2.1G FRESNEL ZONE CONSIDERATIONS FOR REFLECTION AND SCATTER FROM REFRACTIVE INDEX IRREGULARITIES

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INTRODUCTION

Several different echoing mechanisms have been proposed to explain VHF/UHF scatter from clear air. GAGE and BALSLEY (1980) suggest three: (1) anisotropic scatter; (2) Fresnel reflection, and (3) Fresnel scatter, in order to account for the spatial (angle and range) and temporal dependence of the echoes. ROTTGER (1980) proposes the term "diffuse reflection" to describe the echoing mechanism when both scatter and reflection coexist. We present a unifying formulation incorporating a statistical approach that embraces all the above mechanisms and gives conditions under which reflection or scatter dominates. Furthermore, we distinguish between Fraunhofer and Fresnel scatter and present a criterion under which Fresnel scatter is important.

Scatter from anisotropic irregularities of refractive index n has, for many years, been thought to be principally responsible for microwave echoes from the clear air. Existing formulations assume that the correlation length of n irregularities generated by turbulence. are small compared to the Fresnel length. But there is experimental evidence that the contrary may be true. This paper extends the existing formulations for the case where the Fresnel zone radius is comparable to or smaller than the correlation length. LIU and YEH (1980) recognized the limitations of the existing formulations which are based upon first-order expansion of the phase term in the integral for the scattered (or reflected) electric field intensity and WAKASUGI (1981) suggested expansion to second order.

THE FRESNEL TERM IN THE INTEGRAL FOR ECHO POWER

TATARSKI (1961, sect. 4.2) derived a formula for the field scattered from a volume V_s with dimensions that implied the size of V_s cannot be determined by the radar's resolution volume V_6 (DOVIAK and ZRNIC', 1983). (The subscript 6 is used to denote a resolution volume circumscribed by the surface giving a weight, to the scatterers, 6 dB less than the peak at the volume origin; DOVIAK et al., 1979.) In a later publication, TATARSKII (1971, sect. 2.8) extended his earlier formulation so that V_s could equal V_6. It can be shown that for backscatter the condition assumed in this extension is

$$\rho_t << \sqrt{\lambda r_s/2\pi} = f/\sqrt{\pi} \qquad (1)$$

where r_s is the range to an element of the scatter volume λ, the radar wavelength, f the first Fresnel zone radius, and ρ_t is the correlation length of refractive index irregularities for lags transverse to r_s. Inequality (1) imposes the condition that constant phase surfaces of the incident wave are planes over the distance ρ_t, and the receiver is in the far field of this correlation length (i.e., $r_s > 2\rho_t^2/\lambda$).

We now develop the scatter equations which allow correlation length to be larger than that specified by (1). Assuming the Born approximation (i.e., single scatter theory), the field intensity E_1, backscattered by refractive index irregularities Δn in the antenna far field is

$$E_1(\vec{r}_o,t) = k_o^2\sqrt{\frac{P_t\eta_o g}{(2\pi)^3}} \int_{V_s} \frac{f_\theta(\vec{r})\ \Delta n(\vec{r},t)}{r_s^2} \exp\{-j2k_o r_s\} dV \qquad (2)$$

where r_s is the range to Δn at $\vec{r}$ (see Figure 1), $f_\theta(\vec{r})$ is the angular pattern of the incident electric field intensity assumed to be circularly symmetric about the beam axis, η_o is the free space wave impedance, $k_o=2\pi/\lambda$, P_t is the transmitted peak power, g the antenna gain, and $\vec{r}$ the distance from the origin of V_6 to the scattering element. V_s is a spherical shell of thickness $c\tau/2$ where τ is the transmitted pulse width and therefore $E_1(\vec{r}_o,t)$ is the intensity of echoes sampled at a range-time delay $(2r_o/c)+\tau$ after the transmitted pulse. Assume Δn to be a zero mean random variable. In a matched filter receiver having an internal resistance R, the increment of current magnitude $|dI|$ produced by the scattering element is

$$|dI| = |dE_1|\ \lambda W(\vec{r})\sqrt{\frac{g f_\theta^{\,2}(r)}{4\pi\eta_o R}} \qquad (3)$$

where $W(\vec{r})$ is the range weighting function (ZRNIC' and DOVIAK, 1978; DOVIAK and ZRNIC', 1979). The integration now extends over all $\vec{r}$ for which $W f_\theta\ \Delta n$ has significant value. For a receiver filter matched to a rectangular transmitted pulse, the range weighting function is

$$W(\vec{r}) = \begin{cases} 1 - \dfrac{2|\vec{r}\cdot\vec{a}_o|}{c\tau} & ;\ |\vec{r}\cdot\vec{a}_o| \le c\tau/2 \\ = 0 & ;\ \text{otherwise} \end{cases} \qquad (4)$$

where $\vec{a}_o$ is the unit vector from the origin of V_6 to the radar.

The received power, time averaged over a cycle of the transmitted frequency, is:

$$P_r = \frac{1}{2} II^*R \qquad (5)$$

where * denotes the conjugate.

