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Effects of Unsteady Free Stream Velocity and Free Stream Turbulence on Stagnation Point Heat Transfer

Rama S. R. Gorla

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Rama S. R. Gorla
Cleveland State University
Cleveland, Ohio

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TABLE OF CONTENTS

EFFECTS OF UNSTEADY FREE STREAM VELOCITY AND FREE STREAM TURBULENCE ON STAGNATION POINT HEAT TRANSFER

SUMMARY

An analysis is presented to study the combined effects of transient free stream velocity and free stream turbulence on heat transfer at a stagnation point over a cylinder situated in a crossflow. An eddy diffusivity model has been formulated and the governing momentum and energy equations are integrated by means of the steepest descent method. The numerical results for the wall shear stress and heat transfer rate are correlated by a turbulence parameter. It has been found that the wall friction and heat transfer rate increase with increasing free stream turbulence intensity.

NOMENCLATURE

C_f = skin friction

D = diameter of cylinder

f = frequency of disturbance

$\hat{f}$ = dimensionless frequency of oscillations ($fD/2U_\infty$)

k = thermal conductivity

K = constant ($3.631\ U_\infty/D$)

Nu = Nusselt number

Pr = Prandtl number

Pr_t = turbulent Prandtl number

q = heat flux rate

Re = Reynolds number ($U_\infty D/v$)

T = temperature

Tu = turbulence intensity

u,v = velocity components

U_∞ = reference velocity

x = coordinate in streamwise direction

y = coordinate normal to the surface

ρ = density

α = thermal diffusivity

η = dimensionless distance

τ = dimensionless time

θ = dimensionless temperature

σ_w = wall shear stress

Ω = dimensionless frequency ($\pi fD/2U_\infty$)

Subscripts

s = steady state

w = wall conditions

∞ = ambient conditions

INTRODUCTION

Turbomachinery blades provide some of the most challenging problems of fluid mechanics. The presence of secondary effects in the boundary layers over rotating blades results in the flow chracteristics that are different from the classical two-dimensional boundary layer theory. The extent of three dimensionality depends on the angular velocity, the flow coefficient, space chord ratio, aspect ratio, stagger angle of the blade, etc. Turbine rotor blades rotate at speeds up to 15,000 rpm in the wakes of several stationary vanes. The precise role of the free stream velocity fluctuations superimposed on the complex flows that occur in turbomachinery blades is not yet completely understood.

There exists a need for a systematic and detailed theoretical and experimental study of the effects of the upstream flow disturbances on the flow and heat transfer characteristics of a gas turbine blade. The flow disturbances at the exit of the stationary vanes are pulsatile in character and so the velocity and temperature distribution as well as the local skin friction and heat transfer coefficients become transient. In multistage turbomachinery, the wake of a blade following the first row of blades always evolves under the influence of the turbulence present in the wake of the previous row of blades.

Many modern turbine blade sections, even at the high Reynolds numbers at which they operate, manifest considerable areas over which the boundary layer remains laminar. The superimposition of mainstream velocity flucturations associated with artificially induced mainstream turbulence is known to affect dramatically the laminar fluid flow and heat transfer characteristics. The life of gas turbine blading depends to a large extent on the local heat transfer rates to the blade surface from the free stream fluid. A study of the combined effects of the mainstream turbulence and the unsteady mainstream velocity on the flow and heat transfer from a gas turbine blade is therefore of considerable practical importance.

Sutera et al. [1,2] simulated the free stream turbulence by a distributed vorticity in the free stream. Their model assumed a vorticity component in the oncoming flow to be unidirectional and oriented so that the vortex lines were susceptible for stretching. They found that amplification by stretching of vorticity of sufficiently large scale can occur. As a result, such vorticity, with small intensity in a flow can appear near the boundary layer with an amplified intensity. They also found that the thermal boundary layer is apparently much more sensitive to the induced effects than is the velocity boundary layer. Further elucidation of the mechanism by which the presence of the vortices affects the heat transfer has been provided by Kestin and Wood [3,4]. Smith and Kuethe [5] postulated an expression for the eddy diffusivity induced by the free stream turbulence. Galloway [6] proposed a model on the assumption that the heat transfer in the presence of free stream turbulence is enhanced by Goertler vortices. Traci and Wilcox [7] presented a two-equation model of turbulence. Early theoretical investigators [8,9] predicted the effect of the free stream oscillations on the mean boundary layer heat transfer or shear stress. These studies resulted in a good qualitative understanding of the flow mechanisms involved, but they do not provide the capability for quantitative predictions.

Junkhan and Serovy [10] reported experimental data concerning the effects of free stream turbulence intensity on the flow and heat transfer from an isothermal flat plate with a favorable pressure gradient. Their data indicates that there is no effect of free stream turbulence intensity on heat transfer through a laminar boundary layer with zero pressure gradient.

There exist several practical applications in which wakes evolve in the presence of free stream turbulence. One such situation exists in single or multistage turbomachinery, in which the wake of a blade following the first row of blades always evolves under the influence of the turbulence present in the wake of the previous row of blades. An extensive search of literature on general two- and three-dimensional wakes is reported by Raj [11]. Pal and Raj [12] have investigated the effect of free stream turbulence on the characteristics of a turbulent wake developed from the trailing edge of a thin and smooth flat plate both analytically and experimentally. They found that the free stream turbulence increases the wake recovery and growth rates.

The present study was undertaken in order to investigate the combined effects of free stream turbulence and time dependent free stream velocity on the unsteady flow and heat transfer characteristics of a stagnation point on a circular cylinder. This basic study should enable understanding of one important aspect of the complex flow and heat transfer phenomena in the real turbomachinery environment to a first degree of approximation.

Gorla, Jankowski and Textor [13] investigated the time-mean characteristics of the laminar boundary layer near an axisymmetric stagnation point when the velocity of the oncoming flow relative to the body oscillates. They presented different solutions for the small and high values of the reduced frequency parameter. Gorla [14] proposed a model for the eddy diffusivity induced by free stream turbulence intensity and integral length scale. The eddy diffusivity model was applied to the stagnation point of a ciruclar cylinder situated in a uniform crossflow in the presence of free stream turbulence. A numerical solution of the governing momentum and energy equations yielded results for the skin friction coefficient and the Nusselt number. The effects of free stream turbulence intensity and integral length scale on the local steady state heat transfer rate from a circular cylinder at various angles around its periphery measured from the stagnation point were studied by Gorla and Nemeth [15]. Correlations were provided by Gorla [16] for the experimental data reported by Base et al. [17] concerning the heat transfer at the stagnation point under the combined influence of turbulence intensity and free stream velocity oscillations. A detailed analysis was made of the combined effects of transient free stream velocity and turbulence intensity on the unsteady stagnation point flow characteristics by Gorla [18] while these effects on unsteady stagnation point heat transfer are reported recently by Gorla [19].

Computer time for this project was provided by the Computer Center of the Cleveland State University. A detailed literature review was performed by Mr. Robert Platteter.

GOVERNING EQUATIONS

The flow development and the coordinate system are shown in Figure 1. Assuming an incompressible flow with constant properties and negligible dissipation, the governing equations, within the framework of the boundary layer approximation may be written as

$$\text{Mass:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\text{Momentum:} \quad \frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{\partial U_e}{\partial t} + U_e\,\frac{\partial U_e}{\partial x} + \frac{1}{\rho}\,\frac{\partial \sigma}{\partial y} \tag{2}$$

$$\text{Energy:} \quad \frac{\partial T}{\partial t} + u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = -\,\frac{1}{\rho C_p}\,\frac{\partial q}{\partial y} \tag{3}$$

In the above equations, x and y represent the distances along the streamwise direction and normal direction; u and v the velocity components in x and y directions; $U_e(x,t)$ the external velocity; t the time; ρ the density of the fluid; T the temperature, C_p the specific heat of the fluid; σ the shear stress and q the heat flux.

