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A Possible Solution to the CP
Domain Wall Problem in the
Early Universe

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A POSSIBLE SOLUTION TO THE CP DOMAIN
WALL PROBLEM IN THE EARLY UNIVERSE

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ABSTRACT

In an SU(5) GUT model of phase transitions in the early universe, we show how the degeneracy between vacua with different spontaneously broken CP can be dynamically lifted by creating a bias through the formation of a heavy fermion pair condensate of fermions. The fermion condensate bias drives a transition to a unique vacuum state. The final resulting transition to a true vacuum eliminates the domain walls, solving the domain wall problem in a matter-antimatter domain cosmology.

The question of whether the universe is globally baryon asymmetric or there are "islands" of matter and antimatter on a very large scale depends on the nature of CP violation at the GUT energy scale. If the CP violation is hard, i.e., of the Kobayashi-Maskawa [1] type, one expects the former; if the CP violation in the GUTS is spontaneous [2], the latter situation may occur [3,4]. Sato [5] has shown how the CP domains can grow to astronomical size with moderate supercooling and inflation. Stecker [6] has recently discussed important observational aspects of such a baryon symmetric domain cosmology. Such initial CP domains are separated by domain walls which are very massive and could eventually gravitationally dominate the evolution of the universe in conflict with observation. Thus, they must either not exist or decay in the early universe [7,8].

A very interesting solution to this "domain wall" problem was proposed by Kuzmin, et al. [9]. They have shown that for a wide range of parameters in the Higgs sector, the CP symmetry of the Lagrangian, which is broken at high temperature, is restored again at a lower temperature. As a result, the walls disappear at lower temperature. It has also been argued that the inflationary universe [10] scenario could solve domain wall problem.

In what follows, we shall describe another possible scenario where the domain walls can disappear naturally. By combining the idea of a strongly interacting SU(5) phase [11] with spontaneous CP violation, we show how the degeneracy between the two different vacua with respect to CP symmetry can be lifted dynamically before the transition from the SU(5) phase to the low energy SU(3) X SU(2) X U(1) phase is completed.

Our model of spontaneous CP violation in an SU(5) GUT is closely related to the model of Branco [12] and Nieves [13]. In distinction to their model, we shall not take natural flavor conservation into account. Further, we only

use dimensionful couplings in the Lagrangian and shall use Coleman-Weinberg [14] type of symmetry breaking.

The model contains three Higgs fields H_ξ's ($\xi = 1,2,3$) , all belonging to the $\underline{5}$ of SU(5). The multiplets H_1 and H_2 are assumed to have the following coupling to the fermions:

$$L_y = \bar{\chi}_{Lab} f_1 \ \psi_R^a \ H_1^b + \epsilon_{abcde} \ \bar{\chi}_R^{ab} \ f_2 \ \chi_L^{cd} \ H_2^e + \text{h.c.} \tag{1}$$

Here each generation of fermions is assigned to the fundamental $\underline{5}$ of SU(5) denoted by ψ and the antisymmetric representation $\underline{10}$ is denoted by χ. The Yukawa coupling constants f_1 and f_2 are matrices in generation space and are assumed to be real. In addition we introduce two Higgs fields $\underline{24}$ of SU(5); φ_1 and φ_2 and define a complex field.

$$\varphi = \frac{1}{\sqrt{2}} (\varphi_1 + i\varphi_2) \tag{2}$$

The Higgs potential for the fields Φ's and H_ξ' s ($\xi = 1,2,3$) can be written as

$$V = V_0(\varphi) + V_1(\varphi,H) + V_2(H) \tag{3}$$

where,

$$\begin{aligned}
V_0(\varphi) = &\ \nu_1(\text{Tr}\,\varphi\varphi^+)^2 + \nu_2[(\text{Tr}\,\varphi^2)^2 + \text{h.c.}] \\
&+ \nu_3(\text{Tr}\,\varphi^2)(\text{Tr}\,\varphi^{+2}) + \lambda_1 \text{Tr}(\varphi\varphi^+\varphi\varphi^+) \\
&+ \lambda_2 \text{Tr}(\varphi\varphi^+)^2 + \lambda_3[\text{Tr}\,\varphi^4 + \text{h.c.}]
\end{aligned} \tag{4}$$

