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AND MULTILEVEL OPTIMIZATION

JAROSLAW SOBIESZCZANSKI-SOBIESKI,

JEAN-FRANCOIS M. BARTHELEMY,

AND

GARY L. GILES

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Langley Research Center
Hampton, Virginia 23665

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Jaroslaw Sobieszczanski-Sobieski
NASA Langley Research Center
Hampton, Virginia

Jean-Francois M. Barthelemy
Virginia Polytechnic Institute and State University
Blacksburg, Virginia

Gary L. Giles
NASA Langley Research Center
Hampton, Virginia

Abstract

This paper describes a method for systematic analysis and optimization of large engineering systems, e.g., aircraft, by decomposition of a large task into a set of smaller, self-contained subtasks that can be solved concurrently. The subtasks may be arranged in many hierarchical levels with the assembled system at the top level. Analyses are carried out in each subtask using inputs received from other subtasks, and are followed by optimizations carried out from the bottom up. Each optimization at the lower levels is augmented by analysis of its sensitivity to the inputs received from other subtasks to account for the couplings among the subtasks in a formal manner. The analysis and optimization operations alternate iteratively until they converge to a system design whose performance is maximized with all constraints satisfied. The method, which is still under development, is tentatively validated by test cases in structural applications and an aircraft configuration optimization. It is pointed out that the method is intended to be compatible with the typical engineering organization and the modern technology of distributed computing.

Introduction

A modern aircraft or spacecraft is a complex engineering system composed of many subsystems that are very tightly coupled - a change in one affects many others and the whole. While mastery of that coupling by designers was always important and in the past made the difference between acceptable and excellent performance of the vehicle, now that mastery may make the difference between flying and not being able to take off. An example is aircraft with forward-swept, aeroelastically tailored wings and canard (X-29) which could not fly without aerodynamics, structures, and active control well tuned to each other.

For the designer, mastery of the subsystems couplings requires, as it always did, the uniquely human abilities of creativity, judgment, and intuition fed by numerical information from all the contributing disciplines on all quantifiable aspects of the design. The volume of that information has recently grown very rapidly with the progress in analytical methods in engineering disciplines. In addition to the traditional analysis answering the "what" questions, a new type of analysis - sensitivity analysis - became available to answer the "what if" questions. To make the best possible use of all that information toward the ultimate goal of designing a vehicle that performs "best" in a certain sense clearly requires that a systematic methodology, at the level of sophistication equal to that of the contributing disciplines, should be built into the design process itself, not to the exclusion of the designer's intellect but to its enhancement.

One particular concept for such methodology, called "multilevel optimization by linear decomposition" now under research and development at NASA Langley Research Center is described in this paper. To provide the necessary contrast, the sequential decisionmaking in the prevailing design practice is discussed first. The salient features of multilevel optimization with linear decomposition are introduced next, and a sample of references is provided for background. Experience with the application of the method to engineering problems is reported for initial validation of the concept. In conclusion, it is pointed out that the method is inherently compatible with the way engineers cooperate in a design organization and with distributed computing capabilities provided by modern computer technology.

A Paradox in Sequential Decisionmaking In Design

In a typical design process, major decisions are made sequentially. The example in fig. 1 is for an aircraft design; usually the aerodynamic shape is decided first, and subsequently the airframe is sized for strength, etc. An analogous sequence could be laid out for any other major industrial product, for instance, a ship. The loops in the discipline boxes symbolize iterative design improvements carried out within confines of a single engineering discipline or subsystem. The loops spanning several boxes depict multidisciplinary design improvement iterations. Omitted for graphical simplicity is the parallelism of the disciplinary subtasks. That parallelism is important in order to develop a broad work front necessary to shorten the design time.

If all the intra and interdisciplinary iterations were carried out to convergence, the process could yield a numerically optimal design. However, the process is usually not converged because of time and budget limitations. This is especially true for the interdisciplinary iterations. Thus, the sequential decisionmaking leads to a paradoxical disparity between the volume of information about the object of the design and the design freedom measured by the number of design variables and options still available to the designers. As seen in fig. 2, the former ascends with time because of the analyses and experiments performed while the latter declines because of casting the decisions "in concrete."

The paradox is that the designers are gaining information but losing freedom to act on it.

A simple example will reveal that the paradox shown in fig. 2 leads to a suboptimal design. For the example, we will look into an aircraft design process at the time when the wing planform and structural sizing have already been accomplished to produce a combination of two design variables, the aspect ratio and structural weight, that maximizes a measure of the aircraft performance without violating the constraints. Simplifying the example as much as possible, we can consider a design space formed by the aspect ratio and the minimum structural wing weight. In that design space, shown in fig. 3, the aircraft performance can be depicted by a set of contour lines, each line corresponding to a constant value of the performance measure P. Superimposed on the contour lines are the constraint curves, C1 and C2. Each constraint curve divides the design space into the feasible (constraint satisfied) and infeasible (constraint violated) subspaces (domains). The crosshatching marks the infeasible side. It is not important for the purposes of this discussion to define which particular aircraft characteristic was chosen as a measure of performance (objective function) and what constraints were taken into account in plotting the set of curves P, C1, and C2. The aircraft range for a given takeoff gross weight and payload, and the wing static strength may be thought of as respective examples for measure of performance and constraint. Inspection of the figure shows that the design which maximizes P without violating C1 and C2 is at point O1.

