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AN INVESTIGATION INTO THE PROBABILISTIC
COMBINATION OF QUASI-STATIC AND
RANDOM ACCELERATIONS

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16. ABSTRACT

The development of design load factors for aerospace and aircraft components and experiment support structures, which are subject to a simultaneous vehicle dynamic vibration (quasi-static) and acoustically generated random vibration, require the selection of a combination methodology. Typically, the procedure is to define the quasi-static and the random generated response separately, and arithmetically add or root sum square to get combined accelerations. Since the combination of a probabilistic and a deterministic function yield a probabilistic function, a viable alternate approach would be to determine the characteristics of the combined acceleration probability density function and select an appropriate percentile level for the combined acceleration. The following paper develops this mechanism and provides graphical data to select combined accelerations for most popular percentile levels.

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INTRODUCTION

The development of design load factors for aerospace and aircraft components and experiment support structures, which are subject to simultaneous vehicle dynamic vibration (quasi-static) and acoustically generated random vibration, require the selection of a combination methodology. Typically, the vehicle dynamic vibration is low frequency, transient, and deterministic. The random response is typically narrow band random whose characteristics are described probabilistically with a normal distribution of instantaneous values and a Rayleigh peak distribution. The combination of quasi-static and random response of the single degree-of-freedom system shown in Figure 1 is illustrated in Figure 2.

Since component support structure design limits generally are defined during the simultaneous occurrence of the two loads, the combination methodology is important from weight and qualification status.

The current practice to obtain the combined g level g_t is generally similar to the following.

1) Calculate the quasi-static (QS) maximum response.

2) Calculate the Grms response to random input by Miles relationship and multiply by a factor to get peak. If a factor of 3 is used, this is the same as the 99.87 percentile level of the instantaneous distribution.

3) Add them on a directional basis to get combined g_t.

The percentile level for the combined acceleration developed this way is not calculable. It would seem, however, that a viable alternate approach would be to determine the characteristics of the probability density function of the combined acceleration (QS and random) and select an appropriate percentile level for a combined g_t. The following discussion develops that scenario.

DISCUSSION

If the time history combined functions of Figure 2 can be idealized as $y(t) = A\sin\omega t + r(t)$, where the randomly varying function $r(t)$ is normally distributed about a zero mean with variance σ_r^2 and has a normal probability density function (PDF) of:

$$f(r) = \frac{1}{\sigma_r \sqrt{2\pi}} \exp\left(-r^2/2\sigma_r^2\right) \tag{1}$$

Then the PDF of the random function superimposed on the sinusoid is [1,2,3,4]:

$$p(y) = \frac{1}{\pi\sigma_r\sqrt{2\pi}} \int_{-\pi/2}^{\pi/2} \exp\left\{-\tfrac{1}{2}\left(\frac{y-A\sin\theta}{\sigma_r}\right)^2\right\} d\theta \qquad (2)$$

where σ_r is the standard deviation of the random distribution and A is the peak amplitude of the sinusoid.

Figure 3 illustrates the characteristics of equations (1) and (2). Figure 3A shows a comparison between the combined PDF and the normal distribution f(r) for a random distribution with a variance σ_r^2 equal to 0.09 of the sinusoidal variance σ_s^2. Obviously, as the random variance approaches 0, the combined distribution approaches a sinusoidal distribution.

Figure 3B shows a comparison between the combined distribution p(y) and the normal distribution f(r) for a superimposed random distribution with a variance σ_r^2 equal to the sinusoidal variance σ_s^2. Obviously, as the random variance gets very large in comparison with the sinusoidal variance, the combined PDF approaches the random PDF.

Figures 4 through 10 illustrate the combining of a banded random white noise with a sinusoidal signal. The 100 to 600 Hz band random is illustrated in Figure 4. Its instantaneous distribution is shown in Figure 5. The sine wave and its distribution are shown in Figures 6 and 7, respectively. The RMS of the sine is equal to approximately three times the RMS of the random. The combined signal, its instantaneous distribution, and cumulative probability are shown in Figures 8, 9, and 10 respectively.

For a random distribution superimposed on a sine wave, the variance of the combined distribution is:

$$\sigma_c^2 = \sigma_s^2 + \sigma_r^2 \qquad (3)$$

$$= A^2/2 + \sigma_r^2$$

or, the standard deviation of the combined signal equals the root sum square (RSS) of the sinusoidal RMS and the random RMS values.

$$\sigma_c = \sqrt{\frac{A^2}{2} + (g_{rms})^2} \qquad . \qquad (4)$$

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With the variance of the combined signal and the PDF defined, one can obtain the combined g_c acceleration level appropriate for any selected cumulative probability level.