and

$$V_1(\Phi,H) = \sum_{\xi,\xi'=1}^{3} \alpha_\xi H_\xi H_\xi^+ Tr(\Phi\Phi^+) + \beta_\xi H_\xi^+ \Phi^+ \Phi H_\xi + \beta_{\xi'} H_\xi^+ \Phi \Phi^+ H_\xi$$
$$+ \delta_\xi [H_\xi^+ \Phi^2 H_\xi + h.c.] + \gamma_\xi [H_\xi^+ H_\xi(Tr\Phi^2) + h.c.]$$
$$+ 1/2\ \delta_{\xi\xi'} [H_\xi^+ \Phi^2 H_{\xi'} + h.c.] +$$
$$+ 1/2\ \gamma_{\xi\xi'} [H_\xi^+ H_{\xi'}(Tr\Phi^2) + h.c.] \qquad (5)$$

We stress here that $\delta_{\xi\xi'} \neq \delta_{\xi'\xi}$ and $\gamma_{\xi\xi'} \neq \gamma_{\xi'\xi}$.[1] And further, the
diagonal elements γ_ξ and δ_ξ are not zero.

$$V_2(H) = \sum_{\xi,\xi'=1}^{3} [a_{\xi\xi'}(H_\xi^+ H_\xi)(H_{\xi'}^+ H_{\xi'}) + 1/2\ b_{\xi\xi'} |H_\xi^+ H_{\xi'}^+|^2$$
$$+ 1/2\ c_{\xi\xi'} [(H_\xi^+ H_{\xi'})(H_\xi^+ H_{\xi'}) + h.c.]$$
$$+ 1/2\ d_{\xi\xi'} [(H_\xi^+ H_{\xi'})(H_{\xi'}^+ H_{\xi'}) + h.c.]] \qquad (6)$$

Here a and b, are symmetric in ξ and ξ' and c and d are zero along the
diagonal. All the constants in V_0, $V_1(\Phi,H)$ and $V_2(H)$ as well as f_1 and f_2
are real so that the Lagrangian is invariant under the CP transformation: $H_\xi \to H_\xi^*$,
$\Phi \to \eta\Phi^*$, $\psi_R \to \psi_L^C$, $\chi_L \to \chi_R^C$. Note that Φ_1 and Φ_2 transform under CP
as $\Phi_1 \to \eta \Phi_1$ and $\Phi_2 \to -\eta \Phi_2$, where $\eta = \pm 1$. Thus, when both Φ_1
and Φ_2 develop VEV simultaneously CP is spontaneously broken. In this case
$\langle\Phi\rangle$ is complex and the γ and δ couplings in (5) lead to a complex mass matrix
for the physical color triplets of the H_ξ's which will be denoted as A_ξ's.
The CP invariance is also broken by the H_ξ's when $SU(2) \times U(1)_Y$ breaks to
$U(1)_{em}$. This reproduces the CP violation at electroweak energy scale M_W.

In order to obtain the correct pattern of symmetry breaking $SU(5) \to$
$SU(3)^C \times SU(2) \times U(1)_Y \to SU(3)^C \times U(1)_{em}$, the various VEV must have the form

[1] In Ref. 13 it was assumed that the coupling constants $\delta_{\xi\xi'}$ and $\gamma_{\xi\xi'}$ are
symmetric. However, complex VEV of Φ then will not lead to CP violation as the
authors have claimed.

$$< \Phi > = \rho \, e^{i \vartheta} \, \text{diag} \, [\, 1,1,1,-3/2,3/2 \,]$$

$$< H_1 > = e^{i \vartheta'} \text{column} \, (0,0,0,0,v_1),$$

$$< H_2 > = \text{column} \, (0,0,0,0,v_2), \tag{7}$$

$$< H_3 > = \text{column} \, (0,0,0,0,v_3)$$

Here ρ and v_ξ's are real. In $<\Phi>$, the VEV of the component of Φ which transforms as $(1,3,0)$ under $SU(3)^C \times SU(2)_L \times U(1)$, is of the order M_W^2/M where M_X is the unification mass scale and is therefore ignored. In Eq.(1), the Higgs multiplet H_3 does not couple to the fermions and its phase may be taken to be zero in Eq. (7). The relative phase between H_1 and H_2 is denoted by ϑ'. This phase can not be rotated away since the Higgs fields carry the same $U(1)$ quantum number. Thus, for a range of parameters of the Higgs Potential, there exist CP non-conserving solutions. The ϑ and ϑ' dependence of the potential in general can be written as

$$V = V_0 \, (\rho) + V_1 \, (\rho,H) + V_2(H) \tag{8}$$
$$= A + B \cos 4\vartheta + C \cos(2\vartheta - \vartheta') + D \cos 2\vartheta' + E \cos\vartheta' + F \cos 2\vartheta$$