Suppose now that when the flutter speed is subsequently calculated, the design at O1 turns out to have too low a flutter speed. In fig. 4, it is shown to be on the infeasible side of the flutter constraint plotted as C3. The design has to be modified to increase its flutter speed. If at this point in the design process, the configuration - the aspect ratio - is frozen, then the increase in the flutter speed can be achieved by stiffening the wing structure at the price of a weight penalty required to move from O1 to O2 at a constant aspect ratio. The weight penalty reduces the performance from P1 to P2. If the configuration were not frozen, a new optimal design could be located at O3, whose performance P3, although smaller than P1, exceeds P2 (P2 < P3 < P1). The difference

P3 - P2 is a performance penalty due to the sequential freezing out of design options in a sequential design process. We can say that design O2 is suboptimal relative to design O3. Another look at figs. 3 and 4, and a little reflection, will help realizing that although the magnitude of the performance penalty, P3 - P2, depends on the shapes of the functions involved (P, C1, C2, C3), its existence does not and, consequently, the example reveals that suboptimal results can be expected in a sequential design process in which each additional stage restricts the number of design variables while bringing in new constraints that must be satisfied.

A System Alternative to Sequential Decisionmaking

To demonstrate an alternative based on a system approach, reenter the example at the point where the flutter deficiency of the design, labeled O1 in fig. 4, has been found. The essence of the system approach is decomposition of a large problem into several smaller ones without losing the coupling. Therefore, we recognize in this case that two engineers, or engineering groups, must fix the flutter problem with the least penalty to the performance, P, by cooperating and yet each doing a separate subtask. In fig. 5, the individuals, or groups, are labeled C - for configuration, including aerodynamics and performance, and S - for structures.

The subtask of correcting the flutter problem with a minimum weight penalty, ΔW_{min}, is carried out by S for a particular aspect ratio, temporarily held constant by C; using aerodynamic analysis results (e.g., pressure distribution) and their sensitivity to aspect ratio - all supplied by C. The results produced by S are a flutter-free design at a minimum weight penalty at that aspect ratio, and the sensitivity of that design to aspect ratio. That sensitivity is quantified in the form of derivatives of the weight penalty and cross-sectional dimensions with respect to the aspect ratio.

Completion of the above task moves the design from O1 to O2 in fig. 6, exactly as in the previous discussion. However, group C will now recover a part of the performance penalty by changing the aspect ratio and the weight penalty concurrently. In this operation, the weight penalty is not an independent variable but is tied to the aspect ratio variation by the sensitivity derivative which tells how much the weight penalty must change per unit of aspect ratio variation to keep the flutter constraint satisfied. Such dependence of the weight penalty on aspect ratio is only a linear approximation of a true nonlinear relation and can be depicted by the tangent to C3 at O2 shown in fig. 6. The configuration improvement produced by C calls now for a move along that tangent toward the increasing performance, that is toward O3. The move should stop when the tangent veers off too far (a matter of judgment) from C3 to let group S repeat its subtask in

order to recover from the linearization error by regenerating the minimum weight penalty and its sensitivity derivative at the new value of the aspect ratio. Thus, by alternating subtasks performed by C and S we can improve the design by moving toward the theoretical optimum at O3 in a staircase fashion: O2 to O2A, to O2B, to O2C, and so on, as long as we see that the performance improvement is worth the effort.

Decomposition of a System

Having the idea of decomposition introduced by means of a simple example, we will now formulate the following broad guidelines for performing a decomposition:

1. break the overall large task into a number of smaller, self-contained subtasks, along interdisciplinary lines or the physical divisions among the subsystems.

2. preserve the couplings between the subtasks.

3. carry out concurrently as many subtasks as possible to develop a broad workfront of people and computers.

4. keep the volume of coupling information small relative to the volume of information that needs to be processed internally in each task.

The first three are self evident. The last one deals with the disparity between the large volume of information that is being processed within a subtask and a relatively small volume of information that couples the subtask (subsystem) to other subtasks (subsystems). For example, contrast the mass of data being manipulated in a finite-element analysis of an airframe with the input data of loads, mechanical properties, and geometry, and with the structural weight and critical constraint data which is all that is fed back to the aircraft performance analysis. Generally, it can be expected that the computational and labor cost of performing analysis and optimization will be reduced in a decomposition scheme in which that disparity exists. On the other hand, lack of such disparity will usually indicate that either the decomposition scheme is improper or that the problem is not decomposable.

Decomposition Methods

Several studies have been devoted to the decomposition of large optimization problems. Although none of these studies is truly multidisciplinary, their findings pertain to all areas of engineering as well as economics and management. The following overview describes typical approaches borrowed from the field of structural optimization. For the sake of the discussion, two classes of decomposition methods are defined: formal methods and intuitive, or heuristic, methods. Formal methods use the mathematical structure of the problem to derive a decomposition scheme. Consequently, a rigorous framework exists within which the mathematical properties of the method may be assessed. In intuitive methods, however, an understanding of the behavior of the physical system is apparently the prime factor directing the decomposition. These methods are sometimes referred to as rational methods and their mathematical characteristics can seldom be studied in great detail. An intuitive method provides, of course, the only option for decomposing those problems which do not possess the structure for which a formal decomposition method exists. This division into formal and intuitive methods is somewhat arbitrary as a given approach may very well be shown to belong to both classes, however, it facilitates the discussion.

Formal decomposition methods. A very extensive body of work exists on decomposition in linear programing (LP). The existence of very large problems in the fields of economics and operations research has stimulated efforts aimed at exploiting the special structures of the constraint matrix. The major initial step in that area seems to have been the introduction of the Dantzig-Wolfe decomposition principle in 1960. By 1970, developments in this active field were so numerous as to warrant the publication of a textbook by Lasdon.[1] In the area of structural design, problems involving collapse design of trusses and frames were solved successfully using the LP decomposition techniques.[2] Optimization algorithms have been devised for separable nonlinear problems using coordination techniques developed in the context of multilevel decisionmaking processes. Once a problem has been decomposed into smaller subproblems, the main task is to coordinate the design of the different subproblems. Essentially, the subproblems are grouped in the lower levels of a hierarchy and an additional lower-level subproblem is added to select the coordinating variables so as to force the other subproblems into choosing designs corresponding to improved overall performance. The coordination subproblem is itself cast in the form of an optimization problem. A study of coordination in hierarchical systems is given in the 1970 monograph by Mesarovic and coauthors.[3] Kirsch and coworkers have used both the model coordination technique and the goal coordination technique to solve various structural design problems ((4), chap. X).

