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A RADIOMETRIC BODE'S LAW: PREDICTIONS FOR URANUS

by

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The magnetospheres of three planets, earth, Jupiter, and Saturn, are known to be sources of intense, nonthermal radio bursts. Recent studies¹⁻³ have shown that the emissions from these sources undergo pronounced long-term intensity fluctuations that are caused by the solar wind interaction with the magnetosphere of each planet. Determinations by spacecraft of the low-frequency radio spectra and radiation beam geometry now permit a reliable assessment of the overall efficiency of the solar wind in stimulating these emissions. We find that earlier estimates⁴ of how magnetospheric radio output scales with the solar wind energy input must be revised greatly, with the result that, while the efficiency is much lower than previously thought, it is remarkably uniform from planet to planet. This result has prompted the formulation in the present paper of a 'radiometric Bode's Law' from which a planet's magnetic moment can be estimated from its radio emission output. (This terminology is by analogy with the 'magnetic Bode's law', sometimes called Blackett's law⁵.) Applying the radiometric scaling law to Uranus, we estimate the low-frequency radio power likely to be measured by the Voyager 2 spacecraft as it approaches this planet. We also show how measurements of the Uranus radio flux by Voyager 2, which will probably be made in early to mid 1985, can be used to estimate the planetary magnetic moment and solar wind standoff distance prior to the in situ measurements.

We define the efficiency ϵ as the ratio of the median power radiated in magnetospheric emissions, P_r , to the incident solar wind power, P_i , under average solar wind conditions. The median radiated power is typical of that observed under average solar wind conditions.

The starting point for the calculation of P_r is integration over frequency of the median flux density spectra shown in Figure 1. The total radiated power (watts) is then the resulting integral multiplied by the area into which the emission is beamed. Thus in the case of Saturn and the earth, the integral is over the entire known flux spectrum since the auroral (terrestrial) kilometric radiation (AKR) and Saturn kilometric radiation (SKR) are both solar wind controlled. For Jupiter, only the hectometer-wavelength emission (HOM), which is known to be solar wind controlled², contributes significantly to the total solar-wind driven output. Therefore only that portion of the spectrum, from about 200 kHz to about 3 MHz (undotted portion of the Jovian spectrum in Figure 1), is relevant in this analysis. In order to compute the area into which the emission is radiated, the beam geometry of each source must be known. Saturn's northern-hemisphere emission has been shown⁶ to radiate into a filled 2π steradian (sr) conical beam and earth's into a more narrow 3.5 sr beam⁷. By contrast, while Jupiter's highest frequency burst emission, the decameter-wavelength emission (DAM), is very narrowly beamed, the HOM is more moderately beamed into a wide, relatively thin, $\sqrt{\pi}/2$ sr surface⁸.

Total radiated powers of the terrestrial⁹, Jovian¹⁰, and Saturnian⁶ radio sources have been estimated in previous studies. However, only the value for AKR derived by Kaiser and Alexander⁹ applies without modification in the present analysis since they used the median flux radiated into the actual emission beam. For Jupiter, we have modified an earlier 3×10^{10} W isotropic (4π sr) calculation¹¹ to reflect the modelled ($\pi/2$ sr) HOM radiation beam cited above. Similarly, we have divided by 2 the value used previously for Saturn's SKR⁶, which was for

an isotropic (4π sr), not a 2π sr, radiator. Thus we have taken care throughout to use the measured or inferred beam geometry of each source and to include only that portion of each planet's median burst spectrum that is known to be solar wind driven. Finally, the emitted power was doubled in the case of both Saturn and the earth in order to account for radiation from the opposite (southern) hemisphere. It is not known if the jovian HOM has a southern hemisphere component, and since modelling efforts⁸ have suggested a single source we have not doubled the measured power.

