## 2.2B THE EFFECTS OF PULSE RATE, POWER, WIDTH AND CODING ON SIGNAL DETECTABILITY

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When deciding upon radar and signal processing parameters for MST radars, the quantity that one attempts to maximize within existing constraints is the signal detectability. For Doppler spectral analysis the detectability can be defined (see BALSLEY, 1978 or GAGE and BALSLEY, 1978) as the ratio of the amplitude of the largest spectral peak of the received signal to the noise level fluctuation,  $D = S_r / \Delta S_N$  (see Figure 1).

This paper will examine the effects on the detectability of varying the pulse repetition rate (PRF), peak pulse power ( $P_{pk}$ ) and pulse width ( $\tau_p$ ). Both coded and uncoded pulses will be considered. During this discussion the following quantities will be assumed to be constant: antenna area, echo reflectivity, Doppler shift, spectral width, spectral resolution, effective sampling rate, and total incoherent spectral averaging time. The detectability will be computed for two types of targets: 1) discrete target (i.e., a single echoing region smaller than the smallest pulse width).

First let us examine the effects of coded pulses. The received signal from a coded pulse is decoded by convolving the received voltage with the code. The phase of the received signal from the echoing region will be the mirror image of the transmitted code. Since the autocorrelation function of a code of length  $L_c$ has a peak value of  $L_c$ , the decoding process enhances the echo signal power by a factor of  $L_c^2$ . For white noise which is uncorrelated between each bit of the code, the convolution will add the power incoherently and thus the noise power will be increased by a factor of  $L_c$ . The above is true regardless of the type of code used. For multicode processing (using complementary codes or pseudorandom codes) the signals from successive codes must be added coherently to obtain the desired autocorrelation sidelobe response. However, as long as the number of codes used is less than the normal number of coherent averages, multicode processing will not have any additional effect on the signal detectability. Of course, the sidelobes of the code autocorrelation functions will affect range contamination of signals and influence the choice among various codes.

Now we will determine the signal detectability for coded and uncoded pulses as a function of PRF, transmitter power, pulse width, and code length. Let us define the fundamental bit length or resolution pulse width of a coded pulse to be  $\tau_0$  and the total pulse length to be  $\tau_p = L_c \tau_0$ . The same symbols can be used for uncoded pulses by letting  $L_c = 1$  and  $\tau_p = \tau_0$ . The returned signal power,  $P_s$ , is proportional to peak transmitter power for discrete targets and to peak power and pulse width for diffuse targets. Specifically, for both coded and uncoded pulses,

$$P_s \propto P_{pk} L_c^2$$
 (discrete targets)  
 $P_s \propto P_{pk} \tau_o L_c^2$  (diffuse targets).

The noise power can be written as

 $P_N \propto B L_c/m \propto L_c/\tau_o m$ 

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Figure 1. Typical Doppler spectrum (after BALSLEY, 1978a).

where B  $\propto 1/\tau_0$  is the receiver bandwidth and m is the number of coherent averages. The signal-to-noise ratio, SNR =  $P_S/P_N$ , then becomes

SNR  $\propto P_{pk} m \tau_o L_c$  (discrete) SNR  $\propto P_{pk} m \tau_o^2 L_c$  (diffuse)

Following the derivation of signal detectability, D, used by BALSLEY (1978) and eliminating dependencies on the fixed quantities given previously, it can be shown that

$$D \propto P_{pk}$$
 (PRF)  $\tau_0 L_c = P_{AV}$  (discrete)  
 $D \propto P_{pk}$  (PRF)  $\tau_0 L_c = P_{AV} \tau_0$  (diffuse).

Table 1 summarizes these results for 4 cases with varying values of PRF,  $P_{pk}$ ,  $\tau_0$ , and  $L_c$  for a constant average transmitter power ( $P_{AV} = P_{pk}$  (PRF)  $L_c \tau_0$ ) The "reference" values of each quantity are PRF = f,  $P_{pk} = P$ ,  $\tau_0 = \tau$ , and  $L_c = 1$ . Each quantity is multiplied in turn by an integer constant N, keeping the average power constant in each case. The receiver bandwidth, B, is set to  $1/\tau_0$  and the number of coherent averages  $n_c$  is adjusted to maintain a constant effective sampling rate, PRF/ $n_c$ . The resulting dependencies of the signal-tonoise ratio and detectability are shown in the last two columns.