Intuitive decomposition methods. The first attempts at developing intuitive decomposition schemes for large structural design problems were extensions of the fully stressed design method. In these approaches, the structure is seen as a combination of elements (substructures). Given an initial design for all the elements, an analysis of the structure is made to determine interelement forces. Then, each element is optimized separately on the assumption that changes in that element design do not change interelement forces. Once all the elements have been designed, the structure is analyzed again and the process is repeated until convergence is achieved. Giles[5] and Sobieski[6] performed the design of airplane wings under constraints on stresses and element stability using that approach. Kirsch and coworkers[7] designed frameworks using a

similar approach but reanalyzing the structure after each substructure optimization, in order to account better for load redistribution. When optimizing a structure one substructure at a time, it is difficult to handle global constraints, that is constraints affected by variables belonging to more than one substructure. Sobieski and Loendorf[8] described a procedure for structural sizing of airplane fuselages under local constraints on stresses and local instabilities and global constraints on fuselage elastic displacements. The optimization was first carried out with the local constraints, as described above. If necessary, the resulting design was subsequently modified to satisfy the displacement constraints using a unit load method to determine the impact that changes in element design have on the violated displacements. A generalized, fully-stressed design approach to large problems is certainly economically appealing. However, it presents two difficulties. Minimization of individual component masses does not guarantee minimization of the total mass; this situation is caused by the inability to control the load path on the assembled structure level. As mentioned earlier, it also makes it difficult to handle global constraints. Schmit and coworkers[9] used a two-level approach to design trusses and aircraft wings. At the global level, the distributions of stiffnesses, the global level variables, were chosen so as to minimize the total structural weight, while satisfying global displacement constraints and also some local constraints on stresses and structural element buckling. For known stiffnesses, the end forces on the various structural elements were calculated. At the local level, these structural elements were optimized separately with respect to their detail design, the local level variables, so that the changes in element stiffnesses were minimized, while the local constraints were still satisfied. The process was repeated until convergence was achieved. In this decomposition, the introduction of the global level problem was a key factor in overcoming both difficulties attributed to the generalized, fully-stressed design approach. By placing the minimization of the weight at the global level, the opportunity was kept to trade structural mass between the elements in order to improve the load paths while reducing the total weight. Also, the ability to explicitly handle global constraints was retained.

The literature survey summarized above has not revealed any general method that would be capable of accounting for the couplings among the system and subsystems without having to reoptimize the subsystems for every variation of the parent system design variables and that would apply to general nonlinear programing problems. Since such repeated reoptimizations would be cost-prohibitive in most large-scale engineering applications, a new approach that accounts for system-subsystem couplings without repetitive subsystem reoptimizations has been developed at the NASA Langley Research Center and is now at the stage of testing and verification. The approach is called "linear decomposition" for reasons that will become apparent soon.

Linear Decomposition

For generality, fig. 7 shows a generic system decomposed to form a hierarchical, three-level tree. If the system were a structure, the top level would represent the assembled structure, each subsystem at the middle level would correspond to a substructure, and the bottom level subsystems would simulate individual structural components, e.g., stiffened panels. Thus, three levels is the minimum we need to have each level qualitatively different for generality of the discussion. We assume that the system has been initialized so that physical characteristics are completely defined at each level. It is not necessary for the initialized system to be feasible.

Analysis. For simplicity of the presentation, it is convenient to assume that the analysis proceeds from top to bottom so that output from the analysis of a parent subsystem becomes input for the analysis of the subordinated subsystems. For an example, consider a structure assembled of substructures and loaded by forces applied at the substructure boundary nodes. For the analysis of such substructure, one needs an internal, local input and an external input. An example of the former is a set of the material properties and cross-sectional dimensions, and an example of the latter are the boundary forces. The internal input has to be initialized by the "best" trial design, the external input comes from the parent subsystem analysis. In substructuring, the substructure boundary forces from the assembled structure analysis are fed into the substructure analysis as loads, and the output-input chain continues by entering the internal forces from substructure analysis into the analysis of a substructure of the next lower order, or into an individual structural component analysis.

In many engineering applications, the decomposition must account for the fact that inputs to the analysis of a given subsystem may be coming not only from its parent but from any other subsystem at the same or even a different level, including inputs from the subordinated subsystems to their parent. An example of the latter can be drawn from the substructuring analysis in the case where the loads applied to substructure interior nodes are reduced to equivalent loads applied at the substructure boundary nodes. This requires an analysis of the substructure before commencing the assembled structure analysis. In other words, a system decomposition may lead to a network rather than the "top-down" graph shown in fig. 7. However, we will limit this discussion to the case depicted in fig. 7 in order to keep it as simple as possible for a clear introduction of the basic approach. Extension of the approach necessary to handle the network systems is presented in reference (10).

It is important that analyses at each level include the sensitivity analysis necessary to produce derivatives of the output quantities with respect to the input quantities. These derivatives measure sensitivity of behavior (response) to the input variations.

Obviously, if there are several subsystems at a given level they can be analyzed concurrently.