The boundary conditions for the wall and for the outer edge of the boundary layer are:

$$\begin{aligned} y = 0&: \ u = v = 0 \text{ and } T = T_w \\ y \rightarrow \infty&: \ u = U_e(x,t) \text{ and } T \rightarrow T_\infty \end{aligned} \tag{4}$$

To complete the formulation of the problem, initial conditions must be specified in the (t,y) plane for $x = 0$ and in the (x,y) plane for $t = 0$. Consideration will be given here for a flow in which at time $t = 0$, the flow field is given by steady-state conditions. At time $t \to 0$, the external velocity $U_e(x,t)$ begins to deviate from the steady state velocity $U_0(x)$

$$U_e(x,t) = U_0(x) \cdot G(t) \tag{5}$$

In the above equation, $G(t)$ represents unsteadiness of the free stream. It is assumed that $G(t) = 1$ for $t < 0$.

The shear stress and the heat flux in equations (2) and (3) may be written as

$$\sigma = \rho(\nu + \varepsilon_m) \frac{\partial u}{\partial y} \tag{6}$$

$$q = - \rho C_p(\alpha + \frac{\varepsilon_m}{Pr_t}) \cdot \frac{\partial T}{\partial y} \tag{7}$$

where ε_m = momentum eddy diffusivity

Pr_t = turbulent Prandtl number

It has been assumed that the conventionally used value of the turbulent Prandtl number $Pr_t = 0.9$ is applicable to the present problem.

Proceeding with the analysis, we define a stream function ψ such that $u = \frac{\partial \psi}{\partial y}$ and $v = - \frac{\partial \psi}{\partial x}$. It may be verified that the continuity equation is automatically satisfied. We further define

$$\eta = y\sqrt{KG/\nu}$$

$$\tau = Kt$$

$$\psi = x\sqrt{KG\nu}\ f(\tau,\eta)$$

$$\theta = \frac{T-T}{T_w-T_\infty} \tag{8}$$

Substitution of expressions in (8) into the momentum and energy equations yields the following nondimensional equations for the velocity and temperature fields:

$$(1+s)f''' + (s'+f-\frac{\dot{G}}{G^2})f'' = \{\frac{\dot{G}}{G^2}(f'-1)+\frac{\dot{f}'}{G}+(f')^2-1\} \tag{9}$$

$$\frac{1}{Pr}[(1+\frac{Pr\cdot S}{Pr_t})\ \theta']' = (\frac{\eta\dot{G}}{2G^2}-f)\theta' + \frac{\dot{\theta}}{G} \tag{10}$$

where

$$S = (\varepsilon_m/\nu).$$

The transformed boundary conditions are

$$\begin{aligned} &f'(\tau,0) = f(\tau,0) = 0,\ f'(\tau,\infty) = 1 \\ &\theta(\tau,0) = 1,\ \theta(\tau,\infty) = 0 \end{aligned} \tag{11}$$

In the above equations, primes indicate differentiation with respect to η and the dot with respect to τ.

EDDY DIFFUSIVITY FORMULATION

Upon substituting $\frac{\partial}{\partial \tau} = 0$ and $G = 1$ into equations (9) and (10), one obtains governing equations for the velocity and temperature fields under steady state conditions. The momentum and energy equations for the steady-state problem are given by

$$[(1+s)f'']'+ff''+[1-(f')^2] = 0 \tag{12}$$

$$[(\frac{1}{Pr} + \frac{s}{Pr_t})\cdot\theta']'+ f\theta' = 0 \tag{13}$$

The appropriate boundary conditions are given by

$$f(0) = f'(0) = 0 \text{ and } f'(\infty) = 1$$

$$\theta(0) = 1 \text{ and } \theta(\infty) = 0 \tag{14}$$

The steady-state friction factor C_f and Nusselt number N_u may be calculated after solving equations (12) and (13) and may be written as

$$C_f = \frac{\sigma w}{(\frac{1}{2}\rho U_\infty^2)} = 13.838 \frac{x}{D} Re^{-\frac{1}{2}} f''(0) \tag{15}$$

$$Nu = - 1.906 Re^{\frac{1}{2}} \theta'(0) \tag{16}$$

where

$$Re = (\frac{U_\infty D}{\nu})$$

In equations (12-16), it must be noted that $f = f(\eta)$ and $\theta = \theta(\eta)$ only.

Equations (12) and (13) are solved on a computer using the fourth-order, Runge-Kutta numerical procedure. The double precision arithmetic was used in all the computations. The selection of the integration step size depends upon the desired level of accuracy. After some computational trials, a step size of $\eta = 0.001$ was chosen.

In the present paper, we restrict ourselves to the case of homogeneous and isotropic free-stream turbulence. We have assumed the following expression for the eddy diffusivity in the present work.

$$\frac{\varepsilon m}{\nu} = s = s_1(\eta+\eta^2) \qquad (17)$$

where s_1 = constant proportional to the turbulence intensity

and η = dimensionless distance normal to the surface.

A review of the literature suggests that there are several uncertainties in the published experimental data including tunnel blockage, variation of physical properties, effect of turbulence scale, and inaccuracies in the measurement of turbulent intensity. Restricting the discussion to the stagnation point, it has been found convenient to use the single correlation parameter $(Tu \cdot Re^{\frac{1}{2}})$. This parameter has also been suggested by Smith and Kuethe [5] on the basis of a semi-empirical theory.

Figures 2 and 3 show the results for the steady-state friction factor and heat transfer rate, respectively, as functions of the correlation parameter $(Tu \cdot Re^{\frac{1}{2}})$. The constant s_1 in the present eddy diffusivity model has been adjusted so as to have a best fit of the present numerical predictions with experimental data available in the literature. It has been found that the following expression for s_1 makes the predictions of the present analysis to be in good agreement with the published data over the entire range of $(Tu \cdot Re^{\frac{1}{2}})$:

$$s_1 = 0.018\ (Tu \cdot Re^{\frac{1}{2}}) \qquad (18)$$

The results from the present analysis are seen to agree with the experimental data within 15 percent.

SOLUTION

The governing momentum and energy equations (9) and (10) for the unsteady problem are integrated by the steepest descent method.

Considering the momentum equation first, we let

$$f(\tau,\eta) = \sum_{n=2}^{\infty} a_n(\tau) \cdot \frac{\eta^n}{n!} \tag{19}$$

Substituting the expression for $f(\tau,\eta)$ into equation (9) and collecting coefficients of like powers of η, we find that

$$\begin{aligned} a_3 &= -(s_1 a_2 + 1 + \frac{\dot{G}}{G^2}) \\ a_4 &= \frac{\dot{a}_2}{G} + \frac{3}{2}\frac{\dot{G}}{G^2} \cdot a_2 + 2s_1{}^2 a_2 + 2s_1(1 + \frac{\dot{G}}{G^2}) \\ a_5 &= 3s_1 a_4 + a_2 + \frac{2\dot{G}a_3}{G^2} + \frac{\dot{a}_3}{G} \\ a_6 &= -4s_1 a_5 + \frac{5}{2}\frac{\dot{G}}{G^2} a_4 + \frac{a_4}{G} + 2a_2 a_3 \end{aligned} \tag{20}$$

etc.