The use of this methodology is shown in Figure 11, for the pre-selected popular percentile values. The following scenario applies.

1) Determine maximum quasi-static response acceleration A in g's.

2) Determine the g_{rms} response of the article to the random input.

3) Select the combined instantaneous probability level desired (0.95 to 0.99865).

4) Determine the ratio of peak quasi-static A to random g_{rms} σ_r.

5) From the appropriate curve, read the ordinate corresponding to A/σ_r and multiply by σ_r to get the quantile pertinent to the combination.

This process will give a combined acceleration (gs) value for the selected no-exceedance probability.

In summary, the PDF of a combined sine/random signal has been determined. This along with the known peak quasi-static and random response g_{rms} allows one to describe the combined signal in probabilistic terms. One then can describe g_c to any probability level desired. A rational selection of a combined quasi-static/random response acceleration can now be selected for any prescribed probabilistic percentile level.
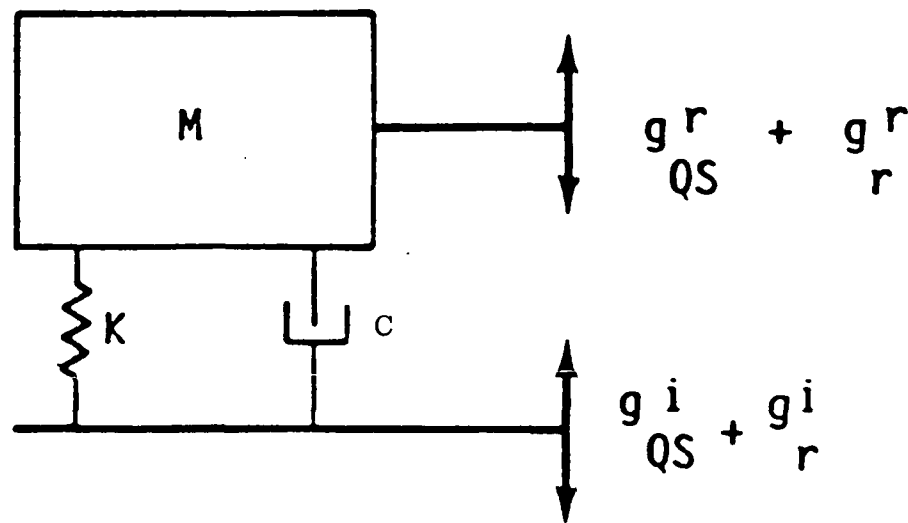

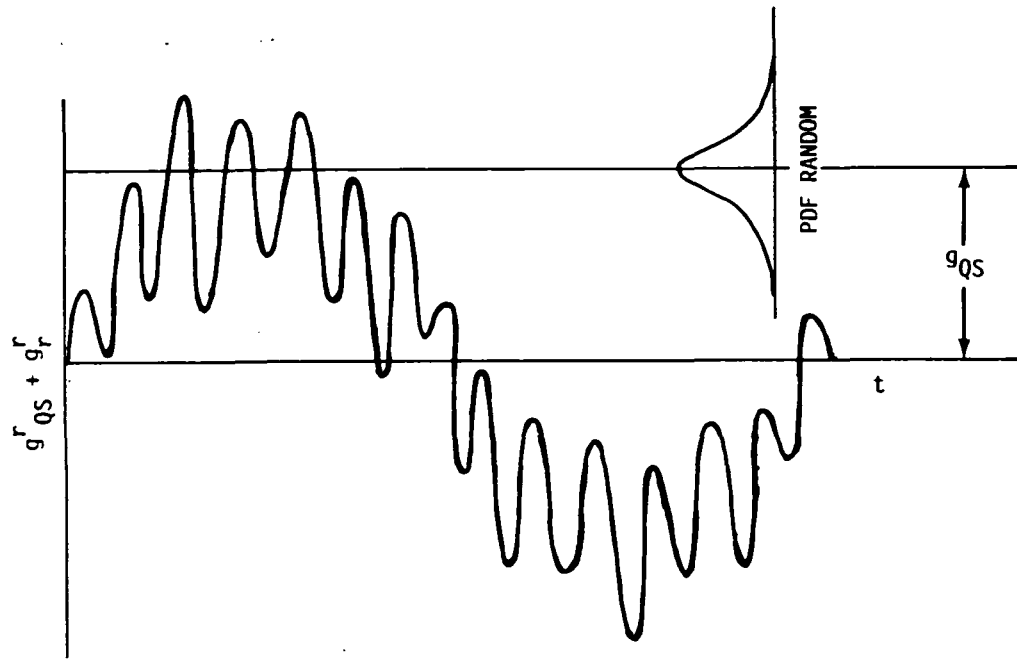Figure 1. Single degree-of-freedom component idealization.

