3.1A TECHNIQUES FOR MEASUREMENT OF VERTICAL AND HORIZONTAL VELOCITIES: MONOSTATIC VS BISTATIC MEASUREMENTS

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First we distinguish three techniques of direct measurement from which velocities may be obtained: (1) Doppler frequency shift, (2) spaced antenna drift, and (3) spatially modulated transverse beam measurements. Although in a fundamental sense these may be considered equivalent, they differ in their experimental implementation and thus warrant the distinction. Our discussion here will concentrate on the Doppler-frequency-measurement approach. It will compare bistatic with monostatic configurations as regards received power (or sensitivity), spatial resolution, Doppler shift and avoidance of ground clutter.

SENSITIVITY AND RESOLUTION

In the monostatic case the scattering volume is delineated by antenna beam width and pulse length. In the bistatic case the scattering volume may also be so delineated, but, since the geometry is different, so also are several other quantities: the shape and size of the scatter volume, the operative scale size in the atmospheric spectrum, and the Doppler relations. The power received bistatically may be expressed approximately as

$$P_{bi} \simeq \frac{P_T G_1}{4\pi R_1^2} \eta \, [b \; w \; h] \, \frac{A_2}{4\pi R_2^2} (\sin\xi)^2 \tag{1}$$

in which

P_T = transmitted power

G_1 = gain of the narrower-beam antenna

R_1, R_2 = distances from antennas to scale volume

η = radar reflectivity (cross section per unit volume)

b, w, h = base, width, and height of trapezoidal shaped scatter volume, V (Figure 1)

A_2 = receiving aperture of the wider-beam antenna

ξ = angle between electric vector of incident wave and scattering direction.

We have assumed the volume to be relatively restricted by narrow beams and a short pulse, and to be uniformly filled with the scattering medium. For the various quantities we can write, noting Figure 1 and 2,

$$b \simeq R_1 \, (\Delta\alpha_1) \csc\,(\theta/2) \tag{2}$$

$$w \simeq R_1 \, (\Delta\beta_1) \tag{3}$$

$$h \simeq \frac{C\tau}{2} \csc\,(\theta/2) \tag{4}$$

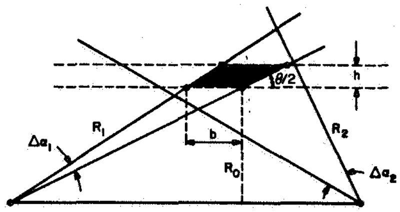

Figure 1. Scatter-volume geometry.

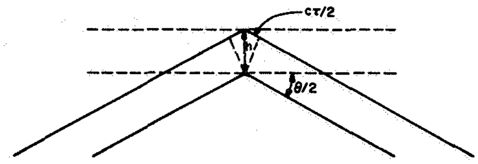

Figure 2. Delination of delay-shell width, h.

$$G_1 \simeq \frac{4\pi}{(\Delta\alpha_1)(\Delta\beta_1)} \tag{5}$$

$$\eta = 0.38\ C_n^{\ 2}\ \lambda^{-1/3}\ [\csc\ (\theta/2)]^{11/3} \tag{6}$$

Here we have also assumed for purposes of illustration a Kolmogorov turbulence spectrum. Substituting (2) through (6) in (1), we obtain

$$P_{bi} \simeq 0.015\ P_T\ C_n^{\ 2}\ \frac{A2}{R_2^{\ 2}}\ \lambda^{-1/3} C\tau\ [\csc\ (\theta/2)]^{17/3} \tag{7}$$

where we have picked a polarization perpendicular to the plane of scattering in order to make $\xi = \pi/2$. By letting the scatter angle θ go to π, this expression reduces to the power received in the monostatic case:

$$P_{mo} \simeq 0.015\ P_T\ C_n^{\ 2}\ \frac{A}{R_0^{\ 2}}\ \lambda^{-1/3}\ C\tau \tag{8}$$

