

NASA Technical Memorandum 83608

Factors That Affect the Fatigue Strength of Power Transmission Shafting and Their Impact on Design

Stuart H. Loewenthal
*Lewis Research Center
Cleveland, Ohio*

Prepared for the
Fourth International Power Transmission and Gearing Conference
sponsored by the American Society of Mechanical Engineers
Cambridge, Massachusetts, October 8-12, 1984

NASA

**FACTORS THAT AFFECT THE FATIGUE STRENGTH OF POWER TRANSMISSION SHAFTING
AND THEIR IMPACT ON DESIGN**

Stuart H. Loewenthal
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

ABSTRACT

A long-standing objective in the design of power transmission shafting is to eliminate excess shaft material without compromising operational reliability. A shaft design method is presented which accounts for variable amplitude loading histories and their influence on limited life designs. The effects of combined bending and torsional loading are considered along with a number of application factors known to influence the fatigue strength of shafting materials. Among the factors examined are surface condition, size, stress concentration, residual stress and corrosion fatigue.

INTRODUCTION

The reliable design of power transmitting shafts is predicated on several major elements. First, the fatigue (stress-life) characteristics of the given shaft in its expected service environment must be established. This can be accomplished from full-scale component fatigue test data or approximated using test specimen data. Some of the influencing factors to be considered are the surface condition of the shaft, the presence of residual stress or points of stress concentration and certain environmental factors such as temperature or a corrosive atmosphere. Secondly, the expected load-time history of the shaft must be obtained or assumed from field service data and then properly simulated analytically. The effects of variable amplitude loading, mean stress and load sequence are potential important factors to include in a description of the loading history. Finally, a reliable mathematical model is needed which rationally considers both the fatigue characteristics of the shaft and its loading history to arrive at the proper shaft diameter for the required service life and reliability. One last step is to check shaft rigidity and critical speed requirements, since these and other nonstrength factors can occasionally dictate an increase in shaft diameter. This is often the case for lightweight, high speed machinery.

While the above considerations have often been addressed in fatigue analysis of structural members [1 to 5], their application to the design of power transmission shafting has only been partially accomplished. Traditional shaft design methods [6 and 7] do consider the effects of combined stress loading, usually through the distortion energy theory of failure, but rarely take into account the effects of variable amplitude loading, mean stresses or limited life design. More recent approaches [8 and 9] adapt traditional methods to computer-aided design procedures but still neglect some of these other important factors.

The principal objective of this present investigation is to develop a more complete approach to shaft design from a strength standpoint. The proposed method will emphasize the influence of these aforementioned operating variables

on shaft diameter and life. Other application factors such as surface condition, stress concentration and size will also be addressed.

NOMENCLATURE

b	slope of the S-N curve on log-log coordinates or fatigue strength exponent (taken as positive value)
d	shaft diameter m (in.)
d_R	relative diameter, defined in Eq. (23)
FS	factor of safety
K_t	theoretical stress concentration factor
k-factor	product of fatigue life modifying factors, defined in Eq. (7)
k_a	surface factor
k_b	size factor
k_c	reliability factor
k_d	temperature factor
k_e	fatigue stress concentration factor
k_f	press-fitted collar factor
k_g	residual stress factor
k_h	corrosion factor
k_j	miscellaneous effects factor
L_R	relative life, defined in Eq. (12)
M	bending moment, N-M (in.-lb)
N_L	total shaft life in cycles
N_f	number of cycles to failure at σ_f
N_i	number of cycles to failure under load i
n	shaft speed, rpm
n_i	number of loading cycles under load i
q	notch sensitivity
T	torque, N-m (in.-lb)

σ	bending stress, N/m^2 (lb/in. ²)
σ_{ef}	effective nominal stress, N/m^2 (lb/in. ²)
σ_f	corrected bending fatigue limit of shaft, N/m^2 (lb/in. ²)
σ_f^*	bending or tensile fatigue limit of polished, unnotched test specimen without mean stress, N/m^2 (lb/in. ²)
σ_{fm}^*	bending or tensile fatigue limit of polished, unnotched test specimen with mean stress, N/m^2 (lb/in. ²)
σ_f^i	true cyclic fracture strength or fatigue strength coefficient, N/m^2 (lb/in. ²)
σ_u	ultimate tensile strength, N/m^2 (lb/in. ²)
σ_y	yield strength, N/m^2 (lb/in. ²)
τ	shear stress, N/m^2 (lb/in. ²)
τ_u	ultimate shear strength, N/m^2 (lb/in. ²)
τ_y	yield shear strength, N/m^2 (lb/in. ²)

Subscripts:

a	alternating=(max-min)/2
m	mean=(max+min)/2
max	maximum
min	minimum
r	fully-reversing

FATIGUE FAILURE

Ductile machine elements subjected to repeated fluctuating stresses above their endurance strength but below their yield strength will eventually fail from fatigue. The insidious nature of fatigue is that it occurs without visual warning at bulk operating stresses below plastic deformation. Shafts sized to avoid fatigue will usually be strong enough to avoid elastic failure, unless severe transient or shock overloads occur.

Failure from fatigue is statistical in nature inasmuch as the fatigue life of a particular specimen cannot be precisely predicted but rather the likelihood of failure based on a large population of specimens. For a group of specimens or parts made to the same specification the key fatigue variables would be the effective operating stress, the number of stress cycles and volume of material under stress. Since the effective stresses are usually the highest at points along the surface where discontinuities occur, such as keyways, splines, and fillets, these are the points from which fatigue cracks are most

likely to emanate. However, each volume of material under stress carries with it a finite probability of failure. The product of these elemental probabilities (the "weakest link" criterion) yields the likelihood of failure for the entire part for a given number of loading cycles.

At present there is no unified, statistical failure theory to predict shafting fatigue. However, reasonable accurate life estimates can be derived from general design equations coupled with bench-type fatigue data and material static properties. Fatigue test data are often obtained on either a rotating-beam tester under the conditions of reversed bending or on an axial fatigue tester. The data generated from these machines are usually plotted in the form of stress-life (S-N) diagrams. On these diagrams the bending stress at which the specimens did not fail after some high number of stress cycles, usually 10^6 to 10^7 cycles for steel, is commonly referred to as the fatigue limit, σ_f . For mild steels it is the stress at which the S-N curve becomes nearly horizontal. This seems to imply that operating stresses below the fatigue limit will lead to "infinite" service life. However, this is misleading since no part can have a 100 percent probability of survival. In fact, fatigue limit values determined from S-N diagrams normally represent the mean value of the failure distribution due to test data scatter. Statistical corrections must be applied for designs requiring high reliabilities as will be discussed. Furthermore, many high strength steels, nonferrous materials and even mild steel in a corrosive environment do not exhibit a distinct fatigue limit [1]. In view of this, it is best to consider that the fatigue limit represents a point of very long life ($>10^6$ cycles).

APPROACH

Traditional shaft analysis generally considers that the nominal loads acting on the shaft are essentially of constant amplitude and that the shaft life is to exceed 10^6 or 10^7 cycles [6]. Sometimes shock or overload factors are applied. However, most shafts in service are generally exposed to a spectrum of service loads. Occasionally, shafts are designed for lives that are less than 10^6 cycles for purposes of economy. Both of these requirements complicate the method of analysis and increase the uncertainty of the prediction. Under these conditions, prototype component fatigue testing under simulated loading becomes even more important.

Short life design. - Local yielding of notches, fillets, and other points of stress concentration are to be expected for shafts designed for short service lives, less than about 1000 cycles. Since fatigue cracks inevitably originate at these discontinuities, the plastic fatigue behavior of the material dictates its service life. Most materials have been observed to either cyclicly harden or soften, depending upon its initial state, when subjected to cyclic plastic strain. Therefore, the cyclic fatigue properties of the material, which can be significantly different than its static or monotonic strength properties, need to be considered in the analysis. For short, low cycle life designs, the plastic notch strain analysis, discussed in detail in [3, 5, and 10] is considered to be the most accurate design approach. This method, used widely in the automotive industry, predicts the time to crack formation based on an experimentally determined relationship between local plastic and elastic strain and the number of reversals to failure.

Intermediate and long life designs. - For intermediate and long life designs both total strain-life and nominal stress-life (S-N curve) methods have been successfully applied [3 and 10]. Although both methods provide

reasonable fatigue life predictions, only the nominal stress-life method will be outlined here.