Cumulative constraint. Fig. 8 introduces the concept of a cumulative constraint that will be needed in further discussion. A cumulative constraint is a single number that is a function of a set of the constraint functions

$$\Omega = f(g_i), \quad i = 1,m \qquad (1)$$

and measures the degree of satisfaction, or violation, of the entire set. We formulate an individual constraint function as

$$g_i = \frac{DEMAND}{CAPACITY} - 1 \leq 0 \qquad (2)$$

which is the negative of the conventional margin of safety.

There are several ways to formulate the cumulative constraint as a function of the individual constraints in the set, for instance, the well known quadratic exterior penalty function is a cumulative constraint. The particular formulation adopted here is a function

$$\Omega = \frac{1}{\rho} LN \left(\sum_i (e^{\rho g_i}) \right) \qquad (3)$$

referred to as the Kresselmeir-Steinhauser function.[11] The function is continuous and differentiable, in contrast to the envelope of the constraint functions which is slope-discontinuous at the constraint function intersections, and, as seen in the graph, it follows the constraint envelope at a distance that is user-controlled by the factor ρ. In effect, it approximates the minimum of the safety margins of the set of individual constraints. Increase of the factor ρ draws the function closer to the envelope. However, the factor should not be set so large that the cumulative constraint function loses numerical differentiability by forming sharp "knees" at the constraint intersections.

Optimization - Having described the analysis and introduced the concept of cumulative constraint, we can discuss the optimization process. This introductory discussion will be limited to one variant of the algorithm under which the process proceeds from the bottom up. Another variant which allows a reversed order, from the top down, has also been developed.[12] Each subsystem optimization at the bottom level, shaded in fig. 9, involves:

1. design variables X_B: physical quantities local to the subsystem, e.g., detailed cross-sectional dimensions of a panel.

2. objective function Ω^B: the cumulative constraint (eqs. 2 and 3) of the subsystem constraints such as local buckling, stress, etc.

3. constant parameters: inputs, I_B, received from the parent subsystem. Denoting the vector of output from the middle-level parent subsystem analysis by O_M, we have

$$I_B = O_M \qquad (4)$$

4. equality constraints h_B: these constraints may be required in order to preserve the constancy of the parameters, for example, if a parameter is a total cross-sectional area of a panel, an equality constraint on the detailed cross-sectional dimension variables is needed.

5. inequality constraints: upper and lower limits on the design variables.

Using the above definitions and eq. 4, the optimization problem for each subsystem at the bottom level can be expressed formally as:

find X_B such that

$$\min_{X_B} \Omega^B(X_B, O_M) \qquad (5a)$$

is determined subject to

$$h_B (X_B, O_M) = 0 \qquad (5b)$$

$$L_B \leq X_B \leq U_B \qquad (5c)$$

The use of a cumulative constraint as the subsystem objective is a logical choice because it is a nondimensional quantity and, therefore, it is comparable among the subsystems regardless of their physical nature which may differ from one to another. The subsystem optimization is followed by sensitivity analysis of the minimum of the objective with respect to the subsystem input quantities (equal to the output from the parent subsystem, eq. 4). This analysis which yields sensitivity information in the form of partial derivatives

$$\frac{\partial \Omega^B_{min}}{\partial O_{Mi}}$$

where subscript "i" refers to an element of the output vector O_M, is called optimum sensitivity analysis to distinguish it from the behavior sensitivity analysis and is carried out not by finite difference but by a special algorithm.[13] Thus, the results from each subsystem optimization are the minimum of the cumulative constraint and its sensitivity to the output from the parent subsystem. These results are now carried upward to the parent subsystem.

If there are several subsystems at a given level their optimizations can be executed concurrently.

Now, moving up one level to the middle level, shaded in fig. 10, we perform a subsystem optimization for each subsystem at that level. The optimization involves:

1. design variables X_M: physical quantities local to the subsystem, e.g., membrane stiffness of the wing box at several locations over the wing.

2. objective function Ω_M: cumulative constraint for a set of constraints that includes two subsets. The first subset consists of the individual constraints g_M intrinsic to the subsystem itself regarded as an entity assembled of the subordinated subsystems, e.g., limit on the tip deflection of a wing box made up of spar beams and cover panels. The second subset includes the minimum values of the cumulative constraints, Ω^B_{min}. These values are approximated by the linear extrapolation estimates $(\Omega^B_{min})_e$ as functions of the middle level design variables

$$(\Omega^B_{min})_e = \Omega^B_{min} + \sum_i \frac{\partial \Omega^B_{min}}{\partial O_{Mi}} \Delta O_{Mi} \qquad (6)$$

using the minimum values of cumulative constraints, Ω^B_{min}, and their optimum sensitivity derivatives transmitted from the subordinated lower level subsystems. These derivatives are taken with respect to the middle-level subsystem output quantities which, in turn, are governed by the subsystem design variables. This linear extrapolation eliminates the need to reoptimize the subordinated bottom-level subsystems for each design variable change introduced in the middle-level parent subsystem and gives the method its name of linear decomposition.

3. constant parameters I_M: inputs received from the parent (top level) system.

For mathematical completeness, the quantities Ω^B_{min} and its partial derivatives that are received from the bottom-level subordinated subsystems and appear in eq. 6 should also be included as parameters. However, these quantities are omitted here in the discussion and in the ensuing equations for simplicity of this introductory presentation and because it has been found that they do not always have a numerical effect on the results. Complete formulation that includes these parameters is given by Barthelemy.[12]

Denoting the vector of output from the analysis at the top level by O_T, we have

$$I_M = O_T \qquad (7)$$

4. equality constraints which are analogous to those defined for the bottom level.

5. inequality constraints, g_M, which include also move limits on the middle level

subsystem design variables that are necessary for control of the extrapolation error introduced by the use of eq. 6.