Table 1 can be simplified by writing the first 3 independent variables in terms of dimensionless quantities PRF/f,  $P_{\rm pk}/p$ ,  $\tau_{\rm o}/\tau$  and thus showing only the dependency on N. This has been done in Table 2. For the discrete targets we can see that all 4 cases have the same signal-to-noise ratio and the same detectability. For the diffuse case, because of the dependence on the resolution pulse width, the long uncoded pulse has a detectability which is a factor of N greater than the other 3 cases.

In those diffuse cases where high resolution is obtained, we note that using coded pulses gives the same signal detectability as using short uncoded pulses with either higher PRF or higher peak power. Pulse coding becomes desirable, then, when high resolution is needed and when the peak power cannot be increased due to transmitter limitations and the PRF cannot be increased, perhaps because of range aliasing problems.

Note that, given a set of Doppler power spectra obtained with any of the high resolution systems (cases 5, 6 or 8 in Table 2) the detectability can be increased by  $N^{1/2}$ , at the expense of range resolution, by averaging the spectra across N range gates. This effect occurs because the spectral noise power

	PRF	Ppk	Res PW T	Code Size L	Tot PW T P	PAN	Ð	ຊິ	Ę, S	N N	SNR	Ą
Discrete Targets 1) High PRF	Nf	E.	1		4	NÉ P.T	1/1	ĥ	8 8	∝ 1/Nr⊞	α NPat	æ NPfT
2) High Peak Power	ч	NP	Ļ	н	ч	NfPT	1/τ	Ø	я NP	∝ 1/Tm	× NPmt	œ NPfr
3) Long Pulse	°н.	р.	NT	н,	NT	NfPT	1/NT	Ħ	я Ч	∝ 1/NTm	∝ NPmT	∝ NPfT
4) Coded Pulse	μ	Ъ	ч	N	NT	NfPT	1/τ	E	$\propto N^2 p$	∝ N/Tm	∝ NPmT	∝ NPfT
Diffuse Taroote												
1) High PRF	Nf	ġ	t	ч	ч	NÉPT	1/τ	Nm	я РŢ	α 1/NTm	∝ NPmτ <sup>2</sup>	$\propto NPf\tau^2$
2) High Peak Power	щ	NP	ч	Ч	ч	NEPT	1/τ	Ħ	∝ NPT	$\propto 1/\tau_m$	∝ NPmτ <sup>2</sup>	$\propto NPf\tau^2$
3) Long Pulse	ч	р. Д	ΝT	÷	NT	NfPT	1/NT	ß	œ NPT	∝ 1/NTm	$\propto N^2 Pm t^2$	$\propto N^2 p f \tau^2$
4) Coded Pulse	Ŧ	, Cu	÷	N	NT	NfPT	1/τ	E	∝ N <sup>2</sup> P1	r ∝ N/TM	∝ NPmt <sup>2</sup>	∝ NPft <sup>2</sup>
PRF = Pulse repetition	on fre	quency		$P_{AV} = Av$	erage tr	ansmitte	ed = (P	RF)P <sub>pk</sub>	L da	P <sub>N</sub> = Noi	se power α	L <sub>c</sub> B/n <sub>c</sub>
P <sub>pk</sub> = Peak transmitte	d powel	н		B = Re	quired r	eceiver	bandwi	dth ∝	1/r	SNR = Sig	nal-to-nois	ie ratio = $P_s/P_N$
T <sub>0</sub> = Resolution puls	e widtl	्रम		n c = Nu	mber of	coheren	t avera	Iges		D = Sig	nal detecta	bility
L <sub>c</sub> = Number of bits .	in codi	<b>O</b>		P_= Re	turned si	ignal po	Wer					
$\tau_{p}$ = Total pulse wid:	th = Ľ	د م		-	«Pk L	2 (d	iscret	(ə				
					* P <sub>pk</sub> to	Le <sup>2</sup> (d	iffuse	~				