The unknown coefficient $a_2(\tau)$ is determined through the use of boundary and initial conditions. When once $a_2(\tau)$ is determined, the remaining features of the flow field such as the transient velocity profiles may be readily evaluated. The time-dependent wall shear stress and friction factor may be written as

$$\begin{aligned} \sigma_w &= \mu(\frac{\partial u}{\partial y})_{x,o,t} \\ &= \mu \cdot k \times G(\frac{KG}{\nu})^{\frac{1}{2}} \cdot a_2(\tau) \end{aligned}$$

$$C_f = \frac{\sigma_w}{(\frac{\rho U_\infty{}^2}{2})} = \frac{2}{x}(\frac{\nu}{KG})^{\frac{1}{2}} \cdot a_2(\tau) \tag{21}$$

We now define an integrating factor: $e^{I(\tau,\eta)}$

where $$I = \int_0^\eta \left(\frac{1}{1+s}\right)\cdot\left(s'+f - \frac{\eta}{2}\frac{\dot{G}}{G^2}\right) d\eta \tag{22}$$

Momentum equation then becomes upon integration

$$f'' = e^{-I} \cdot \alpha(\tau,\eta)$$

where $$\alpha = a_2(\tau) + \int_0^\eta \frac{e^I}{1+s} \cdot \left[\frac{\dot{G}}{G^2}\,(f'-1) + \frac{\dot{f}'}{G} + (f')^2 - 1\right] d\eta$$

In order to arrive at the boundary condition at the outer edge of the boundary layer, momentum equation is integrated from $\eta = 0$ to ∞. Thus

$$1 = \int_0^\infty f''\, d\eta = \int_0^\infty e^{-I(\tau,\eta)} \,.\, \alpha(\tau,\eta)\, d\eta \tag{23}$$

To evaluate the integral in equation (23), the series for $f(\tau,\eta)$ is to be inverted. We now define a new variable γ such that

$$\gamma = I(\tau,\eta) + \frac{\dot{G}}{G^2}\left[\frac{\eta^2}{4} - \frac{s_1\eta^3}{6} + \frac{s_1{}^2\eta^5}{10}\right] - s_1\eta - \left(\frac{2s_1 - s_1{}^2}{2}\right) \cdot \eta^2$$
$$= \sum_{n=0}^{\infty} C_n\, \eta^{n+3} \tag{24}$$

where $$C_0 = \frac{a_2}{3!} - \frac{2}{3}\, s_1{}^2$$

$$C_1 = \frac{a_3}{4!} - \frac{s_1 a_2}{8} + \frac{s_1{}^3}{4}$$

$$C_2 = \frac{a_4}{5!} - \frac{s_1 a_3}{30} + \frac{2}{5}\, s_1{}^3 - \frac{2}{5}\, s_1{}^4 + \frac{s_1{}^5}{5}$$

$$C_3 = \frac{1}{6} \cdot \left[- \frac{s_1 a_4}{4!} + \frac{s_1{}^2 a_2}{2!} + (s_1{}^4 - 2s_1{}^3)\left(2s_1 - \frac{\dot{G}}{2G^2}\right)\right]$$

Inversion of series for γ in equation (24) is given by

$$\eta = \sum_{m=0}^{\infty} \frac{A_m}{(m+1)} \cdot \gamma^{(m+1)/3} \tag{25}$$

Upon applying Cauchy's residue theorem, we get

$$A_m = \frac{1}{2\pi i} \int \gamma^{-(m+1)/3}\, d\eta$$

where $i = \sqrt{-1}$. The integrations are carried out three times in the γ plane to dispose of the fractional powers of γ once in the η plane. We now have

$$1 = \int_0^{\infty} f''\, d\eta = \int_0^{\infty} e^{-\gamma} \cdot \psi(\eta,\tau) \cdot \frac{d\eta}{d\gamma}\, d\gamma \tag{26}$$

where $\psi = \sum_{n=0}^{\infty} \frac{Bn}{n!} \cdot \eta^n$

$$B_o = a_2$$

$$B_1 = -\left(\frac{1+\dot{G}}{G^2}\right)\left(1 + s_1 - \frac{\dot{G}}{4G^2}\right) - s_1 a_2$$

$$B_2 = \left(1+s_1 - \frac{\dot{G}}{4G^2}\right)\cdot\left(\frac{\dot{G}}{G^2} a_2 + \frac{\dot{a}^2}{G}\right) + 2s_1\left(1 + \frac{\dot{G}}{G^2}\right)\left(1+s_1 - \frac{\dot{G}}{4G^2}\right)$$
$$+2a_2\left(\frac{\dot{G}}{4G^2} - s_1 + s_1{}^2\right)$$

$$B_3 = 2\left(1+s_1 - \frac{\dot{G}}{4G^2}\right)\left(\frac{\dot{G}a_3}{2G^2} + \frac{\dot{a}_3}{2G} + a_2{}^2\right) - 3s_1\left(1+s_1 - \frac{\dot{G}}{4G^2}\right)\left(\frac{\dot{G}}{G^2} a_2 + \frac{\dot{a}_2}{G}\right)$$
$$-6\left(1 + \frac{\dot{G}}{G^2}\right)\left(1+s_1 - \frac{\dot{G}}{4G^2}\right)\left(\frac{\dot{G}}{4G^2} - s_1 + s_1{}^2\right)$$
$$+6a_2\left(\frac{s_1{}^2}{2} - \frac{7}{24}\frac{s_1\dot{G}}{G^2} - \frac{5}{12} s_1{}^3\right) \tag{27}$$

etc.

From equation (26) we recall that

$$\psi\,(\eta,\tau)\cdot\frac{d\eta}{d\gamma} = \gamma^{-2/3}\cdot\sum_{m=0}^{\infty} dm\;\;\gamma^{m/3}$$

where

$$d_0 = \frac{B_0}{3}\cdot C_0^{\;-1/3}$$

$$d_1 = \frac{1}{3}\cdot C_0^{\;-2/3}\cdot\left(B_1 - \frac{2}{3}B_0\,\frac{C_1}{C_0}\right)$$

$$d_2 = \frac{1}{3}\left[-B_0C_2C_0^{\;-2} + B_0C_1^{\;2}C_0^{\;-3} - B_1C_1C_0^{\;-2} + \frac{B_2}{2}\,C_0^{\;-1}\right]$$

$$d_3 = \frac{1}{3}\left[\frac{4}{3}B_0C_3C_0^{\;-7/3} + \frac{28}{9}B_0C_1C_2C_0^{\;-10/3}\right.$$

$$-\frac{140}{181}C_1^{\;3}B_0C_0^{\;-13/3} - \frac{4}{3}B_1C_2C_0^{\;-7/3}$$

$$\left.+\frac{14}{9}B_1C_1^{\;2}C_0^{\;-10/3} - \frac{2}{3}B_2C_1C_0^{\;-7/3} + \frac{B_3}{6}C_0^{\;-4/3}\right]$$

etc.

Equation (26) will now be integrated in terms of the complete gamma functions to yield

$$\frac{\partial f}{\partial\eta}\,(\tau,\infty) = \sum_{m=0}^{\infty} d_m\cdot\Gamma\!\left(\frac{M+1}{3}\right) = 1 \qquad (28)$$

Upon applying Euler's summation procedure to the above series, we have

$$1 = \frac{15}{16}\cdot\Gamma\!\left(\frac{1}{3}\right)\cdot d_0(\tau) + \frac{11}{16}\cdot\Gamma\!\left(\frac{2}{3}\right)\cdot d_1(\tau)$$

$$+\frac{5}{16}\cdot d_2(\tau) + \frac{1}{16}\,\Gamma\!\left(\frac{4}{3}\right)\cdot d_3(\tau) \qquad (29)$$

After substituting expressions for d_0, d_1, d_2 and d_3 into the above equation, we obtain a first-order ordinary differential equation for $a_2(\tau)$. Equation (29) has been integrated numerically by means of a fourth-order, Runge-Kutta procedure. A discussion of the results will be presented in the next section.