Obviously, the key to accurate fatigue life prediction is obtaining a good definition of stress-life, S-N, characteristics of the shaft material. Mean bending and/or torsional stress effects should be taken into account if present. Furthermore, a good definition of the loading history is also required. Even when these requirements are met, the accuracy of the prediction is approximate with today's state-of-knowledge. As an example, an extensive cumulative fatigue damage test program was conducted by the SAE to assess the validity of various fatigue life prediction methods [10]. Numerous simple geometry, notched steel plate specimens were fatigue tested in uniaxial tension. Tests were conducted under constant amplitude loading and also under a variable amplitude loading that closely simulated the service loading history. The test specimens' material fatigue properties and the actual force-time history were very well defined. Under these well controlled conditions, predicted mean life from the best available method was within a factor of 3 (1/3 to 3 times) of the true experimental value for about 80 percent of the test specimens while some of the other methods were considerably less accurate [10]. Under less ideal conditions, such as when the loading history and material properties are not as well known or when a multiaxial stress state is imposed, a predictive accuracy within a factor of 10 of the true fatigue life would not be unacceptable with today's state-of-knowledge.

S-N CURVE

In order to determine the proper shaft size for a given number of stress cycles under a variable amplitude loading situation it is necessary to construct an S-N curve for the shaft under the proper mean loading condition. If an experimentally determined S-N curve for the shaft is available then, of course, it is to be used. However, if actual test data is not available, it is still possible to generate a reasonable estimate of the S-N characteristics of the shaft as shown in Fig. 1. In Fig. 1, a straight line connects the fatigue strength coefficient σ_f' at 1 cycle with the shaft's corrected fatigue limit σ_f at 10^6 stress cycles (or 10^7 cycles if applicable) on log-log coordinates [3]. The coefficient σ_f' is the true stress (considering necking) required to cause fracture on the first applied bending stress reversal. It is normally greater than the nominal tensile strength of the material σ_u .

This method assumes that the fracture strength of the shaft is not appreciably affected by the presence of any mean bending or torsional stresses or the presence of a notch. The reason for this is that in a bending or torsional strength test, the outer fibers fracture first. Any initial mean or residual stress or notch effect will be lost to local yielding as the load is applied. This is not the case for an axial strength test, since the whole cross section of the specimen rather than the outer fibers must carry the mean load [3].

Values for σ_f' are not commonly available in the open literature. Table 1 [11 and 12] lists representative values of σ_f' and σ_f along with other strength properties for several steel compositions. For steels not listed in Table 1 with hardnesses less than approximately 500 BHN, reference (5) recommends the following rough approximation:

$$\sigma_f' \approx \sigma_u + 345 \text{ MPa} \quad (1)$$

or

$$\sigma_f \approx \sigma_u + 50\,000 \text{ psi}$$

where σ_u = ultimate tensile strength.

The parameter b appearing in Table 1 is commonly referred to as the fatigue strength exponent [11]. It is the slope of the S-N line on log-log coordinates, taken as a positive value here, where $b = \log(\sigma_f'/\sigma_f)/6$ for a fatigue limit based on $N_f = 10^6$ cycles or

$$b = \log(\sigma_f'/\sigma_f)/7 \text{ for } N_f = 10^7 \text{ cycles} \quad (2)$$

Thus, if σ_f' and σ_f are known or approximated, slope b can be found and an S-N curve can be constructed from the relation:

$$\sigma_{a_i} = \left(\frac{N_f}{N_i}\right)^b \sigma_f \quad (3)$$

where $10^3 \leq N_i \leq N_f$ and where σ_{a_i} is the alternating failure stress corresponding to N_i cycles to failure and σ_f is the fatigue limit strength corresponding to N_f cycles to failure.

As shown in Fig. 2, Eq. (3) together with the simple approximation for σ_f given in Eq. (1) provides a reasonably good correlation with reversed bending fatigue data of different strength steels appearing in [13]. The well known approximation that the fatigue limit σ_f is about half of the tensile strength σ_u seems to hold reasonably well for all the steel test data appearing in Fig. 2, except for that in the 0 to 483 MPa tensile strength range. The reason for this discrepancy is not clear. It does, however illustrate the importance of obtaining actual fatigue life properties rather than relying on simple approximations. Furthermore the high degree of scatter of the test data in Fig. 2 is not uncommon in fatigue testing. The S-N curve represents the mean or average strength characteristics of a population of components. Working stress levels must be reduced to assure higher reliabilities than this 50 percent survival rate as will be discussed next.

FATIGUE LIFE MODIFYING FACTORS

It should be stressed that the fatigue limit σ_f value to be used in constructing the S-N curve in Fig. 1 is that for the shaft to be designed and not that of the test specimen material. The fatigue of the shaft is almost always different from fatigue limit of the highly polished, notch-free fatigue test specimen, listed in material property tables such as in Table 1. A number of service factors that are known to affect fatigue strength have been identified. These factors can be used to modify the uncorrected fatigue limit of the test specimen, σ_f^* , as follows:

$$\sigma_f = k_a k_b k_c k_d k_e k_f k_g k_h k_i \sigma_f^* \quad (4)$$

where

- σ_f corrected bending fatigue limit of shaft
- σ_f^* bending or tensile fatigue limit of polished, unnotched test specimen without mean stress
- k_a surface factor
- k_b size factor
- k_c reliability factor
- k_d temperature factor
- k_e fatigue stress concentration factor
- k_f press-fitted collar factor
- k_g residual stress factor
- k_h corrosion factor
- k_i miscellaneous effects factor

Design data for factors k_a through k_e are relatively available in the open literature [1, 2, and 6] and thus they will be only briefly discussed here. However, factors k_f through k_i , although lesser known and documented, are often quite important to shafting fatigue and therefore will receive greater attention. A more thorough examination of all these factors can be found in [14 and 15].

Surface factor, k_a . - Since the surface of the shaft is the most likely place for fatigue cracks to start, its surface finish and any irregularities such as oxide and scale defects or surface decarburizations can have a major impact on fatigue life. Typical values of k_a range from about 0.9 for turned, ground and polished shafts of low tensile strength (400 MPa) to as low as 0.1 for high strength, forged shafts with significant surface defects [2 and 14].

Size factor, k_b . - Large shafts tend to have lower fatigue strength than small shafts. This is primarily due to the great volume of material under stress and the attendant greater likelihood of encountering a potential fatigue initiating defect in the material's microstructure. Also the metallurgical structure of large parts tends to be coarser and less uniform than small parts. Since the diameter of fatigue specimens tend to be small, typically 8 mm in diameter, a strength reduction factor should be applied for larger shafts. Values of k_b typical range from about 0.9 for 500 mm diameter shafts to approximately 0.65 for shafts 250 mm in diameter [14 and 15].

Reliability factor, k_c . - As previously discussed, published fatigue limit data usually represent an average value of the endurance strength of the sample of test specimens. In the absence of specific test data, the failure distribution of steel specimens is often assumed to follow a Normal or Gaussian distribution with a standard deviation of about 8 percent of the mean. Thus for a 90 percent nominal reliability, k_c is approximately 0.9 and for a 99 percent reliability k_c is approximately 0.8 [14 and 15].

Temperature factor, k_d . - The fatigue limit of carbon and alloy steel is relatively unaffected by operating temperatures between approximately -70° to 300° C. At lower temperatures the bending fatigue strength of steel increases while at temperatures above about 400° C, some steels begin to lose strength [14 and 15].

Fatigue stress concentration factor, k_e . - Experience has shown that shafts almost always fail at a notch, hole, keyway, shoulder or other discontinuity where the effective stresses have been amplified. Fatigue data indicate that low strength steels, due to their ductility, are far less sensitive to the effects of a stress raiser than high strength steels. This is reflected by the notch sensitivity parameter q which is used to modify the theoretical (static) stress concentration factor K_t as follows:

$$k_e = \frac{1}{1+q(K_t - 1)} \quad (5)$$

Reference [16] is an excellent source of design values for both K_t and q .

Press-fitted collar factor, k_f . - A common method of attaching gears, bearings, couplings, pulleys, and wheels to shafts and axles is through the use of an interference fit. The change in section creates a point of stress concentration at the face of the collar. This stress concentration coupled with the fretting action of the collar as the shaft flexes is responsible for many shaft failures in service. A limited amount of fatigue test data have been generated for steel shafts having press-fitted, plain (without grooves or tapers) collars in pure bending. Based on this data from several sources, typical fatigue life reductions range from about 50 to 70 percent [16 and 17]. Therefore, approximate range of press-fitted collar factors:

$$k_f = 0.3 \text{ to } 0.5$$

Larger shafts having diameters greater than about 75 mm (3 in.) tend to have k_f values less than 0.4 when the collars are loaded. Smaller shafts with unloaded collars tend to have k_f values greater than 0.4. The effect of interference pressure over a wide range between collar and shaft has been found to be small, except for very light fits (less than about 28 MPa or 4000 psi) which reduces the penalty to fatigue strength [16]. Surface treatments producing favorable compressive residual stresses and hardening processes such as cold rolling, peening, induction or flame hardening can often fully restore fatigue strength ($k_f = 1$) [1]. Stepping the shaft seat with a generous shoulder fillet radius or providing stress relieving grooves on the bore of the collar can also provide substantial strength improvements.