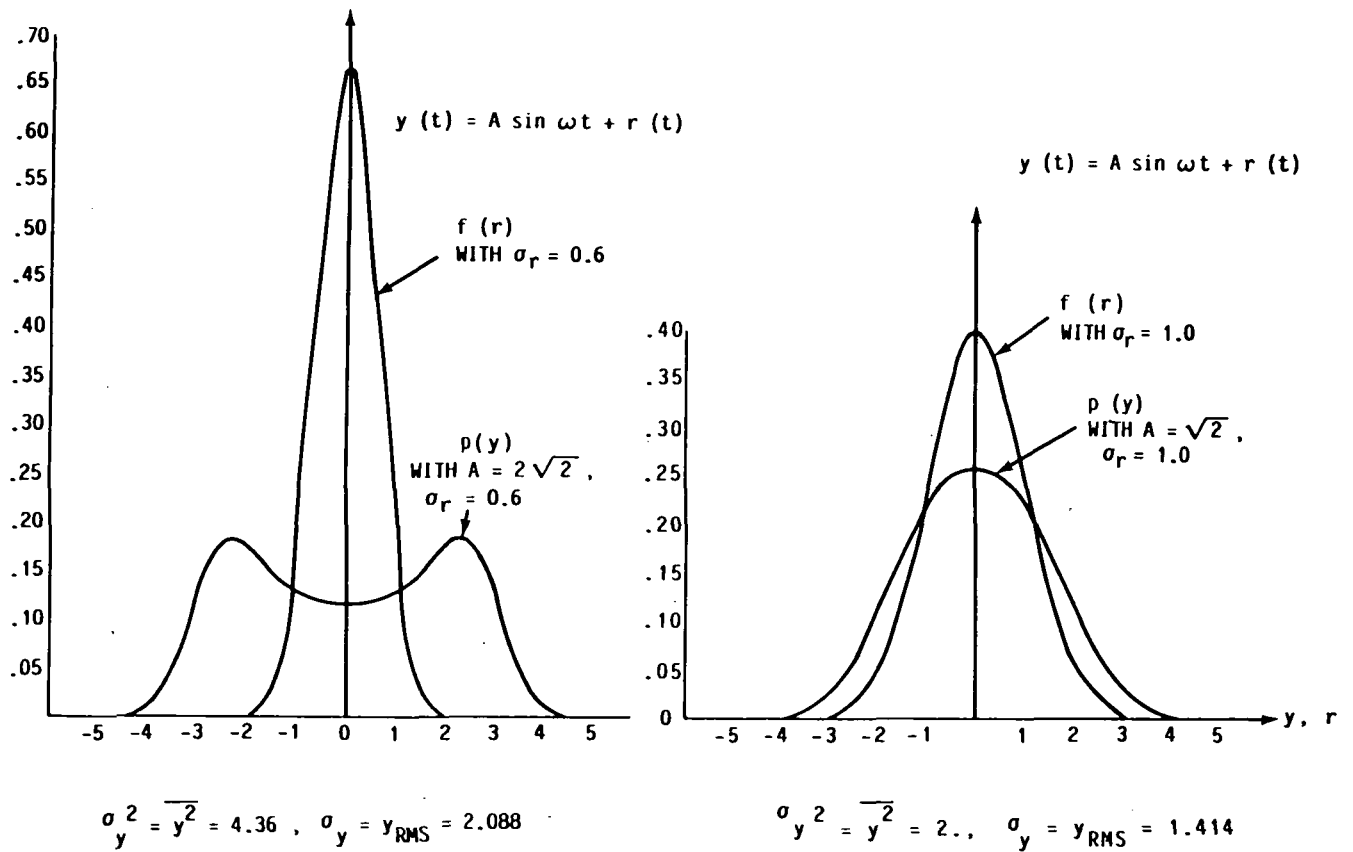Figure 2. Time history combined quasi-static and random response.