Here A, B, C, D, E and F are parameters of the potential which are independent of ϑ and ϑ'. The CP nonconserving solution for ϑ and ϑ', which we formally denote as ϑ_0 and ϑ'_0, clearly has the symmetry $\vartheta_0 \rightarrow - \vartheta_0$, $\vartheta'_0 \rightarrow - \vartheta'_0$. Such discrete symmetries gives rise to the well-known domain structure in the early universe [3,4] and domain walls [7]. The reason is simple: below the transition temperature, the Higgs fields have a non-zero VEV corresponding to some point in the manifold of degenerate vacua. If this manifold is disconnected, a domain structure of the universe will be created.

Thus, as the temperature of the universe decreases below the scale of the superheavy gauge bosons, we expect that separate domains were generated with $\{\vartheta_0, \vartheta'_0\}$ and $\{-\vartheta_0, -\vartheta'_0\}$ phases.

Of course, these phases will affect the details of the mechanism of baryon production. The dominant contribution to the baryon asymmetry comes from the decay of superheavy gauge and Higgs bosons corrected at the one loop level by an exchange of Higgs scalars [15,16] (shown in Fig. 1).

The baryon to photon ratio is given by [15]

$$\frac{n_B}{n_\gamma} = \left(\frac{N_x}{N}\right) \Delta B (r - \bar{r}) \tag{9}$$

where N_x and N are the number of species of superheavy and light particles, respectively and r ($\bar{r}$) are the branching ratios for the production baryons and antibaryon in the decay of the superheavy particles with $\Delta B \neq 0$ (net change in baryon number).

In our model, the decay of the superheavy colored Higgs A_ξ's must mix to give rise to baryon production. The mass matrix for these Higgs fields is computed to be

$$m^2 = \vartheta^2 \, (-5/30) \begin{bmatrix} \tilde{\beta}_1 & \delta_{12}e^{2i\vartheta} & \delta_{13}e^{2i\vartheta} \\ \delta_{21}e^{-2i\vartheta} & \tilde{\beta}_2 & \delta_{23}e^{2i\vartheta} \\ \delta_{31}e^{-2i\vartheta} & \delta_{32}e^{-2i\vartheta} & \tilde{\beta}_3 \end{bmatrix} \tag{10}$$

Here $\tilde{\beta}_\xi \equiv \beta_\xi + \beta_{\xi'}$ ($\xi = 1,2,3$) and we have ignored terms of order M_W^2/M_X^2.

Let $U_{\xi\xi'}$ be the unitary matrix that diagonalizes the above mass matrix

and m_1, m_2 and m_3 denote the masses of the physical Higgs-bosons A_ξ ($\xi=1,2,3$). In reference [13] it has been shown that with the assumption $m_1^2 \gg m_2^2 \gg m_3^2$

$$(r-\bar{r})_2 \simeq \text{Im} \ (U_{12}^* \ U_{23}^* \ U_{22} \ U_{13}) \tag{11}$$

where the subscript 2 indicates that the colored Higgs boson A_2 provides the dominant contribution to $(r-\bar{r})$. Clearly since θ is the only phase that will appear in the unitary matrix U,

$$(r-\bar{r}) \propto \sin\theta \tag{12}$$

as can be verified explicitly using Eq.(10). Domains with matter and antimatter excesses corresponding to the different signs of θ will be formed as the universe goes through the phase transition. This is the same type of matter-antimatter domain structure previously emphasized [3,4].

In order to provide our new solution to this domain wall problem, following Ref.11, we add to our model a $\underline{5} + \underline{\bar{5}}$ of heavy fermions[2], with the Yukawa coupling to the $\underline{24}$ plet of Higgs in the form:

$$L_{Y'} = -G_Y \ \bar{\Psi}_i \ \Phi_{ij} \ \Psi_j + h.c. \tag{13}$$

The Lagrangian clearly has the discrete symmetry $\Phi \to -\Phi$, $\Psi_i \to \gamma_5 \ \Psi_i$. The one loop correction to the potential is given by[3]

[2]E.g., these can appear naturally in higher rank GUTs incorporating fermion generations. The asymptotic freedom is unspoiled [17] for a certain range of the Yukawa coupling constant 'G_y'.

