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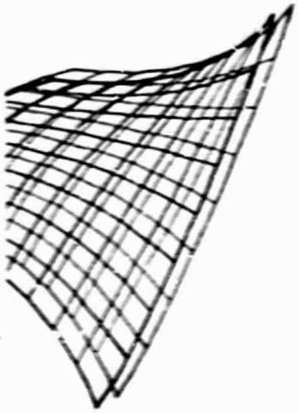

INSTITUTE FOR NUMERICAL METHODS IN ENGINEERING

UNIVERSITY COLLEGE OF SWANSEA, SINGLETON PARK, SWANSEA, U.K., SA2 8PP. TEL. (0792) 205678

AN ADAPTIVE FINITE ELEMENT METHOD FOR HIGH SPEED COMPRESSIBLE FLOW

R. Löhner, K. Morgan & O. C. Zienkiewicz

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AN ADAPTIVE FINITE ELEMENT METHOD FOR HIGH SPEED COMPRESSIBLE FLOW

R. Löhner, K. Morgan and O.C. Zienkiewicz

Institute for Numerical Methods in Engineering, University College of Swansea, Singleton Park, Swansea SA2 8PP, U.K.

Our aim is to solve large 2/3-D compressible fluid flow problems employing finite elements. Other numerical techniques, like finite differerce or finite volume methods have reached a high degree of sophistication [1,2], but it is expected that the finite element method will make significant contributions due to its geometrical flexibility, a factor that is of importance for industrial applications.

The development has so far gone through the following stages:

(a) Definition of the basic algorithm: in order to be competitive, and at the same time to model correctly the physical properties of hyperbolic equations, it was decided to employ explicit time-marching schemes. As the straightforward Galerkin method is suboptimal [3], a modified or 'upwinded' Taylor Galerkin-type [4] procedure was implemented. The complete description may be found in [5], together with numerical examples. This one-step algorithm has now been superseded by an equivalent two-step method, the description of which may be found in [6]. This two-step scheme is 2-3 times faster than the one-step scheme on scalar machines (for two-dimensional problems) and has been vectorized in order to realise the full power of modern supercomputers.

(b) Domain splitting: it is well known that solutions of high speed compressible flow problems exhibit narrow regions of rapid change (e.g. shocks) which are embedded in larger regions where the solution is smooth. Accordingly, large variations in element size are expected in typical discretizations. However, the small elements might then require that a correspondingly small global timestep could be employed in larger elements. The remedy adopted here, and described in detail in [7], is to split the domain into regions in which different timestep-sizes can be used. The domain subdivision is performed completely automatically by the computer code at prescribed time intervals, and allows a time-accurate development of the unsteady solution.

(c) Adaptive mesh refinement: in general, an analyst will have no a priori knowledge of the location of those areas of the domain where more (i.e. smaller) elements should be employed. Therefore, usually, much more elements than necessary will be employed, leading to an inefficient overall procedure. An ideal computational algorithm would require the ability to refine the mesh where necessary as the solution proceeds. The geometric flexibility of the linear triangular element makes it ideally suited for refinement processes of this type. We adopted a posteriori methods [8], as they seem at present more economical, and for the same reason also did not implement hierarchical techniques [9], but the more classic enrichment of adding more elements. At present we are only considering steady state problems, the generalization to transient problems being an obvious extension. This means, that the timestepping scheme is utilized as a relaxation procedure. After a given number of timesteps the solution domain is analysed, and more elements are added where necessary. Generally speaking, there exist three possibilities for approximately determining the error

$$e = u - u^h \tag{1}$$

where u denotes the exact and u^h the discrete solution:

i) Comparison with higher order schemes: The significant derivatives of the partial differential equation (PDE) under consideration are evaluated twice, using in each case difference schemes of different order [10,11,12]. By determining the discrepancy of both approximations an estimation of the error can then be obtained. The problem with this kind of approach is that it fails near boundaries and at singularities or boundary layers (which are common in fluid dynamic problems). At the same time it is not extendable to FEMs, which operate on an element level.