From the integral of (3)

$$I = \frac{\lambda k_o^2 g}{(2\pi)^2}\sqrt{\frac{P_t}{2R}} \int \frac{W(\vec{r}) f_\theta^{\,2}(\vec{r})\ \Delta n(\vec{r},t) e^{-j2k_o r_s}}{r_s^2} dV \qquad (6)$$

For the condition $c\tau \ll r_o$, r_s does not change significantly where $W(\vec{r})$ is appreciable so r_s in the denominator of (6) can be replaced with r_o. Upon substituting (6) into (5) and taking the ensemble average, the expected received power becomes:

$$\langle P_r\rangle = \frac{P_t g^2}{4\lambda^2 r_o^4}\iint R(\vec{r},\vec{r}')\ W(\vec{r})\ W(\vec{r}') f_\theta^{\,2}(\vec{r}) f_\theta^{\,2}(\vec{r}')\ e^{-j2k_o(r_s-r_s')}\ dV dV' \qquad (7)$$

$$R(\vec{r},\vec{r}') = \langle\Delta n(\vec{r})\ \Delta n(\vec{r}')\rangle \qquad (8)$$

Let's assume that the irregularities have homogeneous statistical properties so that $R(\vec{r},\vec{r}') \approx R(\vec{r}-\vec{r}')$ and that the two-way pattern function $f_\theta^{\,2}(\vec{r})$ is given by

$$f_\theta^{\,2}(\vec{r}) = \exp\{-\theta^2/4\sigma_\theta^2\} \qquad (9)$$

where σ_θ^2 is the second central moment of the two-way power pattern and θ is the angular displacement, measured at the radar site, of $\vec{r}$ from the origin of V_6. In terms of the 6 dB angular width θ_6 for the two way pattern f_θ^4, $\theta_6=3.33\sigma_\theta$. For the assumption of narrow beams (9) can be approximated

$$f_\theta^2 \simeq \exp\{-t^2/4r_o^2\,\sigma_\theta^2\} \tag{10}$$

where $t=\sqrt{t_1^2+t_2^2}$ is the projection of $\vec{r}$ onto the plane transverse to the beam axis at $\vec{r}_o$.

The Gaussian matched filter provides the best resolution of all the receivers having the same bandwidth (ZRNIC' and DOVIAK, 1978). Because of this and because practical "matched filters" used in Doppler weather radars are Gaussian we assume that $W(\vec{r})$ is well approximated by

$$W(\vec{r}) \simeq e^{-(\vec{a}_o\cdot\vec{r})^2/4\sigma_r^2} \tag{11}$$

where σ_r^2 is the second central moment of the weighting function $W^2(\vec{r})$ and

$$\sigma_r = 0.35\ c\tau/2 = 0.30\ r_6 \tag{12}$$

for a Gaussian filter "matched" to a rectangular pulse of width τ. The 6 dB range resolution is r_6.

Use the Taylor expansion for r_s

$$r_s \equiv |\vec{r}_o-\vec{r}| \simeq r_o-\vec{a}_o\cdot\vec{r} + \frac{1}{2r_o}\{r^2-(\vec{a}_o\cdot\vec{r})^2\} \tag{13}$$

for terms up to second order in r^2. We note that the second term of this expansion is the projection of $\vec{r}$ onto the $\vec{a}_o$ direction and the third term contains the projection on the transverse plane. Thus, in terms of the ℓ,t coordinates centered in V_6,

$$r_s = r_o + \ell + t^2/2r_o \tag{14}$$

This quadratic expansion is valid (i.e., third order terms in r are negligible) provided that the scatter volume V_s size is by limited by

$$d_t^2 < 2r_o\ \{\sqrt{f^2/\pi+d_\ell^2} - |d_\ell|\} \tag{15}$$

where $2d_t$ and $2d_\ell$ are the dimensions of V_s transverse and parallel to r_o. The condition (15) assumes $d_\ell \ll r_o$. As can be deduced, the farther the integration variable is displaced from the plane $\ell=0$, the smaller must be the scatter volume size perpendicular to the beam axis. However, d_ℓ in (15) need not be larger than the smaller of $r_6/2$ or the longitudinal projection $(d_z/\cos\psi+r_o\theta_6\tan\psi)/2$ of the scattering layer within V_6 (Figure 1). Substituting (14), the integral in (7) becomes

$$I_7 \equiv \iint R(\vec{r}-\vec{r}')\ W(\ell)\ W(\ell')\ \exp\{-\frac{(t^2+t'^2)}{2\sigma_t^2} - j2k_o(\ell-\ell' + \frac{t^2-t'^2}{2r_o})\}\ dVdV' \tag{16}$$

where $\sigma_t=\sigma_\theta r_o\sqrt{2}$ is proportional to the arc length of V_6. We now still find it convenient to define new coordinates:

$$t_1-t_1' \equiv \delta_1;\quad t_2-t_2' \equiv \delta_2;\quad \ell-\ell' \equiv \delta_3 \tag{17a}$$

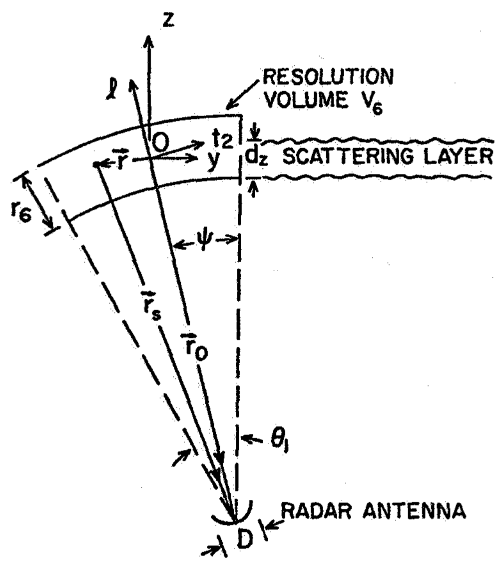

Figure 1. Geometry for backscatter. The distances ℓ and t are measured from the origin 0 of the resolution volume in directions parallel (longitudinal) and perpendicular (transverse) to the beam axis $\vec{r}_0$. t_1 is parallel to the χ axis and perpendicular to t_2.