The solar wind input P_i at each planet was computed from

$$P_i = \pi m_p \rho_o V^3 (\lambda_o / d)^2 \quad (1)$$

which is a measure of the power (W) incident on the cross-sectional area of the magnetosphere due to the bulk motion of the solar wind plasma. Here, m_p is the proton mass, ρ_o is the solar wind number density at the earth, V is the average solar wind bulk speed, λ_o is the solar wind-magnetosphere standoff (Chapman-Ferraro) distance, and d is the planet-sun distance expressed in AU ($1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$). We have taken values at each planet that apply under average solar wind conditions, that is, $\rho_o = (7 \times 10^6 \text{ m}^{-3})$, $V = 4 \times 10^5 \text{ m/sec}$, and λ_o (meters) = $10R_e$ ($1R_e = 6.36 \times 10^6 \text{ m}$), $50R_j$ ($1R_j = 7.14 \times 10^7 \text{ m}$), and $20R_s$ ($1R_s = 6.03 \times 10^7 \text{ m}$), at earth, Jupiter, and Saturn, respectively. The last term, d^2 , accounts for the inverse-distance-squared decrease in the mean solar wind density beyond 1 AU.

Table 1 summarizes the total radiated and total input powers derived in the present study and also the spectral and beam geometry

characteristics of each radio source. The resulting efficiencies ($\epsilon = P_r/P_i$) are extremely small, approximately 0.0005%. These are much smaller than those previously derived⁴, primarily owing to selection in this study of only the solar wind driven emission and to better information, from spaceborne radio experiments, on the actual beaming properties and median emission spectra of each source. Although the efficiencies may be as much as an order of magnitude larger under very active solar wind conditions⁵, they are still extremely small when compared with the overall efficiencies of other magnetospheric phenomena, such as UV auroral output, which in the case of the earth^{1,2} has an efficiency nearer 0.1%. This underscores the fact that radio emission processes represent a relatively insignificant fraction of the total solar-magnetosphere energy budget.

Note also that ϵ hardly varies from planet to planet. For example, while P_r varies by over two orders of magnitude, the efficiencies are all within a factor of 1.7 or less of each other. This empirical result suggests that the solar wind driven components of the planetary radio sources are subject to a simple scaling relationship in which the total radiated power is directly proportional to the solar wind power incident on the planet's magnetosphere, the constant of proportionality being approximately 5×10^{-6} .

If P_r is directly proportional to P_i through the constant of proportionality ϵ , then P_r is also directly related, through equation 1, to $(I_0/d)^2$. Using the data of Table 1, we illustrate this relationship in Figure 2, where the radiated power P_r is plotted as a function of $(I_0/d)^2$ on the left and versus $M^{2/3}/d^{4/3}$ on the right. M is the planet's magnetic moment. It is derived from the solar wind standoff

distance through

$$l_0 = (M^2 / 2\pi m_p \rho V^2)^{1/6} \quad (2)$$

where $\rho = \rho_0/d^2$ and which includes the effect of magnetopause surface currents¹¹. The line is a least-squares fit to the data and we refer to the relationship shown in Figure 2 as the radiometric Bode's Law. With l_0 in km, d in AU and P_r in watts, the fit is given by

$$(l_0/d)^2 = 6.5P_r^{1.13} \quad (3)$$

Equation (3) permits the estimation of a planet's standoff distance and magnetic moment based on measurement of the median power flux from the planet's solar-wind driven radio emission.

Also shown in Figure 2 is a prediction for Uranus based on an extrapolation of the radiometric Bode's Law to small values. Currently it is not known if Uranus is a radio source^{14,15}. However, recent far ultraviolet observations^{16,17} of H Ly α emissions with the IUE spacecraft have been interpreted as due to a Uranus aurora, in which case the planet certainly possesses a magnetosphere and is likely to emit low-frequency radio emission as well. Table 2 summarizes the radio power levels, P_r , that we would expect from a magnetosphere having a given magnetic moment M_U . Uranus' spin axis is tilted near 90 $^\circ$ to its orbit plane, and the Voyager spacecraft observations inbound to the planet will be nearly pole-on. Hence, only the radio emission from one hemisphere of the planet will be detectable and so the tabulated values of P_r are twice those likely to be measured initially by the

Voyager radio astronomy experiment. The standoff distance ℓ_0 and incident solar wind power P_i resulting from each value of M_U are also shown.