Residual stress factor, k_q . - The introduction of residual stress through various mechanical or thermal processes can have significant harmful or beneficial effects on fatigue strength. Residual stresses have the same effect on fatigue strength as mean stresses of the same kind and magnitude. Thus residual tensile stresses behave as static tensile loads that reduce strength while residual compressive stresses behave as static compressive stresses which are beneficial to fatigue strength. Table 2 lists many of the most common manufacturing processes and the type of residual stress they are likely to produce. The extent that the residual tensile stresses from these processes reduce or benefit fatigue strength is dependent on several factors including the severity of the loading cycle and the yield strength of the material in question. Since the maximum residual stress (either compressive or tensile) that can be produced in a part can be no greater than the yield strength of the material minus the applied stress, harder, higher strength

materials can benefit more or be harmed more by residual stresses [3 and 18]. This coupled with an increase in notch sensitivity makes it important to stress relieve welded parts made from stronger steels and increases the need to cold work critical areas. For low cycle fatigue applications it usually does not pay to shot peen or cold roll mild steel parts with relatively low yield strengths since much of the beneficial residual compressive stress can be "washed-out" with the first applications of a large stress.

Cold working of parts or the other means listed in Table 2 to instill residual compressive stress is most often applied to minimize or eliminate the damaging effect of a notch, fillet, or other defect producing high stress concentration or residual tensile stresses. This is clearly illustrated in Fig. 3 where shot peening the notched region of the test specimen has almost entirely eliminated the notch effect. Cold working processes not only generate favorable compressive stresses but also work harden the surface of the part leading to increased fatigue strength. Typical design information and data on the effects of cold working and many of the other residual stress can be found in [1, 3, and 18].

Corrosion fatigue factor, k_h . The formation of pits and crevices on the surface of shafts due to corrosion, particularly under stress, can cause a major loss in fatigue strength. Exposed shafts on outdoor and marine equipment as well as those in contact with corrosive chemicals are particularly vulnerable. Corrosion fatigue cracks can even be generated in stainless steel parts where there may be no visible signs of rusting [1]. Furthermore, designs strictly based on the fatigue limit may be inadequate for lives much beyond 10^6 or 10^7 cycles in a corrosive environment. Metals fatigue tested even in a mildly corrosive liquid like fresh water rarely show a distinct fatigue limit [1]. For example, the S-N curve for mild carbon steel tested in a salt water spray shows a very steep downward slope, even beyond 10^8 cycles. Corrosion fatigue strength has also been found to decrease with an increase in the rate of cycling so both the cycling rate and number of stress cycles should be specified when quoting fatigue strengths of metals in a corrosive environment. Reference [1] contains a wealth of information on the corrosive fatigue strength of metals. Typically, the bending fatigue strength of chromium steels at 10^7 cycles range from about 60 to 80 percent of the air tested fatigue limit when tested in a salt water spray [1]. Surface treatments such as galvanizing, sherardizing, zinc or cadmium plating, surface rolling or nitriding can normally restore the fatigue strength of carbon steels tested in fresh water or salt spray to approximately 60 to 90 percent of the normal fatigue limit in air [1].

Miscellaneous effects factor, k_i . - Since fatigue failures nearly always occur at or near the surface of the shaft, where the stresses are the greatest, surface condition strongly influences fatigue life. A number of factors that are often overlooked but are known to affect the fatigue strength of a part are listed below:

- (1) fretting corrosion
- (2) thermal cycle fatigue
- (3) electro-chemical environment
- (4) radiation
- (5) shock or vibration loading
- (6) ultra-high speed cycling
- (7) welding
- (8) surface decarburization

Although only limited quantitative data has been published for these factors [1 and 2], they should, nonetheless, be considered and accounted for if applicable.

S-N prediction. - Figure 3 illustrates the effects that the above fatigue life modifying factors (k-factor) have on the stress-life relation of Eq. (4). A comparison was made with rotating beam fatigue data generated in [19] for smooth, notched, and notched, shot-peened steel specimens having a tensile strength σ_u of 897 MPa.

From the approximation given in Eq. (1), σ_f at 1 cycle was estimated to be 1241 MPa. The fatigue limit of the test specimens σ_f^* at 10^6 cycles was estimated to be $0.5 \sigma_u$ or 449 MPa. In the case of the smooth, polished test specimen all of the k -factor = 1, so the upper line appearing in Fig. 3 can be drawn.

In the case of the notched specimen having the geometry shown in Fig. 3, $K_t = 1.76$ and $q = 0.79$ according to (16). From Eq. (5), the fatigue stress concentration factor, $k_e = 0.63$ and the fatigue limit of the notched specimen = $0.63 (449)$ or 283 MPa as shown in Fig. 3.

It is instructive to note from Fig. 3, that the compressive residual stress and work hardening provided by shot peening virtually eliminated the detrimental notch effect almost entirely. Secondly, the slope of the S-N curve is steeper, that is b is larger, for the notched shaft. Since shaft life is inversely proportional to stress raised to the $1/b$ power, where $1/b \approx 13.6$ for the smooth shaft versus $1/b \approx 9.3$ for the notch shaft, the notched shaft's life, although lower, is less sensitive to stress amplitude changes than that of the smooth shaft. In fact, slope b increases with a decrease in k -factor or a decrease in tensile strength. This is shown in Fig. 4 where b is plotted from the following approximation derived from Eqs. (1), (2), and (4):

$$b = \frac{1}{6} \log \left(\frac{\sigma_u + 345}{0.5 \sigma_u \times k\text{-factor}} \right) \text{ for } \sigma_u \text{ in MPa} \quad (6)$$

where

$$k\text{-factor} = k_a k_b k_c k_d k_e k_f k_g k_h k_i \quad (7)$$

It should be pointed out that the presence of a mean stress, either applied or residual, will cause a change in endurance strength and therefore affect slope b . Mean torsional, bending or tensile stresses will decrease σ_f and thus increase b while compressive stresses will have the opposite effect. The effect of mean stresses will be discussed later.

VARIABLE AMPLITUDE LOADING

The following is a greatly simplified approach to estimate the required shaft diameter for either a limited or unlimited number of stress cycles under a variable amplitude loading history. It assumes that the loading history can be broken into blocks of constant amplitude loading and that the sum of the resulting fatigue damage at each block loading equals one at the time of failure in accordance with Palmgren-Miner linear damage rule. Great care must be exercised in reducing a complex, irregular loading history into a series of constant amplitude events in order to preserve the fidelity of the prediction. Reference [4] discusses the merits of several cycle counting schemes that are commonly used in practice for prediction purposes.

A shortcoming of Miner's rule is that it assumes that damage occurs at a linear rate without regard to the sequence of loading. There is ample experimental evidence that a virgin material will have shorter fatigue life, that is Miner's sums less than one, when first exposed to high cyclic stress before low cyclic stress [1 and 4]. This "overstressing" is thought to create submicroscopic cracks in the material structure that can accelerate the damage rate. On the other hand, test specimens exposed first to stresses just below the fatigue limit are often stronger in fatigue than when new. This "coaxing" or "understraining" effect which can produce Miner's sums much larger than one is believed due to a beneficial strain aging phenomena. While Miner's sums at the time failure can range from 0.25 to 4 depending on loading sequence and magnitude, the experimental range shrinks to approximately 0.6 to 1.6 when the loading is in a more random manner [19]. This is often acceptable for failure estimates. More complicated cumulative damage theories have been devised to account for "sequencing" effects. In fact reference [19] discusses seven different ones, but none of them have been shown to be completely reliable for all practical shaft loading histories. In most cases, Miner's rule serves almost as well and because of its simplicity it is still preferred by many.

Assuming that the shaft is exposed to a series of 3, in this case, alternating bending moments of constant amplitude M_{a_1} acting for n_1 , loading cycles, M_{a_2} for n_2 cycles, and M_{a_3} for n_3 cycles, then according to Miner's rule:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1 \quad (8)$$

where N_1 is the number of cycles to failure at bending moment M_{a_1} , N_2 is the cycles to failure at M_{a_2} , etc.

From the straight line on the log-log S-N plot of Fig. 1, it is clear that

$$\frac{\sigma_{a_1}}{\sigma_f} = \left(\frac{N_f}{N_1} \right)^b \quad \frac{\sigma_{a_2}}{\sigma_f} = \left(\frac{N_f}{N_2} \right)^b \quad \frac{\sigma_{a_3}}{\sigma_f} = \left(\frac{N_f}{N_3} \right)^b \quad (9)$$

where σ_{a_1} is the alternating bending stress at bending moment M_{a_1} , N_f is the number of stress cycles corresponding to the fatigue limit σ_f of the shaft (usually 10^6 - 10^7 cycles) and b is the slope of the S-N curve taken as a positive quantity.

