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This report summarizes my recent work on the construction of finite-difference models of differential equations having zero truncation errors.[1] The details of the calculations which follow will not be given since a paper is being written on these topics.

I. Unidirectional Wave Equation

The unidirectional, linear wave equation in one-dimension is

(1) $$u_x + u_t = \lambda u$$

$$\lambda = \text{constant.}$$

For the initial value problem, where

(2) $$u(x,o) = f(x) = \text{given function,}$$

the exact, general solution is

(3) $$u(x,t) = e^{\lambda t} f(x-t) \quad .$$

If we define

(4) $$u(x_m, t_n) = u_m^n$$

$$x_m = (\Delta x)m \quad , \quad t_n = (\Delta t)n$$

where (m,n) are integers, then it is easy to prove that the following finite-difference scheme is a zero truncation error model of eq. (1):

(5a) $$u_m^{n+1} = e^{\lambda h} u_{m-1}^n$$

$$\Delta x = \Delta t = h$$

where

(5b) $$u_m^0 = f((\Delta x) m) \quad .$$

It should be pointed out that the conventional use of either the Euler forward- or backward- difference schemes for either of the derivatives in eq. (1) does not lead to eq. (5a).[2] In more detail, the "full" difference scheme associated with eq. (1) which has zero truncation error is easily gotten from eq. (5a) and written out in detail is

$$(6) \qquad \frac{\Delta_t^{(+)} u_m^n}{\left(\frac{e^{\lambda h} - 1}{\lambda}\right)} + \frac{\Delta_t^{(-)} u_m^n}{\left(\frac{e^{\lambda h} - 1}{\lambda}\right)} = \lambda u_{m-1}^n$$

where

$$(7a) \qquad \Delta_t^{(+)} u_m^n = u_m^{n+1} - u_m^n$$

$$(7b) \qquad \Delta_x^{(+)} u_m^n = u_m^n - u_{m-1}^n$$

Note that the conventional application of finite difference techniques to eq. (1) would give

$$(8) \qquad \frac{\Delta_t^{(t)} u_m^n}{\Delta t} + \frac{\Delta_x^{(t)} u_m^n}{\Delta x} = \lambda u_m^n$$

or some similar result, which does have a non-zero truncation error. Direct comparison of eqs. (6) and (8) shows that previous knowledge of finite-difference models for differential equations may not be of great help in the construction of schemes with zero-truncation error.[1,2]

For practical problems where eq. (1) is to be numerically integrated, either eq. (5a) or eq. (6) may be used since they are mathematical equivalent expressions. However, from a computational point of view, eq. (5a) should be used.

Recently, I have been able to obtain a zero truncation error difference model for the following nonlinear, uni-directional wave equation

(9) $u_t + u_x = \lambda u^n$ (n = positive integer) .

The general case is rather difficult to write down; however, for n = z, we have

(10a) $u_t + u_x = \lambda u^2$

and the difference scheme is

(10b) $$u_m^{n+1} = \frac{u_{m-1}^n}{1 - (\lambda h)\, u_{m-1}^n} .$$

where

(10c) $\Delta t = \Delta x$

and

(10d) $u_m^n = u(x_m, t_n)$.