Note that the range of magnetic moments shown in Table 2 is large. Shown in Figure 2, however, is what we consider to be a more plausible range of M_U , based on two constraints. At the low end, we do not consider it likely that M_U is less than about $0.1 G-R_U^3$ for the following reason: If we assume a 1% conversion efficiency of solar wind power into precipitating electron flux, a smaller magnetosphere would not intercept sufficient power to drive the $>10^{10}$ W emitted by the Uranian aurorae¹⁸. At the high end, $4.0 G-R_U^3$ represents the magnetic moment recently derived¹⁹ theoretically from external rotating dynamo considerations. By way of comparison, the magnetic Bode's law³ estimate of the Uranus field strength based on angular momentum considerations is about $1 G-R_U^3$, intermediate of these two extremes.

Reference to the inset scale, labelled " $\ell_0 (R_U)$ " in Figure 2, shows directly how the total median radio power scales with the magnetopause standoff distance at Uranus ($R_U=23800$ km). By way of example, measurement of a Uranus radio source having a median radiated power equal to 5×10^6 W, or 10^7 W total from both northern and southern hemispheres, would require a standoff distance of nearly $20 R_U$. This is equivalent to a $1.9 \times 10^{12} G\text{-km}^3$ ($\approx 0.14 G-R_U^3$) magnetic moment.

At this writing, Voyager 2 is about 6 AU from Uranus and no radio emissions have been detected by the planetary radio astronomy (PRA) instrument²⁰. This fairly conclusively rules out the detections of very strong radio bursts tentatively reported by Brown¹⁴ using the IMP-6 spacecraft, since bursts of the reported magnitude should have been

observed by Voyager 2 when it was as much as 14 AU from Uranus. Assuming that the radiometric Bode's law applies at Uranus, the IMP-6 bursts would have required $M_U = 20 \text{ G} \cdot R_U^3$ (see note 5 of Table 2) and so the possibility of a magnetic field this large is not considered likely. Taking what we believe to be the plausible range of magnetosphere size described above (0.1 to $4 \text{ G} \cdot R_U^3$), we estimate that Uranus radio emission should be easily detectable by the PRA instrument sometime between December 1984 and May 1985. This will permit an assessment of the planet's standoff distance and magnetic moment well in advance of the January 1986 Voyager-Uranus encounter.

TABLE 1
EFFICIENCY COMPARISON

PLANET	P_r (watts)	BEAM (sr)	SPECTRUM USED	P_1 (watts)	ϵ ($\times 10^{-6}$)
Jupiter	4×10^9	$\pi/2$	HOM only	1.1×10^{15}	3.6
Saturn	2×10^8	2π	full	3.8×10^{13}	5.3
Earth	6×10^7	3.5	full	9.6×10^{12}	6.2

TABLE 2
URANUS RADIO EMISSION AND MAGNETIC MOMENTS

M_U ($G-R_U^3$)	$\frac{L_O}{R_U}$	SOLAR WIND POWER (W) ($=1.1 \times 10^{15}$)	P_r^+ (W)	NOTES
1×10^{-4}	<2	1×10^{-5}	$<2 \times 10^5$	(1)
0.01	8	2×10^{-4}	2×10^6	
0.1	18	1×10^{-3}	9×10^6	(2)
1.0	38	5×10^{-3}	3×10^7	(3)
4.0	61	1×10^{-2}	8×10^7	(4)
20.0	104	4×10^{-2}	2×10^8	(5)

NOTES:

- (1) Minimum M_U for Uranus to stand-off solar wind
(2) Minimum M_U to support observed auroral emission
(3) M_U from magnetic Bode's law⁵
(4) Hill, Dessler, and Rassbach¹⁹ magnetic moment
(5) Minimum M_U needed to drive IMP-6 radio bursts¹⁴

* Expected total power from radiometric Bode's law, for a given M_U .

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Figure Captions

Fig. 1. Low-frequency median flux density spectra for the three known radio planets, earth⁹, Jupiter¹⁰, and Saturn⁶, are shown with the solar wind driven components indicated by solid lines. Flux densities are normalized to what an observer would measure at a distance of 1 AU from each planet. For clarity, the kilometer-wavelength components of Jupiter's emission are not shown.