To compare bistatic with monostatic it is convenient to place the latter at the midpoint of the bistatic path, and let the zenith angle χ of its beam-pointing be the tilt angle of the bistatic scattering plane with respect to the great circle plane (Figure 3). If, to simplify the argument, we consider scattering from a region in the transverse midplane, then

$$R_1 = R_2 \tag{9}$$

$$\alpha_1 = \alpha_2 = \theta/2 \tag{10}$$

and so

$$\frac{R_1}{R_0} = \frac{R_2}{R_0} = \csc\ (\theta/2) \tag{11}$$

It will also be helpful to express antenna apertures in terms of dimensions y

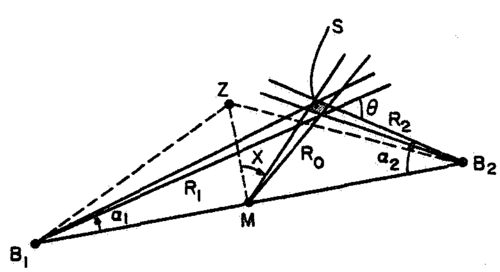

Figure 3. Geometry for bi-mono comparison. B_1, B_2 are bistatic termini. M is mono radar. Z is vertical. S is scatter volume.

(transverse to the bistatic path) and x (tilted up and in the plane of the bistatic path) so that

$$A = xy, \tag{12}$$

and

$$\Delta\alpha \simeq \lambda / x \tag{13}$$

$$\Delta\beta \simeq \lambda / y \tag{14}$$

The base, width, and height of the scattering volume, equations (2), (3), and (4), can then be written

$$b \simeq R_1 \frac{\lambda}{x} [\csc (\theta/2)] \tag{15}$$

$$w \simeq R_1 \frac{\lambda}{y} \tag{16}$$

$$h \simeq \frac{C\tau}{2} \csc (\theta/2) \tag{17}$$

With these preliminaries, we may now make comparisons of the bistatic with the monostatic case. The ratios of received powers, scattering volumes, and volume dimensions are

$$\frac{P_{bi}}{P_{mo}} \simeq \frac{P_{T,bi}}{P_{T,mo}} \frac{x_{bi}y_{bi}}{x_{mo}y_{mo}} \left(\frac{\lambda_{bi}}{\lambda_{mo}}\right)^{-1/3} \frac{\tau_{bi}}{\tau_{mo}} [\csc(\tfrac{\theta}{2})]^{11/3} \tag{18}$$

$$\frac{V_{bi}}{V_{mo}} \simeq \frac{(bwh)_{bi}}{(bwh)_{mo}} = \left(\frac{\lambda_{bi}}{\lambda_{mo}}\right)^2 \left(\frac{x_{bi}y_{bi}}{x_{mo}y_{mo}}\right)^{-1} \frac{\tau_{bi}}{\tau_{mo}} [\csc(\tfrac{\theta}{2})]^4 \tag{19}$$

$$\frac{b_{bi}}{b_{mo}} = \frac{\lambda_{bi}}{\lambda_{mo}} \left(\frac{x_{bi}}{x_{mo}}\right)^{-1} [\csc(\theta/2)]^2 \tag{20}$$

$$\frac{w_{bi}}{w_{mo}} = \frac{\lambda_{bi}}{\lambda_{mo}} \left(\frac{y_{bi}}{y_{mo}}\right)^{-1} \csc(\theta/2) \tag{21}$$

$$\frac{h_{bi}}{h_{mo}} = \frac{\tau_{bi}}{\tau_{mo}} \csc(\theta/2) \tag{22}$$

$$\frac{\ell_{bi}}{\ell_{mo}} = \frac{\lambda_{bi}}{\lambda_{mo}} \csc(\theta/2) \tag{23}$$

This last relation was obtained by noting that the geometry of Figure 2 applies also to the Bragg condition which selects scale size ℓ, if we replace h in the figure by ℓ, and $C\tau$ by λ.

Other things being equal (namely, powers, antennas, wavelengths, and pulse widths), the bistatic configuration has more received power to work with, since csc $(\theta/2)$ is greater than unity and is raised to a moderately high power; this margin increases markedly as the bistatic baseline is lengthened. On the other hand the size of the scattering volume V_{bi} also increases drastically so that spatial resolution suffers.