It should be noted that the transient velocity profiles may be determined when once $a_2(\tau)$ is determined. The velocity profiles are given by the equation

$$\frac{\partial f}{\partial \eta} = \sum_{j=0}^{\infty} dj\ \Gamma_{\gamma}\left(\frac{j+1}{3}\right) \tag{30}$$

where Γ_{γ} is the incomplete gamma function. The friction factor is given by equation (21).

In order to integrate the unsteady energy equation (10), we set

$$\theta(\tau,\eta) = \sum_{n=0}^{\infty} \frac{b_n(\tau)\cdot\eta^n}{n!} \tag{31}$$

where

$$b_2 = -Pr\, s_1\, b_1$$

$$b_3 = Pr\, Q_1\, s_1\, b_1 + \frac{Pr\, \dot{b}_1}{G} + \frac{Pr\dot{G}}{2G^2}\cdot b_1 - 2Prs_1 b_1 + Pr^2 s_1{}^2 b_1 \tag{32}$$

etc.

In equation (32), we have

$$Q_1 = \frac{s_1 Pr}{Pr_t} \tag{33}$$

Proceeding as before, the energy equation reduces to the following form:

$$\frac{9GPr_t\ \underset{\sim}{Co}^{4/3}}{Pr\, s_1\, \Gamma(4/3)} + \frac{db_1}{d\tau} = \frac{1}{3}\cdot\frac{9GPr_t\ \underset{\sim}{Co}^{4/3}}{Pr\, s_1\, \Gamma\ (4/3)}\cdot \{A_0\Gamma(\tfrac{1}{3})$$

$$+ A_1\Gamma(\tfrac{2}{3}) + A_2\}\cdot b_1(\tau) \tag{34}$$

where

$$\underset{\sim}{C_0} = \left[\frac{a_2}{6} - \frac{2Q_1S_1}{3Pr_t} + \frac{Q_1\dot{G}}{6G^2}\right]$$

$$A_0 = \left(\frac{1}{\underset{\sim}{C_0}}\right)^{1/3}$$

$$A_1 = -\frac{24\underset{\sim}{C_1}}{36}\left(\frac{1}{\underset{\sim}{C_0}}\right)^{5/3}$$

$$A_2 = -\frac{\underset{\sim}{C_2}}{(\underset{\sim}{C_0})^2} + 6\frac{(\underset{\sim}{C_1})^2}{(\underset{\sim}{C_0})^2}$$

$$\underset{\sim}{C_1} = \frac{a_3}{24} - \frac{Q_1a_2}{8}$$

$$\underset{\sim}{C_2} = \frac{a_4}{120} - \frac{Q_1a_3}{30} + \frac{Q_1^2}{5}\cdot\left(\frac{2s_1}{Pr_t} - \frac{\dot{G}}{2G^2}\right) + \frac{(Q_1^4 - 2Q^3\, s_1}{5Pr_t} \tag{35}$$

The time-dependent wall heat flux and Nusselt number may be written as

$$q_w = -K_f\left(\frac{dT}{dy}\right)_{x,o,t}$$

$$= -K_f(T_w - T_\infty)\,\frac{\sqrt{KG}}{\nu}\,\theta'(0,\tau)$$

$$N_u = -D\sqrt{KG/\nu}\cdot\theta'(0,\tau) \tag{36}$$

when once $b_1(\tau)$ is calculated, the temperature profile as well as the wall heat transfer rate may be deduced. Equation (34) was numerically integrated by means of the fourth-order, Runge-Kutta procedure.

DISCUSSION

It must be noted that the analysis is performed for predicting the transient response behavior at the stagnation point due to the combined effects of an arbitrary time dependent free stream velocity and homogeneous, isotropic free stream turbulence. No assumption has been made for $G(\tau)$ that represents the unsteadiness of the free stream. In fact, the present analysis is valid for any arbitrarily varying continuous $G(\tau)$. There exist in the literature some experimental data and some predictive methods to analyze separately the wall shear factor at a stagnation point under steady-state conditions in the presence of free-stream turbulence and the friction factor due to transient free stream velocity with no free stream turbulence. Neither experimental nor analytical work has been reported to investigate the combined effects of unsteady free stream velocity and free stream turbulence at a stagnation point.

In order to illustrate the application of the present analysis to investigate the title problem and also to assess its accuracy, we have chosen two different cases of time-dependent, free stream velocity.

The first case corresponds to $G(\tau) = (1 + \tau)$ with no free stream turbulence present. Physically, this implies that at time $\tau = 0$ the inviscid flow begins to change with constant positive temporal acceleration. Yang [20] investigated this problem be approximate integral methods. The results for $a_2(\tau)$ from the present analysis are compared with those of Yang [20] in Fig. 4. The quasi-steady solution given by Yang incorrectly predicts a discontinuity in the wall shear at $\tau = 0$. The results from the present analysis start at $a_2(0) = 1.23259$. After reaching a maximum value of a_2 initially as τ increases, a_2 decreases with increasing τ towards the quasi-steady solution. The present analysis is seen to yield physically more realistic results than Yang's integral analysis for $\tau < 1$, during which time the flow is experiencing its greatest unsteadiness. The agreement between the present results and those of Yang is within 2 percent for $\tau > 1.0$.

The second case corresponds to $G(\tau) = 1 + A \sin(\Omega\tau)$. This represents a stagnation point flow in which the free stream oscillates with amplitude A and dimensionless frequency Ω about a mean velocity. Lighthill [8] studied this problem using a perturbation method and integral approach. Lighthill's analysis was valid for $\Omega \to \infty$. Ishigaki [9] studied the same problem retaining second-order terms in the perturbation analysis. The results from the present analysis for the shear phase advance are compared with those of Ishigaki and Lighthill in Fig. 5. The agreement between the present results and those of Ishigaki was seen to be within 5 percent. Also presented in Fig. 5 are the present results for wall hat flux phase lag. Lighthill's asymptotic result as $\Omega \to \infty$ is also in Fig. 5.

Figures 6 and 7 illustrate variation of the shear stress function $a_2(\tau)$ under the combined effect of transient free stream velocity and free stream turbulence. For the sake of illustration, $G(\tau) = 1 + A \sin(\Omega\tau)$ only was chosen. It is seen that any existing transients die out within 1 oscillation of the free stream. After about 1 oscillation, the shear stress function $a_2(\tau)$ assumes a periodic shape. The amplitude of the oscillations is a strong function of the turbulence parameter $(Tu \cdot Re^{\frac{1}{2}})$. It must be noted that $a_2(\tau)$ increases substantially from the steady-state solution for wall shear stress. Physically, $a_2(\tau)$ corresponds to the wall shear stress at given instant of time as given by equation (15). It may be observed that even after all starting transients have died out the wall shear does not fluctuate with equal amplitude about the steady-state value.

Figure 8 illustrates the variation of the normalized wall heat transfer under the combined effect of transient free stream velocity and free stream turbulence. We have chosen $G(\tau) = 1 + \sin(\Omega\tau)$ for this purpose. It is seen that any existing transients die out within two oscillations of the free stream. After about two oscillation's, the wall heat flux assumes a periodic shape. The amplitude of the oscillations was observed to be a strong function of the turbulence parameter $(Tu \cdot Re^{\frac{1}{2}})$. It may be observed that even after all starting transients have died out the wall heat flux does not fluctuate with equal amplitude about the steady-state value.