Fig. 2. The radiometric Bode's law. The total median radio power P_r (watts) is plotted as a function of $(\lambda_0/d)^2$ on the left and versus $M^{2/3}/d^{4/3}$ on the right. Plausible range for Uranus is shown. Inset scale labelled " $\lambda_0 (R_U)$ " refers to solar wind standoff distance at Uranus in units of $R_U (=23800 \text{ km})$. λ_0 , d , and M are expressed here in units of km, AU, and $G\text{-km}^3$, respectively.

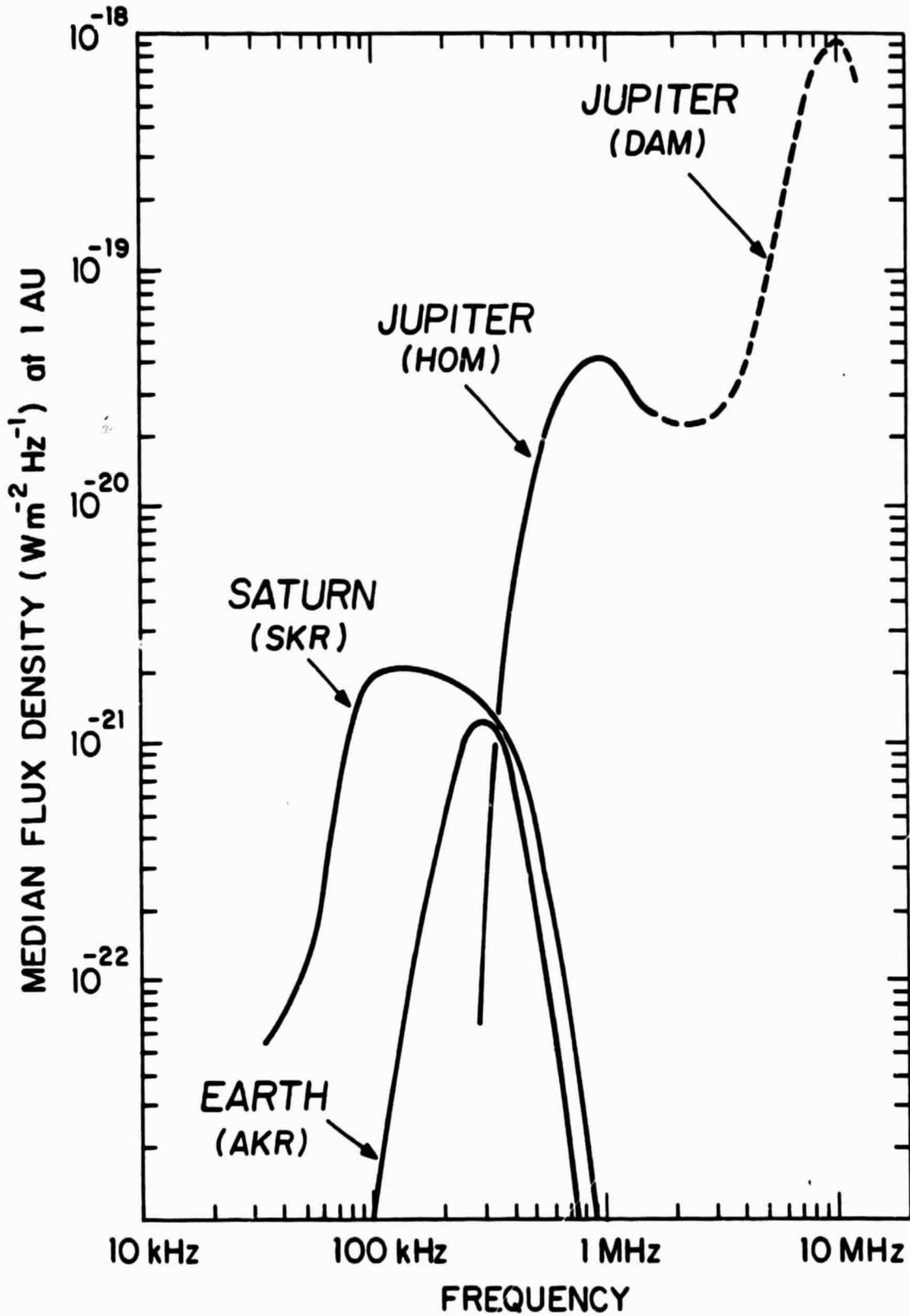


FIGURE 1

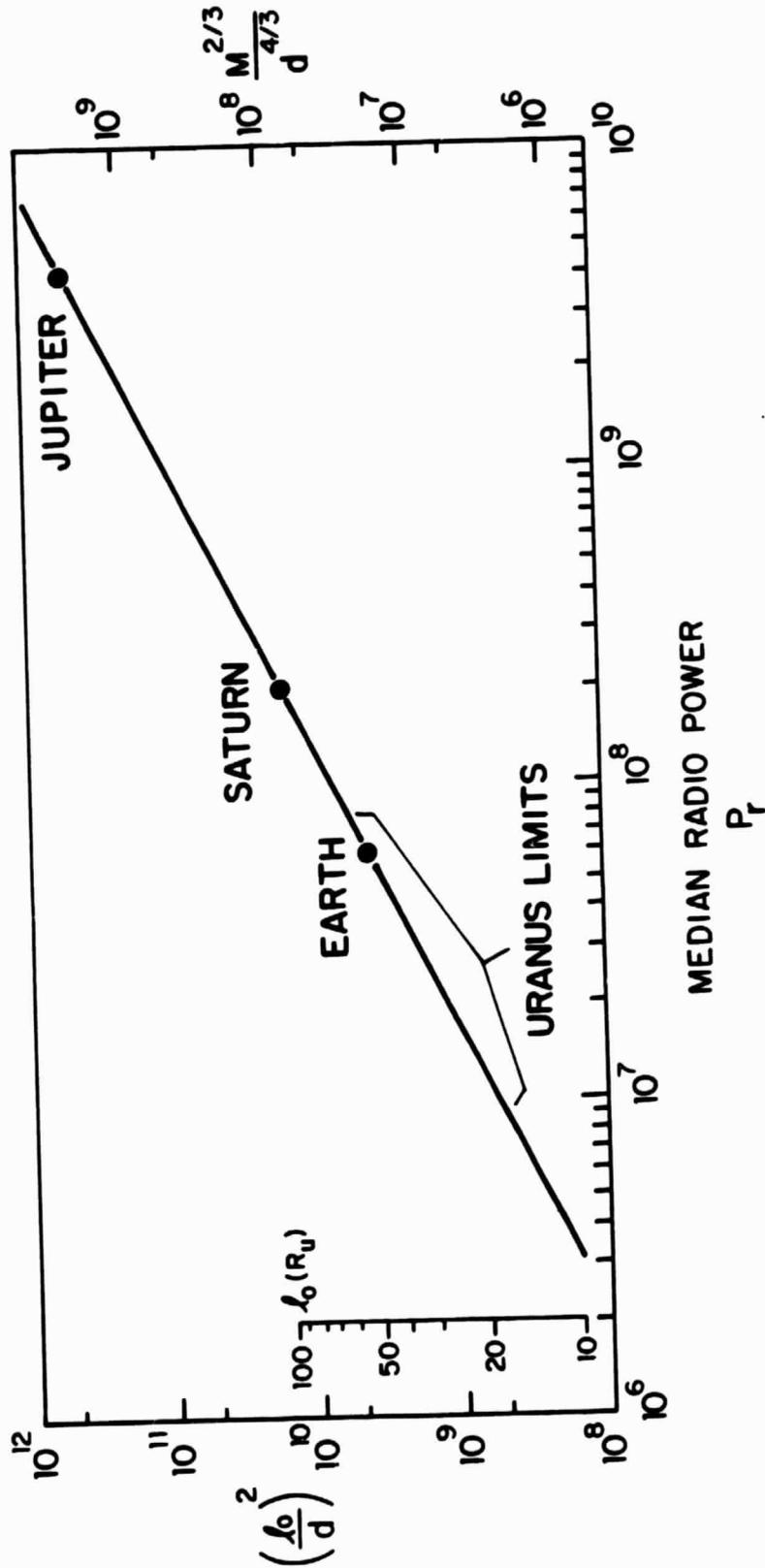


FIGURE 2