$$\frac{t_1+t_1'}{2} \equiv \sigma_1; \quad \frac{t_2+t_2'}{2} \equiv \sigma_2 ; \quad \frac{\ell+\ell'}{2} \equiv \sigma_3 \tag{17b}$$

so that the t_1, t_1' component of (16) can be written as

$$I_7(t_1) = \iint R(\vec{\delta}) \exp \{- \frac{\sigma_1^2+\delta_1^2/4}{\sigma_t^2} - j2k_o\sigma_1\delta_1/r_o\} \, d\sigma_1 d\delta_1 \tag{18}$$

where $\vec{\delta} = \vec{a}_1\delta_1+\vec{a}_2\delta_2+\vec{a}_3\delta_3$ is the lag vector (note $\vec{a}_3=\vec{a}_o$). The transformation from (16) to (18) is valid if, as is assumed here, the limits of integration cover the entire volume where the integrand has significant value. Thus executing the integration over σ_1

$$I_7(t_1) = \sigma_t\sqrt{\pi} \int R(\vec{\delta}) \exp \{-k_o \delta_1 \sigma_t/r_o)^2 - \delta_1^2/4\sigma_t^2\} d\delta_1 \tag{19}$$

Applying similar procedures to the t_2 and ℓ coordinate integrations we obtain

$$I_7 = \sigma_r\sigma_t^2\pi^{3/2}\sqrt{2} \int R(\vec{\delta})\, e^{\underbrace{-(\delta_t^2/4\sigma_t^2 + \delta_z^2/8\sigma_r^2)}_{\text{Resolution Volume Weight}} \; \underbrace{-\pi^2\delta_t^2\sigma_t^2/f^4}_{\text{Fresnel Term}} \; - j2k_o\delta_3} \, dV\delta \tag{20}$$

where $\delta_t^2 = \delta_1^2 + \delta_2^2$. The solution (20) is acceptable if the 2nd order expansion of r_s in (14) is valid. Inequality (15) is the condition on V_s for this expansion to be applicable. However, when the transverse dimension ℓ_t of V_s is large such that (15) is not obeyed, we can still use (20) if $R(\vec{\delta})$ is small when

$\vec{\delta}$ has a magnitude comparable to or larger than the right side of (15). In other words the correlation length ρ_t perpendicular to the beam axis must be

$$\rho_t^2 < 2r_o \{\sqrt{f^2/\pi + d_\ell^2} - |d_\ell|\} \tag{21}$$

If condition (15) is satisfied, then there is no condition on ρ_t. By comparing (21) with (1), it becomes evident that the second order expansion relaxes the limits placed on the scatter volume size and correlation length ρ_t. Now these limits are increased by the factor $(8\pi r_o/\lambda)^{1/4}$. For example, if r_o=10 km and λ=6 m, ρ_t would have to be less than 1.4 km in order for (20) to be applicable whereas ρ_t would have to be less than 100 m for a 1st order theory.

In the integral (20) the correlation is multiplied by two exponential weighting functions: (1) the resolution volume weight which depends solely upon the width σ_t and range resolution σ_r of V_6 and (2) the Fresnel terms which gives a weight in the t direction that depends upon the ratio f/σ_t. Only when the radius of the Fresnel zone is large compared to $\sqrt{\pi\sigma_t\rho_t}$ can the Fresnel term in (20) be ignored. Therefore, both beam width and correlation length enter into the comparison with f. But because σ_t is a function of f, that is

$$\sigma_t = \frac{0.45\ r_o}{D\sqrt{\ell n 2}} = \frac{0.9 f^2}{D\sqrt{\ell n 2}} \tag{22}$$

where D is the antenna diameter, we can simplify the conditions so that the Fresnel term can only be ignored if ρ_t satisfies

$$\rho_t < \frac{D\sqrt{\ell n 2}}{0.9\pi} \tag{23}$$

On the other hand, because f is always smaller than σ_t in the antenna's far field, the Fresnel term in (20) will have more weight than the beam width part of the resolution volume term. Thus situations that allow us to neglect the Fresnel term will also permit us to ignore beam width influence. If (23) is satisfied, we can use (20) (without the beam width and Fresnel terms) to obtain the scattered field, even though V_s is larger than V_6; then we need to sum incoherent echo power from elemental volumes large compared to ρ_t^3 but small compared to V_6 (DOVIAK and ZRNIC', 1983). We call this case incoherent Fraunhofer scatter. But HODARA (1966) shows that within the lower troposphere, the correlation length has the following height dependence

$$\rho \simeq 0.4h/(1+0.01h) \quad (m) \tag{24}$$

where h is in meters. Furthermore, VHF backscatter data analyzed later in this paper suggest that $\rho_t \simeq 20$ m for irregularities in the lower stratosphere. Thus, unless the antenna diameter is of the order of 100 m or more, the Fresnel term will be important in determining the field scattered by refractive irregularities. If the scattering volume contains many subvolumes for which (20) applies, but (23) is not satisfied, we have a situation of incoherent Fresnel scatter. When $d_t < \sqrt{2r_o f}$ (from Equation 15), then we have coherent Fresnel scatter. If $d_t < f$, then signal is coherent irrespective of the transverse reshuffling of refractive index irregularities.

THE SPECTRAL SAMPLING FUNCTION

Because it is common to describe the statistical properties of refractive index irregularities by the spectral density function the effects of the resolution volume and Fresnel terms on echo power can be examined conveniently by introducing a spectral sampling function. Equation (20) can be expressed in terms of the Fourier transform of $R(\vec{\delta})$ multiplied by the lag weighting function $H(\vec{\delta})$ where

$$H(\vec{\delta}) = \exp - \left\{ \left(\frac{1}{4\sigma_t^2}\right) + \frac{\pi^2\sigma_t^2}{f^4} \quad \delta_t^2 + \frac{\delta_3^2}{8\sigma_r^2} \right\} \tag{25}$$