Figure 3. Combined sinusoidal/random distributions VRS random only.
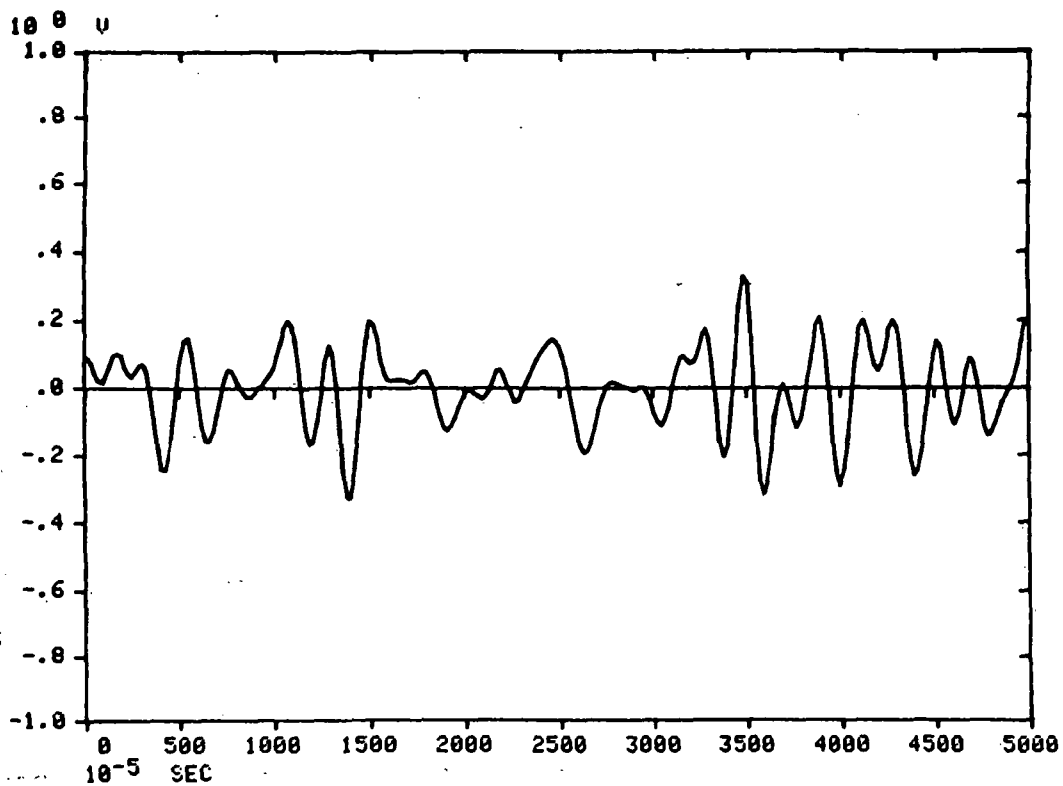