Using the above definitions and eq. 7, the optimization problem for each subsystem at the middle level may be expressed formally as: find X_M such that

$$\min_{X_M} \Omega^M (X_M, O_T) \qquad (8a)$$

is determined, where

$$\Omega^M = \Omega^M(g_M, \Omega^B_{min}) \qquad (8b)$$

with Ω^B_{min} approximated as a function of X_M by means of eq. 6, subject to

$$h_M(X_M, O_T) = 0 \qquad (8c)$$

$$L_M \leq X_M \leq U_M \qquad (8d)$$

The solution of the above problem is subsequently analyzed for sensitivity with respect to parameters O_T. The minimum of the cumulative constraint and its derivatives with respect to the system output O_T are carried to the top level.

Optimization at the top level, shaded in fig. 11, involves:

1. design variables X_T: physical quantities that govern the entire system, for an aircraft example: configuration geometry, structural weight prescribed for the airframe, etc.

2. objective function F: a measure of the system performance, e.g., fuel consumption or direct operating cost.

3. two sets of inequality constraints:

(a) g_T: system performance limitations, e.g., takeoff field length.

(b) the cumulative constraints Ω^M_{min} from each middle level subsystem "i" approximated as functions of X_T by means of a linear extrapolation based on the optimum sensitivity derivatives:

$$(\Omega^M_{min})_e = \Omega^M_{min} + \sum_i \frac{\partial \Omega^M_{min}}{\partial O_{Ti}} \Delta O_{Ti} \qquad (9)$$

Inspection of eqs. 6 and 9 reveals that eq. 9 is recursively related to eq. 6.

4. the upper and lower limits on the design variables, including the move limits to control the linearization error introduced by eq. 9.

Using the above definitions, the optimization problem at the top level (system level) is formally defined as:

find X_T such that

$$\min_{X_T} F(X_T) \qquad (10a)$$

subject to

$$g_T \leq 0 \qquad (10b)$$

$$(\Omega^M_{min})_e < 0 \qquad (10c)$$

(where $(\Omega^M_{min})_e$ is evaluated by eq. 9)

$$L_T < X_T < U_T \qquad (10d)$$

The top level optimization deals with the system performance directly and has embedded in it an approximation to all the subsystem constraints in the form of the linear extrapolations (eq. 6 and eq. 9) based on the subsystem optimum sensitivity derivatives. These derivatives quantify the design trade-offs among the subsystems and account for their couplings. In other words, the optimum sensitivity derivatives carry the information (with the accuracy of a first-order approximation) about the effects that the variations of the top level (system level) design variables will have on the critical safety margins of all the subsystems in the system.

Referring to the terminology introduced in the section on decomposition literature, we will recognize the optimization of a particular subsystem at the middle level as a coordination problem for the cluster of subordinated subsystems at the bottom level. By the same token, the optimization of the top level is a coordination problem for the middle level subsystems directly, and for the bottom level subsystems indirectly. Although the presentation is limited to three levels, its generalization to n-levels is straightforward by inserting more levels between the top and bottom ones. In substructuring, for instance, it would call for dividing the substructures into sub-structures of a lower rank, while placing the individual structural components at the bottom level.

The analysis and the optimizations constitute one cycle of the iterative procedure which continues until the extremum of the system objective is found and all the system constraints and the subsystem cumulative constraints are satisfied. For more algorithmic detail, consult Sobieszczanski-Sobieski.[10]

Heuristic or formal decomposition. While the procedure described in the previous five figures is generic, the decomposition of the system is problem dependent. It can be done by a common sense inspection and judgment, as illustrated in the top of fig. 12. It can also be done formally by inspecting the matrix relating the design variables to the objective and constraint functions, as shown in the lower part of fig. 12. A dot at the row-column intersection means that the variable corresponding to the column appears in the equation corresponding to the row, and a blank means that the variable does not appear in that equation. The three examples show typical patterns.

Adaptability to the design stage. Once the decomposition tree is established, it can be grown with respect to the number of subsystems and the depth and detail of analysis, as depicted in fig. 13 (see the fuselage finite-element model changing from a "stick" model to a stiffened shell model). This adaptability allows the same overall logic of approach to be used at various stages of design while changing the modules in that logic - a desirable feature from the standpoint of the process integration.

Issues under research. The multilevel procedure described here is still being developed toward a state of maturity required for industrial applications. To achieve that state, a research program continues to investigate several fundamental issues.

One of these issues is the convergence at the bottom, middle, and top levels of optimization, and includes the basic question whether the procedure converges to at least a local minimum of the nondecomposed problem. Closely related to this issue is the economy of the method measured by the labor cost and computational cost. Another question is that of accuracy, relating to both the linear extrapolations based on the sensitivity derivatives and to the consistency of the level of analysis applied to different subsystems. It may not be easy to assure the latter, especially when the subsystems are physically dissimilar.

A particularly intriguing problem appears to be the synchronization of the analysis and optimization tasks being performed concurrently. Obviously, it may not be possible to achieve the decomposition in such a way that each subsystem analysis and optimization at a given level would take the same time (again, especially if the subsystems are physically dissimilar). If these times are not equal, the optimization of the parent subsystem can either wait for returns from all the subsystems, or it may proceed with old information from the "slow" subsystems until the new information is available. The overall convergence will probably be affected by the course of action chosen in this regard.

Probably, the most important issue is that of incorporating human judgment, control, and creativity into the entire procedure. Deliberately, the procedure is set up to facilitate these human contributions because each subsystem analysis and optimization is a self-contained, black box operation and, in principle, it can be carried out by any mixture of human judgment, analysis, and even experiment. A problem may arise if human judgment suggests a discrete change in the subsystem optimization. Such discrete change, e.g., shifting to another design concept in a subsystem, may have an, as yet unknown, impact on the overall convergence.