Substituting Eq. (9) back into Eq. (8), noting $\sigma_{a_1} = 32 M_{a_1} / \pi d^3$ (solid circular shaft) and simplifying yields, the following expression for calculating shaft diameter, d :

$$d^3 = \frac{32 (FS)}{\pi \sigma_f} \left[\frac{n_1}{N_f} (M_{a_1})^{1/b} + \frac{n_2}{N_f} (M_{a_2})^{1/b} + \frac{n_3}{N_f} (M_{a_3})^{1/b} \right]^b \quad (10)$$

where the factor of safety term (FS) has been introduced.

Equation (10) can also be rearranged to find the life N_L of a shaft of given diameter for a prescribed operating duty cycle. Multiplying both sides of Eq. (10) by $\left(\frac{1}{N_L}\right)^b$ where $N_L =$ total shaft life in cycles $= n_1 + n_2 + n_3$, etc. and solving for N_L gives:

$$N_L = \frac{\left(\frac{\pi d^3}{32(FS)} \sigma_f\right)^{1/b} N_f}{\frac{n_1}{N_L} (M_{a_1})^{1/b} + \frac{n_2}{N_L} (M_{a_2})^{1/b} + \frac{n_3}{N_L} (M_{a_3})^{1/b}} \quad (11)$$

where the terms $\frac{n_1}{N_L}$, $\frac{n_2}{N_L}$, and $\frac{n_3}{N_L}$ are the fraction of time spent at each bending load M_{a_1} , M_{a_2} , and M_{a_3} , respectively.

Effect of duty cycle. - Equation (11) can be used to illustrate the large detrimental effects that high loads have on shaft life. Consider the case where a shaft is exposed to two blocks of alternating bending moments, where a bending moment of amplitude M_{a_1} acts for n_1/N_L fraction of the time and M_{a_2} acts for the remainder according to the schematic appearing in Fig. 5. Defining relative life L_R , to be shaft life when $M_{a_2} \neq M_{a_1}$ divided by shaft life when $M_{a_2} = M_{a_1}$, then from Eq. (11) L_R is found to be:

$$L_R = \frac{N_L}{N_L} \Big|_{M_{a_2}=M_{a_1}} = \frac{1}{\frac{n_1}{N_L} + \left(1 - \frac{n_1}{N_L}\right) \left(\frac{M_{a_2}}{M_{a_1}}\right)^{1/b}} \quad (12)$$

Plotting Eq. (12) in Fig. 5 it is clear that even a 20 percent overload $\left(\frac{M_{a_2}}{M_{a_1}} = 1.2\right)$ acting only 20 percent of the time ($n_1/N_L = 0.8$) will cause a 30 percent life reduction for $b = 0.16$ or a 64 percent life reduction for $b = 0.08$ relative to a shaft with only constant amplitude loading. In practice, the life reduction would be closer to 30 rather than 64 percent since a b - value of 0.16 is more representative of a machined, mild steel shaft with stress concentration while $b = 0.08$ would be representative of a smooth, notch free (k -factor ≈ 1), high strength shaft. (see Fig. 4). However, in any case, this example points out that the high fluctuating loads acting on a structural element, such as a shaft, tend to dictate its service life.

EFFECT OF MEAN STRESSES

The analysis presented is predicated on the knowledge of the S-N characteristics of the shaft under the anticipated loading conditions. Modifying factors have been identified in Eq. (4) to correct specimen fatigue data for

certain geometric and environmental factors that can affect fatigue strength. The effects of mean stresses will be addressed next.

Since most shafts transmit power and rotate with gear, sprocket or pulley loads, mean torsional stresses are invariably present. Also mean bending stresses can be developed such as those due to rotating unbalance forces. These mean stresses cause a reduction in fatigue strength. Residual stresses, induced deliberately or unattentionally (see Table 2) behave like mean stresses and can either benefit or reduce strength depending on whether they are compressive or tensile [3 and 18].

The effects of mean stresses on long term fatigue strength are sometimes available in the form of experimentally determined constant life diagrams [20]. In these diagrams the amplitude of the fluctuating stress is plotted versus the magnitude of the mean stress at 10^4 , 10^5 , etc. cycles to failure. Sometimes notched specimen data is included. When specific data is unavailable, mean stress effects are often approximated by certain mathematical failure relations, such as Soderberg, Gerber and Modified Goodman failure lines [2, 3, and 6]. When specific test data is available then, obviously, this is preferred. However, the following discussion outlines how mean stresses can be reasonably accounted for by knowing only the fatigue limit, yield and ultimate strengths of the material.

Mean bending stress. - For the case when only bending loads are acting on the shaft, that is zero torque, the loading is considered to be "simple" since only one kind of stress is present. For simple loading several failure relations have been proposed, but the modified Goodman line is, perhaps, the most widely used. It is given by:

$$\frac{\sigma_a}{\sigma_f^*} + \frac{\sigma_m}{\sigma_u} = 1 \quad (13)$$

where σ_a and σ_m are, respectively, the alternating and mean components of the simple bending stress, σ_u is the ultimate tensile strength and σ_f^* is the fatigue limit of the shaft material as determined from specimen fatigue tests with no mean stress present.

Since $\sigma_a = \sigma_f^*$ when $\sigma_m = 0$ according to Eq. (13), σ_a can be interpreted to be the bending fatigue limit strength in the presence of a mean bending stress, say σ_{fm}^* . The asterick is used to denote test specimen rather than shaft fatigue properties. Thus the reduction of fatigue strength with mean strength takes the form:

$$\frac{\sigma_{fm}^*}{\sigma_f^*} = 1 - \frac{\sigma_m}{\sigma_u} \quad (14)$$

In other words, the bending fatigue strength of the material decreases linearly with mean bending stress, becoming zero when the ultimate strength is reached (immediate fracture failure).

Combined stress. - Most power transmitting shafts are not simply loaded, but are subjected to combined stresses. The most common situation is a combination of reversed bending stress (a rotating shaft with constant moment loading) and steady or nearly steady torsional stress. Although a large body of test data has been generated for the simple stress condition, such as pure tensile, flexural or torsional stress, little information has been published for the combined bending and torsional stress condition. However, some cyclic bending and steady torsional fatigue test data for alloy steel analyzed in [21]

shows a reduction in reversed bending fatigue strength with mean torsional stress according to the elliptical relation:

$$\left(\frac{\sigma_r}{\sigma_f^*}\right)^2 + \left(\frac{\tau_m}{\tau_y}\right)^2 = 1 \quad (15)$$

where σ_r is the reversed bending stress, that is, the alternating bending stress component with no mean bending stress present, and where τ_m and τ_y are, respectively, the applied mean shear stress and yield shear strength of the test specimen. Since $\sigma_r = \sigma_f^*$ at $\tau_m = 0$, Eq. (15) represents the reduction of reversed bending fatigue strength with mean torsional stress or, in other words,

$$\frac{\sigma_{fm}^*}{\sigma_f^*} = \sqrt{1 - \left(\frac{\tau_m}{\tau_y}\right)^2} \quad (16)$$

where σ_{fm}^* has been introduced, as before, to represent the fatigue limit in the presence of a mean stress.

Superimposing the effects of both mean bending and torsional stresses on fatigue strength, that is combining Eqs. (14 and 16), results in

$$\sigma_{fm}^* = \sigma_f^* \left(\sqrt{1 - \left(\frac{\tau_m}{\tau_y}\right)^2} - \frac{\sigma_m}{\sigma_u} \right) \quad (17)$$

In the case of solid, circular shafts, the mean stress levels are

$$\sigma_m = \frac{32 M_m}{\pi d^3} \quad \text{and} \quad \tau_m = \frac{16 T_m}{\pi d^3} \quad (18)$$

and since, for steels,

$$\tau_y \approx \left(\frac{\sigma_y}{\sqrt{3}}\right) \quad (19)$$

then substituting Eqs. (18 and 19) back into (17) yields:

$$\sigma_{fm}^* = \sigma_f^* \left[\sqrt{1 - 77.8 \left(\frac{T_m}{\sigma_y d^3}\right)^2} - 10.2 \left(\frac{M_m}{\sigma_u d^3}\right) \right] \quad (20)$$

Thus, if a mean bending moment M_m and/or mean torsional load T_m are present, the fatigue limit of the specimen σ_{fm}^* can be found from Eq. (20) and the fatigue limit of the shaft σ_f is then:

$$\sigma_f = k_a k_b k_c k_d k_e k_f k_g k_h k_i \sigma_{fm}^* \quad (21)$$

The above value of σ_f can then be substituted back into Eqs. (10 and 11) to find shaft diameter and/or life.

The Appendix contains an example to illustrate how the proposed method is to be applied. As a precautionary note, it is good practice to check if the shaft diameter calculated from Eq. (10) is sufficiently large to withstand static failure considering the combination of peak bending moment, $M_a + M_m$, and mean transmitted torque T_m . Standard, static strength equations found in [2, 6, 14, and 15] and elsewhere can be used.