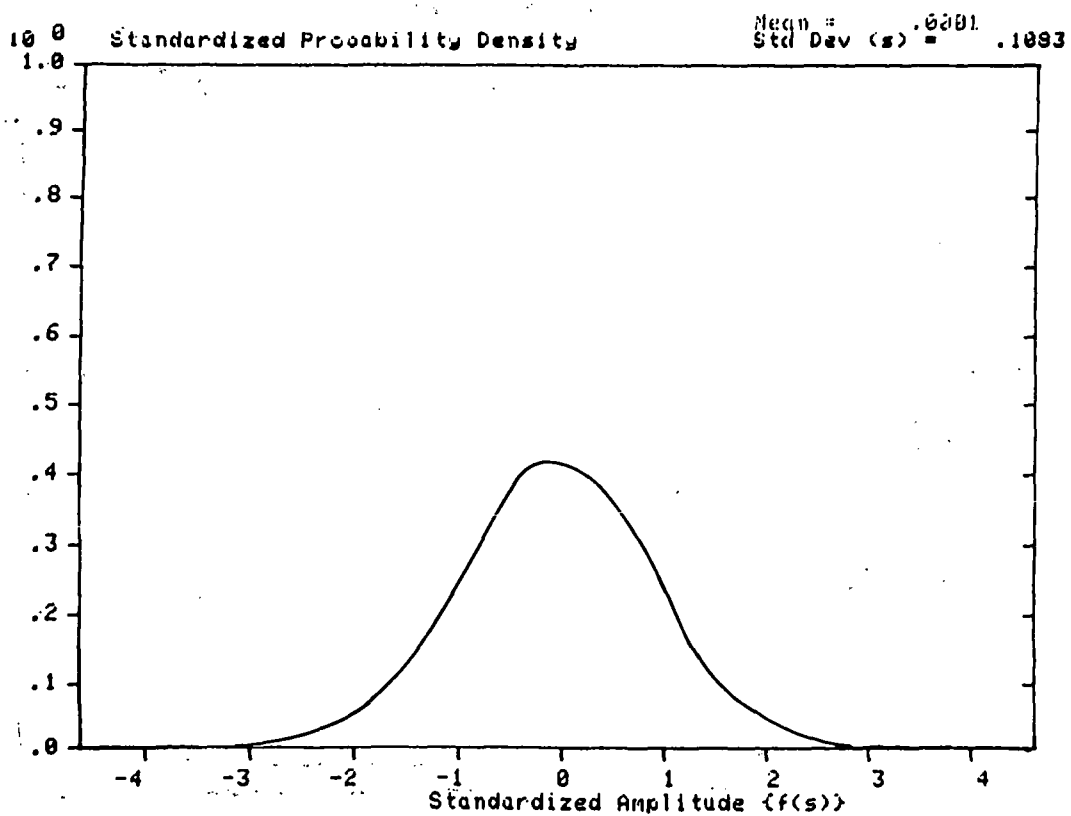
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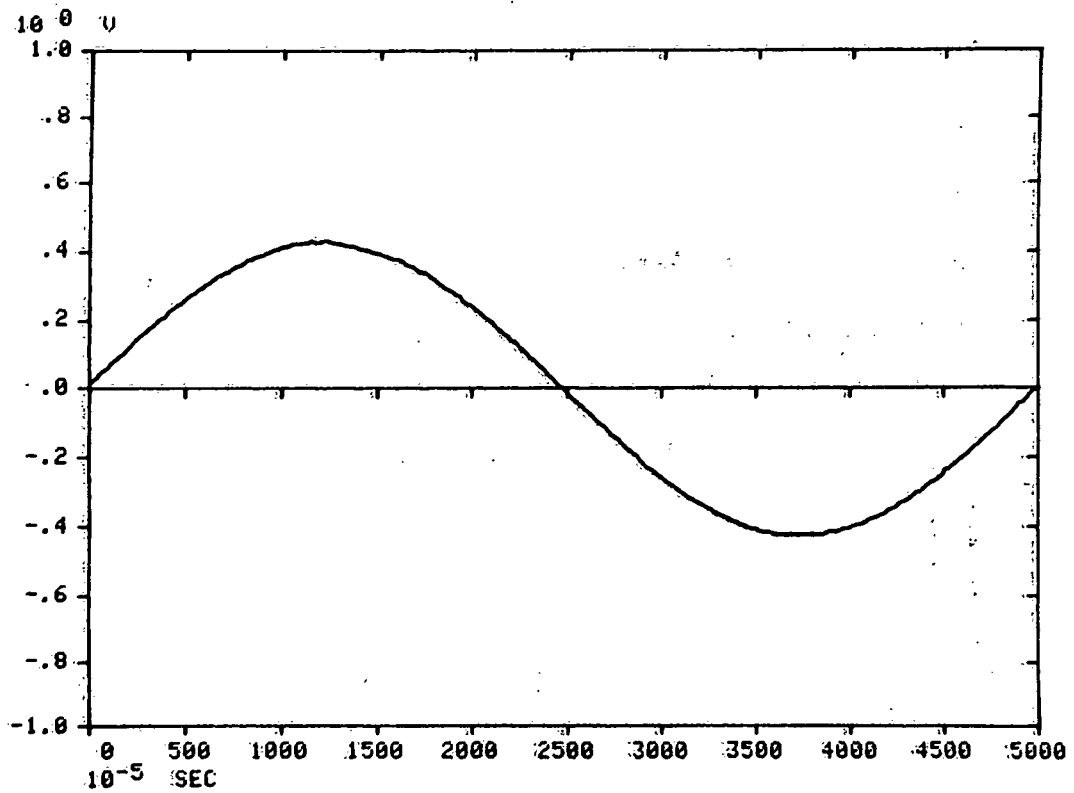