The development toward maturity involves continuation of the literature survey whose condensation was included previously, numerous tests of several variations of the algorithm using very simple test cases, and a fairly large test case of a framework structure. A multidisciplinary test is underway for reconfiguration of a transport aircraft wing treated as a part of an aircraft system. The latter two cases are outlined in a subsequent section; in addition, the series of test cases includes, a wing separated from the aircraft, and a high performance sailplane wing design.[12] The issues of computational parallelism and synchronization among the subtasks are being explored using a network of "desk top" microcomputers connected to a central hard disk.[14]

Application Experience

This section provides a brief account of the numerical experiments undertaken to validate multilevel optimization by linear decomposition.

Structural Applications

A two-level structural optimization of a framework shown in the upper left of fig. 14 has been successfully carried out and reported.[15] The decomposition illustrated by the schematic in fig. 14 exploits the fact that the end forces shown in fig. 14 acting on each separate beam in the framework can be calculated using the cross-sectional area, A, and moment of inertia, I, for the beams without directly using the beam design variables which are shown for the I-shaped cross-section in the inset to the right in fig. 14. Furthermore, the constrained quantities, e.g., stresses, in an individual beam can be calculated using only the beam's detailed cross-sectional dimensions and the applied end forces. The local constraints for each beam guard against overstress and local buckling. There is also a constraint at the system level to prevent an excessive horizontal displacement of the upper right-hand corner of the framework where the load is applied.

Thus, we have a case of a decomposition consisting of the top level (system level) corresponding to the assembled framework and the bottom level corresponding to the isolated beams. In this decomposition scheme, the A's and I's are the system design variables and the detailed dimensions are the subsystem design variables. The beam is optimized by reducing the cumulative constraint to a minimum (maximizing the safety margin). In the process, the beam cross-section is reproportioned while preserving the A and I prescribed for the beam at the system level. This is a reverse of a conventional approach of resizing structural components to a minimum weight for the given end forces.

Optimization at the top level manipulates the framework stiffness distribution by means of the A and I variables moving the design toward the minimum structural weight that can be attained while satisfying the system level constraint for displacement and the subsystem level constraints for stress and local buckling.

A detailed report[15] indicated that this test showed good correlation with a reference obtained by conventional optimization without decomposition. The minimum structural weight values obtained for various starting points were within 2 percent of the reference value.

The two-level framework structure has been extended to three levels, as illustrated in fig. 15, by replacing the I-shaped beam cross-sections with box beams made up of stringer-reinforced panels (cross-section shown in Detail B). The panels add the third, bottom level of subsystems. This makes the test more general because it contains now all three level categories: top, middle, and bottom. At the time of this writing, the tests are still in progress, and preliminary results are promising.

Multidisciplinary Application

The wing of a transport aircraft (Lockheed L-1011), depicted in fig. 16, is to be reconfigured to minimize fuel consumption for a given mission.

To achieve this, a three-level decomposition has been applied, as shown in fig. 17. The procedure that has been introduced in the generic terms in the preceding sections of this paper acquires now a specific, physical meaning in each of its elements.

At the top level, we consider the aircraft as an assembled system that should have positive safety margin (all constraints satisfied) and a minimum fuel consumption for a given mission. The configuration dimensions noted in fig. 16 and the wing box structural weight are design variables at this level. The constraints are the aircraft performance constraints such as maximum takeoff field length, minimum range for a given payload, etc. Analysis at the top level is standard aircraft performance analysis[16] that includes calculation of the lift and drag data needed for performance evaluation. Parameters passed to the middle level are the structural weight of the wing box, the wing geometry, and the data needed to calculate aerodynamic loads at the middle level.

Since only the wing is to be reconfigured in this particular case, the middle level consists

of only one subsystem - a wing structural box
built up of spar beams, ribs, and cover panels.
The panels are sheet metal reinforced by
stringers which cause the stiffness properties
to be orthotropic. Consequently, the wing cover
membrane stiffnesses in the spanwise and chord-
wise directions can be controlled by thickness
of the panel sheet material and an equivalent
thickness of the stringer material. To provide
a smooth distribution of stiffness over the
wing, the two thicknesses for each panel are
defined by an assumed distribution function
extending over several panels. One such
function used for initial testing is shown for
the sheet metal thickness in fig. 18. The
function is a quadratic polynomial in a
dimensionless spanwise coordinate, β, and has
two, separate branches: one outboard and one
inboard, with the engine supporting rib being
the dividing line. The thickness is set
constant chordwise, and is the same for the
upper and lower covers. The middle level design
variables are the coefficients, C, in the
polynomial shown in fig. 18, one may regard this
stiffness distribution as an example of variable
linking. By prescribing a different function,
including in it the chordwise coordinate, and
using separate functions for the upper and lower
covers, it is possible to introduce more
realistic spanwise and chordwise variability in
the thickness distribution. The constraints
local to the wing box subsystem guard against
excessive wingtip displacements and are included
in a cumulative constraint which represents also
the panel constraints (eq. 6). There is also an
equality constraint to maintain the constant
structural weight value prescribed for the wing
box as a parameter received from the top level.

Analysis at the middle level includes
aerodynamic load calculation on the wing, and a
finite-element analysis of the displacements and
internal forces. The parameters passed to the
bottom level are geometrical dimensions for each
panel, the sheet material thickness, the
equivalent thickness of the stringer material,
and the edge loads (N_x, N_y, N_{xy}) obtained
as internal forces from the analysis of the
finite-element model in which the panels appear
as orthotropic finite elements.