Unlimited life design. - For the special case of an unlimited life design of a shaft having a constant amplitude bending moment M_a with both mean bending and torsional stresses present, the required shaft diameter d can be found by substituting Eq. (20) back into Eq. (10) and setting $n_f = N_f$ and $M_{a_i} = M_a$. This gives:

$$d^3 = \frac{32(FS)}{\pi} \sqrt{\left(\frac{M_a}{\sigma_f} + \frac{M_m}{\sigma_u}\right)^2 + \frac{3}{4} \left(\frac{T_m}{\sigma_y}\right)^2} \quad (22)$$

Eq. (22) with $M_m = 0$ is the basic shaft design equation proposed for the soon-to-be-issued ASME Standard B106.1 M, Design of Transmission Shafting [21].

DISCUSSION OF RESULTS

As previously discussed, Fig. 5 shows the effects of a duty cycle consisting of two blocks of cyclic bending moments on relative shaft life. In a similar manner, Eq. (11) can be used to study the influence of duty cycle on relative shaft diameter. This is illustrated in Fig. 6. A representative slope, b , value of 0.13 was selected which approximately corresponds to a steel shaft of about 690 MPa (100 000 psi) tensile strength with k -factor = 0.5 according to Fig. 4. It is clear from Fig. 6 that a relatively small increase in shaft diameter is needed to accommodate a modest overload without sacrificing shaft life. Furthermore, the relative constancy of this increase in diameter, particularly at the higher overload values, indicates that shaft size is basically dictated by the highest bending load, even if it is only present for a relatively small percentage of the duty cycle.

The effect of a mean torsional load T_m and cycles to failure on relative shaft diameter is illustrated in Fig. 7. To normalize this data, relative shaft diameter has been arbitrarily set equal to 1.0 at 10^6 cycles to failure and $T_m = 0$. These predicted curves were derived by substituting Eq. (20) back into Eq. (10) and considering that only a single cyclic bending moment load of amplitude M_a is present. When this is done, the following expression for relative shaft diameter d_R results:

$$d_R = \left[\left(\frac{n}{N_f}\right)^{2b} + \frac{3}{4} \left(k\text{-factor} \cdot \frac{\sigma_f^*}{\sigma_y}\right)^2 \left(\frac{T_m}{M_a}\right)^2 \right]^{1/6} \quad (23)$$

For purposes of illustration, a representative case was selected where $\sigma_u = 690$ MPa (100 000 psi), k -factor = 0.5, $b \approx 0.13$, $\sigma_f^* \approx 0.5$ (σ_u) and $\sigma_y \approx 0.85$ (σ_u).

Several general observations can be made about the trends appearing in Fig. 7. First, the savings in shaft diameter for a limited life design at 10^3 cycles to failure versus that for an unlimited life (fatigue limit) design at 10^6 cycles becomes smaller as the transmitted or mean torque is increased. For example, a 26 percent smaller shaft can be used at $T_m = 0$ while a diameter reduction of just 12 percent is possible at $T_m = 3 M_a$. Secondly, the required increase in shaft diameter to accommodate an increase in transmitted torque at constant shaft life is relatively modest for high cycle fatigue life designs. For example, an increase in shaft diameter of only 8 percent is needed to accommodate a transmitted torque that is 3 times the bending moment amplitude ($T_m = 3 M_a$) at 10^6 cycles. However, at lower cycles to failure, this increase in diameter with transmitted torque becomes greater, being about 28 percent at 10^3 cycles for $T_m = 3 M_a$.

Finally, the sensitivity of shaft diameter to changes in the required cycles-to-failure, N_f , although not shown in Fig. 7, is greater for higher values of slope b according to Eq. (23). Thus the diameter of highly notched (low k -factor), low strength steel shafts (see Fig. 4) will exhibit a greater reduction with a decrease in design life than will those of smooth, high strength shafts. Also, highly notched, high strength shafts (low k -factor, high σ_y) will exhibit a smaller increase in diameter with an increase in transmitted torque than will smooth, low strength shafts according to Eq. (23).

SUMMARY AND CONCLUSIONS

A shaft design method is presented which can be used to estimate the diameter required to survive a specified number of stress cycles under a variable amplitude loading history. The analysis is based on a nominal stress-life method in which a straight line connects the true fracture strength at 1 cycle to the fatigue limit of the shaft at 10^6 or 10^7 cycles-to-failure on log-log coordinates. A number of fatigue life modifying factors have been identified to correct test specimen fatigue strength data for geometric and environmental conditions which the actual shaft will likely encounter in service. Among such factors are surface condition, size, reliability, temperature, stress concentration, press-fitted collars, residual stress and corrosion fatigue. The effects of variable amplitude loading were incorporated into the analysis using a Palmgren-Miner linear damage approach. Mean bending stresses were accounted for using a Modified Goodman failure relationship. The influence of a steady transmitted torque was considered through an elliptical reduction in reversed bending fatigue strength with mean torsional stress exhibited by previously published fatigue test data.

The method presented was used to determine the effects of certain key material and operating variables on shaft diameter and fatigue life. The following results were obtained:

(1) The amplitude of the peak cyclic bending moment from a variable amplitude loading history, even briefly applied, has a large influence on shaft diameter and/or fatigue life.

(2) The sensitivity of shaft fatigue life to bending stress is primarily a function of tensile strength and the value of the fatigue life modifying factor. For example, life typically varies with stress to about the -14 power for small, smooth, high strength shafts and to the -5 power for large, rough, heavily notched, low strength shafts.

(3) The sensitivity of shaft diameter to the presence of a mean or steady transmitted torque is relatively small for high cycle fatigue life designs but steadily increases as the desired cycle life is reduced.

(4) The savings in shaft diameter from a reduction in the required number of cycles to failure is greater at lower transmitted torque levels. This savings becomes relatively small for shafts that carry a relatively high amount of torque.

APPENDIX - APPLICATION EXAMPLE

To illustrate application of the proposed method consider that a shaft is to be designed with safety factor of 2 from SAE 1045 steel, quenched and tempered Q&T (225 BHN, $\sigma_u = 724$ MPa and $\sigma_y = 634$ MPa from Table 1) for 100 000 cycles under a steady torque of 3000 N-M and the following variable bending moment schedule:

M_{a_i} , N-M	Percent time	Number of cycles, n_i	Fraction of N_f n_i/N_f
2000	15	15 000	0.015
1500	35	35 000	.035
1000	50	50 000	.050
	100	100 000	

The fatigue limit of a smooth 1045 steel specimen without mean stress σ_f^* is listed as 323 MPa at $N_f = 10^6$ cycles in Table 1. (Note this is somewhat smaller than the approximation $0.5 \sigma_u$ or 362 MPa.)

Start with an initial shaft diameter guess of $d = 0.055$ m (a good starting point is to calculate d from Eq. (10) assuming that no mean load is present). The effect of the mean torque of 3000 N-M on σ_f^* can be found from Eq. (20) as follows:

$$\begin{aligned}\sigma_{fm}^* &= 323 \times 10^6 \sqrt{1 - 77.8 \left(\frac{3000}{0.055^3 \cdot 634 \times 10^6} \right)^2} \\ &= 313 \times 10^6 \text{ N/m}^2\end{aligned}$$

Let's assume that in this example that the product of all the k-factors described by Eq. (4) is equal to 0.4, so the shaft's corrected bending fatigue limit according to Eq. (21) is

$$\begin{aligned}\sigma_f &= 0.4 (313 \times 10^6) \\ &= 125 \times 10^6 \text{ N/m}^2\end{aligned}$$

For this material σ_f' is given as 1227×10^6 N/m², so the S-N curve slope is

$$\begin{aligned}b &= \log(1227/125)/6 \\ &= 0.165 \text{ or } 1/b = 6.05\end{aligned}$$

Finally, for a FS = 2.0, the required shaft diameter d can be found from Eq. (10) to be:

$$\begin{aligned}d^3 &= \frac{32 (2.0)}{\pi \cdot 125 \times 10^6} \left[0.015 (2000)^{6.05} + 0.035 (1500)^{6.05} + 0.05 (1000)^{6.05} \right]^{0.165} \\ &= 1.71 \times 10^{-4} \text{ m}^3\end{aligned}$$

or

$$d = 0.056 \text{ m or } 2.2 \text{ inch}$$

It is instructive to note that if the calculation were repeated considering that only the maximum bending moment of 2000 N-M acted 15 percent of the time and that if the shaft ran unloaded the rest of the time, that is

$$M_{a_2} = M_{a_3} = 0$$

then

$$d = 0.054 \text{ m or } 2.1 \text{ inch}$$

The insignificant reduction in shaft diameter from ignoring the lower loads clearly illustrates the dominant effect that peak loads have on fatigue life. This is also apparent from Eq. (3) where life is inversely proportional to the $1/b$ power of stress amplitude. The exponent $1/b$ typically ranges from about 5 for heavily notched shafts to about 14 for some polished, unnotched steel test specimens without mean stresses (see Table 1). Even at a modest $1/b$ value of 6, 64 times more fatigue damage is caused by doubling the alternating bending moment or bending stress amplitude. This underscores the necessity of paying close attention to overload conditions in both shaft and structural element fatigue designs.