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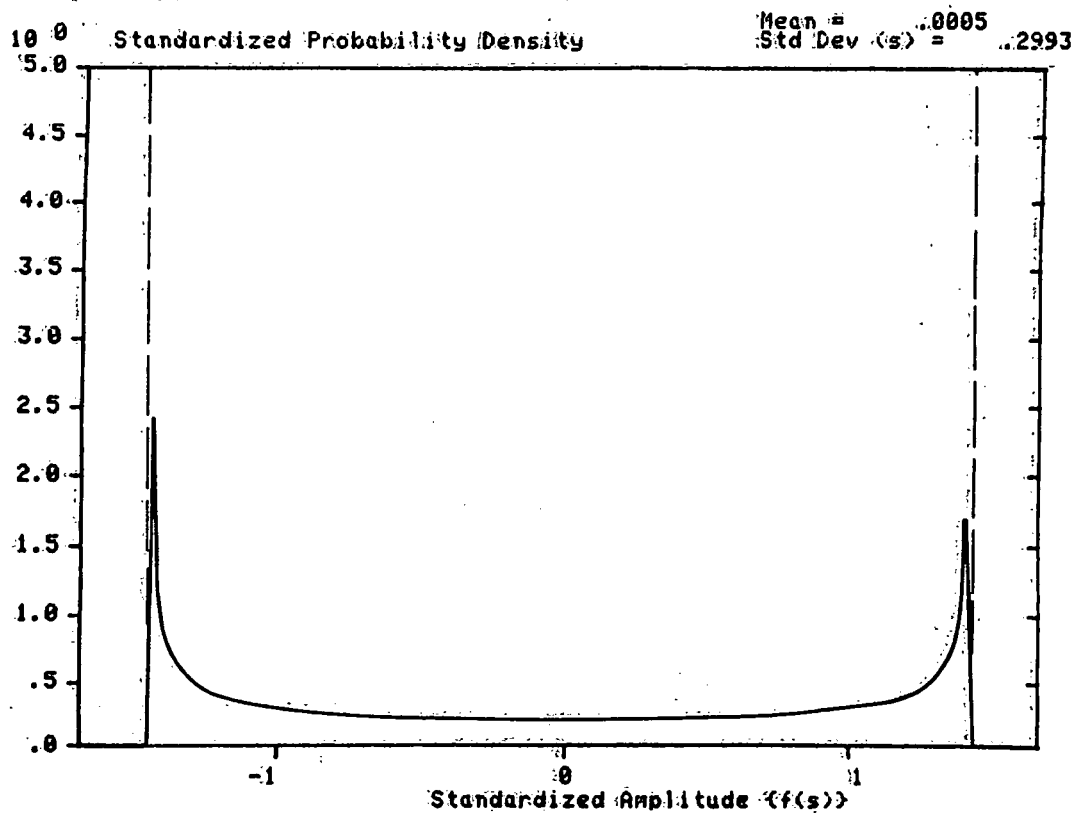
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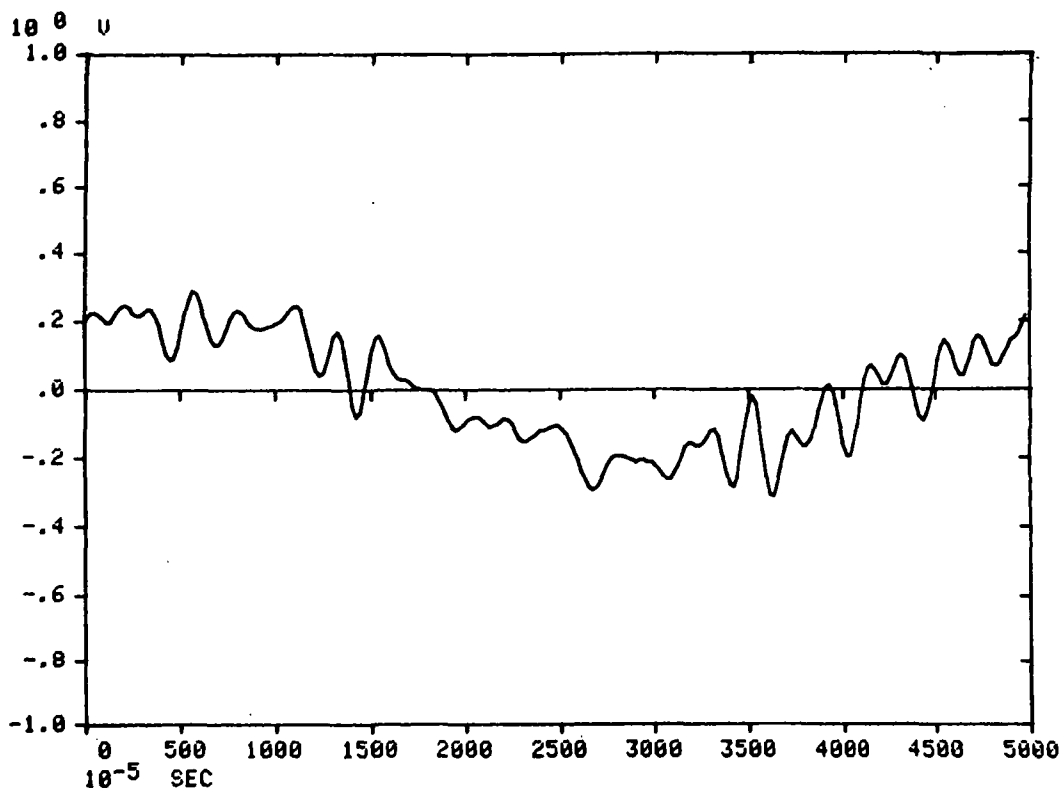Figure 7. PDF of sinusoidal signal.