It must be noted that the dimensionless frequency parameter Ω may be written as $(\pi f R / U_\infty)$ where f is the frequency of disturbance, R the cylinder radius and U_∞ the free stream velocity. The grouping

(fR/U_∞) is the Strouhal number. Ishigaki [21] derived from his theoretical analysis that the parameter $\Omega^{\frac{1}{2}}Tu^2$ is a controlling parameter to describe heat transfer in turbulent oscillating flows. Bayley and Priddy [22] reported heat transfer measurements around a gas turbine blade by controlling the frequency and amplitude of turbulence. The Nusselt number data was correlated by then in terms of a parameter $(fC/U_\infty)^{\frac{1}{2}}$. $Tu^2 \cdot Re$ where C was the chord length of the blade. For the title problem including both unsteady free stream velocity and free stream turbulence, $a_2(\tau)$ could not correlate well with the parameter $(Tu \cdot Re^{\frac{1}{2}})$. It may be recalled that the steady-state friction factor data has been successfully correlated by the parameter $(Tu \cdot Re^{\frac{1}{2}})$ as shown in Fig. 2. Utilizing the analogy between heat and momentum transfer mechanisms, it was hypothesized that $(\Omega^{\frac{1}{2}}Tu^2 Re)$ may be a suitable correlation parameter.

Figure 9 illustrates a comparison between the present predictions and the experimental results of Base et al. [17] for the dimensionless heat transfer rate versus the correlating parameter $\hat{f}^{\frac{1}{2}}Tu^2Re$. Although the present results underestimate the Nusselt number at high values of $\hat{f}^{\frac{1}{2}}Tu^2Re$, the agreement between the two seems to be more encouraging at lower values of the correlating parameter.

CONCLUDING REMARKS

The method of approach used in this paper to the analysis of the combined effects of free stream transient velocity and free stream turbulence at a stagnation point reduced the governing equations to first-order ordinary differential equations. The method is capable of yielding results for any arbitrary time-dependent nature of the free stream velocity. The solutions are not restricted by small perturbation assumptions. A successful formulation has been performed to obtain an expression for the eddy diffusivity induced by the free stream turbulence intensity. The solution of the governing unsteady momentum and energy equations with the proposed eddy diffusivity model yielded predictions for the skin friction coefficient and heat transfer rate. A correlation parameter is suggested to correlate the friction factor and heat transfer results for the title problem. It has been found that the wall friction and heat transfer rate increase with increasing free stream turbulence intensity. In the case of flows involving unsteady free stream velocity, the friction factor and heat transfer rate increase with increasing values of the reduced frequency of oscillations.

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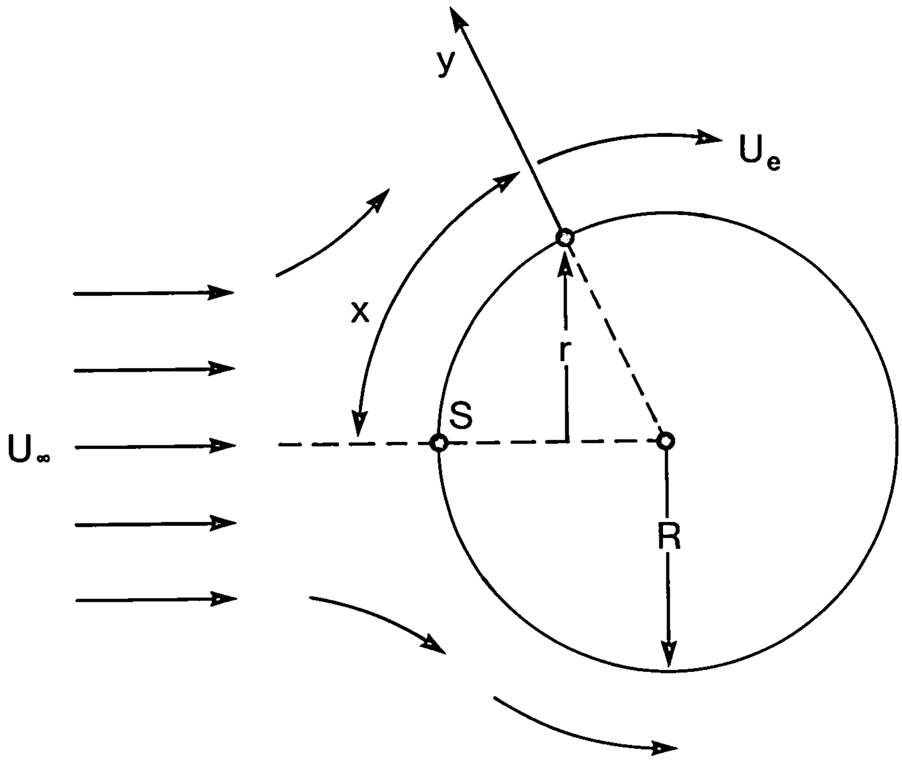

FIGURE 1. Flow Development and Coordinate System.

