OPTICAL ANALYSIS OF CASSEGRAINIAN POINT FOCUS CONCENTRATORS

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ABSTRACT

A Cassegrainian solar concentrator, using a 7-meter diameter primary reflector, is analyzed in three forms: 1) an unmodified Cassegrainian, 2) the Ritchey-Chretien configuration, and 3) the unmodified Cassegrainian with a nonimaging tertiary reflector. Performance was not significantly improved with the Ritchey-Chretien; however, the tertiary resulted in significant improvement in intercept factor and optical efficiency.

INTRODUCTION

The Cassegrainian optical configuration consists of a parabolic primary reflector and a hyperbolic secondary mirror. A solar concentrator designed using this configuration can benefit by allowing a more flexible receiver design, since it no longer has to be supported at the primary focal point. The main disadvantages of the Cassegrainian configuration are the additional reflection and blocking due to the secondary.

In addition to the "true" Cassegrainian described above, a variant, referred to as the Ritchey-Chretien, has been studied. The Ritchey-Chretien (R-C) has a slightly hyperbolic primary with the secondary adjusted accordingly. These modifications correct the system for the off-axis aberration referred to as coma. Since the sun is not a point source, much of the incoming insolation is off of the optical axis, causing coma. Elimination of coma should decrease the overall spot size at the focal plane, increasing the intercept factor for a given concentration ratio.

A nonimaging tertiary reflector, added at the focal point of the system, can improve the optical performance of the Cassegrainian. The configuration considered for this application is the hyperbolic flowline concentrator, as described by Winston (1). This design has the advantage over other nonimaging concentrators such as the compound elliptic concentrator (CEC), of affecting only the edges of the beam, thus reducing the overall reflection losses.

METHOD OF ANALYSIS

This study has used a Monte-Carlo ray trace computer program originally developed by Honeywell (2). This code is modular in nature, and allows modeling of any concentrator system by writing appropriate subroutines describing the geometry of the system to be studied. It has the capability to include the effects of RMS surface imperfections on system performance and a finite sun size with nonuniform flux distribution.

The approach used in this study was to perform a parametric study on the true Cassegrainian, compare the performance of the R-C to the true Cassegrainian for selected parameters, and then analyze a tertiary reflector added to the Cassegrainian system. Finally, an analysis was performed to determine the sensitivity of the optical performance to misalignment of the secondary and tertiary reflectors.

GEOMETRY

As stated previously, the Cassegrainian consists of a confocal parabola and hyperbola, as shown in figure 1. The convex secondary reflector increases the focal length of the optical system, and thus reduces the angle of the extreme rays reflected from the secondary to the system focal plane. Since the optical extent of the system must be conserved, an increased focal length reduces the maximum concentration ratio that can be achieved. The theoretical concentration ratio for a system of axial symmetry is defined in equation 1.

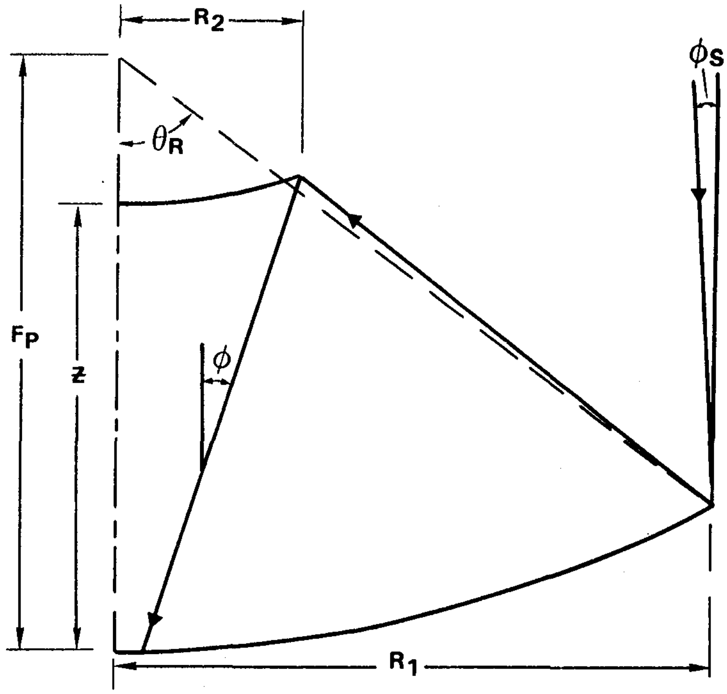

Figure 1. Cassegrainian Geometry

$$CR = \left(\frac{\sin\ \phi}{\sin\ \phi_s}\right)^2 \tag{1}$$

where ϕ is the entrance angle of the extreme rays and ϕ_s is the sun angle. As can be seen, ϕ determines the maximum CR possible.