Thus

$$I_7 = \Phi_{nw}(0,0,2k_o) \tag{26}$$

where $\Phi_{nw}(\vec{K})$ is the three dimensional transform of $R(\vec{\delta})$ multiplied by the lag weighting function. Now Φ_{nw} is the spectrum Φ_n of refractive index irregularities convolved with the spectrum Φ_w of $H(\vec{\delta})$:

$$\Phi_{nw} = 8\sigma_r\sigma_t^2\pi^{9/2} \quad \sqrt{2}\ \Phi_n * \Phi_w \tag{27}$$

where * denotes convolution and

$$\Phi_w = \frac{1}{8\pi^3}\int H(\vec{\delta})\ \exp(-j\vec{K}\cdot\vec{\delta})\ dV \tag{28}$$

is the normalized spectral sampling function. Substituting (25) into (28) and evaluating:

$$\Phi_w = \frac{\pi^{-3/2}\sigma_t^2\ \sigma_r\ \sqrt{2}}{(1+4\pi^2\sigma_t^4/f^4)} \quad \exp\left\{-2\sigma_r^2\ K_\ell^2 - \frac{\sigma_t^2 K_t^2}{(1+4\pi^2\sigma_t^4/f^4)}\right\} \tag{29}$$

where $K_t = \sqrt{K_1^2 + K_2^2}$. The second order phase term has contributed the factor $4\pi^2\sigma_t^4/f^4$ in the above equation. Thus the first order expansion is valid only if this factor is small relative to unity. However for V_6 in the antenna far

$$4\pi^2\sigma_t^4 f^{-4} >> 1 \tag{30}$$

For remote sensing with radar it is commonn to have V_6 in the antenna far field, thus the Fresnel term in $\Phi_w(\vec{K})$ cannot be ignored. As discussed earlier, this conclusion is a result of the fact that the Fresnel radius is always less than the beam width so that the Fresnel term always dominates the beam width weighting function. Thus $\Phi_w(\vec{K})$ can be well approximated by

$$\Phi_w(\vec{K}) \simeq \frac{0.44D^2\sigma_r \ell n2}{\pi^{7/2}} \exp\left\{-\ 2\sigma_r^2 K_\ell^2 - \frac{D^2 K_t^2 \ell n2}{3.24\pi^2}\right\} \tag{31}$$

in which we have substituted (22) for σ_t. Equation (31) shows that the larger is the antenna diameter, the narrower is the spectral sampling function. It is surprising that the sampling function shape and size is independent of r_o and, for a given antenna diameter, the spectrum $\Phi_n(\vec{K})$ of irregularities is weighted equally for all resolution volumes in space. This result differs from that derived by TATARSKII (1971) who only considered first-order phase expansion in which case Φ_w is a function of r_o. By combining (7), (26) and (27) the back-scattered power is given by

$$\langle P_r \rangle = \frac{2\sqrt{2}(0.45)^2\pi^{9/2}\ \sigma_r P_t g^2}{r_o^2 D^2\ \ell n2} \int \Phi_n(\vec{K})\ \Phi_w(\vec{a}_3 2k_o - \vec{K})\ dV_k \tag{32}$$

In the atmosphere it is usual for the horizontal correlation length ρ_n to be larger than the vertical one ρ_z so $\Phi_n(\vec{K})$ will be more sharply peaked along the K_x,K_y directions and less so along the K_z axis. If the irregularities have shapes that are roughly described as oblate spheroids, then the correlation $R(\vec{\delta})$ would also have a similar form but $\Phi_n(\vec{K})$ would be prolate spheriodal in shape (Figure 2a). Equation (31) reveals that whenever range resolution $r_6=3.33\sigma_r$ is larger than 0.34D, as is usual, the sampling function (Figure 2b) along K_ℓ will be narrower than along K_t. If the beam axis is rotated by ψ degrees from the vertical, $\Phi_w(\vec{K})$ will also be rotated by ψ from the K_z axis.

If echo power decreases significantly as ψ is increased, then we have specular type reflection. The sharpness of the angular dependence is a function both of ρ_h and D. Referring to Figures 2a,b and Equation (31), we see that a necessary condition to observe a specular type echoing mechanism is for $0.54\lambda/D<<1$. That is, narrow beams are required which is consistent with simple physical arguments. Assuming space is filled with Δn, specular type echoes will then be observed only if $\rho_h>>\rho_z$. However, we must be cautious in applying these criteria because we have used a specific model (i.e., Gaussian) to