[3] $A = (3/64\pi^2) \ (25/8)^2 g^4 - 1/64\pi^2 (105/8) G_y^4$, keeping only gauge boson and fermion contributions. The scalar contribution is of order g^8 (V.A. Kuzmin, M.E. Shaposhnikov and I.I. Tkachev, Proc. Intl. Seminar on Quantum Gravity, Moscow, 1981).

$$V(\phi) = A \phi^4 \left[\ln(\frac{\phi^2}{\sigma^2}) - 1/2 \right] \tag{14}$$

where σ is a minimum of the potential and

$$\phi^2 = 2/15 \; Tr(\Phi\Phi^+) \tag{15},$$

This one loop correction term retains the symmetry $\Phi \rightarrow -\Phi$
and CP symmetry $\Phi \rightarrow \eta \; \Phi^*$. The temperature correction terms

$$V(\phi,T) = 5/8 \; g^2 \; T^2 \; Tr(\Phi\Phi^+) + \sigma_\phi T^2 Tr(\Phi\Phi^+) + q_H T^2 \; \Sigma \; (H_\xi^+ H_\xi) \tag{16}$$

and the higher order terms obey these symmetries as well. The second and
third temperature correction terms are $O(g^4)$ and can be neglected (see ref. to
Kuzmin, et al., footnote #3).

Let us now recall that we have employed a Coleman-Weinberg type of
potential where there is no characteristic mass term. If a large negative
mass term were present in the potential given in Eq. (4), the phase transition
would take place when the mass term and the term in Eq.(15) are equal. In
such a case where the perturbative potential respects the CP symmetry, the
domains will inevitably appear. However, in a Coleman-Weinberg type of Higgs
potential, the transition proceeds very slowly due to the flatness[#4] of the
potential. Most of the universe is trapped in the symmetric SU(5) phase and
supercooling results. As the universe cools to the temperature $\sim 10^7$ GeV, the

#4 : recent calculation by Sher [17] bears out the evidence that the presence
of these heavy fermions help create such a potential without any fine tuning.

coupling constant grows stronger and stronger[#5] and finally we enter the region where the non-perturbative effects come into play.

Following Ref. [11] we assume that, in this strong coupling region, SU(5) instanton effects give rise to SU(5) singlet condensates of the form $<\bar{\psi}_i \psi_i>$.[#6] It is evident that once such condensates are formed, quadratic and cubic terms like

$$G_Y^2 \, m(T)^2 e^{2i\beta} \, Tr\phi^2 \propto \cos 2(\theta+\beta) \tag{17}$$

and

$$G_Y^3 m(T)e^{3i\beta} \, Tr\phi^3 + n.c. \propto \cos 3(\theta+\beta) \tag{18}$$

will be induced[#7]. (See Fig. 2 below). Here β denotes the non-absorbable phase of the SU(5) singlet $<\bar{\psi}_i \psi_i>$ condensate. The phase angle is calculable β for the strongly interacting SU(5) phase and is dependent on the fermion masses as in the QCD case [19]. Since β is non-zero, the CP degeneracy will be lifted in the presence of such an induced term in the Lagrangian. It can be shown that in the absence of any extra U(1) symmetry, as is the case here, the phase β of the condensates $<\bar{\psi}_i \psi_i>$ of these heavy fermions cannot be rotated away.

The fact that the induced terms given in Eq.(17) and Eq.(18) create an energy difference between the two degenerate vacua is not hard to explain

[#5] The SU(5) running gauge coupling constant has the temperature dependence $g^2(\cdots)/4\pi \sim 2\pi/b \, \ell n \, (T/\Lambda)$ where b=12, $\Lambda =10^{5-6}$ GeV, for $g^2(M_X)/4\pi = 1/42$.

[#6]One can obtain similar condensation effects at a higher temperature with the assumption that there are other fermions belonging to higher representations, e.g. for a 10 of SU(5), $\Lambda \sim 10^{12}$ GeV.