For many purposes it would be desirable in the bistatic case to trade sensitivity for resolution. This can most readily be done by going to smaller wavelengths and shorter pulses -- i.e., by decreasing the ratios of λ_{bi} to λ_{mo} and τ_{bi} to τ_{mo}. Since the monostatic antenna is probably already as large as feasible, it may be unrealistic to increase y_{bi} or x_{bi} over y_{mo} or x_{mo}. Decreasing the pulse width τ_{bi} will help restore the height (or slant height) resolution h_{bi} for a given scatter angle θ. Similarly, decreasing the wavelength λ_{bi} will help restore the lateral resolutions b_{bi} and w_{bi}, for a given scatter angle and antenna size.

To illustrate with an example, if the bistatic pulse width were reduced to half the monostatic ($\tau_{bi} = \tau_{mo}/2$), then by equation (22) the scatter angle θ could drop as low as 60° before the bistatic height resolution became worse than the monostatic. Then, referring to equation (18), and still holding $\tau_{bi} = \tau_{mo}/2$, if we wish to keep the bistatic received powers no less than the monostatic for all scatter angles from 180° to 60°, we must make $\lambda_{bi} = \lambda_{mo}/8$ (assuming transmitter powers and antenna apertures are the same in the two cases). This would improve the spatial resolution, making $b_{bi}/b_{mo} = 1/2$, from equation (20), and $w_{bi}/w_{mo} = 1/4$, from equation (21). Thus if we do not insist on better resolution in the bistatic case, we can relax the size of its antennas, by a factor of at least 1/2 in elevation, and 1/4 in width. This will reduce the received power, but, since it is detectability rather than received power that is important, we are in no trouble because the antenna temperature at the higher frequency will be lower by a more-than-compensatory factor. (We are presuming that the monostatic radar operates in the VHF or UHF region and the bistatic in the UHF or SHF region).

In short, without indulging in any optimization studies, it appears safe to say that a bistatic system can retain the same spatial resolution as a monostatic, without impairment of sensitivity, if the pulse length and wavelength are shortened and the scatter angle does not get too small.

DOPPLER SHIFT

Now we turn to Doppler measurement, keeping, for purposes of comparison, the same configuration sketched in Figure 3. The monostatic radar measures the radial component of wind velocity:

$$f_{D,mo} = -\frac{2}{\lambda}(v \sin\chi + w \cos\chi) \tag{24}$$

where v and w are horizontal and vertical wind components. The bistatic radar measures the component of wind normal to the elliptical constant-delay shells. Near the midpath transverse plane, where the contribution from the along-the-path wind component u is small, the Doppler frequency is

$$f_{D,bi} = -\frac{(\sin\alpha_1 + \sin\alpha_2)}{\lambda}(v \sin\chi + w \cos\chi)$$

$$= -\frac{2\sin(\theta/2)}{\lambda}(v \sin\chi + w \cos\chi) \tag{25}$$

When both bistatic and monostatic are looking at the same general scattering region, the ratio of the Dopplers is

$$\frac{f_{D,bi}}{f_{D,mo}} = \left(\frac{\lambda_{bi}}{\lambda_{mo}}\right)^{-1} \sin(\theta/2) \qquad (26)$$

For the numbers chosen above, which keep the spatial resolutions comparable,

$$\frac{f_{D,bi}}{f_{D,mo}} = \left(\frac{1}{8}\right)^{-1} \left(\frac{1}{2}\right) = 4 \qquad (27)$$

at $\theta = 60°$. For the same wind, the bistatic Doppler shift is greater owing to the shorter wavelength; less Doppler resolution is needed to achieve a given velocity resolution, and therefore a shorter measurement time is permissible. Data from two or more zenith angles (or tilt angles) permit separation of v and w components.

In order to obtain the orthogonal wind component u, the monostatic radar merely has to change its azimuthal pointing. For the bistatic system, the situation is a bit different, if the same two path terminals are to be used. When the scatter volume lies in the great-circle plane, the bistatic Doppler is given by

$$f_{D,bi} = -\frac{u}{\lambda}(\cos\alpha_1 - \cos\alpha_2) - \frac{w}{\lambda}(\sin\alpha_1 + \sin\alpha_2) \qquad (28)$$

To measure u, α_1 and α_2 must differ appreciably -- i.e., the scatter volume must be well removed from the center of the path (Figure 4). It is not immediately apparent how best to make a comparison between bistatic and monostatic in this case, though one expects the advantage to lie with the latter.

