# 8.3B A NOTE ON THE USE OF COHERENT INTEGRATION IN PERIODOGRAM ANALYSIS OF MST RADAR SIGNALS

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## ABSTRACT

The effect of coherent-integration on the periodogram method of estimating the power spectra of MST radar signals is examined. The spectrum estimate usually is biased, even when care is taken to reduce the aliasing effects. Due to this bias, the signal power for Doppler shifted signals is underestimated by as much as 4 dB. The use of coherent integration in reducing the effect of aliased power-line harmonics is pointed out.

## INTRODUCTION

In experiments used to probe the atmosphere with sensitive high power radars, the autocovariance functions (ACF) or power spectra of atmospheric returns are estimated by a processor. Coherent integration is a technique that reduces the data input rate to the processor without unduly distorting the information-bearing part of the returns. In experiments that use phase codes and decoding to obtain a fine range resolution, it is possible to cut down on computations in the processor by relegating the decoding operation till after coherent integration. The use of coherent integration has been discussed for ACF processing by WOODMAN and GUILLEN (1974), and SCHMIDT et al. (1979). Discrete Fourier transform (DFT) methods, especially for estimation of power spectra as time-averaged periodograms (OPPENHEIM and SCHAFER, 1975; COOLEY et al., 1979), increasingly are being used in atmospheric radar experiments (GAGE and BALSLEY, 1978; WOODMAN, 1980; BALSLEY, 1981). In this note, the use of coherent integration in periodogram analysis of MST radar signals is examined.

## SPECTRAL WINDOW AND ALIASING

Let $z(t)$ be the complex signal after coherent demodulation for a given range. Samples of $z(t)$ are input at discrete intervals T, usually the pulse repetition interval (PRI), to the processor. The processor can handle only a finite number N of samples for each range. For these samples the processor obtains first their N-point DFT $Z(f)$, and then estimates the power spectrum $S(f)$ of the atmospheric returns as the time-averaged periodogram

$$P(f) = \langle Z(f)\, Z^*(f) \rangle \tag{1}$$

The periodogram estimate is obtained over the frequency range $(-F,F)$, where $F = 1/2T$, at N points with a frequency resolution $1/TN$. At each point in the above range $P(f)$ is obtained as a discrete convolution of $S(f)$ with the standard periodogram window $H(f)$ (COOLEY et al., 1979).

$$H(f) = \frac{1}{N}\{\sin(\pi fN)/\sin(\pi f)\}^2 \tag{2}$$

For large N, $H(f)$ approximates a delta function. For modest values of N, however, smearing of a strong undesirable feature such as the ground clutter can mask the atmospheric signal. Components of $S(f)$ outside the range $(-F,F)$ are aliased or folded-in. For most MST radar experiments, this range is considerably wider than a baseband containing features of interest in $S(f)$. With coherent integration of $z(t)$ over several radar sweeps, it is possible to reduce the frequency range and to improve the frequency resolution.

With coherent integration, M successive samples of z(t) are accumulated and it is these accumulated samples that are fed to the processor. Thus coherent integration is equivalent to first obtaining a process g(t) that is the moving average of z(t).

$$g(kT) = \frac{1}{M} \sum_{m=0}^{m=M-1} z(kT - mT) \tag{3}$$

and then coarsely sampling g(t) at multiples of MT. The processor now obtains the time-averaged periodogram from N-point sequences of samples of g(t), in the way outlined above for z(t).

The effect of the moving average operation in (3) is to weight S(f) with a frequency window

$$G(f) = \frac{\sin^2 (\pi\ f\ T\ M)}{M^2 \sin^2 (\pi\ f\ T)} \tag{4}$$

This window is a comb function (SCHMIDT et al., 1979) with maxima at multiples of 1/T. Between any two maxima there are M-1 minima with zero weight at multiples of 1/MT. Due to sample time MT, the basic frequency range is now (-F/M, F/M), sampled at N points with a frequency resolution 2F/MN. Th components outside this range are weighted by G(f) and aliased. To reduce undesired effects of this aliasing it is necessary to ensure that the expected Doppler shifts of the atmospheric returns remain smaller than F/M.

## DISCUSSION

The behavior of the frequency window G and the folded parts is illustrated for two values of M in Figure 1. The frequency axis is normalized as fTM. The parts of S(f) outside the normalized range (-0.5,+0.5) are aliased.

