4.4A DETERMINATION OF VERTICAL AND HORIZONTAL WAVELENGTHS OF GRAVITY WAVES

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The determination of horizontal and vertical wavelengths of gravity waves obviously relies on measurements of wave parameters in horizontal and vertical directions, if one does not want to use the dispersion relation for model calculations. A very suitable parameter, measured fairly easily with MST radars, is the fluid velocity.

Average velocities and superimposed turbulent fluctuations are much larger in the horizontal than in the vertical direction. Vertical and horizontal fluid velocities due to wave-like events are mostly about equal in magnitude. Vertical fluid velocities due to waves therefore can be more reliably detected than horizontal velocities. This is confirmed by MST radar observations.

Other parameters, measured by MST radars, also give information about wave parameters. The radar reflectivity for instance can be modulated, viz. by the turbulence intensity or the undulation of reflecting surfaces. This indirectly could be used to determine spatial and temporal scales. However, from experience, the vertical fluid velocity appears to be most suitable.

To investigate wave events, we at least have to detect a full period of a wave, but longer wave trains of course would be much more suitable. For most reliable estimates the wave event should be fairly monochromatic and stationary. It may be possible to filter different waves from a continuous spectrum if these are separated widely enough in frequency and have amplitudes significantly different from the amplitudes of background turbulence. Only these conditions will allow to deduce acceptable estimates of vertical and horizontal wavelengths.

The vertical velocity w due to gravity wave is proportional to

$$\exp[i(k_x x+k_y y+k_z z-wt)],$$

where $k_x = \frac{2\pi}{\lambda_x}$, $k_y = \frac{2\pi}{\lambda_y}$, $k_z = \frac{2\pi}{\lambda_z}$

and x, y, z denote a proper coordinate system (e.g., east, north, vertical), t is the time, and $w = \frac{2\pi}{T}$, where T is the period of the wave. We have to take into account that the radar measurements of w are in a fixed frame of reference and actually $w_o = w - \underline{k} \cdot \underline{U}o$ is Doppler shifted, if a background wind $\underline{U}o$ is observed. Thus, also the deduced wavelengths λ_x, λ_y and λ_z are given in this fixed reference frame and have to be converted if one would know $\underline{k} = (k_x, k_y, k_z)$ and $\underline{U}o$.

Keeping x, y and z constant (i.e., measuring in a fixed portion) allows determination of ω. If a significant amplitude of w is detected at frequency ω, measuring the phase of its oscillation as function of x, y and z allows determination of λ_x, λ_y and λ_z. This schematically is depicted in Figure 1. Assume a wave in the x-z plane and pointing the antenna beams to two directions; this yields radial velocities w_I and w_{II} being caused by vertical fluid velocities of the wave and a background wind Uo. Assuming that the background wind is the same at both beam positions, and neglecting horizontal fluid velocities (this is permitted as long as the beam directions are close to the zenith), the vertical fluid velocities can directly be deduced from the measurements. Swinging the beam can most appropriately be done with the interferometer technique, described in another workshop paper. Of course it also can be done by the traditional method pointing transmitter beams at different directions and receiving with the same beam direction. Briefly speaking, complex data are taken at 3 antennas (only 2 are shown in Figure 1 for clearness) and phase shifts $\Delta\gamma$ are introduced during the data analysis to swing the receiver antenna beam. Complex addition of the 3 data sets yields the velocities according to the different antenna beam directions.