ii) Determination of the relative importance of further degrees of freedom: Further degrees of freedom are introduced on an element by element basis, and the relative importance of these further degrees gives an error estimate [13,14,15]. The problem with this kind of approach is that it is relatively expensive in CPU-time requirements, so that for transient problems a considerable percentage of runtime will be spent on error estimation.

iii) Use of error norms: Here the classic theoretical error estimates are employed locally [16,17]. Thus, no further degrees of freedom are introduced and only first or second derivatives need to be evaluated. Our experience indicates that this type of error indicator works satisfactorily, and, as it is very economical, it is regarded as a good algorithm for transient problems as well. For elliptic problems the appropriate error norms appear naturally whereas for hyperbolic problems the theory is far from complete. Nevertheless one can assume

$$\|u - u^h\|_k < c\, h^{\ell-k} |u|_\ell, \tag{2}$$

where h is a representative element length. Using the L_2-norm (k=0) yields

$$|u - u^h|_o < c\, h^\ell |u|_\ell. \tag{3}$$

The aim of any refinement is to obtain a reduction of errors according to some criterion, e.g. at a certain point, surface or evenly throughout the field. Particularly for hyperbolic problems the error at one point may influence the accuracy of the solution in the whole field (e.g. the root of an expansion fan), so that an even distribution of errors seems to be the only possible practical choice.

Therefore, at each refinement level all elements satisfying

$$h^{\ell}|u|_{\ell} > a \max_{e=1,nelem} h^{\ell} |u|_{\ell} \tag{4}$$

are refined. Since the exact solution u is unknown, the practical requirement becomes

$$h^{\ell}|u^h|_{\ell} > a \max_{e=1,nelem} h^{\ell} |u^h|_{\ell}. \tag{5}$$

Only the cases $\ell=1$ or $\ell=2$ appear to be of practical interest, and both have been studied (see examples). For the case $\ell=2$ the first derivatives of u are evaluated inside the elements, and hereafter the nodal values for the second derivatives are recovered variationally as follows:

$$\int N^i N^j \, dV \, \hat{u}^j_{,xx} = - \int N^i M^k \, dV \, \bar{u}^k_{,x} \quad . \tag{6}$$

where M^k is constant and $\bar{u}^k_{,x}$ is defined on an element basis. It has been found that a-values of the order

$$a = 0.6 - 0.9 \tag{7}$$

yield the most effective refinement strategy. This is in contrast to [13], where the factor $a = 0.1$ was reported as optimal. A possible explanation for the discrepancy of these values may be found in the nature of the PDEs treated in both cases: whereas here the PDEs are hyperbolic - and this means th-t small disturbances propagate far into the field - , in [12] the effective solution of elliptic PDEs was pursued - and this means that small disturbances decay rapidly.

Results

(a) Supersonic flow past a wedge: the successive stages of the domain discretization as well as the solution obtained are shown in figure 1 . In this case the mesh was enriched according to equation (5) with $\ell=1$ and $a=0.6$.

(b) Prandtl-Meyer expansion fan: the problem statement, as well as the successive stages of the domain discretization and the corresponding solutions are depicted in figure 2. The improvement in solution quality is readily seen. In this case the mesh was enriched according to equation (5) with $\ell=2$ and $a=0.8$.

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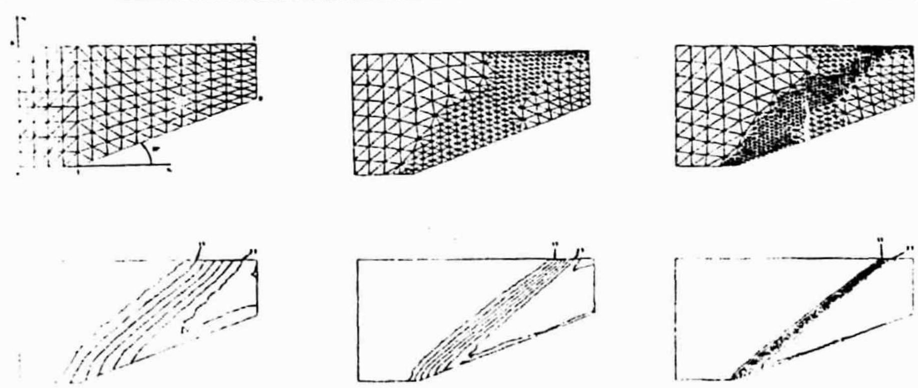