When care is taken to avoid the aliasing effect discussed above, the averaged periodogram provides a reasonable estimate of the signal spectrum S(f). Over the basic window (-F/M, F/M), these estimates are still biased by the window G(f). Usually the Doppler-shifted returns have a bandwidth small compared to the folding frequency F/M. The effect of frequency weighting is therefore negligible on the estimates of Doppler shift and spectral width. The signal power, however, depends on the Doppler shift. Figure 2 shows the bias in signal power as a function of the normalized frequency fTM. For unaliased signals this bias may cause the signal power to be underestimated by as much as 4 dB. For signals that are aliased only once, a Doppler profile can usually be constructed, but the signal power is underestimated by an even larger amount.

In most radar experiments, the computational advantage with the use of coherent integration outweighs the small bias in the spectrum estimates. In some experiments, however, this bias can be critical. This is especially true of the aspect-sensitivity experiments (GAGE and GREEN, 1978; FUKAO et al., 1980) that have been used to detect enhanced specular reflection at quasi-vertical incidence. Unless the Doppler shifts are small compared to F/M, a 2-4 dB reduction of signal power estimates for off-vertical echoes is expected as a result of this bias alone.

The occurrence of nulls in the part of the spectrum that is folded can be used to reduce the contribution of the aliased power-line harmonics. When the number M of coherently integrated samples is large, both M and the PRI T can be adjusted to ensure that the power-line harmonic frequencies coincide with (or are close to) the nulls in G(f).

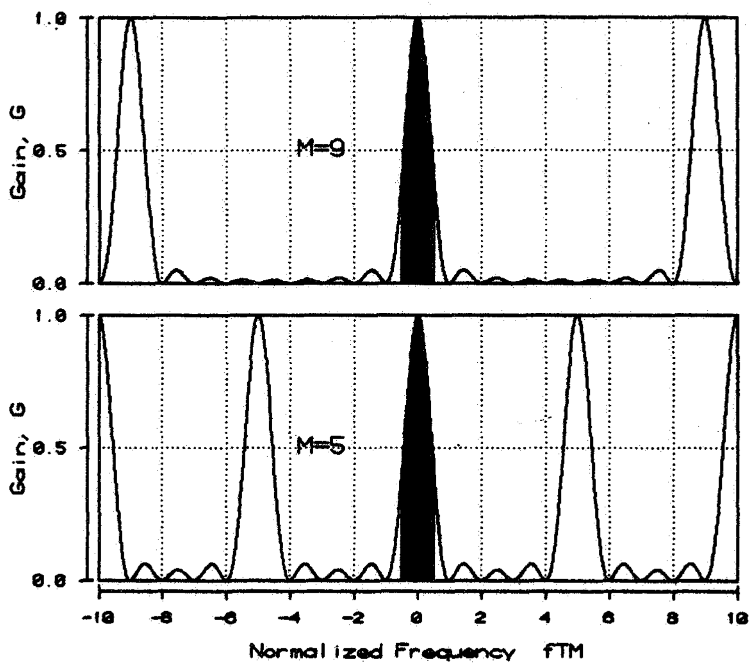


Figure 1. Gain of the frequency window G due to coherent integration over M samples as a function of the normalized frequency fMT, where T is the sampling rate. The frequency components outside (-0.5,+0.5) are folded in.

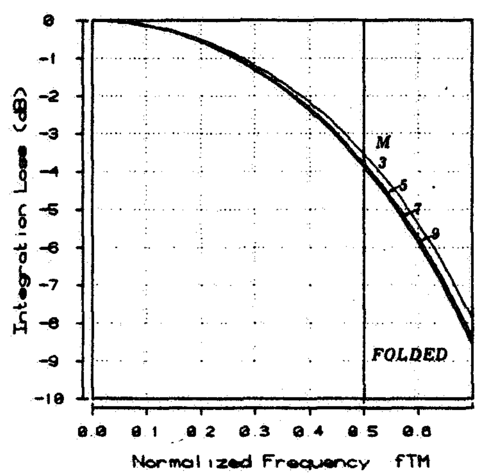


Figure 2. Periodogram bias due to coherent integration as a function of the normalized frequency fMT for different values of M.

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