The variation in system focal length is related to the primary focal length (F_p) and the eccentricity (e) of the secondary by the magnification factor (M), defined as

$$M = \frac{e_2 + 1}{e_2 - 1} \tag{2}$$

The system focal length is the product of M and F_p. Likewise, the extreme ray angle ϕ is related to the primary rim angle. These relationships can be used to determine M and the theoretical concentration ratio.

There are several factors that do not allow the theoretical concentration ratio to be reached. In a perfect optical system (i.e., free from errors on the reflector surfaces) the limitations are caused by various aberrations.

It is possible to design an optical system to reduce or eliminate some of these aberrations. One such design is the Ritchey-Chretien, which is corrected for spherical aberration and coma. There have been several derivations of the relationships required for this configuration. The one chosen for this study is by Wetherell, et al (3). This development results in a "sag" equation for the two surfaces as a function of the primary reflector focal length, the system aperture area, the system focus position, and the vertex-to-vertex spacing between the two reflectors.

Another major source of degradation in concentrator performance is imperfections on the reflector surfaces. They have the effect of increasing the size of the cone of light reflected off of each surface.

It is advantageous from a cost standpoint to design a concentrator system with large slope errors on the reflectors and a small primary rim angle. Also, from an efficiency standpoint it is advantageous to reduce the size of the secondary to reduce the blocking factor by increasing the spacing between the primary and secondary reflectors. Unfortunately, these design decisions tend to spread the beam of radiation impinging on the focal plane, reducing the concentration ratio possible for a given intercept factor.

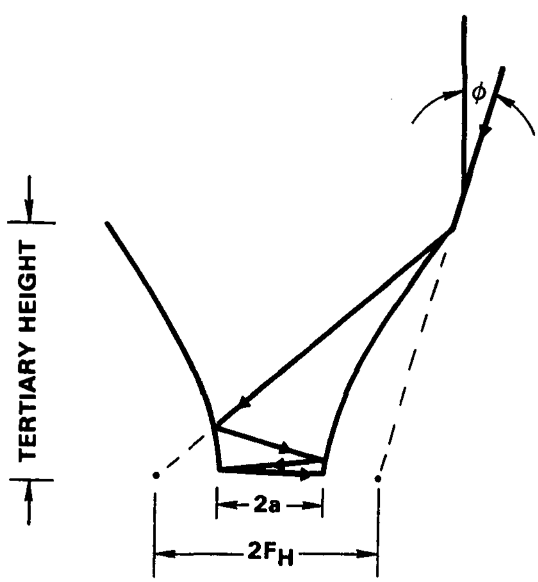

Figure 2. Tertiary Geometry

To increase the concentration ratio for a given intercept factor, a hyperbolic tertiary reflector may be added at the focal plane of the system. The geometry for this non-imaging concentrator is shown in figure 2. One property of a hyperbola is that any ray directed at one of the focal points will be reflected towards the other focal point. After an infinite number of reflections, the ray will exit through the bottom of the concentrator. Rays that would intersect the focal plane within the $2F_H$ diameter exit with correspondingly fewer reflections, while those outside $2F_H$ would be rejected out the top of the concentrator. The concentration ratio of the tertiary is defined as:

$$CR = \frac{F_H^2}{a^2} \tag{3}$$

For a given spot radius "F_H", there exists a family of hyperbolas with varying concentration ratios. The rule that governs the shape of the hyperbola is:

$$\sin^{-1}\left(\frac{a}{F_H}\right) \leq \phi \tag{4}$$

This restriction is the limiting factor for the maximum concentration ratio possible for this concentrator. However, to intercept the entire beam at the limit, the required height of the concentrator would be infinite.