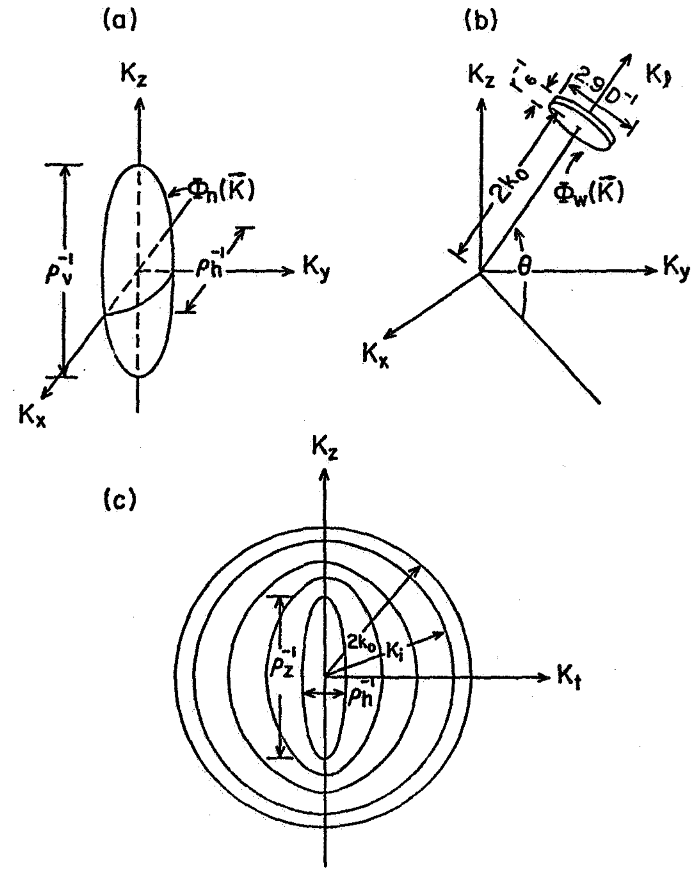

Figure 2. (a) Contour surface of constant spectra intensity $\Phi_n(\vec{K})$ for irregularities having symmetric correlation lengths along x and y that are longer than the correlation length along z. (b) Contour surface of the spectral sampling function $\Phi_w(\vec{K})$ for beam axes at elevation angle $\theta_e = \pi/2 - \psi$. (c) Contours of constant $\Phi_n(\vec{K})$ for which the small-scale irregularities produce isotropic scatter.

describe the statistical properties of n and because ρ_h, ρ_z only characterize the most intense irregularities of refractive index. Thus, if the contours of constant $\Phi_n(\vec{K})$ have the dependence sketched in Figure 2c, scatter could be independent of ψ (i.e., isotropic scatter) if $2k_o \gg \rho_z^{-1}$ and $2k_o \gg \sqrt{2}r_6^{-1}$. The $\Phi_n(\vec{K})$ depicted in Figure 2c can be represented by a sum of isotropic Φ_i and anisotropic Φ_a parts where K_i is the wave number beyond which $\Phi_i \geq \Phi_a$ (independent of direction of $\vec{K}$). $\Phi_i(\vec{K})$ could have the -11/3 power law dependence on K deduced from turbulence theories.

BACKSCATTERING FROM ANISOTROPIC IRREGULARITIES

As an example, let us consider the angular dependence of echo power when the scattering medium can be decomposed into isotropic and anisotropic components (i.e., $R(\vec{\delta})=R_i+R_a$). We further assume that R_a is isotropic in the horizontal plane. To obtain an order of magnitude estimate, we take R_a of the form:

$$R_a = \langle \Delta n^2 \rangle_a \exp \left\{ -\frac{\delta_h^2}{2\rho_h^2} - \frac{\delta_z^2}{2\rho_z^2} \right\}$$

where

$$\delta_h = \sqrt{\delta_x^2 + \delta_y^2} \tag{33}$$

The resolution volume coordinates are related to the natural coordinates x,y,z via:

$$\delta_x=\delta_1; \ \delta_y=\delta_2 \cos\psi-\delta_3\sin\psi; \ \delta_z=\delta_2 \sin\psi+\delta_3 \cos\psi \tag{34}$$

After introducing (34) into (33) and the result into (20), integration is performed giving the formula for echo power from anisotropic irregularities as being proportional to

$$I_a(\psi) = \frac{2\sqrt{2}\sigma_r\sigma_t^2\pi^3\langle \Delta n^2 \rangle_a}{a\sqrt{4b^2d^2-c^4}} \exp \{-4k_o^2 b^2/(4b^2d^2-c^4)\} \tag{35}$$

where for V_6 in the antenna far field:

$$a^2 \simeq \frac{1}{2\rho_h^2} + \frac{\pi^2\sigma_t^2}{f^4} \tag{36a}$$

$$b^2 \simeq \frac{\sin^2\psi}{2\rho_z} + \frac{\cos^2\psi}{2\rho_h^2} + \frac{\pi^2\sigma_t^2}{f^4} \tag{36b}$$

$$c^2 = \left(\frac{1}{\rho_h^2} - \frac{1}{\rho_z^2}\right) \sin\psi\cos\psi \tag{36c}$$

$$d^2 = \frac{\cos^2\psi}{2\rho_z} + \frac{\sin^2\psi}{2\rho_h^2} + \frac{1}{8\sigma_r^2} \tag{36d}$$

Now for laminae of Δn such that ρ_z is smaller than the smallest of:

$$\frac{2\sqrt{2}\pi\sigma_t\sigma_r\rho_h}{f^2}, \text{ or } \frac{\rho_h f^2}{2\sqrt{2}\pi\sigma_t\sigma_r}, \text{ or } \frac{\sqrt{2}\sigma_r f^2}{\pi\rho_h\sigma_t} \tag{37}$$

we can simplify (35)

$$I_a(\psi) \approx \frac{4<\Delta n^2>_a \rho_z \rho_h{}^2 \sigma_r \sigma_t{}^2 \pi^3}{\sqrt{TQ}} \exp \{-2k_o{}^2 \rho_z{}^2 \frac{S}{Q}\} \tag{38}$$

where

$$S(\psi) = \frac{\rho_h{}^2}{\rho_z{}^2} \sin^2\psi + \cos^2\psi + 2\pi^2 \sigma_t{}^2 \rho_h{}^2/f^4 \tag{39a}$$

$$T = 1 + 2\pi^2 \sigma_t{}^2 \rho_h{}^2/f^4 \tag{39b}$$

$$Q(\psi) = 1 + (2\pi^2 \sigma_t{}^2 \rho_h{}^2 \cos^2\psi/f^4) + \rho_h{}^2 \sin^2\psi/4\sigma_r{}^2 \tag{39c}$$

When height of V_6 is constant and laminae are infinitesimally thin, the ratio for echo powers at angle ψ and zenith (ψ=0) is:

$$\frac{<P_a(\psi)>}{<P_a(o)>} = \sqrt{\frac{Q(o)}{Q(\psi)}} \cos^2\psi \exp \{-2k_o{}^2 \frac{\rho_h{}^2 \sin^2\psi}{Q(\psi)}\} \tag{40}$$