An example of such an experiment is shown in Figure 2, where the interferometer beam was pointed at 1.2° zenith angle to an azimuth 207° (closed circles and 27° (open circles). This corresponds to an average horizontal probing distance of 800 m at 20 km altitude. It is very evident that the oscillations of the vertical velocity w are displaced in phase at the two positions. This indicates that a horizontally propagating wave was observed. Making use of three beam positions, one can calculate the horizontal wave vector $k_h = (k_x, k_y)$ and find the horizontal wavelength λ_h. This was done by applying a cross spectrum analysis, which of course also yields the mean amplitude of the vertical velocity $\bar{w}$ and the mean period $\bar{T}$ of the wave. Results are shown in Figure 3. In this diagram additionally the horizontal width b of the volume probed with the applied beam widths is indicated. Since b is much smaller than the horizontal wavelength λ_h, there are no objections that this method of beam swinging is applicable. It also proves that obviously phase velocities of gravity waves cannot be measured with fixed vertical beams as often was used as an argument against the spaced antenna drifts method.

Knowing k_h from these measurements, one can find the horizontal phase velocity V_p and the propagation direction α_v of the wave. These are shown in Figure 4, where additionally the average background wind speed Uo and its direction α_u, measured with the same interferometer set-up are inserted. It shall be mentioned that Uo and α_u are in agreement with radiosonde data. When investigating further wave parameters one can make use of the measured wind speed and direction as well as the measured frequency and propagation vector of the gravity wave to correct for the Doppler shift.

From the phase shift of the vertical velocity oscillations with height, the vertical wavelength can be deduced. Since we observed coherent wave events only over a few kilometers height range and the phase differences are not too pronounced, the measurement of the vertical wavelength is not so consistent. Firstly, the vertical wavelength may change with altitude, secondly the mean frequency of the wave changes with altitude and we always observed much less than one cycle of the wave in the vertical direction. Anyhow, by filtering the different spectral amplitudes, one could deduce an average phase shift with height as is shown in Figure 5 for a shorter observation period than in Figures 3 and 4. From the average phase shift of the dominating component at 4.6 min, one can find a mean vertical wavelength of about 15 km. This value has to be taken with care because of the indication of non-coherency of these wave events.

In summary we find from the analyses of these wave events between 15 km and 25 km altitude that

1) horizontal wavelengths are about 10-20 km, and increasing with height,
2) vertical wavelengths are fairly similar to horizontal wavelengths,
3) wave periods increase with altitude from about 250 s to 300 s,
4) mean vertical velocity amplitude increases with height from about 8 cm s^{-1} to 10 cm s^{-1},
5) direction of wave propagation is almost constant with altitude,

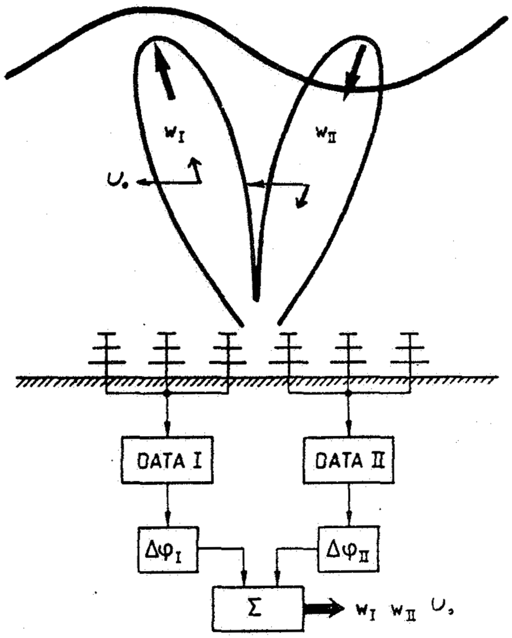

Figure 1. Principle of an MST radar interferometer. Complex data are taken independently with at least 2 antenna sets, introducing a phase shift allows swinging the antenna beam to observe the horizontal velocity Uo and vertical velocities w at different positions of a gravity wave.