There are two additional restrictions that are in effect for integrating the tertiary into the Cassegrainian design. They are:

The radius of the tertiary at the truncation height must not block any rays that are reflected from the primary towards the secondary.

The radius of the tertiary, at the truncation height must intersect all rays reflected from the secondary.

These two restrictions place a maximum and minimum height restriction on the tertiary, respectively. This, together with the required shape of the tertiary for a given concentration ratio, limits the maximum concentration ratio that can be attained.

RESULTS

Cassegrainian Only

A parametric study was performed for the Cassegranian concentrator. Figure 3 illustrates the existance of an optimum Z/F for a particular geometric concentration ratio. The optical efficiency is defined as the product of the intercept factor and 1 minus the blocking factor.

As was discussed earlier, the beam incident on the focal plane spreads as Z/F increases, reducing the intercept factor for a given CR. The shift of optimum optical efficiency due to rim angle is caused by the reduction in primary focal length at higher rim angles, which increases the angle of extreme rays (ϕ) reflected from the secondary. This increase in ϕ reduces the beam size at the focal plane, according to conservation of optical extent, thereby increasing the intercept factor for a given CR. Reducing Z/F also results in an increase in ϕ, and therefore an increased intercept factor, but at the expense of an increased blocking factor, reducing the optical efficiency. Increasing the primary surface error results in a shift of the optimum Z/F to the left, in essence trading increased blocking to obtain a higher optical efficiency.

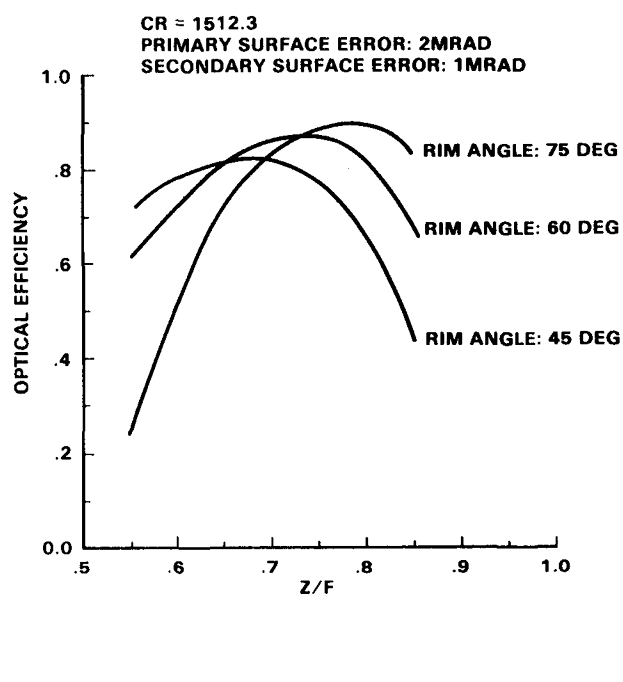

Figure 3. Optical Efficiency Versus Z/F

Table 1 lists the performance of the true Cassegrainian at maximum optical efficiency and maximum intercept factor at a rim angle of 60 degrees for various combinations of primary and secondary surface errors. Results for 45 and 75 degrees are not shown since performance at 45 degrees is low, and 75 degrees results in higher primary costs. The optical efficiency listed assumes 100 percent reflectivity on the primary and secondary reflectors, and 95 percent reflectivity on the tertiary.

Several results are apparent from this table. They are:

Primary surface errors (σ_p) affect the performance of the system much more than errors on the secondary (σ_s).

It is not possible to achieve the acceptance criteria of CR = 1200 at IF $\geq$ .96 at maximum optical efficiency except for the case of σ_p = 2 mr.

A primary surface error of 8 mr will not produce an acceptable IF.