Table 1. Signal detectability for constant average transmitter power

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Table 2. Relative signal detectability for constant average transmitter power

Discrete Targets 1) High PEF N 1 1 1 1 1 1 1 N 1 1 1 N 1 1 N 1 N N N 2) High Peak Power 1 N 1 1 N 1 1 N 1 1 N 1 N N N 3) Long Pulse 1 1 1 N 1 N N 1 1 1 N 1 N N N N 4) Coded Pulse 1 1 1 N 1 1 N 1 1 N 1 N N N N N Diffuse Targets 5) High Peak Power 1 N 1 1 N 1 1 N 1 N 1 N N N N N 5) High Peak Power 1 N 1 1 N 1 N 1 N 1 N N N N N N 5) High Peak Power 1 N 1 1 N N 1 N 1 N N N N N N N 5) High Peak Power 1 N 1 1 N N 1 N 1 N N N N N N N N 5) High Peak Power 1 N 1 1 N N 1 N N N N N N N N N N N N	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		PRF	Ppk	Res PW T	Code Size L	Tot PW T	PAV	n,	ບີ	م م	A.	SNR	A
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$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	1) High PRF	N	r.	Ē	1	T	N	-	N	-i	1/N	N	N
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	2) High Peak Power	н	N	н	F	г	N	ч	н	N	1	N	N
4) Coded Pulse I I I N N N I N N N N N N N N N N N N	4) Coded Fulse       1       1       N       N       N       1       N	3) Long Pulse	T	Т	N	I	N	N	N/T	ч	г	1/N	N	N
Diffuse Targets 5) High Peak Power 1 N 1 1 1 N 1 N 1 1/N N N N 6) High Peak Power 1 N 1 1 N 1 N 1 N 1 N 1 7) Long Pulse 1 1 1 N 1 N 1 N 1/N N <sup>2</sup> N <sup>2</sup> 8) Coded Pulse 1 1 1 N N 1 1 N <sup>2</sup> N N N N 1 1 N <sup>2</sup> N <sup>2</sup> PNF = Pulse repetition frequency $P_{AV} = Average transmitted = (PRP)P_{pk}T_{p} P_{N} = Noise power \pi_{L_{c}} B/n_{c}P_{pk} = Peak transmitted power B = Required receiver bandwidth \approx 1/T_{0} SNR = Signal-to-noise ratio = P_{g}/P_{1}C_{0} = Resolution pulse width n_{c} = Number of coherent averages D = Signal detectability \Gamma_{c} = Number of bits in code Pgk = Returned signal power T_{0} = resolution pulse width = L_{c}T_{0}\pi_{P} P_{R} r_{D} L_{c}^{2} (discrete)$	Diffuse Targets 5) High Park Power I N I I I N I N N N N N 6) High Peak Power I N I I N I N I N I N N N N 7) Long Pulse I I I N I N N N I N N N N N N N N N N	4) Coded Pulse	П	ч	ч	N	N	N	ч	-1	и <sup>2</sup>	N	N	N
5) High PRF       N       1       1       1       1       1       1       1       N <t< td=""><td>5) High PRF       N       1       1       1       1       1       1       1       N       <t< td=""><td>Diffuse Targets</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<></td></t<>	5) High PRF       N       1       1       1       1       1       1       1       N <t< td=""><td>Diffuse Targets</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	Diffuse Targets												