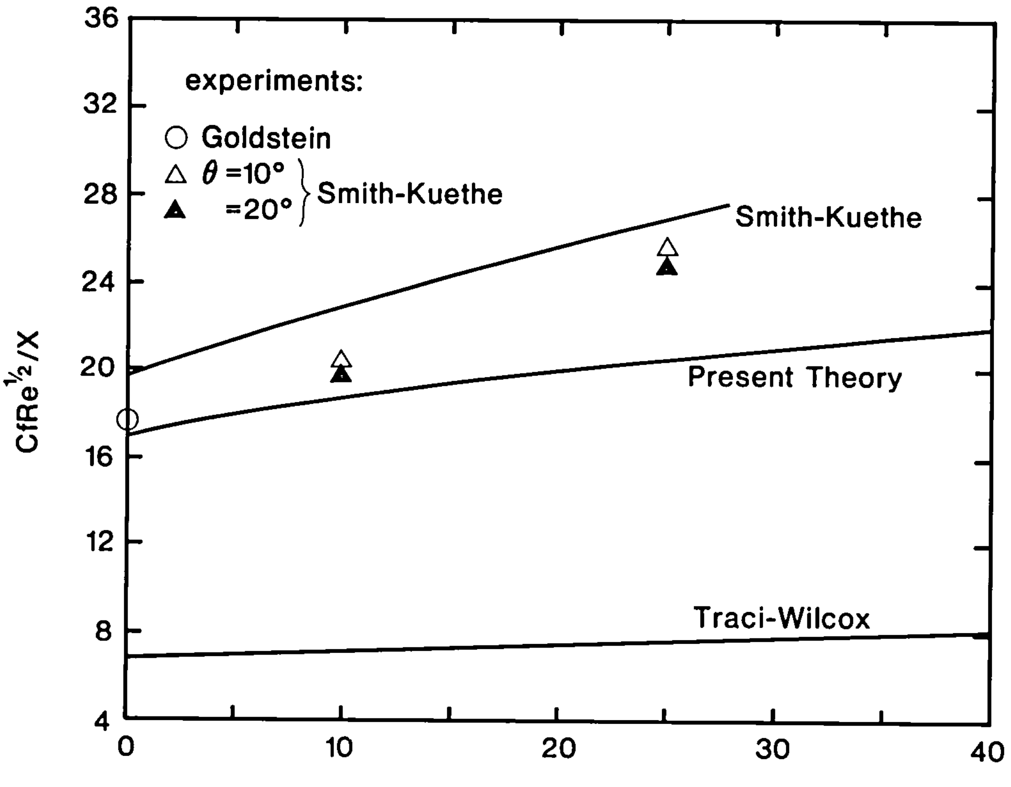

FIGURE 2. Effect of Turbulence Intensity on Friction Factor

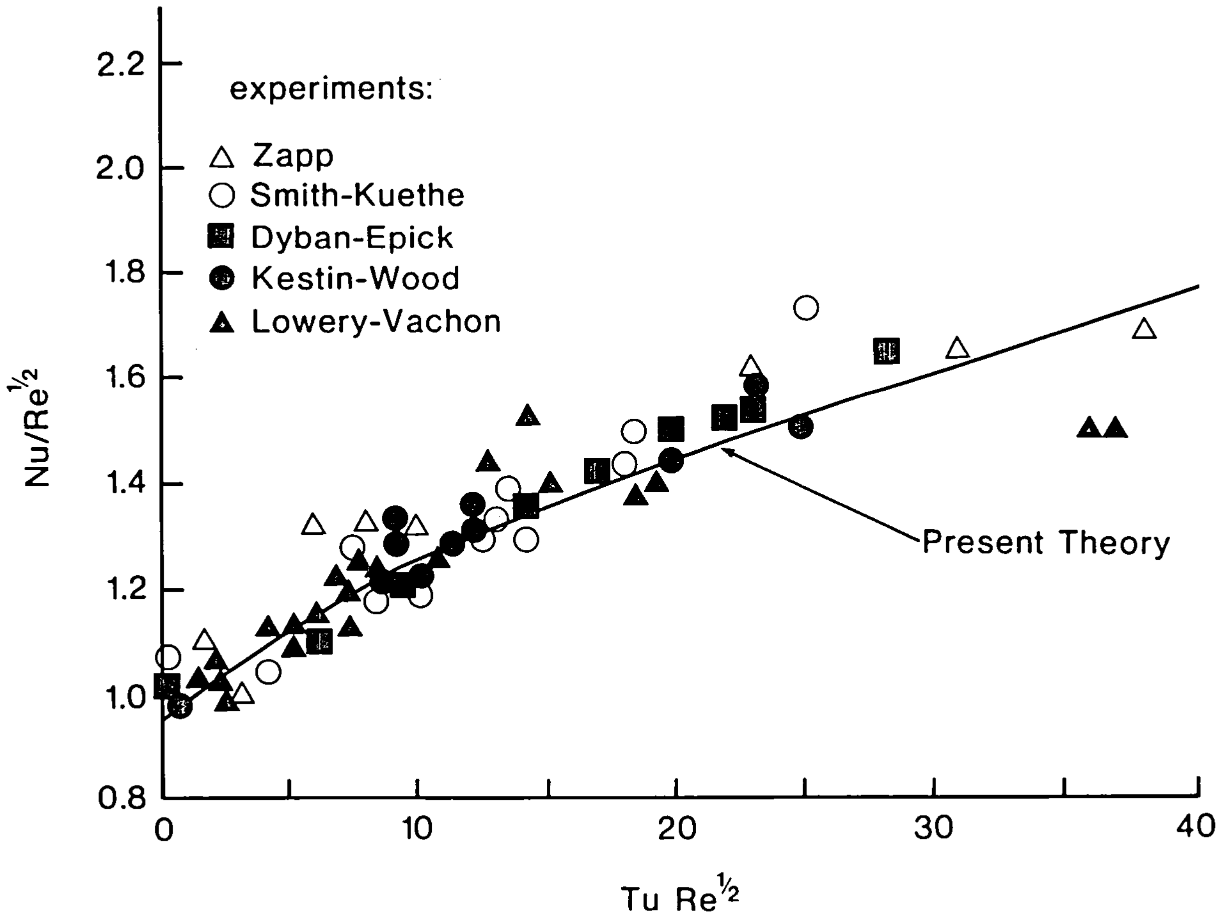

FIGURE 3. Effect of Turbulence Intensity on Heat Transfer Rate

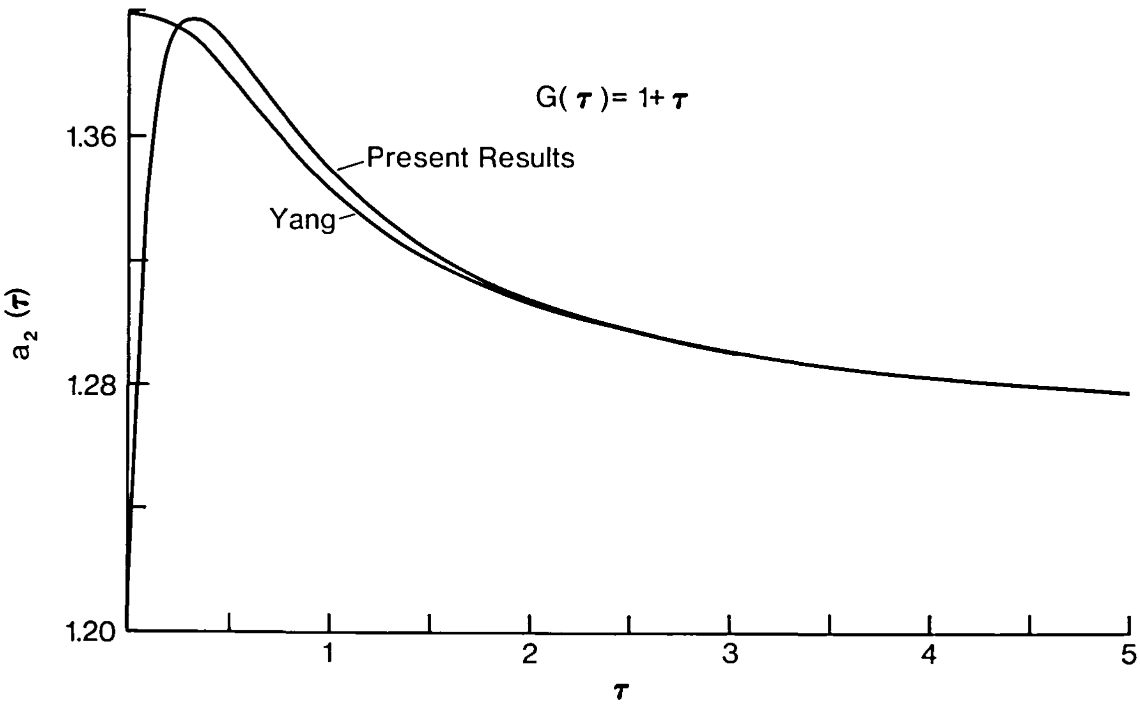

FIGURE 4. Sheer Stress Function Versus τ for $G(\tau) = (1+\tau)$.