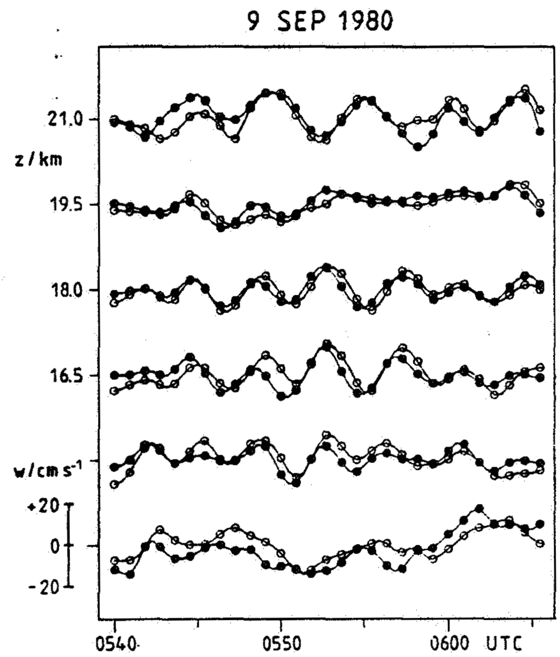

Figure 2. Vertical velocities w, measured at zenith angles of ± 1.2°.

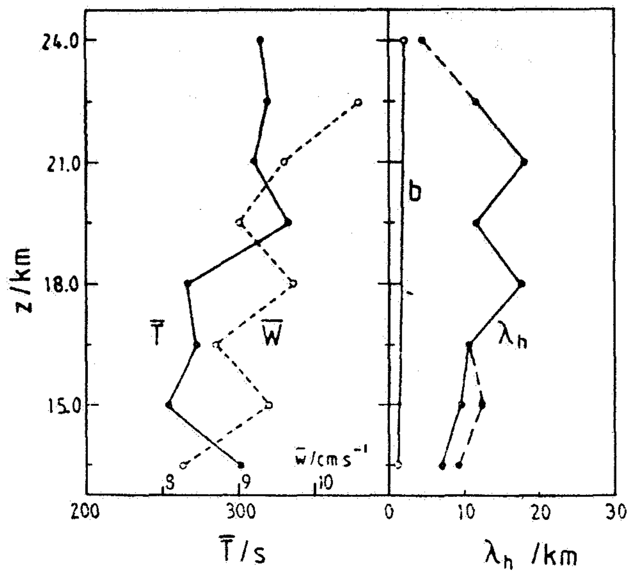

Figure 3. Average period $\overline{T}$, average vertical velocity amplitude $\overline{w}$, and horizontal wavelength λ_h. Averaging was done over the period range 3.5-7.5 min, with the amplitude as weighting function. Observation period is 0446-0616 UTC, on 9 September 1980.

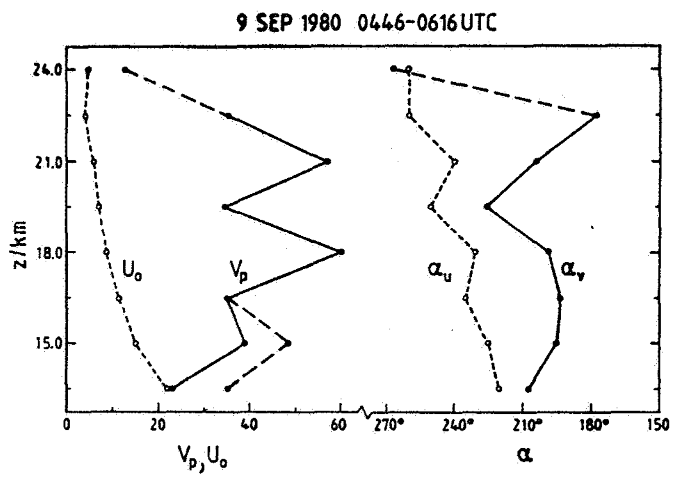

Figure 4. Height profiles of wave's phase velocity v_p and propagation direction α_v as well as background wind speed Uo and direction α_u.

