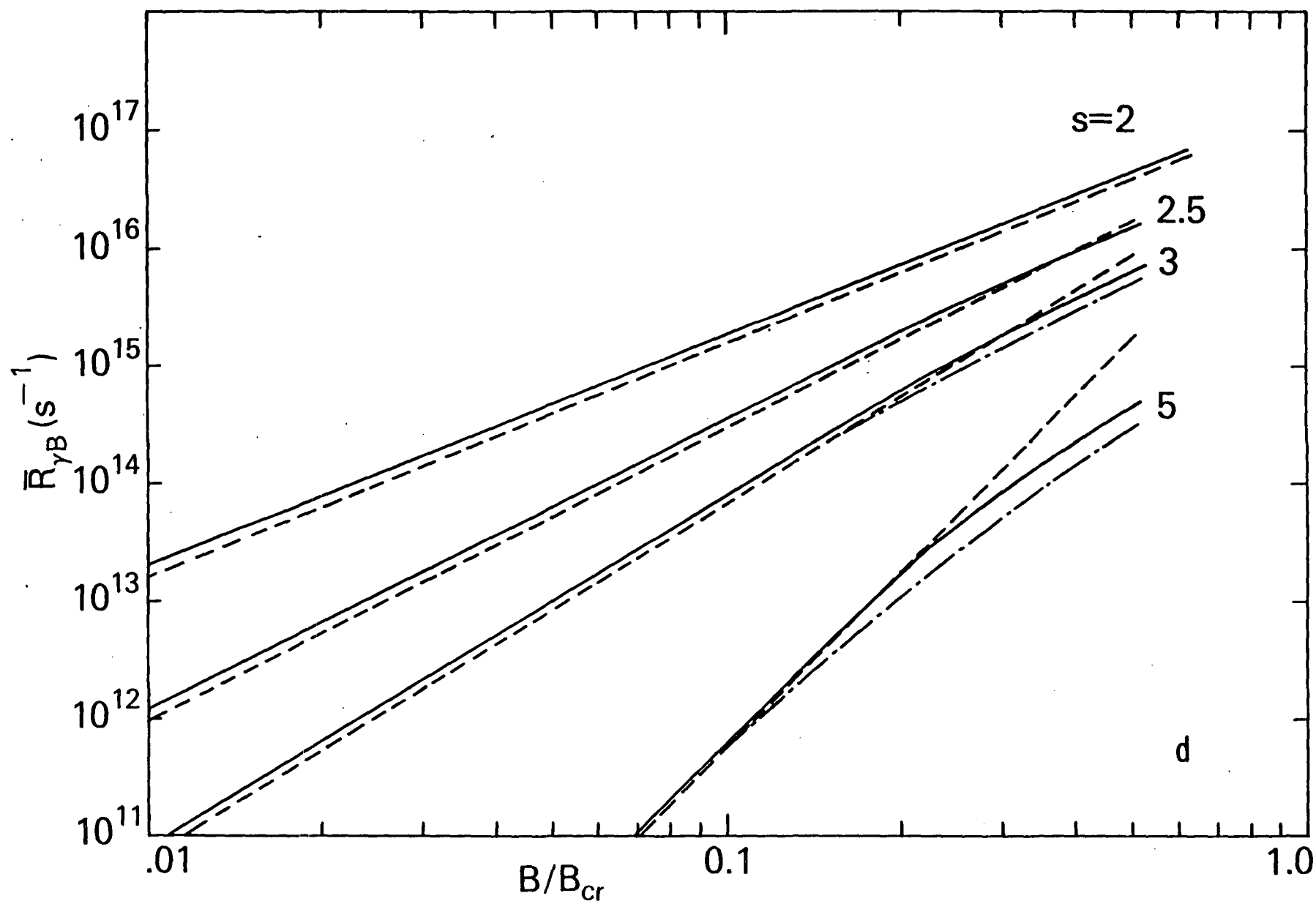
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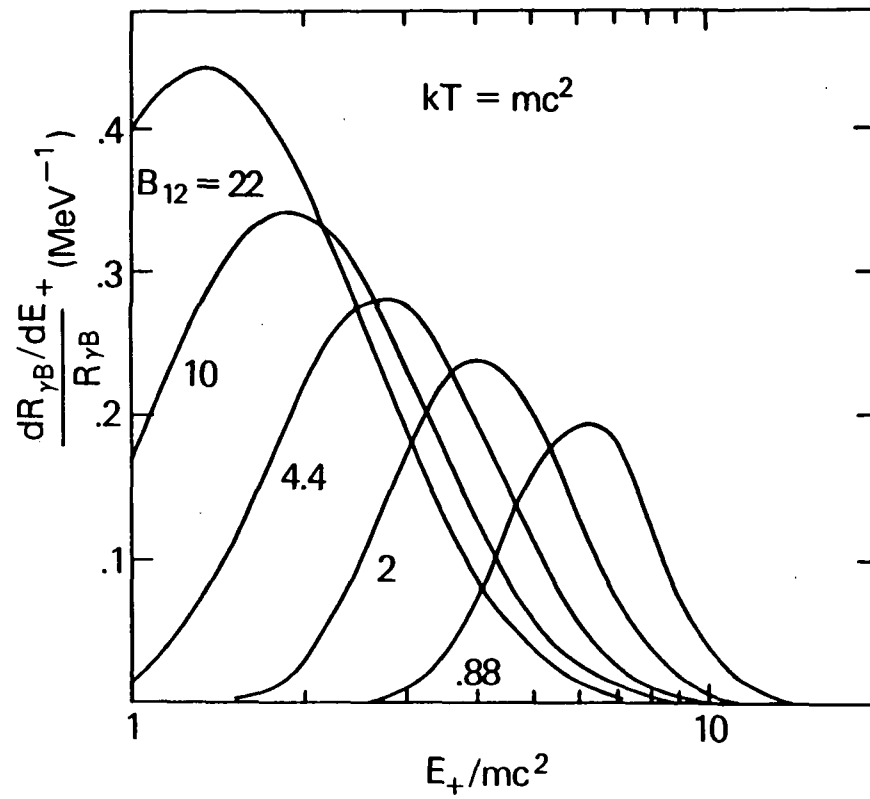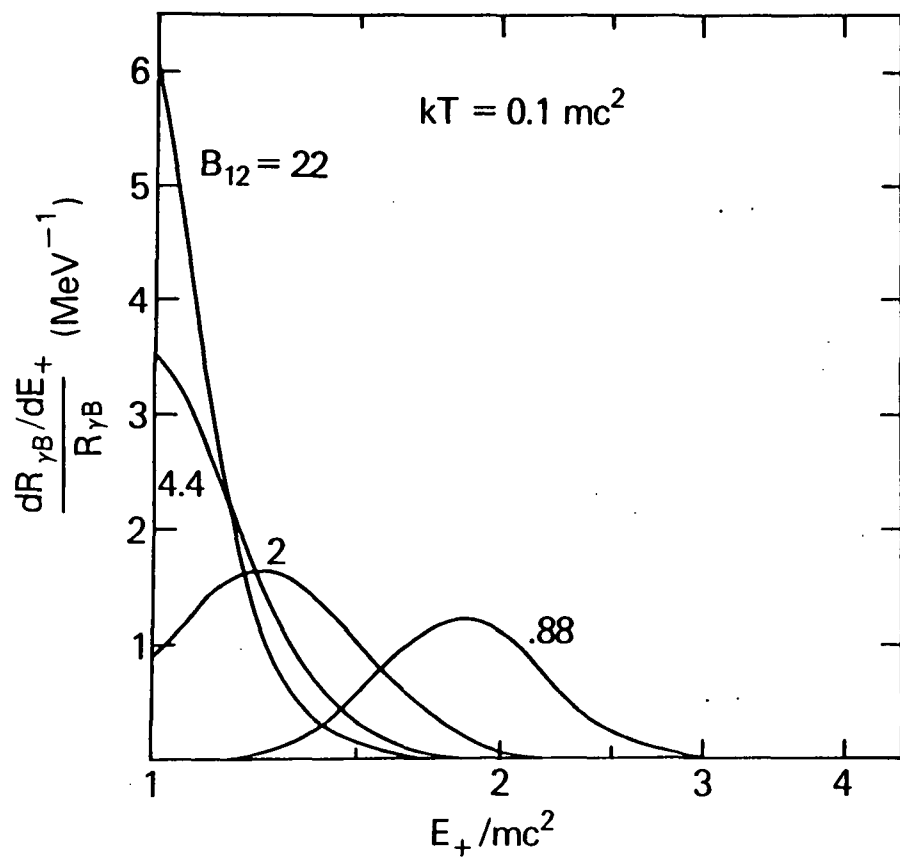
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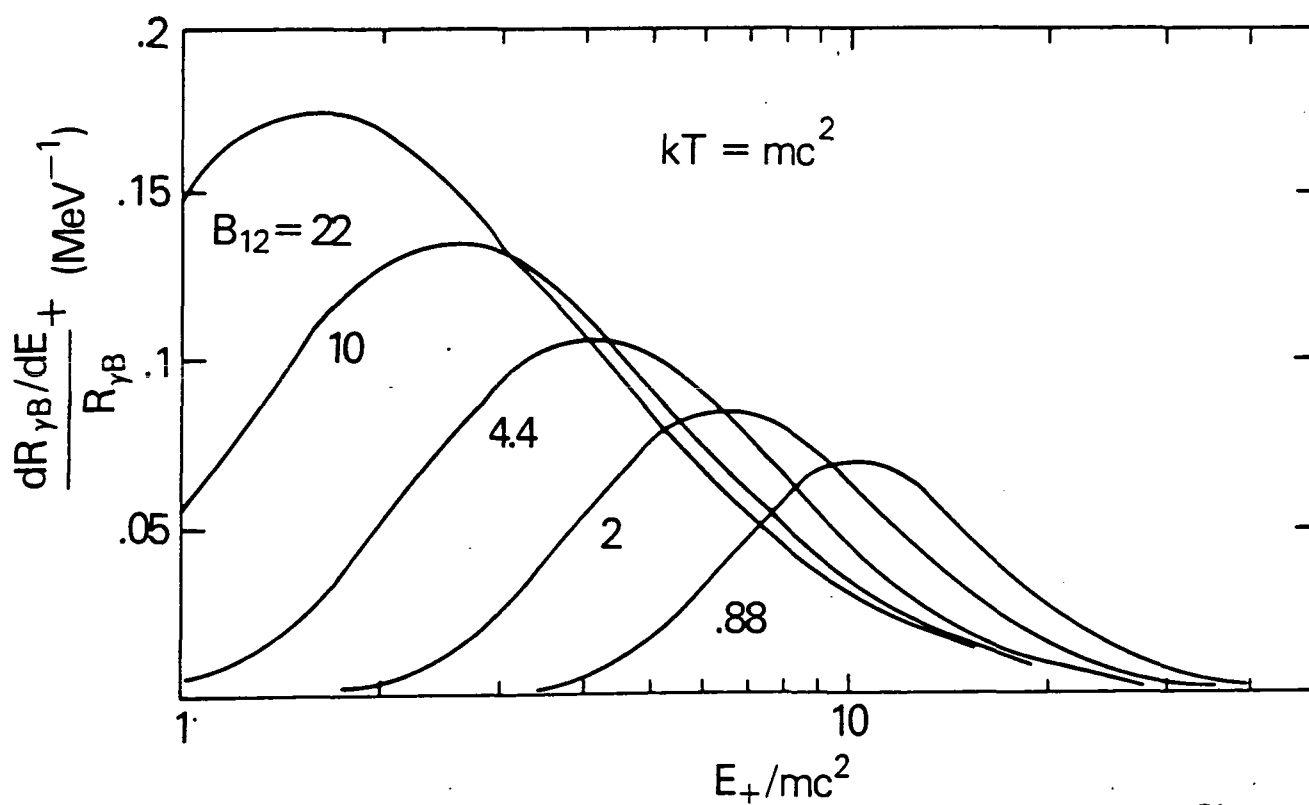