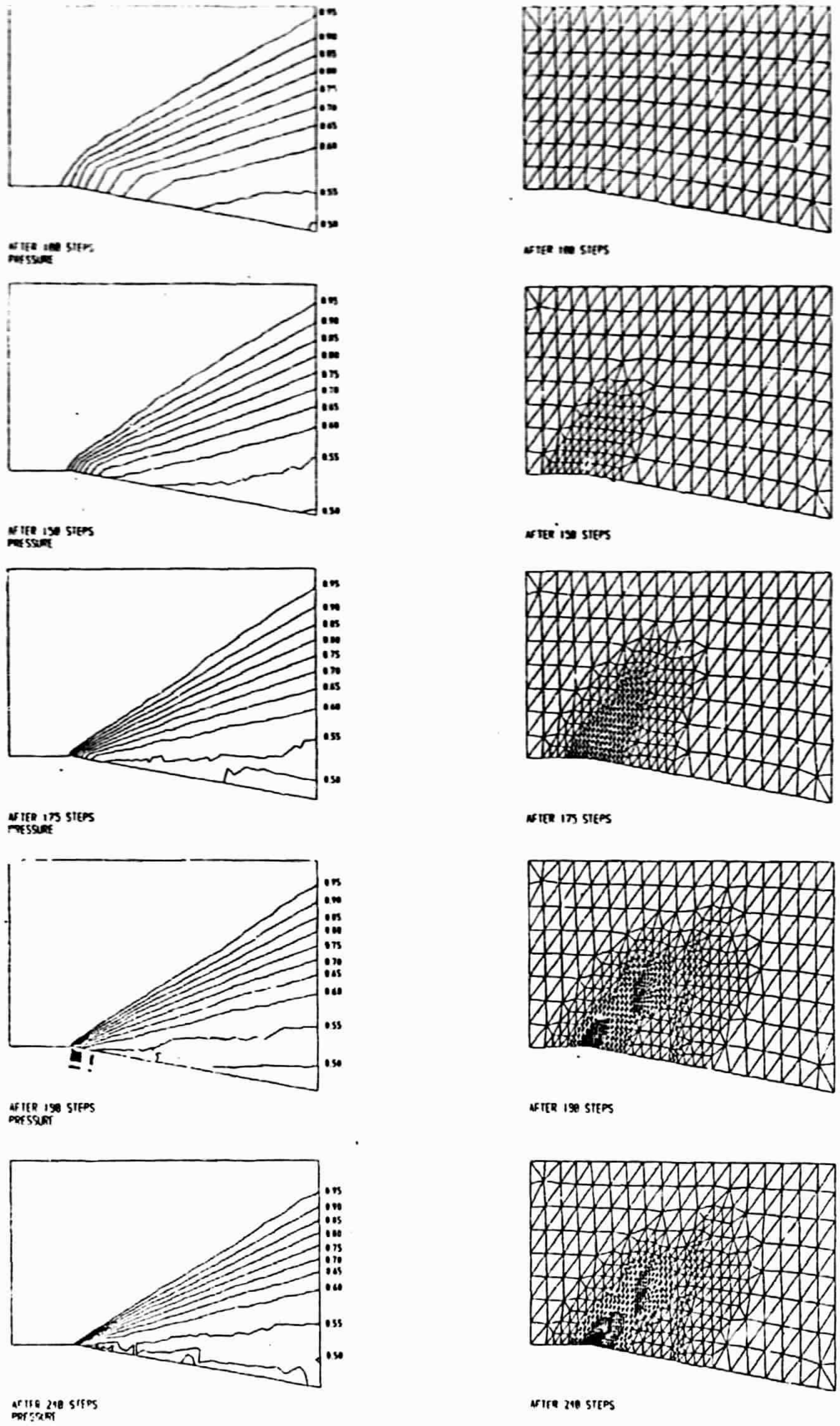
AFTER 100 STEPS
PRESSURE
AFTER 100 STEPS
AFTER 150 STEPS
PRESSURE
AFTER 150 STEPS
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AFTER 198 STEPS

Figure 2