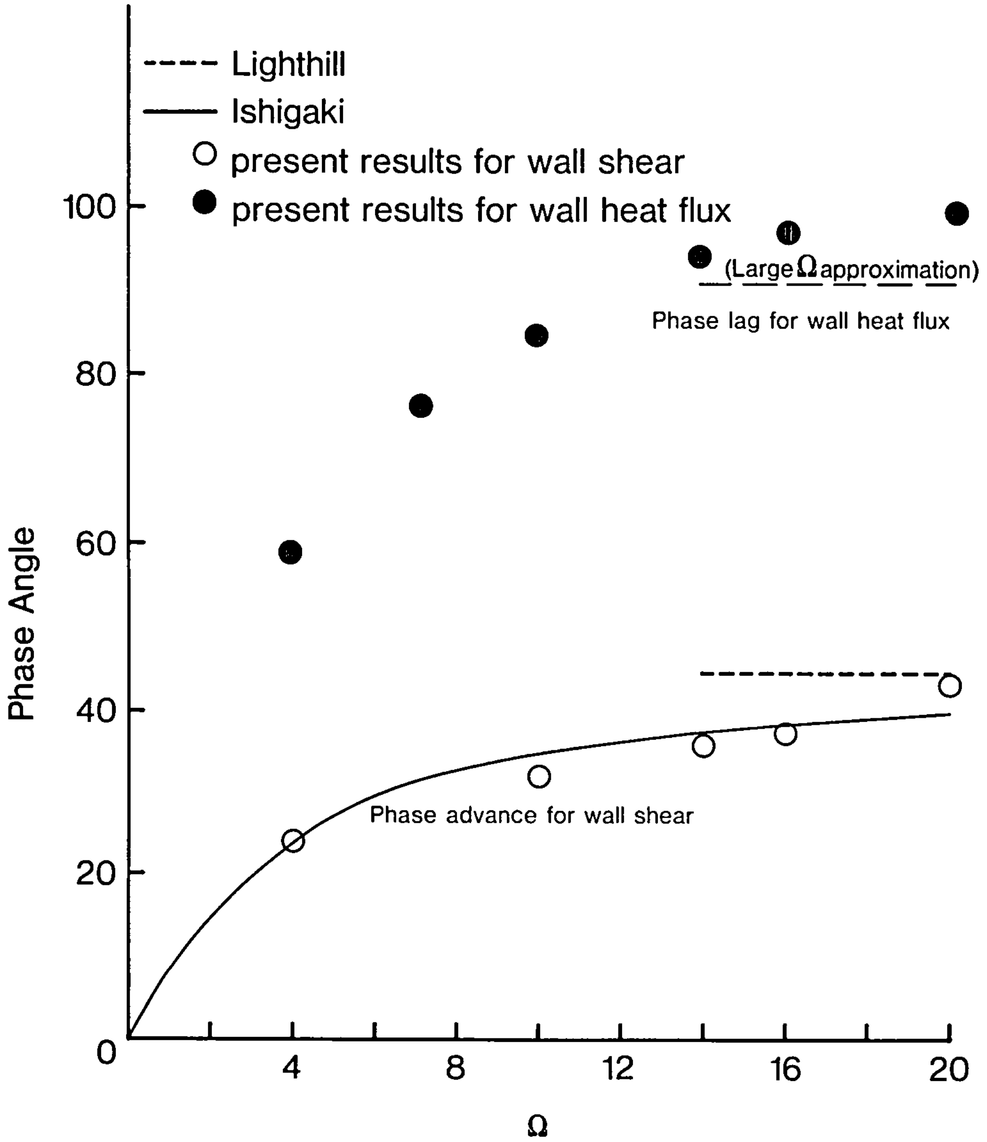

FIGURE 5. Sheer Phase Advance and Wall Heat Flux Phase Lag for $G(\tau) = 1 + A \sin(\Omega\tau)$