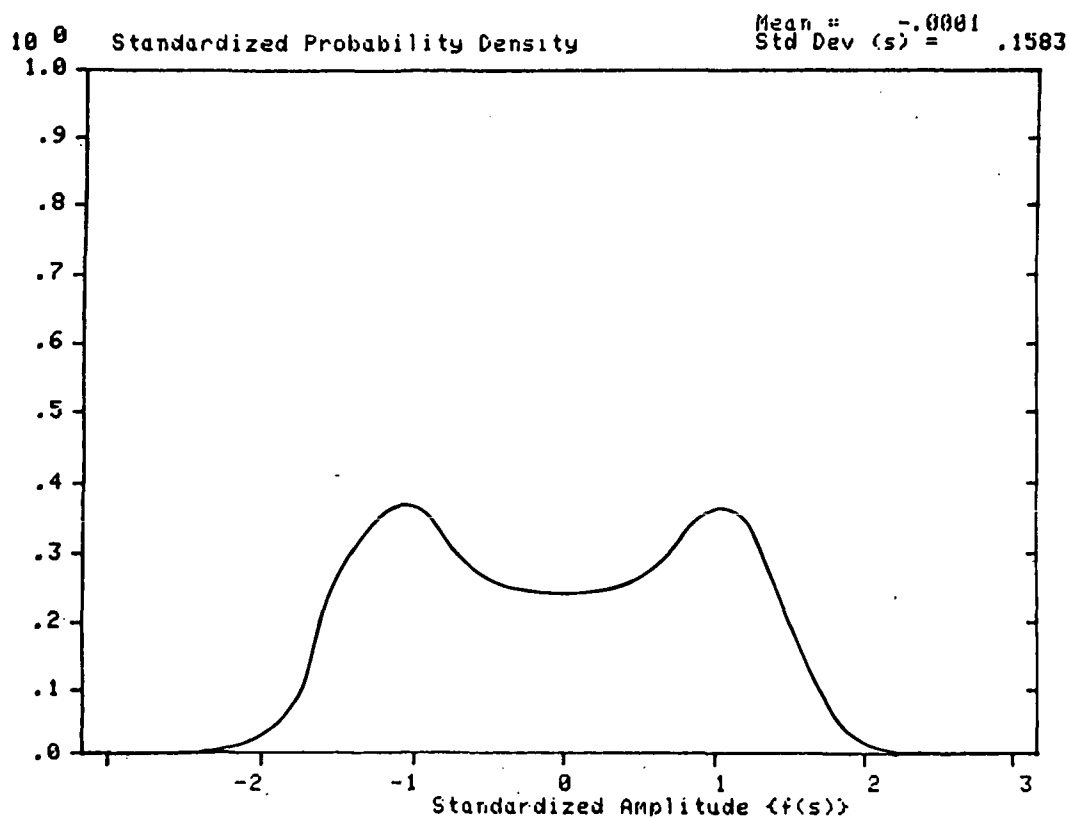Figure 8. Combined sinusoidal/random signal.

