

## 3.7A RADAR INTERFEROMETER MEASUREMENTS

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## INTRODUCTION

WOODMAN (1971) appears to have been the first to use the interferometer technique in scatter probing of the ionosphere. His observations allowed him to determine the position at which the radar beam was normal to the magnetic field (and hence the dip angle) above the Jicamarca Observatory with very high accuracy. More recently the technique has been extended, with the inclusion of Doppler information, and applied to studies of equatorial E-region electrojet plasma turbulence (FARLEY et al., 1981; KUDEKI et al., 1982), equatorial F-region irregularities (KUDEKI et al., 1981), and auroral electrojet irregularities (PROVIDAKES et al., 1983). There have been a few references to interferometer observations in the MST radar literature also, but the observations have been of various types, usually different from those in the ionosphere. One purpose of this paper is to try to clear up any possible confusion. As yet there do not appear to have been any successful 'true' (in the sense that we describe below) MST radar interferometer observations, but only a few brief attempts have been made.

## THE BASIC IDEA

In its simplest form, a radar interferometer consists of two separated receiving antennas, each with its own receiver, and a single transmitting antenna, which could be either a third antenna or one of the two receiving antennas. Suppose all the antennas are pointed vertically (extending the results to oblique observations is trivial) and that there is a single small (~ point) target located at some small angle  $\theta$  from the zenith in the plane defined by the vertical and the line joining the phase centers of the two receiving antennas. If the range to the target is much greater than the separation,  $L$ , between the receiving antennas (as is always the case in practice), then there will be a small time delay, given by  $L\sin\theta/c$ , between signal reception at the two antennas. This delay translates into a phase difference of

$$\Delta\phi = kL\sin\theta \approx kL\theta \quad (1)$$

where  $k=2\pi/\lambda$  is the radar wave number. In the absence of noise, this phase difference can be measured easily by forming the complex cross product of the two signal voltages; i.e.,

$$F = \frac{\langle V_1(t)V_2^*(t) \rangle}{\left[ \langle |V_1|^2 \rangle \right]^{1/2} \left[ \langle |V_2|^2 \rangle \right]^{1/2}} = e^{i\Delta\phi} \quad (2)$$

where  $\langle \rangle$  represents an ensemble or time average. This phase measurement determines the angular position of the target; changes in time of this angle determine an angular velocity, to which range information can be added to give a linear velocity component in the direction of the interferometer baseline. In order to avoid ambiguities of multiples of  $2\pi$  in the phase measurement, it is desirable to have the beamwidth of one or more of the antennas narrower than the interferometer lobe spacing. As a rough rule of thumb, this is accomplished if the two receiving antennas, for example, are 'touching' (their linear dimension in the baseline direction is equal to  $L$ ).

## COMPLICATING FACTORS

There are a number of effects encountered in practice which also must be kept in mind. For example:

1. Receiver noise. Since the noise in the two receivers is uncorrelated, it will not contribute to the cross product in (2). It will change the normalization constant, but this is not important.

2. Cosmic noise and/or other correlated interference. These are common to both interferometer channels and so will cause problems. They must be measured separately (with the transmitter off, say) and subtracted out. If the signal is weak and the correlated interference changes with time and/or is somewhat changed by the presence or absence of the transmitter, the subtraction is somewhat changed by the presence or absence of the transmitter, the subtraction may be difficult to do accurately.

3. Multiple targets. If there are several targets in the beam at once, the cross product in (2) will be a vector sum of phasors whose magnitude and direction represent the strength and angular position of the separate targets, and the magnitude of  $F$  will be less than unity even if the signal/noise ratio is high. If all the targets move with the same velocity and maintain the same relative strengths, the velocity can still be determined from the rate of change of the mean phase angle, but if the situation is more complicated it may become impossible to interpret the phase changes.

4. Broad target. Much the same arguments apply to a target with an appreciable angular width. The magnitude of  $F$  gives a measure of the width; i.e., if we can describe the target as having an angular variance  $\sigma^2$ , the magnitude of  $F$  is roughly  $\exp(-k^2 L^2 \sigma^2 / 2)$ .