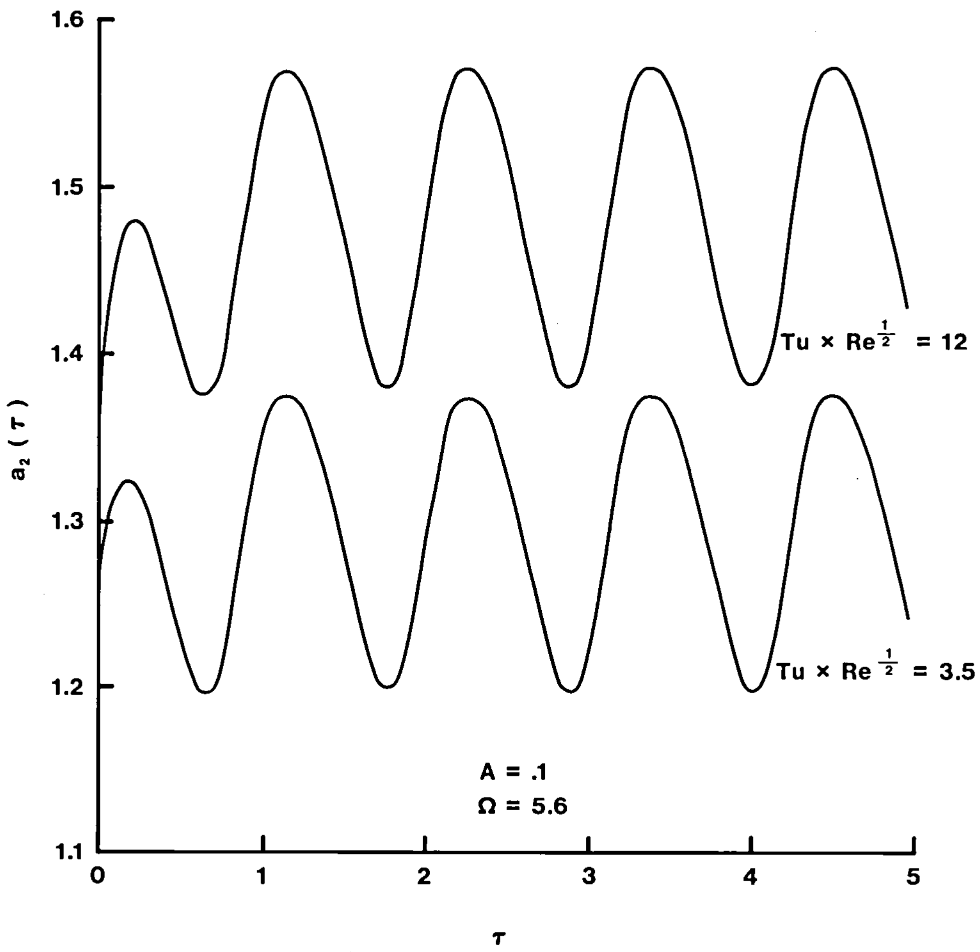

FIGURE 6 Shear Stress Function Versus τ for $G(\tau) = 1 + A \sin(\Omega\tau)$

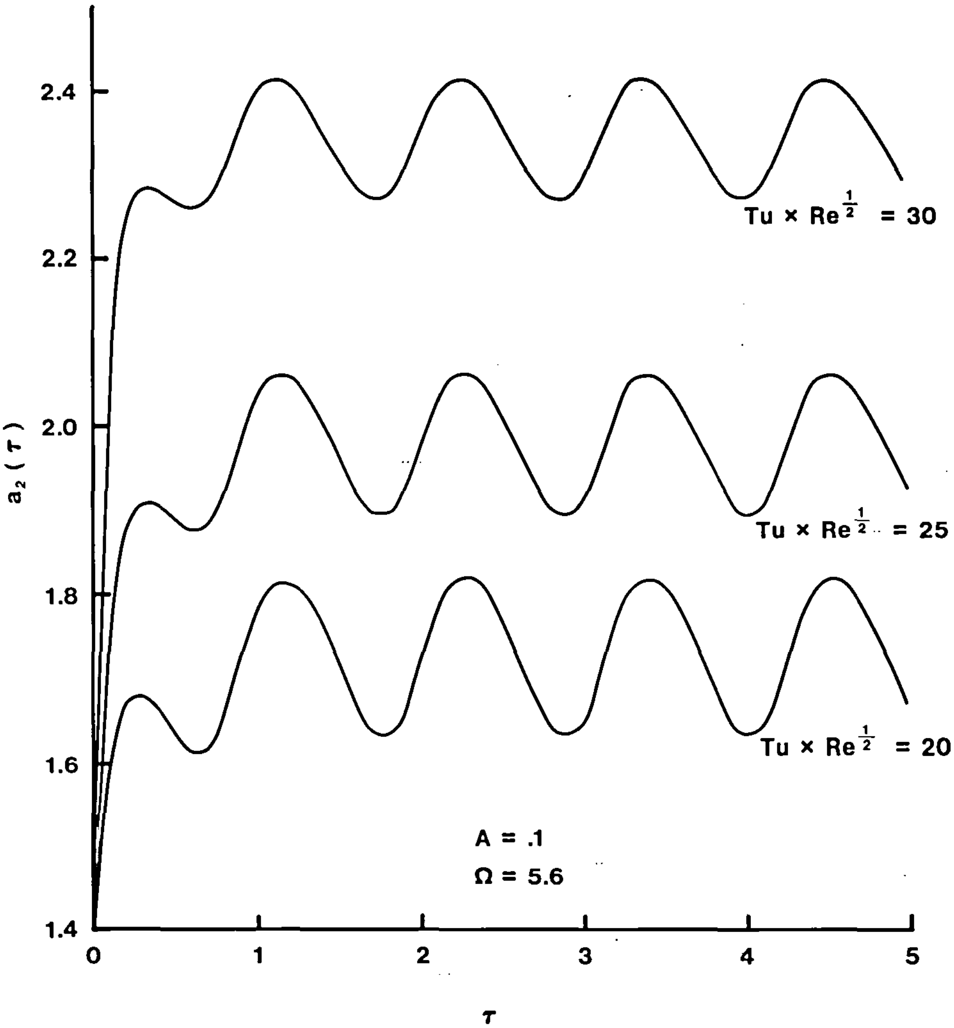

FIGURE 7 Shear Stress Function Versus τ for $G(\tau) = 1 + A \sin(\Omega\tau)$

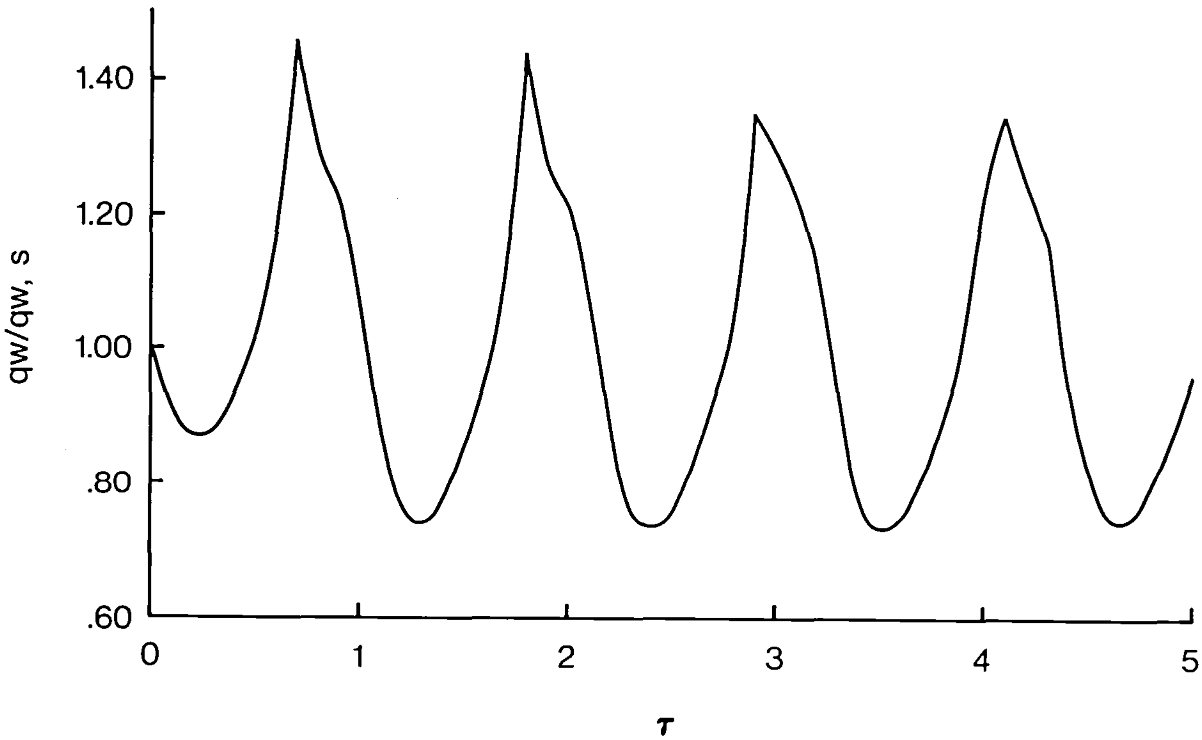

FIGURE 8 Normalized Heat Transfer Versus τ

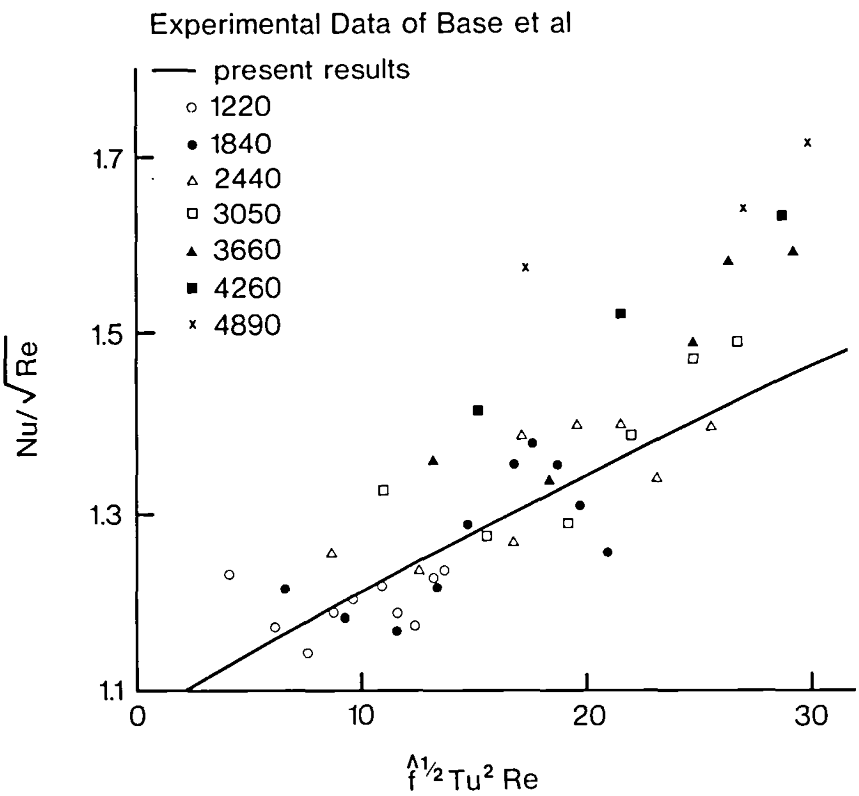

FIGURE 9
Nusselt Number Versus the Turbulence Parameter

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16. Abstract

An analysis is presented to study the combined effects of transient free stream velocity and free stream turbulence on heat transfer at a stagnation point over a cylinder situated in a crossflow. An eddy diffusivity model has been formulated and the governing momentum and energy equations are integrated by means of the steepest descent method. The numerical results for the wall shear stress and heat transfer rate are correlated by a turbulence parameter. It has been found that the wall friction and heat transfer rate increase with increasing free stream turbulence intensity.

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