GROUND CLUTTER

In measuring the vertical velocity component w there is one respect in which the bistatic has an advantage. That advantage lies in the avoidance of ground clutter. Consider a bistatic configuration with the scatter volume located at midpath in the great-circle plane. Referring to Figure 3 with χ set equal to zero, the two Doppler frequencies are

$$f_{D,bi} = -\frac{2w}{\lambda}\sin(\theta/2) \qquad (29)$$

$$f_{D,mo} = -\frac{2w}{\lambda} \qquad (30)$$

We expect the vertical velocity w to be small and to vary about zero. Ground returns are also likely to have zero Doppler or nearly so. Confusion can arise when the ground echo has the same delay as the atmospheric. A ground feature of altitude z will extend above the transmitter's (and receiver's) horizon when (see Figure 5)

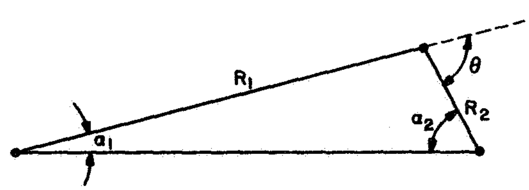

Figure 4. Off-center bistatic scattering.

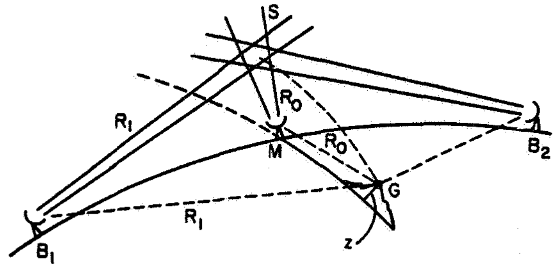

Figure 5. Interference from ground scatter into sidelobes. Ground feature at G and height z relative to radio horizons at M, B_1 and B_2.

$$z \geqq \frac{R_1^2}{2r_0} \text{ , in the bistatic case} \tag{31}$$

and

$$z \geqq \frac{R_0^2}{2r_0} \text{ , in the monostatic case} \tag{32}$$

Using equation (11), the ratio of these heights is

$$\frac{z_{bi}}{z_{mo}} = \left(\frac{R_1}{R_0}\right)^2 = (\csc(\theta/2))^2 \tag{33}$$

Thus, to follow the example used earlier, for $\theta = 60°$ this ratio is 4. A ground feature has to be four times as high in the bistatic, as compared with the monostatic, case before it causes the same interference problems. This advantage is of importance both for measuring vertical wind and for studying quasi-specular reflection at and near vertical incidence.

A HYBRID SUGGESTION

This discussion should not conclude without mention of a hybrid case. Consider a transmitter located at T in Figure 6 and a scatter volume located at S. There are two bistatic receivers, located at R_A and R_B. The planes specified by TSR_B and TSR_A slope toward each other and constitute two sides of a tetrahedron. There is a third receiver, at the transmitter location T, forming a monostatic radar. Doppler frequencies measured at the three receivers yield three wind components from which the vector wind can be determined. All three measurements are made on essentially the same volume of air at the same time, in contrast to most other techniques (except a triple Doppler). Futhermore the scatter volume can be moved vertically upward and downward, so that a true vertical profile of vector wind can be obtained.

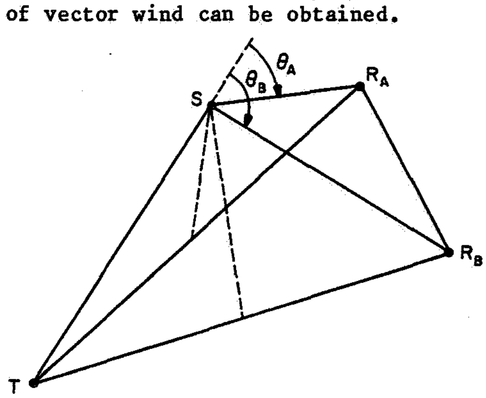

Figure 6. A combination monostatic and bistatic system.