The term $\cos^2\psi$ accounts for the decrease in power due to the range and σ_t increase with tilt away from the vertical because V_6 remains at constant height. To $<P_a(\psi)>$ we add the power $P_i(\psi)$ due to isotropic irregularities to obtain:

$$\{\frac{<P_a(\psi)>}{<P_a(o)>} + A\cos^2\psi\} \div \{1+A\} = \frac{<P(\psi)>}{<P(o)>} \tag{41}$$

where $A=P_i(0)/P_a(0)$. Equation (41) was fitted to data (Figure 3) from ROTTGER et al. (1981). Pertinent parameters for the Rottger et al. data are: λ=6.4 m; D≈260 m; heights h=16.9→18.1 km near the tropopause; beam width θ_1=1.7°; and range resolution = 300 m. We find ρ_h=20 m for the horizontal correlation lengths, and A=0.04 fits well these data. Comparing terms in (39) it is seen that the Fresnel term (i.e., the 2nd term in $Q(\psi)$) does not contribute significantly. Although we have not distinguished any one of the mechanisms discussed in the introduction as being responsible for the echo power, we see that scattering from anisotropic irregularities can account for the observed angular dependence which is sufficiently peaked that one might believe a reflection mechanism is acting.

For sake of simplicity, it is preferable to label the echoing mechanism as scatter whenever there are several or more scattering irregularities for which only a statistical description of their properties (e.g., size, intensity, etc.) is practical. Thus, we do not need to invoke a reflective process to explain observations in this case; the scatter formulation presented here can explain all the features of the received field if indeed the medium is comprised of many irregularities of refractive index for which only statistical properties are known.

Equation (2) is the starting point for our formation for scatter from refractive irregularities. Although we refer to (2) as the scatter integral, it can be used as well in situations (i.e., ρ_t>>f) which might be interpreted as reflective. In order to determine echo power when irregularities have horizontal dimensions large compared to the Fresnel radius, GAGE et al. (1981) have used the general formula

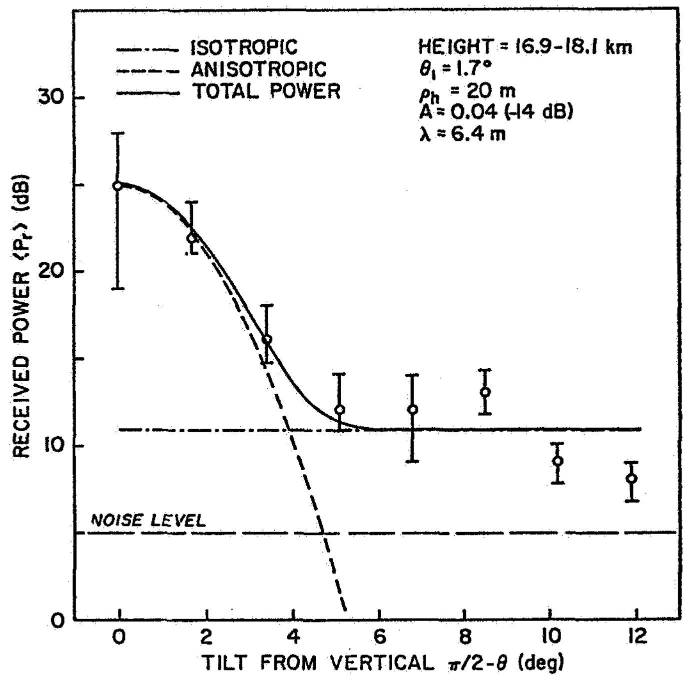

Figure 3. Angular dependence of observed mean backscatter power (open circles) from anisotropic irregularities as the radar beam axis is tilted away from the vertical (ROTTGER, 1981). Fitted to the data is a model that consists of anisotropic turbulence with a two-dimensional (horizontal isotropy) correlation function in an isotropic background.

$$|R|^2 = \frac{1}{4} \left| \int_{-\ell/2}^{\ell/2} \frac{1}{n}\frac{dn}{dz} \exp\{-j2k_o z\}\, dz \right|^2 \tag{42}$$

for the power reflection coefficient where ℓ is the thickness of the partially reflecting layer. In this form variations of n along the horizontal are ignored and, if the scattering layer is in the antenna far field, the echo power P_r is easily found by considering an image source which gives

$$P_r = P_t A_e^2 \, |R|^2/4\lambda^2 r_o^2 \tag{43}$$

where A_e is the effective area of the antenna ($A_e = g\lambda^2/4$). For exactly the same assumptions on n, the solution of (2) should produce an identical echo power. In Appendix A we prove this contention by simply using the second-order phase terms; this shows the wide applicability of the solution presented earlier.

Figure 4 illustrates the type of scatter that would be effective versus the location of the sampling wave number $2k_o$ for the case $\rho_h \gg \rho_z$. The location of boundaries are functions of the parameters ρ_h, D, r_o and the relative strengths of Φ_a and Φ_i, and thus there could be a different order than presented on Figure 4. For example, if $K_i < 2r_o/\rho_h^2$, then Fraunhofer scatter could be either

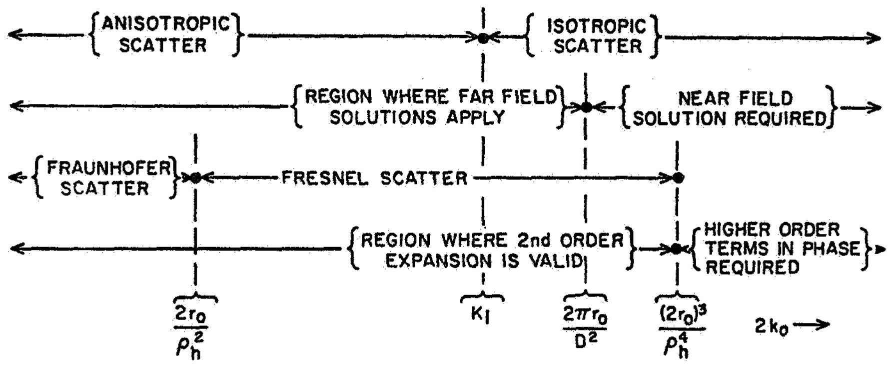

Figure 4. Types of echo mechanisms versus the location of the sampling wave number $2k_0$ for a particular ordering of boundaries. $\rho_h >> \rho_z$, and V_6 is uniformly filled with irregularities.