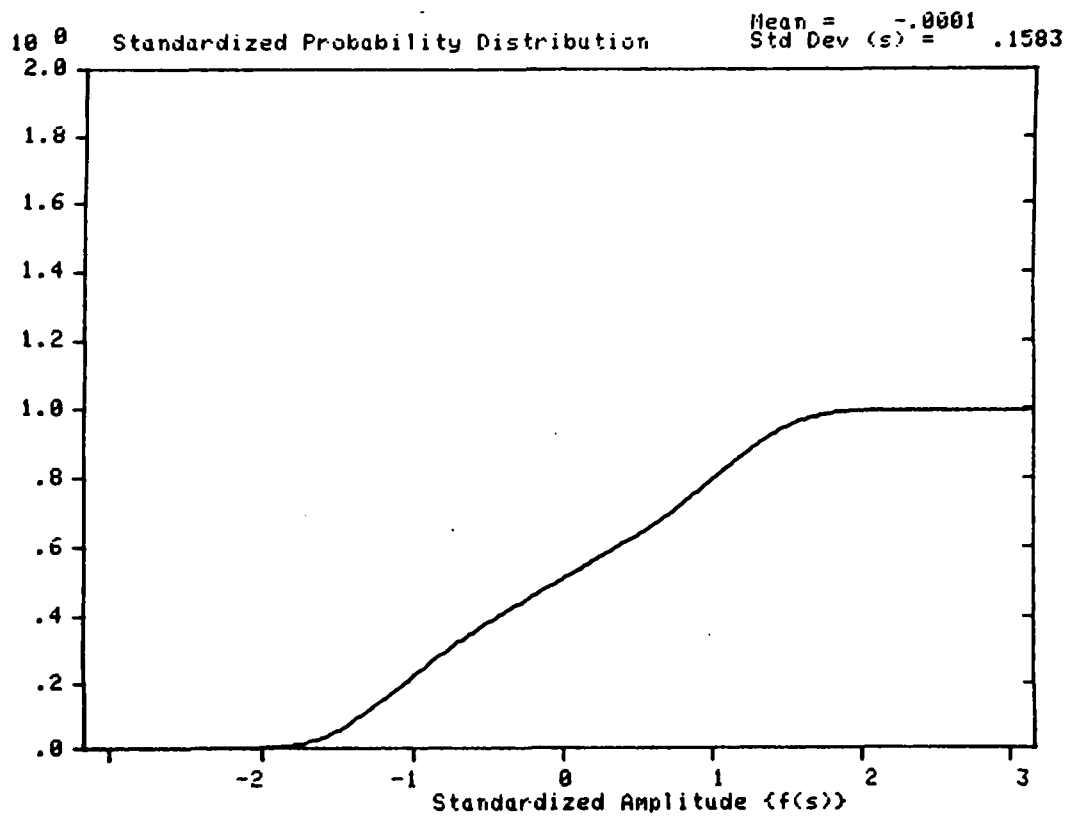
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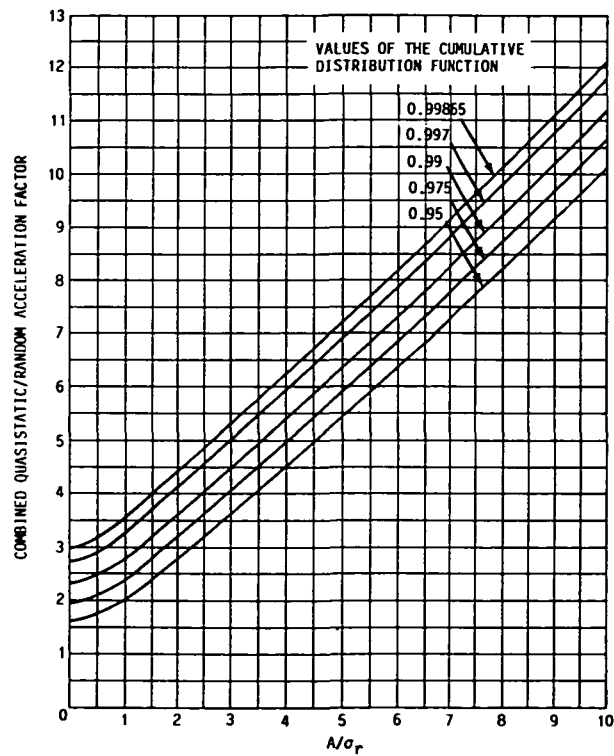
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Figure 10. CDF of combined signal.

Figure 11. Combined acceleration g level for selected
values of the CDF.

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APPROVAL

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The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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