## RADIAL MOTION AND CROSS-SPECTRAL ANALYSIS

So far we have neglected target motion in the direction of the radar beam. Such motion will produce Doppler shifts in the received signals. The Doppler shifts thus give additional information about the scattering medium and also often can be used to separate the contributions to the signal from multiple targets, if the individual targets have different radial velocities as well as different angular positions. The analysis of the signals in this case is similar to that given above, but we introduce the additional step of first Fourier transforming the two signals. Next, we replace the simple voltage cross product with the cross spectrum of the two individual spectra; i.e., we compute

$$S_{12}(\omega) = \frac{\langle V_1(\omega) V_2^*(\omega) \rangle}{\langle |V_1(\omega)|^2 \rangle^{1/2} \langle |V_2(\omega)|^2 \rangle^{1/2}} \quad (3)$$

where  $V_1(\omega)$  and  $V_2(\omega)$  are the (voltage) Fourier transforms of  $V_1(t)$  and  $V_2(t)$ . By analogy with (2) and from the discussion above of broad targets, it is fairly easy to see that, in terms of the target parameters, the cross spectrum is

$$S_{12}(\omega) = e^{ikL\bar{\theta}_\omega} \exp(-\frac{1}{2}k^2 L^2 \sigma_\omega^2)$$

where  $\bar{\theta}_\omega$  and  $\sigma_\omega$  are the mean angular position and spread of the target giving a Doppler shift of  $\omega$ . This result neglects noise contributions, etc. For a more complete discussion see FARLEY et al. (1981).

This measurement and analysis procedure has proved to be a very powerful

tool for investigating plasma turbulence in the ionosphere, as mentioned in the introduction, but full cross spectral analysis does not appear to be useful for MST applications. To see why, let us assume a typical vertical velocity to be  $\sim 5$  m/s (e.g., WOODMAN and GUILLEN, 1974), which would give a Doppler shift of 0.1-0.2 Hz at a radar frequency of 50 MHz. To achieve a reasonable frequency resolution of  $\sim 0.1$  Hz, say, would require an observation time of at least 1 min for each Fourier transform, and several computations of the cross spectrum must be averaged to obtain a meaningful result. In the required integration time of several minutes, a scattering center traveling with a horizontal velocity of say 10 m/s will move several kilometers, probably enough to move the scatterer out of the radar beam and certainly enough to give a phase change of many radians. Operating at higher radar frequencies will improve the situation, but probably not by enough.

In practice, then, one must revert to the analysis of (2), which is also much simpler computationally, and ignore the vertical velocity information in the interferometer calculations. It is still of course possible to obtain mean estimates of the vertical velocity by calculating the power spectrum of either of the received signals, or even the magnitude of the cross spectrum, which is essentially the same thing. It is only the phase information in the cross spectrum which is of no use in MST applications.

#### MST INTERFEROMETER OBSERVATIONS

Some attempts to observe horizontal velocities at Jicamarca in the way just described have been mentioned by RUSTER and WOODMAN (1976) and RUSTER et al. (1978). The attempts were unsuccessful, however. The successive phase angles were more or less randomly distributed, indicating that scattering centers were distributed throughout the radar beam and that the scattering could not be modeled as coming from one or two discrete moving centers. The horizontal velocities shown in RUSTER et al. (1978) were obtained from a version of Spaced Antenna Drift (SAD) analysis. The two time series of the vertical velocities obtained from Fourier analysis of the signals received on the two separated antennas were cross correlated and the horizontal velocity was determined from the delay at which the cross correlation maximized. The attempts at the 'true' interferometer analysis were not very exhaustive, however, and there is some reason to hope that they might be successful at times in the mesosphere at least (WOODMAN, private communication, 1983). Some more recent observations at Jicamarca have led to similar conclusions (CORNISH, private communication, 1983). (Added note: One successful mesospheric observation was described at this workshop by M. Ierkic and J. Rottger.)

Some work using the SOUSY radar with spaced antennas and phase coherent receivers has been described by ROTTGER and VINCENT (1978) and VINCENT and ROTTGER (1980). The voltages were cross correlated, and the lag at which the magnitude of the cross correlation was greatest was used to determine the velocity (the SAD technique). In contrast, the technique discussed here utilizes the phase of the cross correlation at zero lag to determine position. The SOUSY work did involve using phase information to determine the character of the scattering medium. The 'radiation pattern', so to speak, of the medium was measured by steering the lobe pattern of the interferometer array numerically in the data processing. In one example it was found that the echoes from an altitude of 2.44 km corresponded to isotropic scatterers but those from 3.79 km closely approximated a specular partial reflection. This sort of analysis is related to, but not quite the same as, measuring the magnitude of  $F$  in (2) to determine in some sense the angular width of the distribution of scatterers. This width would be very small for a quasi-specular reflection, but would roughly equal the antenna beam width for isotropic and uniformly distributed scatterers.

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