anisotropic or isotropic depending upon the value of $2k_o$. Because we had assumed infinitesimally thin laminae ($K_i \to \infty$ in that case), scatter will be anisotropic no matter how large is $2k_o$. However, it is more likely that $K_i < 2\pi r_o/\rho_h^2$ so that if $2k_o$ was larger than 2 m^{-1} we could pass into a region of isotropic scatter. Data collection at various $2k_o$'s (i.e., multiple wavelength radar) could establish the correlation length ρ_z and a value for K_i. At UHF wavelengths $2k_o$ is so large that the peak of $\Phi_w(\vec{K})$ is expected to fall most of the time in the tail of $\Phi_n(\vec{K})$ at wave numbers where turbulence is mostly isotropic and Φ_n is expected to have the same 11/3rds dependence on K as does the velocity fluctuations. However, at the longer wavelengths in the VHF band, $2k_o$ is much smaller so it can place the $\Phi_w(\vec{K})$ peak in a region where Φ_a may sometimes be larger than Φ_i or smaller than it.

ECHO POWER DEPENDENCE ON RANGE AND RANGE RESOLUTION

GAGE et al. (1981) propose a model for which echo intensity varies as the inverse square power of range but has a range resolution dependence that can vary from zero to a square law. BALSLEY and GAGE (1981) introduce the concept of a scatter volume defined, transverse to the antenna beam, by a correlation radius to derive an echo intensity that depends on the fourth power of range. It is improper to form such a condition because the scatter volume V_s is defined by either the spatial distribution of intensity of Δn fluctuations or by the resolution volume V_6, whichever is smaller. We shall use the solutions derived here to determine the conditions under which various dependences can occur. Recently HOCKING and ROTTGER (1983) have critically reviewed the interpretations of Balsley and Gage.

Assume vertical incidence and use (31) and (32) to obtain

$$\langle P_r \rangle = \frac{C\sigma_r}{r_o^2} \Phi_n(\vec{K}) * \Phi_w(\vec{K}) \tag{44}$$

where C is a constant independent of σ_r and r_o, and $\Phi_w(\vec{K})$ can be expressed as:

$$\Phi_w(\vec{K}) - \sigma_r \Phi_w(K_t) \exp\{-2\sigma_r^2 K_z^2\} \tag{45}$$

where now $K_t^2 = K_x^2 + K_y^2$. Consider two cases: (1) $\Phi_n(\vec{K})$ broad and (2) narrow compared to $\Phi_w(\vec{K})$ along the wave number $\vec{K}_z$ coordinate (Figure 5).

(a) Φ_n Broad

Integration along K_z gives a $\langle P_r \rangle$ approximated by

$$\langle P_r \rangle = \frac{C\sigma_r}{r_o^2} \iint \Phi_w(K_t)\Phi_n(K_x,K_y,2k_o)dK_x dK_y \tag{46}$$

which illustrates that the expected echo power is proportional to range resolution (assuming a uniformly filled V_6) and inversely proportional to the square of range r_o. This is the usual dependence expected when scatter is from irregularities produced by turbulence. The r^{-2} dependence occurs irrespective of whether ρ_h is large or small compared to f.

(b) $\Phi_n(\vec{K})$ Narrow

In this case (44) can be reduced to

$$\langle P_r \rangle \simeq \frac{C\sigma_r^2}{r_o^2} e^{-2\sigma_r^2(K_{sz}-2k_o)^2} \int \Phi_w(K_t)\Phi_n(\vec{K})\, dV_K \tag{47}$$

Again $\langle P_r \rangle$ depends upon the inverse square of r_o, a result which is independent of ρ_h. If $\Phi_w(\vec{K}_t)$ is also broad compared to $\Phi_n(\vec{K})$ along K_t then (47) reduces to

$$\langle P_r \rangle \simeq \frac{C\sigma_r^2}{r_o^2} - \Phi_w(0) e^{-2\sigma_r^2(K_{sz}-2k_o)^2} \int \Phi_n(\vec{K}) dV_K \tag{48}$$

in which $\Phi_n(\vec{K})$ is assumed to have a peak at $K_t=0$. This case occurs when refractive index irregularities have $\rho_h > f$ and strong Fourier components clustered about $2k_o$. Only if $2k_o = k_{sz}$ will $\langle P_r \rangle$ be proportional to σ_r^2; otherwise, we could have other range resolution dependencies. The integral in (48) is the variance $\langle \Delta n^2 \rangle$.

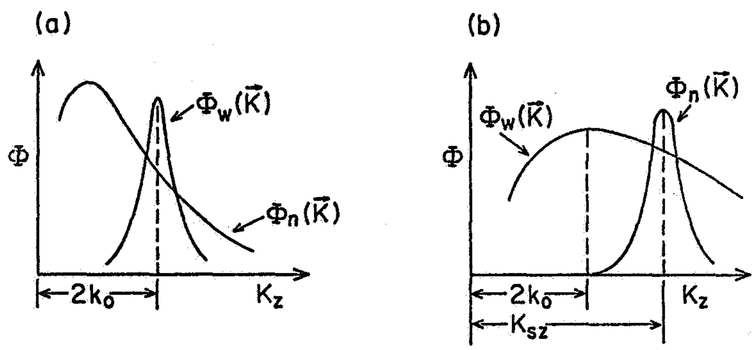

Figure 5. Cases in which Φ_n is broad (a) and narrow (b) compared to Φ_w.