REFERENCES

1. Forrest, P. G., Fatigue of Metals, Pergamon Press, New York, 1962.
2. Juvinall, R. C., Engineering Considerations of Stress and Strain, and Strength, McGraw-Hill, New York, 1967.
3. Fuchs, H. O. and Stephens, R. I., Metal Fatigue in Engineering, Wiley, New York, 1980.
4. Collins, J. A., Failure of Materials in Mechanical Design: Analysis, Prediction Prevention, John Wiley and Sons, New York, 1981.
5. Graham, J. A., ed., Fatigue Design Handbook, SAE, New York, 1968.
6. Deutschman, A. D., Michels, W. J., and Wilson, C. E., Machine Design: Theory and Practice, MacMillan, Inc., New York, 1975. pp. 331-360.
7. Borchardt, H. A., "Shortcuts for Designing Shafts," Machine Design, Vol. 45, Feb. 8., 1973, pp. 139-141.
8. Bagci, C., "Computer-Aided Fatigue Design of Power Transmission Shafts with Strength Constraints using a Finite Line Element Technique and a Proposed Fatigue Failure Criterion," ASME Paper 79-DET-103, 1979.
9. Maschino, S. and Kaftanoglu, B., "Computer-Aided Design of Power Shafts," ASME Paper No. 83-DET-63, 1983.
10. Wetzel, R. M., ed., Fatigue Under Complex Loading: Analyses and Experiments, Vol. 6, Society of Automotive Engineers, Warrendale, Pa., 1977.
11. "SAE Information Report Technical Report on Fatigue Properties-SAEJ1099," SAE Handbook, Pt. 1, Society of Automotive Engineers, Warrendale, Pa., 1978, p 4.44.
12. "SAE Information Report: Estimated Mechanical Properties and Machinability of Hot Rolled and Cold Drawn Carbon Steel Bars-SAE J414," SAE Handbook, Pt. 1, Society of Automotive Engineers, Warrendale, Pa., 1978, p. 3.14.
13. Heywood, R. B., Designing Against Fatigue of Metals, Reinhold, New York, 1962, p. 30.
14. Loewenthal, S. H., and Orthwein, W. C., Shafts, Couplings, Keys, etc., Mechanical Design and Systems Handbook, H. A. Rothbardt, ed., 2nd edition McGraw-Hill Book Co., New York, 1984.
15. Loewenthal, S. H., Design of Power Transmitting Shafts, NASA Reference Publication to be published, 1984.
16. Peterson, R. E., Stress Concentration Factors, Wiley, New York, 1974.
17. "Shafts with Interference-Fit Collars, Part IV: Fatigue Strength of Plain Shafts," ESDU-68005-Pt. 4, Engineering Sciences Data Unit, London, 1968.
18. Almen, J. O. and Black, P. H., Residual Stresses and Fatigue in Metals, McGraw-Hill, New York, 1963.
19. Harris, W. J., Metallic Fatigue, Pergamon Press, New York, 1961, p. 48.
20. "Metallic Materials and Elements for Flight Vehicle Structures," M11 - Handbook 5, Department of Defense, Washington, D.C., 1962.
21. Loewenthal, S. H., "Proposed Design Procedure for Transmission Shafting Under Fatigue Loading," NASA TM-78927, 1978.

TABLE 1. - REPRESENTATIVE STRENGTH AND FATIGUE PROPERTIES OF SELECTED STEELS
 BASED ON TEST SPECIMEN DATA WITHOUT MEAN STRESSES FROM REFERENCES 11 AND 12

SAE spec	BHn	Process description	σ_u Ult Str, ksi (MPa)	σ_y Yield Str, ksi (MPa)	σ_f Fat Str Cof, ksi (MPa)	b Fat Str, Exp	σ_f^* 10 ⁶ cycles, ksi (MPa)
1005-1009	125	Cd Sheet	60 (414)	58 (400)	78 (538)	0.073	35 (244)
1005-1009	90	HR Sheet	50 (345)	38 (262)	93 (641)	0.109	29 (202)
1015	80	Normalized	60 (414)	33 (228)	120 (827)	0.11	27 (186)
1018	126	CD Bar	64 (441)	54 (372)			
1020	108	HR Plate	64 (441)	38 (262)	130 (896)	0.12	30 (208)
1022	137	CD Bar	69 (476)	58 (400)			
1040	170	CD Bar	85 (586)	71 (490)			
1040	225	As Forged	90 (621)	50 (345)	223 (1538)	0.14	33 (233)
1045	225	Q&T	105 (724)	92 (634)	178 (1227)	0.095	47 (323)
1045	390	Q&T	195 (1344)	185 (1276)	230 (1586)	0.074	79 (547)
1045	500	Q&T	265 (1827)	245 (1689)	330 (2275)	0.08	104 (715)
1045	595	Q&T	325 (2241)	270 (1862)	395 (2723)	0.081	122 (843)
1050	197	CD Bar	100 (690)	84 (579)			
1140	170	CD Bar	88 (607)	74 (510)			
1144	305	Drawn at Temp	150 (1034)	148 (1020)	230 (1586)	0.09	66 (454)
1541F	290	Q&T Forging	138 (951)	129 (889)	185 (1276)	0.076	63 (435)
4130	258	Q&T	130 (896)	113 (779)	185 (1276)	0.083	59 (404)
4130	365	Q&T	207 (1427)	197 (1358)	246 (1696)	0.081	77 (532)
4140	310	Q&T Drawn at Temp	156 (1076)	140 (965)	265 (1827)	0.08	90 (619)
4142	310	Drawn at Temp	154 (1062)	152 (1048)	210 (1448)	0.10	53 (366)
4142	380	Q&T	205 (1413)	200 (1379)	265 (1827)	0.08	83 (574)
4142	450	Q&T and Deformed	280 (1931)	270 (1862)	305 (2103)	0.09	83 (572)
4340	243	HR Annealed	120 (827)	92 (634)	174 (1200)	0.095	49 (337)
4340	409	Q&T	213 (1469)	199 (1372)	290 (1999)	0.091	80 (550)
4340	350	Q&T	180 (1241)	170 (1172)	240 (1655)	0.076	82 (567)
5160	430	Q&T	242 (1669)	222 (1531)	280 (1931)	0.071	103 (709)

Note: Values listed are typical. Specific values should be obtained from the steel producer.

Symbols:

- CD = cold drawn
- HR = hot rolled
- Q&T = quenched and tempered

TABLE 2. - MANUFACTURING PROCESSES THAT PRODUCE RESIDUAL STRESSES

Beneficial residual compressive stress	Harmful residual tensile stress
Pre-stressing or overstraining Shot or hammer peening Sand or grit blasting Cold surface rolling Coining Tumbling Burnishing Flame or induction hardening Carburizing or nitriding	Cold straightening Grinding or machining Electro-discharge machining (EDM) Welding Flame cutting Chrome, nickel, or zinc plating

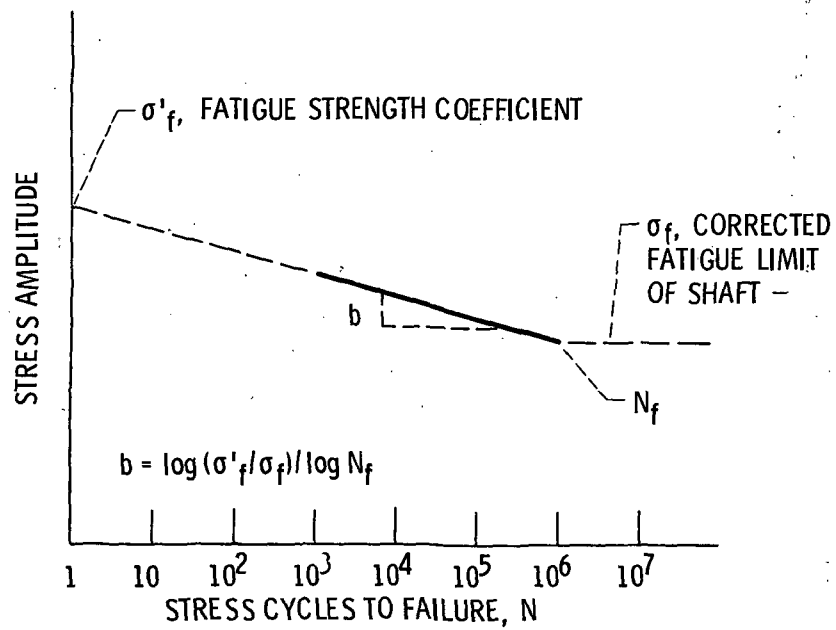


Figure 1. - Generalized S-N curve constructed for σ'_f and σ_f .