6) High Feak Power 1 N 1 1 N 1 N 1 N 1 N 1 N 1 N 1 N 1 N	(6) High Peak Power1N11N1NNN7) Long Pulse11NNN1/N1NNN8) Coded Pulse111NNNNNNN8) Coded Pulse111NNNNNNNPr= Pulse reperition frequency $P_{AV}$ = Average transmitted = (PRF)P_{PK} T_P $P_N$ NNNN $P_{Pk}$ = Peak transmitted powerB= Required receiver bandwidth $\approx 1/\tau_O$ SNRSignal detectability $T_O$ = Resolution pulse width $n_c$ NNNNNN $L_c$ = Number of bits in code $P_g$ Returned signal power $P_{C}^2$ (discrete) $T_p$ = Total pulse width = $L_c$ $T_O$ $P_g$ $P_g$ $P_g$ $P_g$ $T_p$ = Total pulse width = $L_c$ $T_O$ $P_g$ $P_g$ $P_g$ $T_p$ = Total pulse width = $L_c$ $T_O$ $P_g$ $P_g$ $P_g$ $T_p$ = Total pulse width = $L_c$ $T_O$ $P_g^2$ $P_g$ $P_g$ $T_p$ = Total pulse width = $L_c$ $T_O$ $P_g^2$ $P_g$ $T_p$ = Total pulse width = $L_c$ $T_O$ $P_g^2$ $P_g$ $T_p$ = Total pulse width = $L_c$ $T_O$ $P_g^2$ $P_g$ $T_g$ = Total pulse width = $L_c$ $T_O$ $P_g^2$ $P_g$ $T_g$ = Total pulse width = $L_c$ $T_O$ $P_g^2$ $T_g$ <td>5) High PRF</td> <td>N</td> <td>н</td> <td>Ħ</td> <td>T</td> <td>н</td> <td>N</td> <td>г</td> <td>N</td> <td>ч</td> <td>1/N</td> <td>N</td> <td>N</td>	5) High PRF	N	н	Ħ	T	н	N	г	N	ч	1/N	N	N
7) Long Fulse 1 1 1 N 1 N 1 N 1 N 1 N 1/N $N^2$ $N^2$ 8) Coded Pulse 1 1 1 N N N N 1 1 N 2 N N N N FRF = Fulse repetition frequency $P_{AV} = Average transmitted = (PRF)P_{pk} T_{p}$ $P_{N} = Noise power \alpha L_{c} B/n_{c}P_{pk} = Peak transmitted power B = Required receiver bandwidth \alpha 1/\tau_{0} SNR = Signal-to-noise ratio = P_{s}/PT_{c} = Resolution pulse width n n_{c} = Number of coherent averages D = Signal detectability L_{c} = Number of bits in code P_{s} = Returned signal power T_{p} = Total pulse width = L_{c} \tau_{0}\alpha P_{pk} T_{0} L_{c}^{2} (discrete)$	7) Long Pulse 1 1 1 1 N 1 N N 1/N 1 N 1/N N <sup>2</sup> N <sup>2</sup> 8) Coded Pulse 1 1 1 1 N N N 1 1 N <sup>2</sup> N N N N N 1 1 N <sup>2</sup> N <sup>2</sup> N N N N N N N N N N N N N N N N N N N	6) High Peak Power	r,	N	٦	1	<b>,</b>	N	ч	ч	N	Ч	N	N
8) Coded Pulse 1 1 1 N N N 1 1 N <sup>2</sup> N N N N N N N N N N N N N N N N N N N	8) Coded Pulse 1 1 1 N N N 1 1 N <sup>2</sup> N N N N N N N N N N N N N N N N N Pyk <sup>2</sup> P Pyk <sup>2</sup> P N = Pulse repetition frequency $P_{AV} = Average transmitted = (PRF)P_{pk} T_{p} P_{N} = Noise power \propto L_{c} B/n_{c}P_{pk} = Peak transmitted power B = Required receiver bandwidth \propto 1/T_{o} SNR = Signal-to-noise ratio = P_{s}/P_{1}T_{o} = Resolution pulse width n_{c} = Number of coherent averages D = Signal detectability L_{c} = Number of bits in code P_{s} = Returned signal power T_{p} = Total pulse width = L_{c} T_{o}T_{p} = Total pulse width = L_{c} T_{o}T_{p} P_{pk} T_{o} L_{c}^{2} (discrete)\pi_{pk} T_{o} L_{c}^{2} (diffuse)$	7) Long Pulse	-1	ч	N	Ţ	N	N	N/T	п	N	1/N	N <sup>2</sup>	N <sup>2</sup>
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fluctuations are proportional to  $N^{-1/2}$ . Thus by using two types of postprocessing, these high resolution systems can give range resolution improved by a factor of N in regions of good SNR and, in regions of low SNR, a detectability degraded only by  $N^{1/2}$ , compared to the long pulse case.

**REFEREN CES** 

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