The bottom level consists of 216 wing cover
panel optimization problems. They are all
independent of each other and could be processed
concurrently, if a network of computers or a
multiprocessor computer were available. The
design variables are cross-sectional dimensions
of the stringers that are to be varied within
the equality constraints of the constant equiva-
lent stringer thickness and sheet material
thickness. The object of the optimization is to
minimize a cumulative constraint that represents
the constraints of stress and local buckling in
the panel. These constraints are evaluated by
closed form formulas commonly used in
engineering practice. In contrast to the
conventional approach, it is not the panel
weight that is minimized in the bottom level
optimization but the cumulative constraint,
while the weight is maintained constant as
prescribed by the parameters of sheet material

thickness and stringer material equivalent
thickness set at the middle level optimization.

Optimization at all levels is performed by
means of nonlinear mathematical programing
techniques. The technique of usable-feasible
directions is used at the bottom and middle
levels, while a SUMT with a Davidon-Fletcher-
Powell algorithm is used at the top level.

At the time of this writing, all levels of
the decomposition shown in fig. 17 were imple-
mented on the computer and tested individually.
In addition, the middle and bottom levels
(shaded in fig. 17) were connected and tested
together. All these intermediate tests have
been satisfactory as reported in reference (17).

Reference results for judging the performance
of the new method are provided by the test
aircraft manufacturer.[18] The reference
reports on the solution of the same design
problem obtained by a conventional parametric
study method using well-established and
experimentally validated computer programs.

Concluding Remarks

A particular approach to decomposition of a
large engineering problem into a set of smaller
subproblems organized in several levels has been
outlined. The approach uses linear extrapola-
tions based on optimum sensitivity derivatives
as a means to quantify the design trade-offs.
The method is intended as an aid to human
intellect for application to large engineering
system design problems (e.g., aircraft) which
are dominated by quantitative, computable
considerations.

In its present development, the method's
theory has been initially established.
Validation tests are underway using structural
and multidisciplinary test cases; the latter
includes an aircraft configuration for which
industry-generated data are available to gage
the method's performance. The test results to
date have been satisfactory and the development
continues toward the maturity necessary for
industrial applications.

Implementation of the proposed multilevel,
linear decomposition in design will fit the
existing organization of professionals and will
exploit the new technology of distributed,
parallel computing, as illustrated in fig. 19.
The approach introduces the new element of
mathematical quantification of the design
trade-offs and will establish a precise
definition for information exchange among the
specialists working on their subtasks. Under
the proposed scheme, the decisionmakers at each
level will know the consequences of their
decisions on other coupled subsystems. Based on
this knowledge, it should be possible to improve
the design integration toward higher performance
and lower cost.

References

[1]Lasdon, L. S.: Optimization Theory for Large Systems. MacMillan Series in Operations Research, MacMillan Publishing Co., Inc., 1970.

[2]Woo, T.-H.: Decomposition in Structural Synthesis. Ph. D. Dissertation, University of California, Los Angeles, California, 1977.

[3]Mesarovic, M. D.; Macko, D.; and Takahara, Y.: Theory of Hierarchical, Multilevel, Systems. Academic Press, N.Y., 1970.

[4]Kirsch, U.: Optimum Structural Design. McGraw-Hill Book Co., N.Y., 1981.

[5]Giles, G. L.: Procedure for Automating Aircraft Wing Structural Design. Journal of the Structural Division, ASCE, Vol. 97, January 1971, pp. 97-113.

[6]Sobieszczanski-Sobieski, J.: An Integrated Computer Procedure for Sizing Composite Airframe Structures. NASA TP 1300, February 1979.

[7]Kirsch, U.; Reiss, M.; and Shamir, U.: Optimum Design by Partioning into Substructures. Journal of the Structural Division, ASCE, ST1, January 1972, pp. 249-267.

[8]Sobieszczanski, J.; and Loendorf, D.: A Mixed Optimization Method for Automated Design of Fuselage Structures. Journal of Aircraft, Vol. 9, December 1972, pp. 805-811.

[9]Schmit, L. A., Jr.; and Mehrinfar, M.: Multilevel Optimum Design of Structures with Fiber-Composite Stiffened-Panel Components. AIAA paper 80-0723, presented at the 21st AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, May 12-14, 1980, Seattle, Washington.

[10]Sobieszczanski-Sobieski, Jaroslaw: A Linear Decomposition Method for Large Optimization Problems - Blueprint for Development. NASA TM 83248, February 1982.

[11]Kresselmeir, G.; and Steinhauser, G.: Systematic Control Design by Optimizing a Vector Performance Index. Proceedings of IFAC Symposium on Computer Aided Design of Control Systems, Zurich, Switzerland, 1971.

[12]Barthelemy, J.-F.: Development of a Multilevel Optimization Approach to the Design of Modern Engineering Systems. Ph. D. Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, June 1983. Also NASA CR-172184, August 1983.

[13]Sobieszczanski-Sobieski, J.; Barthelemy, J.-F.; and Riley, K. M.: Sensitivity of Optimum Solutions to Problem Parameters. AIAA Paper 81-0548R. AIAA Journal, Vol. 20, No. 9, September 1982, pp. 1291-1299.

[14]Rogers, J. L., Jr.; and Sobieszczanski-Sobieski, J.: Initial Experience with Distribution of Structural Calculations Among Concurrently Operating Computers. NASA-GWU Symposium on Structural Analysis Trends, Proceedings of, October 1984.

[15]Sobieszczanski-Sobieski, Jaroslaw; James, Benjamin; and Dovi, Augustine: Structural Optimization by Multilevel Decomposition. AIAA paper 83-0832-CP, AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference, Lake Tahoe, Nevada, May 2-4, 1983.