Table 2 lists the performance of the Ritchey-Chretien configuration in the same format as above. Comparing the Ritchey-Chretian to the true Cassegraninian shows no significant improvement in optical efficiency. The major reason for this is that the improvement resulting from elimination of coma is masked by the effects of surface errors. Figure 4 shows the intensity distribution for the true Cassegrainian and the R-C for a typical set of parameters. The most noticeable difference between the two configurations is the higher intensity of radiation in the center for the Ritchey-Chretien due to the elimination of coma. If this is a desirable feature in the overall design of a concentrating system, then perhaps the Ritchey-Chretien should be considered. If not, then there is little, if any advantage to using the Ritchey-Chretien.

<u>Cassegrainian with Tertiary Reflector</u>

The main parameter required to integrate the tertiary into the Cassegrainian design is the radius of the spot on the focal plane, F_H. This parameter, along with the desired concentration ratio, defines the required shape of the tertiary. However, it is not necessarily advantageous to set F_H equal to the maximum radius of the spot, since this would require a relatively tall, narrow concentrator. Examination of figure 4 reveals that the intensity distribution is very close to a normal distribution. This fact can be used to determine an appropriate F_H. The procedure for determining F_H is as follows:

Determine the standard deviation of the flux distribution (σ_D).

Calculate F_H by determining the desired fraction of the total energy available to be captured. These calculations have been performed for a capture ratio of .995 and .975. It is apparent that as the capture ratio decreases, the required tertiary height decreases.

Blockage of rays reflected from the primary reflector must also be avoided. At high tertiary heights, this becomes a problem, and limits the concentration ratio of the system.

TABLE 1. CASSEGRAINIAN PERFORMANCE

Rim Angle: 60°

σ_p = 4mr , σ_s = 2mr

CR	IF	η_{max}	z/F	r(cm)*	P(kW)**	IFmax	z/F	η_o	P(kW)
1200	0.9	0.77	0.7	10.2	30	0.98	0.55	0.58	22.5
1500	0.85	0.73	0.675	9.0	28	0.96	0.55	0.55	22

σ_p = 8mr , σ_s = 2mr

CR	IF	η_{max}	z/F	r(cm)	P(kW)	IFmax	z/F	η_o	P(kW)
1200	0.69	0.54	0.65	10.2	20.7	0.79	0.55	0.45	17.5
1500	0.63	0.49	0.65	9.0	18.8	0.73	0.55	0.42	16.5

σ_p = 4mr , σ_s = 1mr

CR	IF	η_{max}	z/F	r(cm)	P(kW)	IFmax	z/F	η_o	P(kW)
1200	0.90	0.77	0.70	10.2	30.0	0.98	0.55	0.58	22.5
1500	0.89	0.73	0.675	9.0	28.5	0.965	0.55	0.57	22.5

σ_p = 2mr , σ_s = 1mr

CR	IF	η_{max}	z/F	r(cm)	P(kW)	IFmax	z/F	η_o	P(kW)
1200	0.98	0.90	0.75	10.2	34.4	0.997	0.7	0.87	33.4
1500	0.96	0.87	0.75	9.0	34.0	0.99	0.65	0.81	31.0

*r = receiver radius
**p = power entering the receiver plane in kilowatts

TABLE 2. RITCHEY-CHRETIEN PERFORMANCE

Rim Angle: 60°

σ_p = 4mr , σ_s = 2mr

CR	IF	η_{max}	z/F	r(cm)	P(kW)	IFmax	z/F	η_o	P(kW)
1200	.91	0.77	.70	10.2	29.5	.94	.65	.71	27.3
1500	.87	0.73	.70	9.0	28.0	.90	.65	.68	26.0

σ_p = 2mr , σ_s = 1mr

CR	IF	η_{max}	z/F	r(cm)	P(kW)	IFmax	z/F	η_o	P(kW)
1200	.986	0.89	.75	10.2	34.4	.998	.70	.85	32.9
1500	.967	0.88	.75	9.0	33.7	.993	.70	.85	32.7

Analysis of the radiation on the secondary reflector indicates that the diameter can be reduced with a small gain in performance. This is because the flux of reflected energy on the outer ring of the secondary is very low, and by not intercepting that energy, it is possible to decrease the blocking factor. Reducing the secondary diameter also benefits the tertiary reflecter design. Since the "source" for the radiation that the tertiary intercepts is now smaller, the height of the tertiary can be reduced for a given concentration ratio, or conversely, a higher concentration ratio can be achieved for a given height.