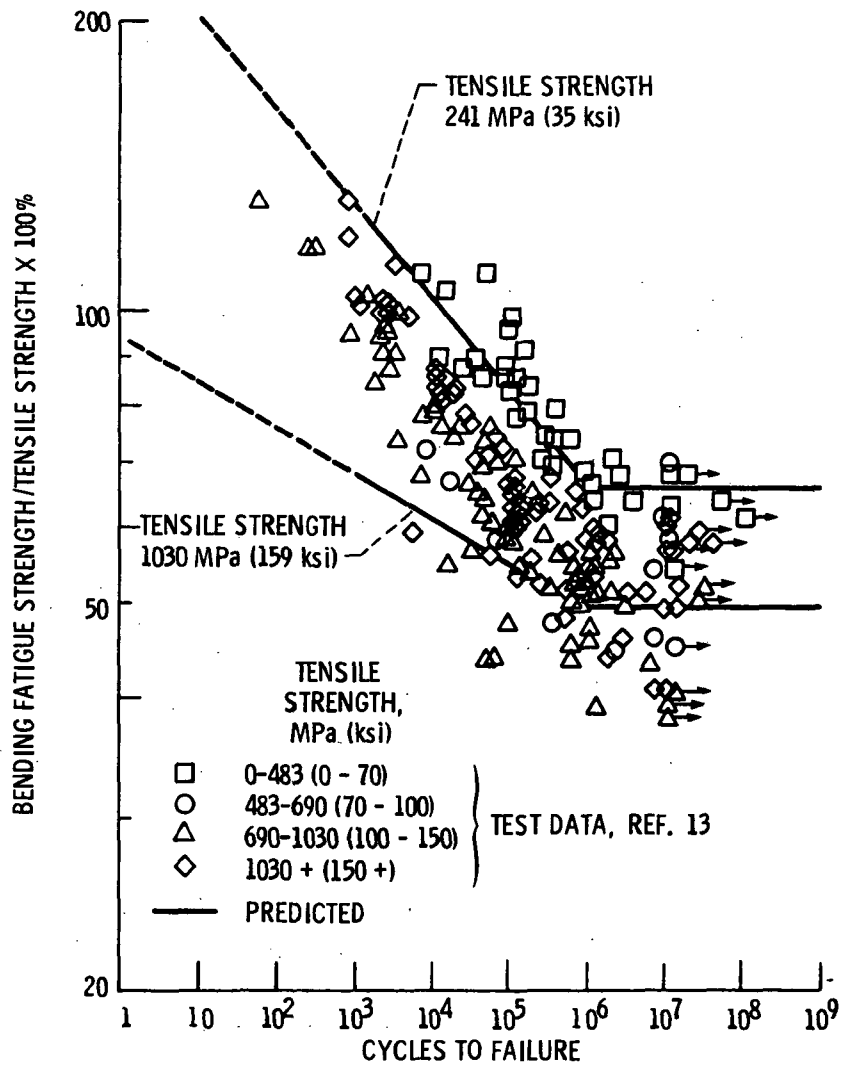


Figure 2. - Predicted and measured effect of tensile strength on the stress-life characteristics of steels in reversed bending (from ref. 13).

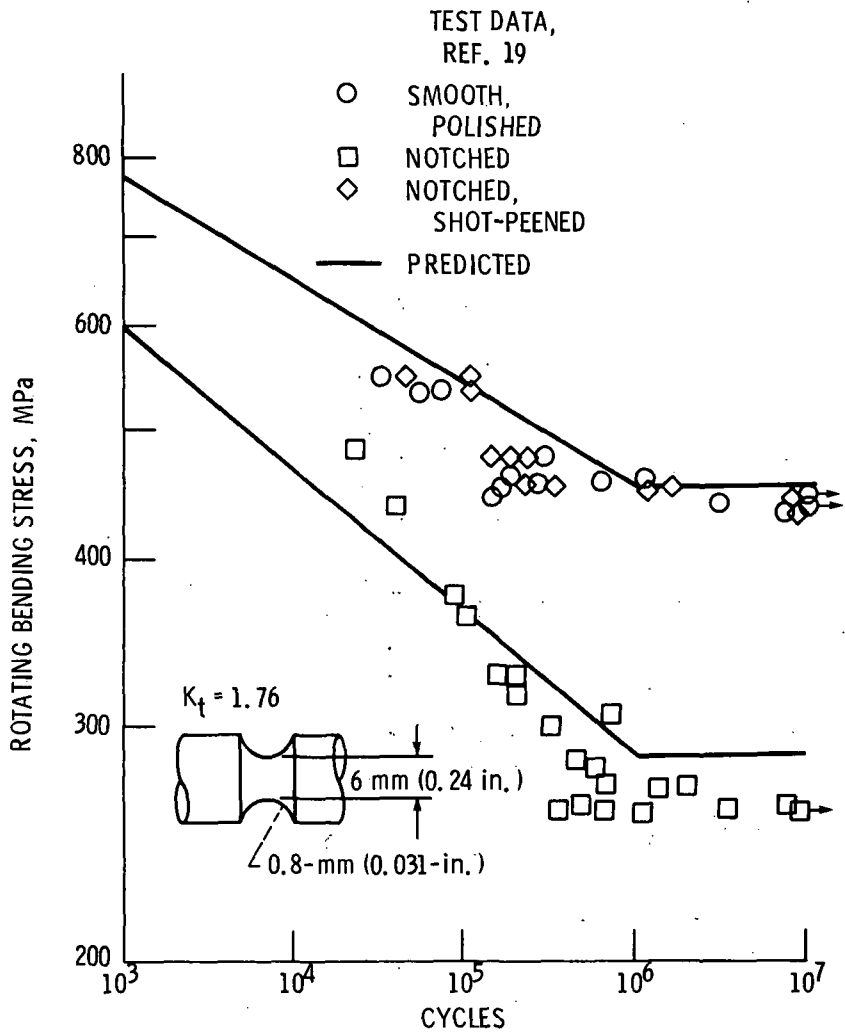


Figure 3. - Comparison of predicted and experimental stress-life characteristics of smooth, notched and notched/shot peened, steel rotating beam specimens (from ref. 19).

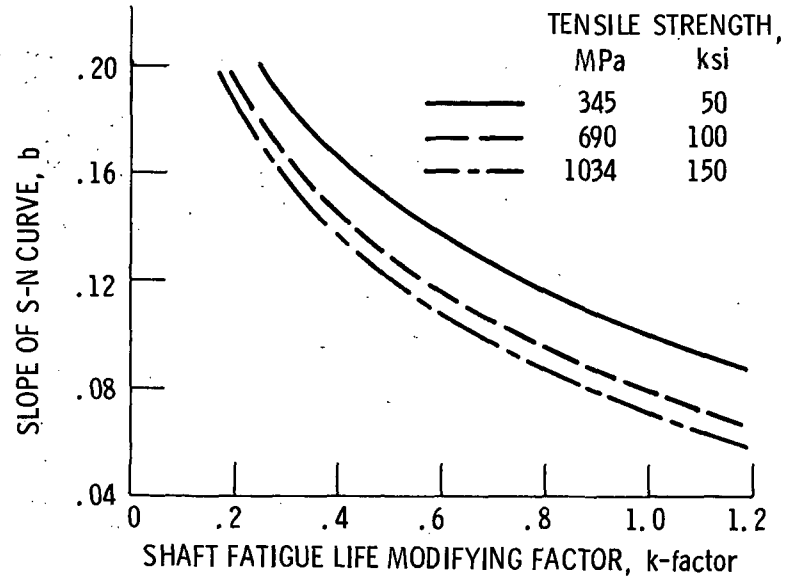


Figure 4. - Effect of fatigue life modifying factor (k-factor) and tensile strength on S-N curve slope, b.