Figure 3

Figure 4

Figure 5a
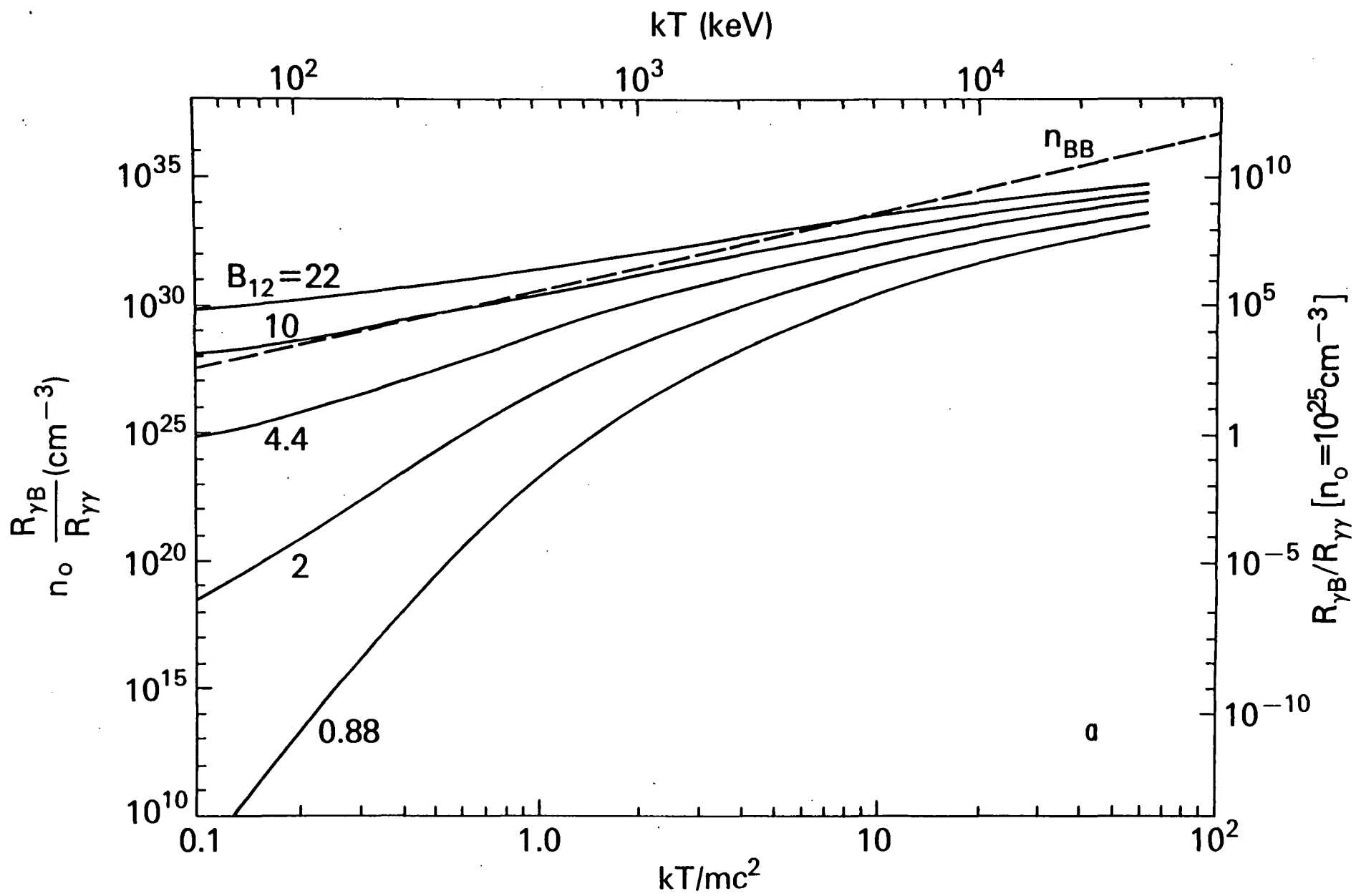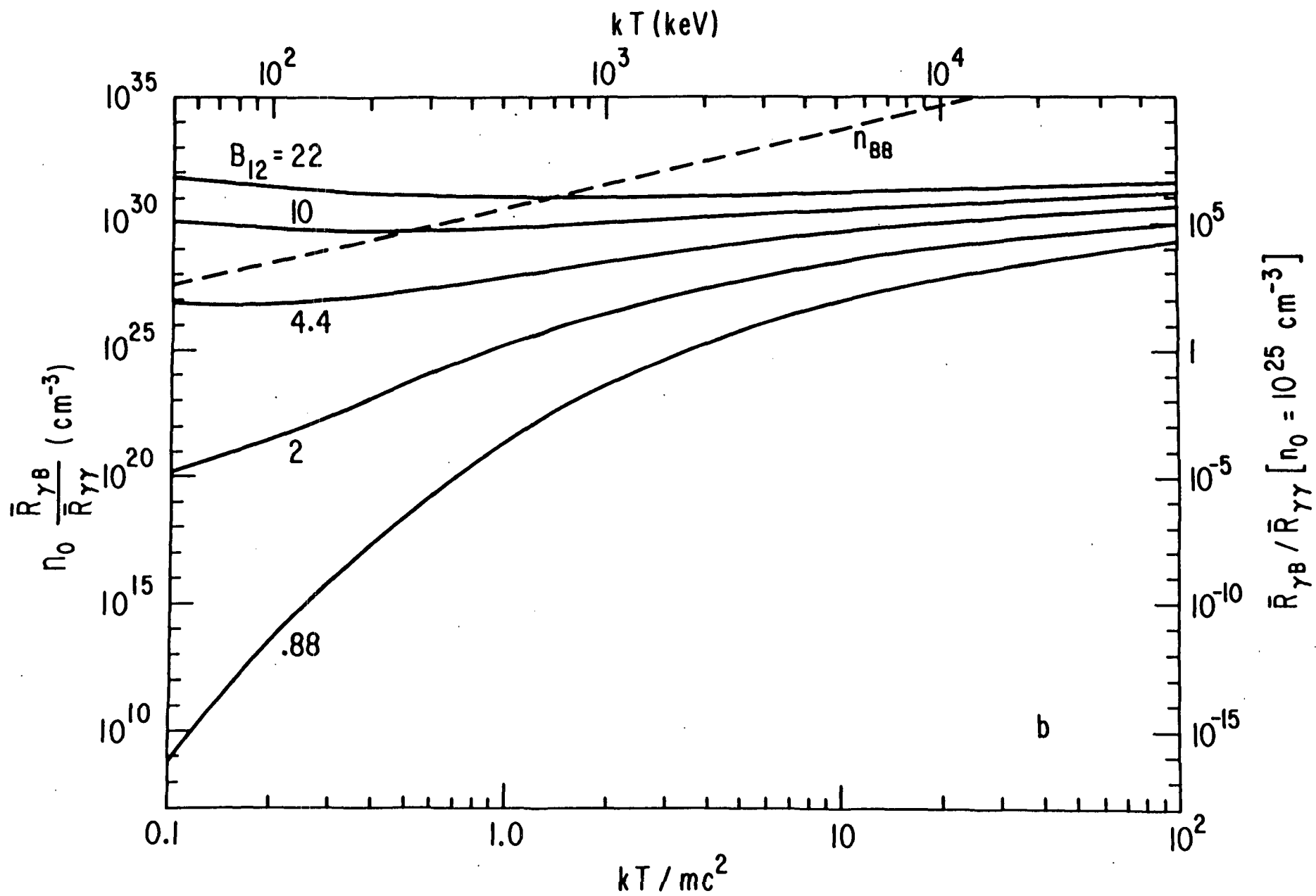
Figure 5b

Figure 5c

Figure 6a

Figure 6b
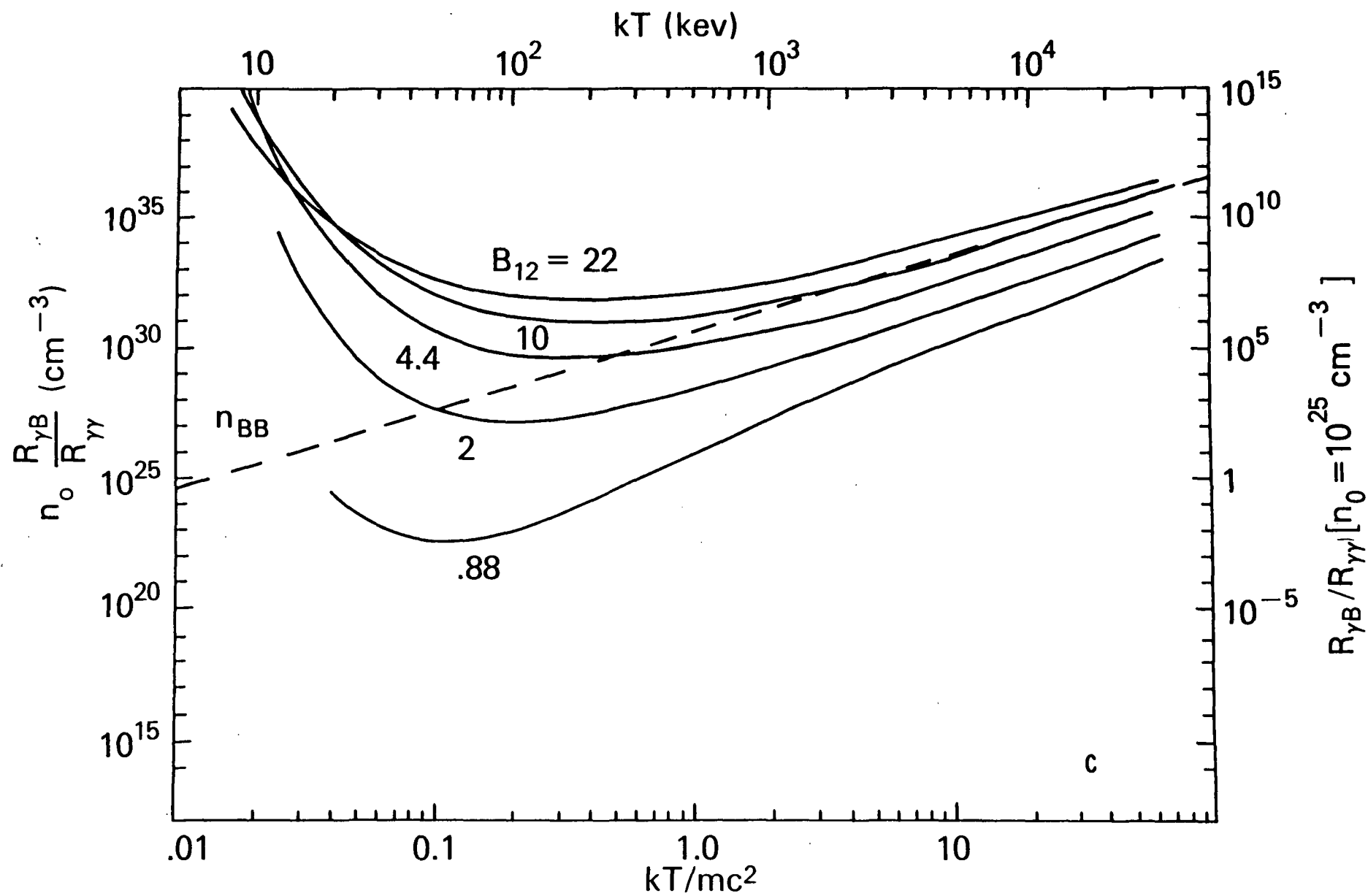