Results for the Cassegrainian with the tertiary reflector are listed in table 3 for a reduced secondary diameter. The most significant result is that the optical efficiency does not peak as a function of z/F. The tertiary reflector redirects the beam to the desired receiver aperture regardless of the size of the beam on the tertiary. The major penalty for redirecting a large beam is a tall tertiary.

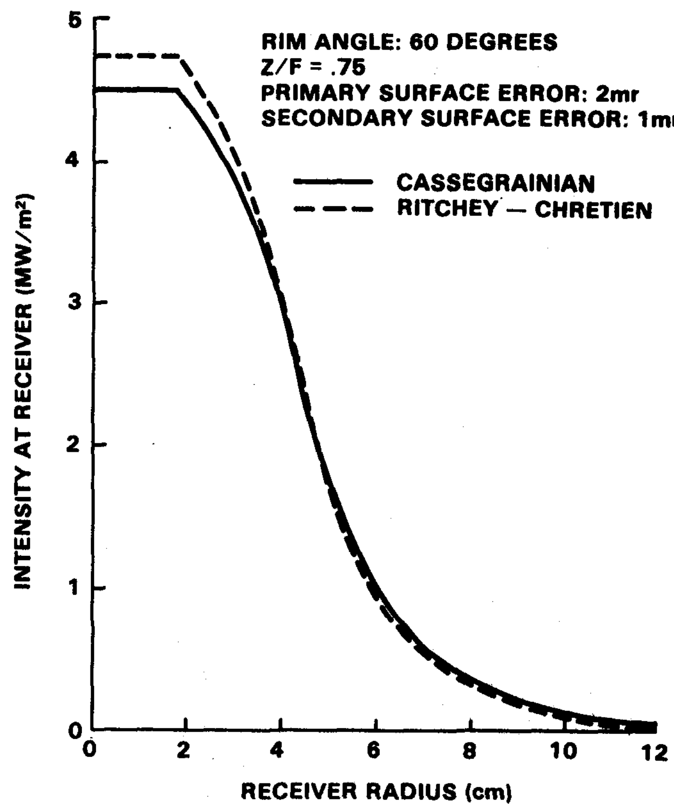

Figure 4. Comparison of Intensity Distributions

TABLE 3. CASSEGRAINIAN PLUS TERTIARY PERFORMANCE

Rim Angle: 60° σ_p = 4mr σ_s = 2mr σ_T = 2mr z/F = 0.75
Secondary Diameter = 1.96 m

Capture Ratio: .995

CR	r(cm)	IF	η_{max}	Z_T(cm)*	P(kW)
1200	10.1	.98	.89	99.0	34.4
1300	9.7	.98	.89	114.0	34.4
1500	9.0	-	-	-	-
2200	7.5	-	-	-	-

Capture Ratio: .975

CR	IF	η_{max}	Z_T(cm)	P(kW)
1200	.97	.88	42	33.7
1300	.97	.88	47	33.7
1500	.96	.87	60	33.6
2200	.96	.87	118	33.3

*z_T = height of the tertiary

Comparing the maximum efficiencies for the Cassegrainian only and the Cassegrainian plus tertiary yields a 17 percent increase at CR = 1200 and a 19 percent increase at CR = 1500. These increases in efficiency result from increasing the intercept factor. Additionally, it is possible to increase the concentration ratio without significantly degrading the efficiency when using the tertiary.

Effects of Misalignment

There are three loss mechanisms that affect the performance of the tertiary reflector. They are:

Rejection of rays through the inlet aperture.

Absorption of energy caused by multiple reflections.

Non-interception of radiation by the inlet aperture.

Another loss mechanism, introduced by the reduction of the secondary diameter, is non-interception of radiation by the secondary reflector.

Rotational misalignment has the largest effect on increased loss, followed by axial alignment. Positive axial misalignment reduces the percentage of radiation not intercepted by the secondary. Perhaps the secondary diameter could be increased, resulting in less sensitivity to axial misalignment.