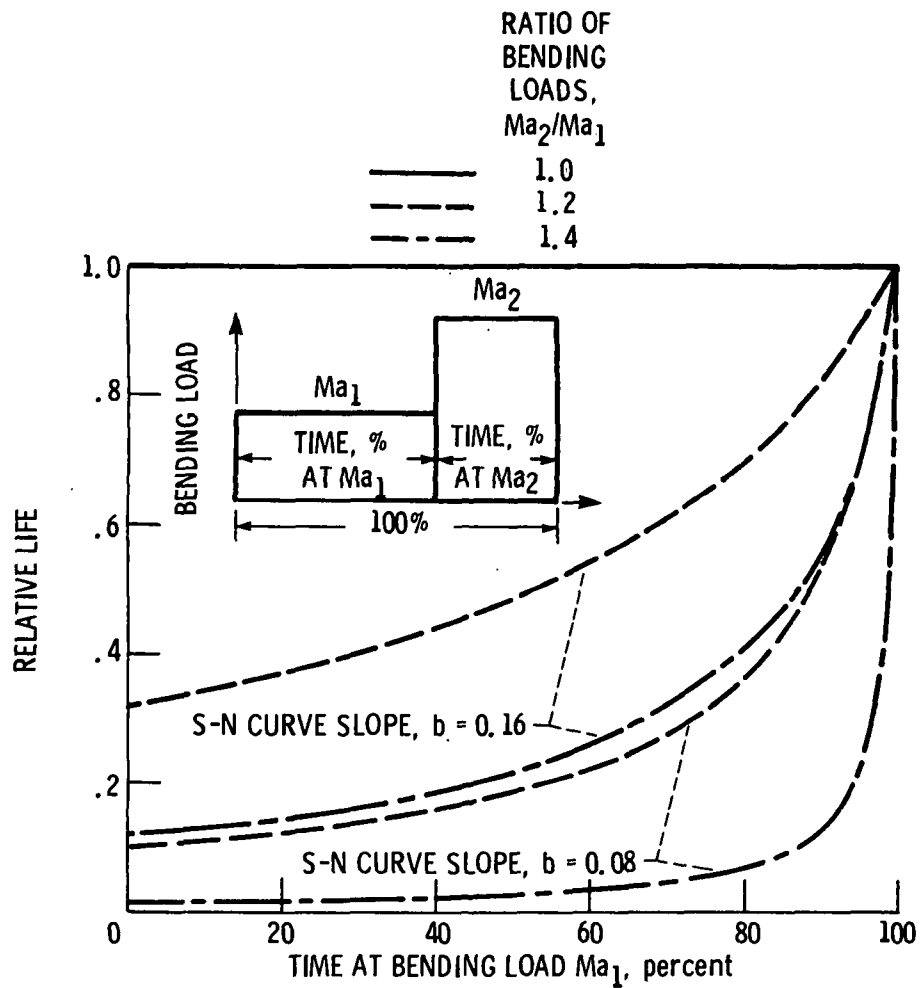


Figure 5. - Effect of duty cycle on shaft fatigue life at some combination of bending moments Ma_1 and Ma_2 .

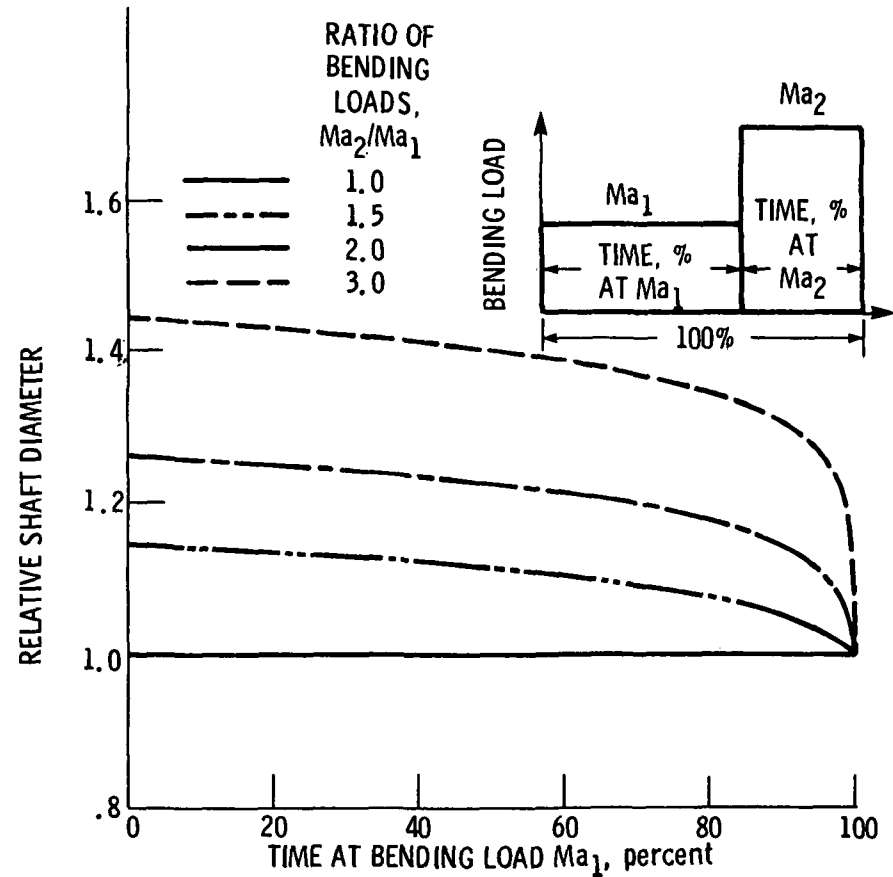


Figure 6. - Effect of duty cycle on shaft diameter at constant life (notched shaft having k-factor = 0.5, tensile strength = 690 MPa, slope $b = 0.13$, FS = 1).

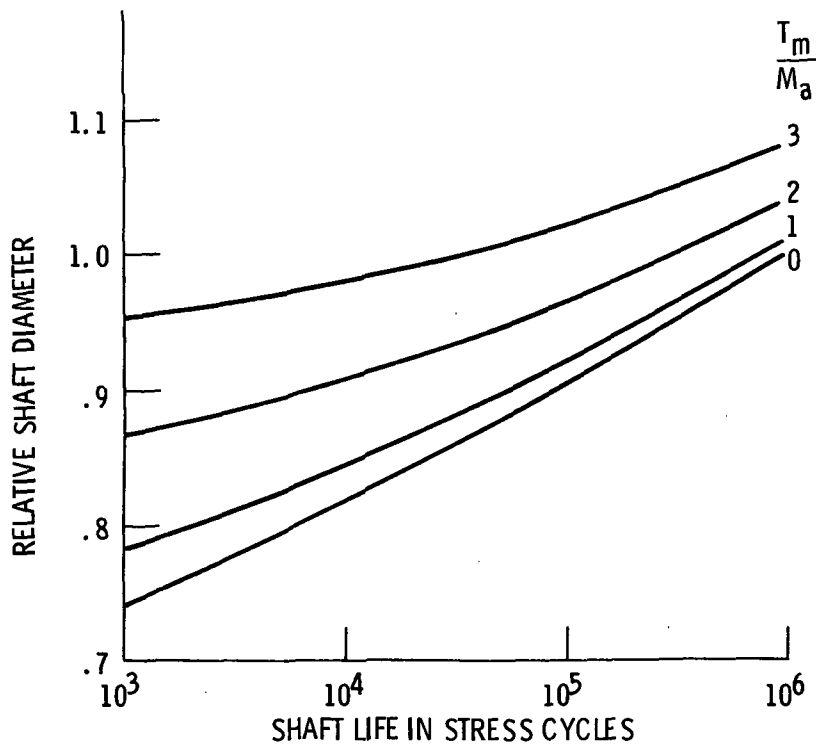


Figure 7. - Effect of transmitted torque and life on shaft diameter (notched shaft having k-factor = 0.5, tensile strength = 690 MPa, slope $b = 0.13$, FS = 1).

1. Report No. NASA TM-83608	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle Factors That Affect the Fatigue Strength of Power Transmission Shafting		5. Report Date	
		6. Performing Organization Code 505-40-42	
7. Author(s) Stuart H. Loewenthal		8. Performing Organization Report No. E-1984	
		10. Work Unit No.	
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135		11. Contract or Grant No.	
		13. Type of Report and Period Covered Technical Memorandum	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code	
15. Supplementary Notes Prepared for the Fourth International Power Transmission and Gearing Conference sponsored by the American Society of Mechanical Engineers, Cambridge, Massachusetts, October 8-12, 1984.			
16. Abstract A long-standing objective in the design of power transmission shafting is to eliminate excess shaft material without compromising operational reliability. A shaft design method is presented which accounts for variable amplitude loading histories and their influence on limited life designs. The effects of combined bending and torsional loading are considered along with a number of application factors known to influence the fatigue strength of shafting materials. Among the factors examined are surface condition, size, stress concentration, residual stress and corrosion fatigue.			
17. Key Words (Suggested by Author(s)) Shafts Shafting Design Transmission shafting		18. Distribution Statement Unclassified - unlimited STAR Category 37	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages	22. Price*

**National Aeronautics and
Space Administration**

**Washington, D.C.
20546**

**Official Business
Penalty for Private Use, \$300**

**SPECIAL FOURTH CLASS MAIL
BOOK**



**Postage and Fees Paid
National Aeronautics and
Space Administration
NASA-451**

NASA

**POSTMASTER: If Undeliverable (Section 154
Postal Manual) Do Not Return**