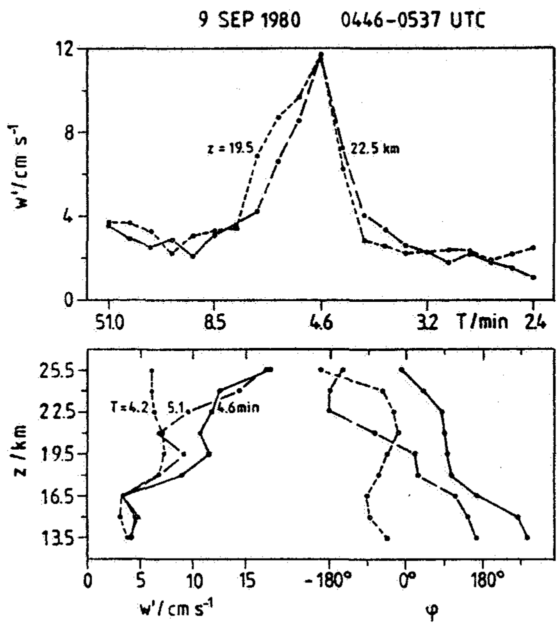

Figure 5. Spectra of vertical fluid velocity w^1, measured at 19.5 km and 22.5 km altitude (upper diagram). Height profile of vertical velocity amplitude and phase of oscillations at T = 4.2, 4.6 and 5.1 min.

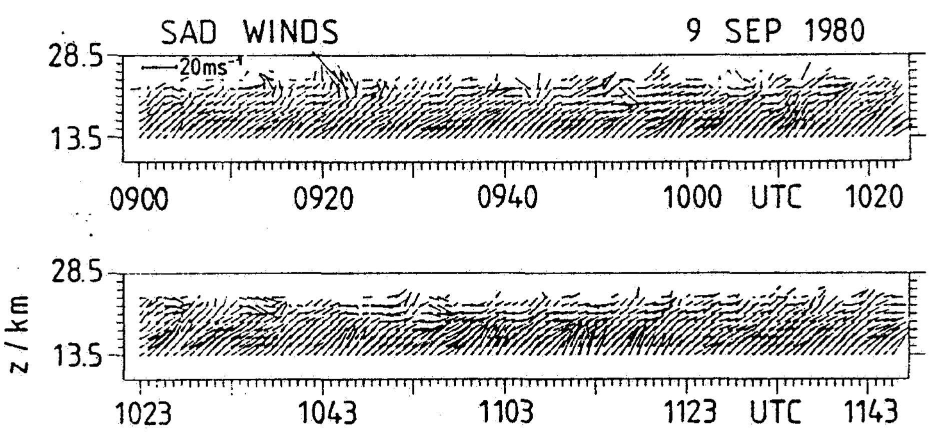

Figure 6. Vector field of lower stratospheric winds measured with the spaced antenna drifts method.

6) phase speed of waves ($\approx$40 ms^{-1}) is almost constant with altitude,
7) phase speed and direction are similar to wind speed and direction close to the lowest heights observed ($\approx$ 13.5 km),
8) wave phases propagate downward,
9) from 7) and 8) it is deduced that these waves are probably generated at wind shear regions close to 13.5 km.

Horizontal fluid velocities occasionally also could be used to detect gravity waves, as is for instance shown in Figure 6. The wind arrows in the lower altitudes obviously indicate some oscillation in direction at periods of 4-8 minutes (e.g., around 1000 UTC and 1115 UTC) which can be explained by atmospheric gravity waves. These measurements would have been fairly difficult with the Doppler method which always measures a superposition of vertical and horizontal velocities. These cannot easily be separated for wave events since the beam would point to different phase locations of the wave.

Waves with shorter vertical wavelength, such as mountain lee waves may have a substantial variation of the horizontal velocity which then could be used to determine the wavelength. In Figure 7 an example of wind speed and direction measured during a lee wave event is presented (from ROTTGER et al., 1981). One clearly summarizes an oscillating pattern with altitude which also was observed in the vertical velocity. Lee waves are standing waves and stationary as long as the background wind does not change. As the background wind increases with height, also the vertical wavelength does. To find a relation of the wavelength and the background wind, a polar plot as shown in Figure 8, may be useful to measure the vertical wavelength (e.g., by determining the height difference between the crossovers of the velocity phasor), one also can easily obtain the sense of rotation of the wave vector with height from this polar plot. Analyses of vertical wavelengths of tides are done in a similar way.