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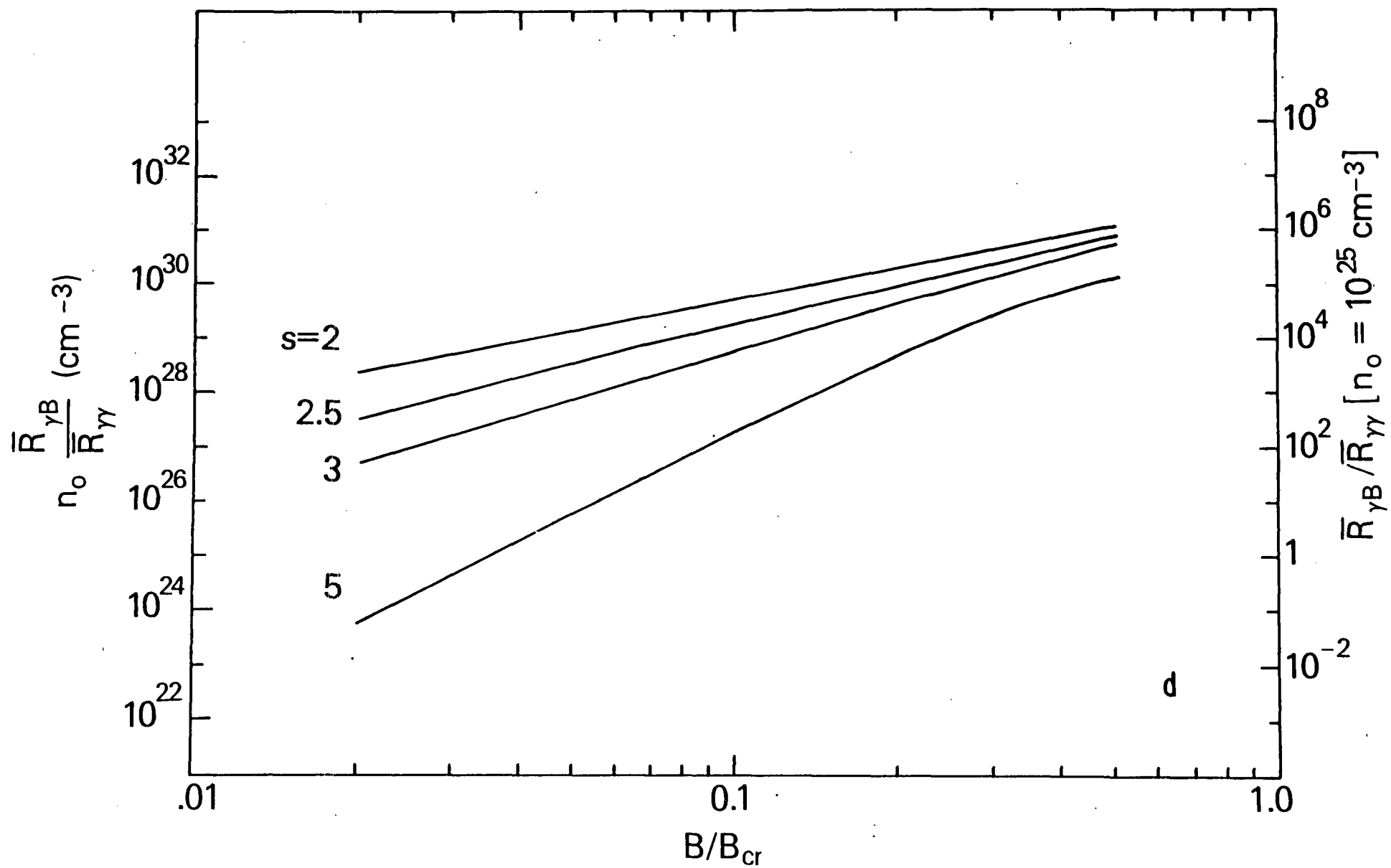
Figure 6d

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15. Supplementary Notes

16. Abstract   Electron-positron pairs may be produced by either one or two photons in the presence of a strong magnetic field. In magnetized plasmas with temperatures $kT \sim mc^2$, both of these processes may be important and could be competitive. We calculate the rates of one-photon and two-photon pair production by photons with Maxwellian, thermal bremsstrahlung, thermal synchrotron and power law spectra, as a function of temperature or power law index and field strength. This allows a comparison of the two rates and a determination of the conditions under which each process will be a significant source of pairs in astrophysical plasmas. We find that for photon densities $n_\gamma < 10^{25}$ cm^{-3} and magnetic field strengths $B \geq 10^{12}$G, one-photon pair production dominates at $kT \gtrsim mc^2$ for a Maxwellian, at $kT \gtrsim 2 mc^2$ for a thermal bremsstrahlung spectrum, at all temperatures for a thermal synchrotron spectrum, and for power law spectra with indices $s \lesssim 4$.

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