[#7]The induced mass term has an approximate temperature dependence of the form $m(T) \propto T \, C_5 \, [2\pi/\alpha(T)]^{10} \, exp[-2\pi/\alpha(T)]$,where $C_5 \cong 7.5 \times 10^{-4}$ [18] at T=0. The correction term from scalar fields is $<10^{-2}$ at a transition temperature of $\sim 10^6$ GeV and thus does not significantly affect our conclusion.

physically. The phase β indicates the alignment of the vacuum, namely in the direction of the condensates $\langle \bar{\psi}_i \psi_i \rangle$. As the universe supercools, the direction of the CP symmetry, which is initially different in different domains, is influenced by the condensates to become effectively aligned in their direction. This is very much like the alignment of ferromagnetic domains in the presence of the external magnetic field. The same cubic term which dynamically breaks the degeneracy due to the discrete symmetry $\phi \rightarrow -\phi$ [11], also removes the vacuum degeneracy owing to the initial CP symmetry in our case where the CP symmetry is broken spontaneously.

The fact that the domain walls never dominate the evolution of the universe implies that [11]

$$T \, C_5 \, [2\pi\alpha(T)]^{10}\alpha^2(T) \, [\exp(-2\pi/\alpha(T)) \, > \, M_X^3/M_P^2, \qquad (19)$$

where $M_P \simeq 1.2 \times 10^{19}$ GeV is the Planck mass. This condition follows from eq.(18) and is satisfied for $T_1 \sim O(1) \, \Lambda$ with $\Lambda \sim 10^{5-6}$ GeV.

The Higgs-boson decay into fermions can produce net zero baryon number. This can be achieved either through the decay of the heavier 5 of Higgs boson into a Higgs boson of the lighter 5-plet [20], or through the direct decay of a Higgs scalar of the 24-plet into Higgs boson of the 5-plet [21]. In order to insure that enough baryons are produced after the phase transition, the reheated temperature of the universe must be at least $\simeq 10^{11-12}$ GeV. A reasonable baryon number density can be produced if certain constraints are satisfied (See Ref. [20,21] for details). If the universe passes through intermediate phases SU(5) $\rightarrow$ SU(4) x U(1) $\rightarrow$ SU(3) x SU(2) x U(1) or SU(5) $\rightarrow$ SU(4) $\rightarrow$ SU(3) x SU(2) x U(1), then the net baryon number could be generated while the universe is in an intermediate phase such as the SU(4) x

U(1) or SU(4) phase. In the SU(4) phase, where SU(2) flavor symmetry is broken, the superheavy fermions [20] could decay to produce sufficient baryon asymmetry.

In summary, we have shown how the vacuum degeneracy resulting from spontaneous CP violation can be broken dynamically by a condensate of heavy fermion pairs.[8] This can occur before the universe goes through a supercooled first order phase transition. We have also demonstrated that such a scenario solves the domain wall problem by creating an energy difference between the two CP degenerate vacua, driving the phase transition to a true vacuum state of unique CP. This transition occurs at $T \ll M_{GUT}$. Such a phase transition will therefore result in monopole suppression. Our scenario also allows a sufficient baryon asymmetry to be produced in the early universe. However, in our case, this asymmetry is local rather than universal owing to the initial CP domain structure which can persist through th supercooling phase. The mechanism suggested by Sato [5] can then act to produce fossil "domains" of baryon and antibaryon asymmetry of survivable size at reheating, after the inflation of the CP domains which occurs during the supercooling phase. The elimination of the CP domain wall problem suggested here thus allows for the possibility of a viable baryon-symmetric domain cosmology.[9]

[8]The scalar self couplings ~ $O(g^4)$ are too weak to allow a scalar-scalar condensate at $T \gtrsim 10^6$ GeV.

[9]This model can be easily extended to $SO(10)$. E.g., we can introduce complex antisymmetric scalar fields which couple to heavy fermions in 16 and $\overline{16}$ representations. Thus, cubic terms can be generated with complex phases such that CP symmetry can again be dynamically broken (in preparation).