If the irregularities are contained in a thin layer having a vertical dimension small compared to σ_r, then echo power would be independent of σ_r, when the resolution volume is centered on the layer. For resolution volumes displaced from the layer height, echo power would have a strong dependence on σ_r even exceeding the square law one! If the scattering irregularities are confined to horizontal dimensions small compared to beam width, then a fourth power range dependence would be obtained. However, the correlation length does not determine the scatter volume dimension as stated by BALSLEY and GAGE (1981). In the cases discussed in (a) and (b) the echo power depends on the inverse square of range because we have assumed uniformly filled V_6.

CONCLUSIONS

When scattering layers are in the far field of an antenna, the Fresnel term is a more important weighting function than the antenna pattern because the width of the antenna pattern is always larger than the Fresnel radius f. Only in the case where the correlation length ρ_t of refractive index irregularities Δn perpendicular to the beam is much smaller than f will the first-order truncation of the Taylor series expansion for phase be valid. Then Fraunhofer scatter is considered to be effective. However, when retention of second-order phase terms is necessary a Fresnel term (see Equation 20) is introduced. The criterion for keeping the second-order phase term depends both upon beam width and the Fresnel radius. Thus, the condition under which incoherent Fraunhofer scatter is effective becomes solely a function of antenna diameter D (i.e., ρ_t<0.29D). When the Fresnel term needs to be included in the solution we have the situation of Fresnel scatter or reflection. It is suggested that unless the antenna diameter is of the order of 100 m or more, the Fresnel term is important in determining the field scattered by refractive irregularities.

The formulas derived here establish the conditions under which a scatter or reflection mechanism can be distinguished. However it is important to have the proper statistical description of the irregularities in order to obtain the spatial and temporal dependence of echo intensity. A multiple wavelength radar, in which its beam position can be scanned, could supply invaluable data to characterize the spectrum of refractive index irregularities and help to explain the properties of the echoes. Only when irregularities have a spatial spectrum form that concentrates variance $\langle \Delta n^2 \rangle$ at wave numbers near $2k_o$ does echo power depend upon the square of range resolution. Echo power depends upon the inverse square of range r_o independent of whether ρ_t is less than or greater than f. However the resolution volume must be uniformly filled with Δn.

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APPENDIX A

In this appendix we shall demonstrate that (42) and (2) and identical formulations for the situation considered in this paper (i.e., a scattering layer in the far field of an antenna, and range resolution sufficiently narrow so that the $1/r^2$ term can be brought out of the integral (2)).

For pulsed transmissions the height interval that contributes to the echo sample is determined by the pulse shape if refractive index irregularities are distributed throughout the vertical. Therefore, in this case (dn/dz)/n in (42) must be multiplied by the range weighting function (e.g., (11)) and then, for pulse widths small compared to r_o, the limits on z in (42) can be increased to infinity without significant error. Thus the reflection coefficient takes the form:

$$R = \frac{1}{2} \int_{-\infty}^{+\infty} e^{-z^2/4\sigma_r^2} \frac{d}{dz} (\ell nn) \exp \{-j2k_o z\} \, dz \tag{A1}$$

Using integration by parts and noting that $n = 1 + \Delta n$ where $\Delta n << 1$, (A1) can be reduced to

$$R = \int_{-\infty}^{+\infty} (\frac{z}{4\sigma_r^2} + jk_o) \, \Delta n \exp \{- \frac{z^2}{4\sigma_r^2} - j2k_o z\} \, dz \tag{A2}$$

Now for range resolution many wavelengths long (i.e., $k_o \sigma_r >> 1$) the term $z/4\sigma_r^2$ can be ignored in the integral without adding appreciable error to ρ. Then

$$R = jk_o \int_{-\infty}^{\infty} W(z)\ \Delta n(z)\ \exp\ \{-j2k_o z\}\ dz \tag{A3}$$

We note here that W(z), as defined in this paper, also contains the weight associated with the frequency transfer function of the receiver's filter. Although this function does not rigorously belong in the integral for ρ, we are primarily interested in the received echo power which is dependent upon the filter function. For a linear system we could, if we ignore receiver noise, just as well consider the filter at the transmitted output thus modifying the pulse shape to give the equivalent weight W(z) considered herein. With similar consideration we can also express (2) in the form

$$E_1 = \frac{k_o^2}{r_o^2} \sqrt{\frac{P_t g \eta_o}{(2\pi)^3}} \int W(z) f_\theta^2(r) \Delta n\ \exp\ \{-j2k_o r_s\}\ dV \tag{A4}$$

We now consider Δn to depend upon z as in the reflection formula and using (10) for $f_\theta^2(r)$, the second-order expansion (14) for r_s, and integrating over the horizontal we obtain

$$E_1 = \frac{\pi k_o^2}{r_o^2} \sqrt{\frac{P_t g \eta_o}{(2\pi)^3}}\ e^{-j2k_o r_o} \left(\frac{1}{2\sigma_t^2} + j\frac{k_o}{r_o}\right)^{-1} \int W\ \Delta n\ \exp\ \{-j2k_o z\} dz \tag{A5}$$

Now for r_o in the antenna far field the term jk_o/r_o has a magnitude larger than $1/2\sigma_t^2$. The echo power P_r, in terms of E_1, is:

$$P_r = A_e |E_1|^2 / 2\eta_o \tag{A6}$$

substitution of (A5) and (A3) into (A6) reveals

$$P_r = \frac{P_t A_e^2 |R|^2}{4\lambda^2 r_o^2}$$

which is identical to (42) derived from the reflection formula.