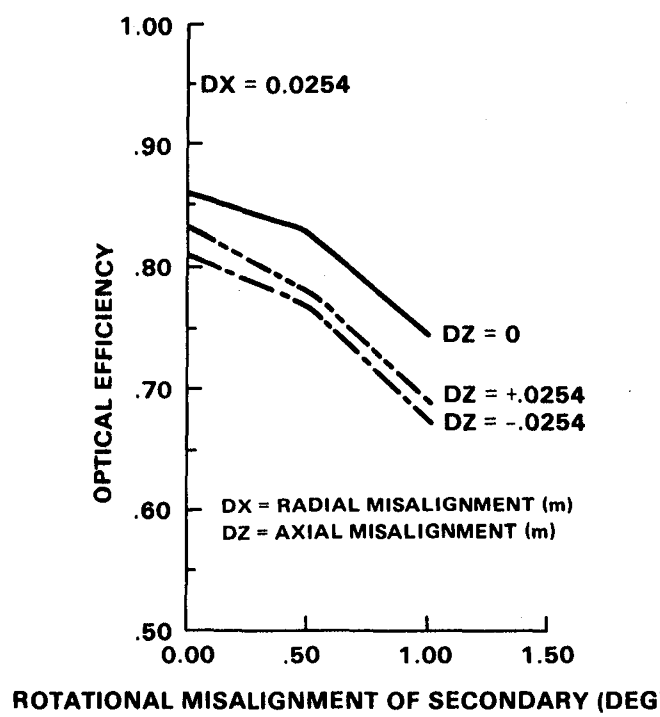

Figure 5. Effect of Misalignment on Optical Efficiency

Figures 5 and 6 show the effects of multiple misalignments on optical efficiency and intercept factor. The criteria chosen for determining the maximum amount of misalignment permissible was to set the minimum optical efficiency equal to the maximum efficiency attainable without the tertiary with perfect alignment. This is an arbitrary decision, although it does give a rational bound on system efficiency. Using this criteria, the maximum combined misalignment is:

Axial:	± .0254m (1")
Radial:	± .0254m (1")
Rotational:	± .5°

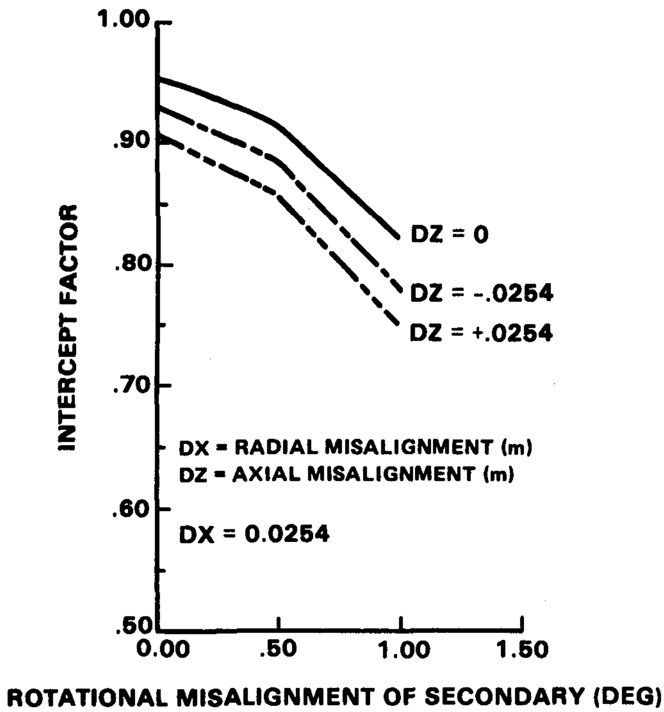

Figure 6. Effect of Misalignment on Intercept Factor

This results in a decrease in optical efficiency and intercept factor of approximately 11 percent, such that the minimum optical efficiency is 77 percent, with an intercept factor of 85.5 percent.

CONCLUSIONS

The Cassegrainian concentrator is a viable system for the 7-meter dish studied.

The Ritchey-Chretien modification does not significantly improve the performance of the Cassegrainian configuration due to the presence of surface slope errors that largely mask improvements produced by the elimination of coma.

A non-imaging tertiary reflector significantly improves the optical performance of the Cassegrainian, and should be integrated into the design.

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