In summary we recognize that a single MST radar can contribute substantially to investigations of many parameters of atmospheric gravity waves by applying the interferometer technique or the traditional beam steering technique. Investigations of long period gravity waves may need multistatic systems, but just one radar can be used to determine some essential parameters of standing waves, such as lee waves.

REFERENCE

Rottger, J., T. Y. Kang, and M. Y. Zi (1981), Mountain lee waves in radar wind profiles, MPAe-Report, W-00-81-36.

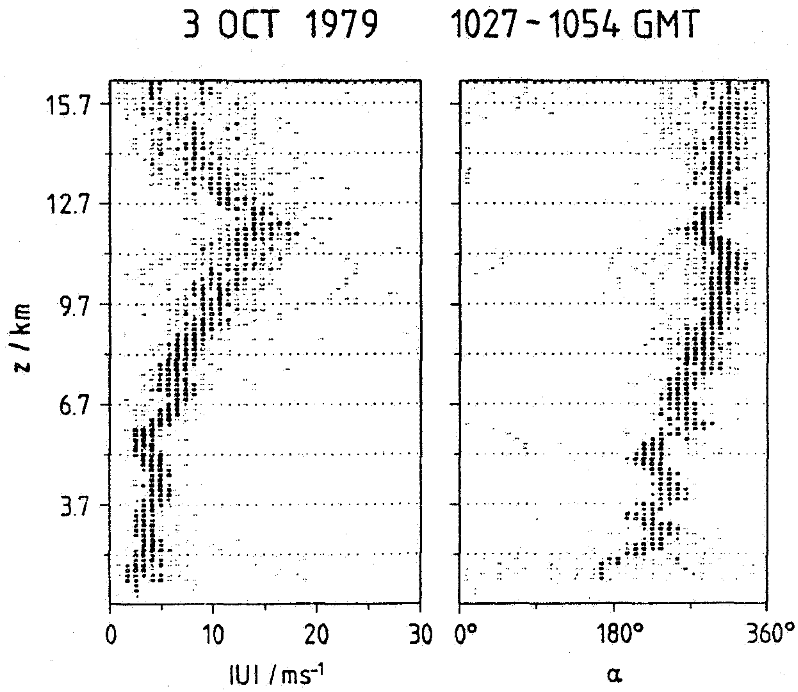

Figure 7. Height profiles of speed /U/ and direction α of the horizontal wind measured during a lee wave event over the Harz mountains.

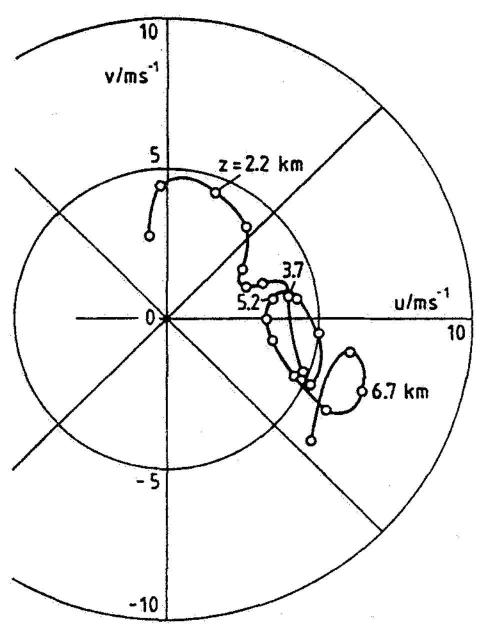

Figure 8. Polar plot of horizontal wind velocity measured during a lee wave event. The open circles denote height increments of 300 m.