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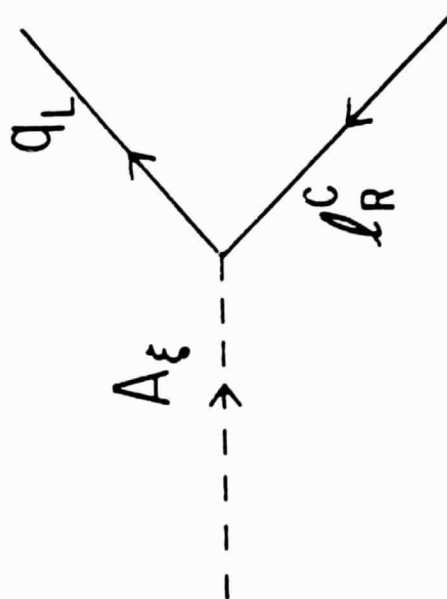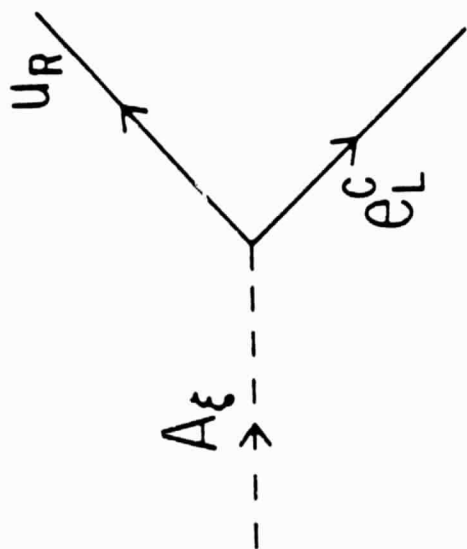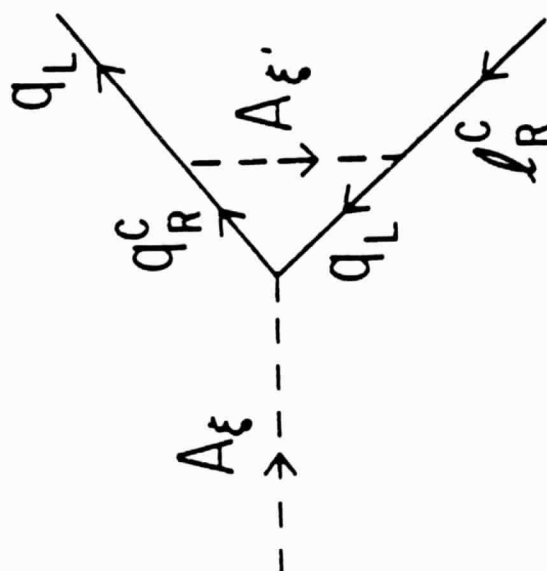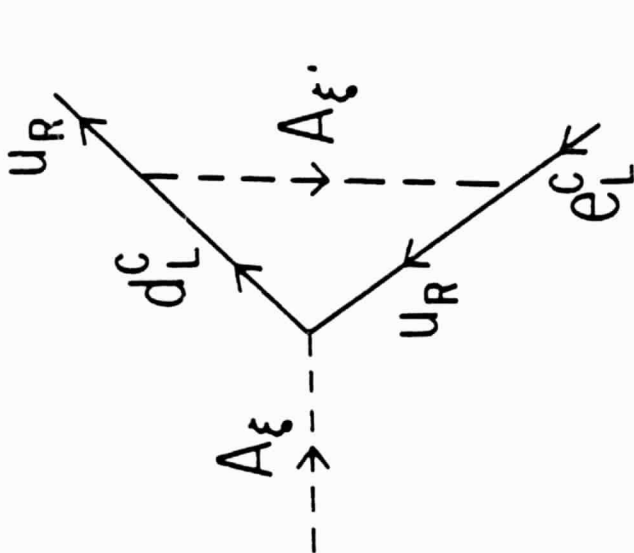[14] S. Colman and E. J. Weinberg, Phys. Rev. D7 (1973) 1888.

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FIGURE CAPTIONS

1.   Dominant diagrams at the one loop level contributing to the production
     of baryon asymmetry in the early universe. A's denote the superheavy
     Higgs scalars (the indices ξ and ξ' count different sets of such
     multiplets needed in this model).

2(a). The diagram contributing to the dynamically induced term
       $\propto G_Y^3 \, m(T) \, Tr\phi^3$ which breaks the CP as well as $\phi \rightarrow -\phi$ degeneracy.

 (b). The diagram contributing to the dynamically induced mass
       term $\propto G_y^2 m^2(T) Tr\phi^2$.