[16]McCullers, L. A.: Aircraft Configuration Optimization Including Optimized Flight Profiles. Presented at NASA Symposium on Recent Experiences in Multidisciplinary Analysis and Optimization, Langley Research Center, Hampton, Virginia, April 24-26, 1984.

[17]Giles, G. L.; and Wrenn, G. A.: Multidisciplinary Optimization Applied to a Transport Aircraft. Presented at NASA Symposium on Recent Experiences in Multidisciplinary Analysis and Optimization, Langley Research Center, Hampton, Virginia, April 24-26, 1984.

[18]Radovcich, N. A.: Some Experiences in Aircraft Aeroelastic Design Using Preliminary Aeroelastic Design of Structures (PADS). Presented at NASA Symposium on Recent Experiences in Multidisciplinary Analysis and Optimization, Langley Research Center, Hampton, Virginia, April 24-26, 1984.
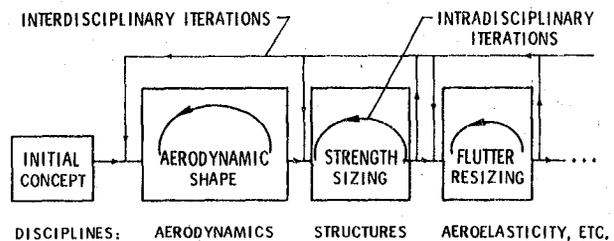
FIGURE 1 SEQUENTIAL DECISIONS IN DESIGN
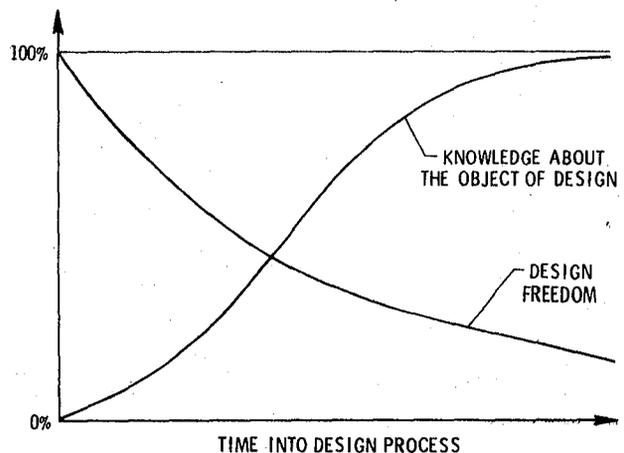
FIGURE 2 PARADOX IN THE SEQUENTIAL DESIGN
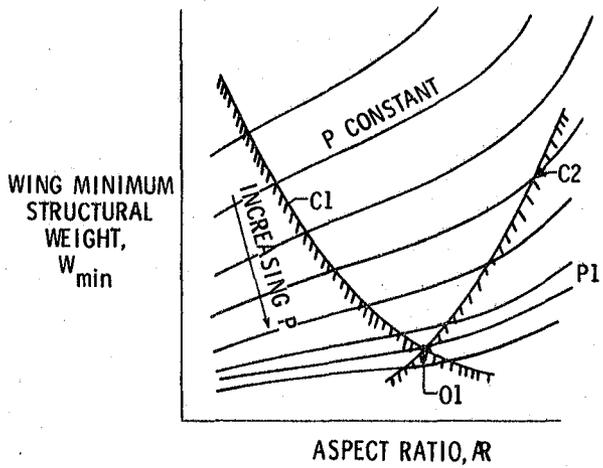
FIGURE 3   CONSTRAINED MINIMUM FOR
TWO CONSTRAINTS AND TWO
DESIGN VARIABLES
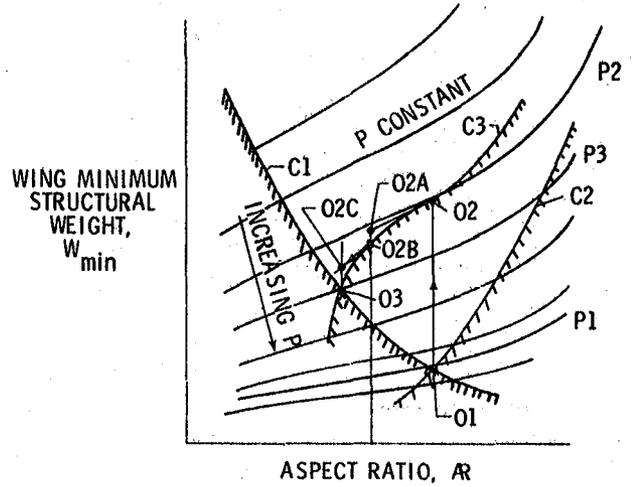
FIGURE 6   A PATH TOWARD NEW
CONSTRAINED MINIMUM
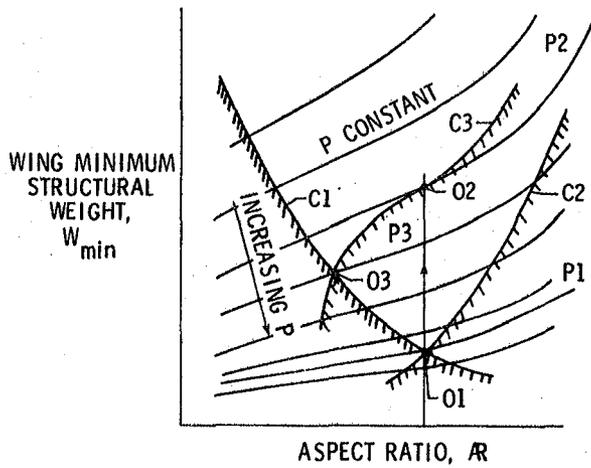
FIGURE 4   A NEW CONSTRAINT ADDED
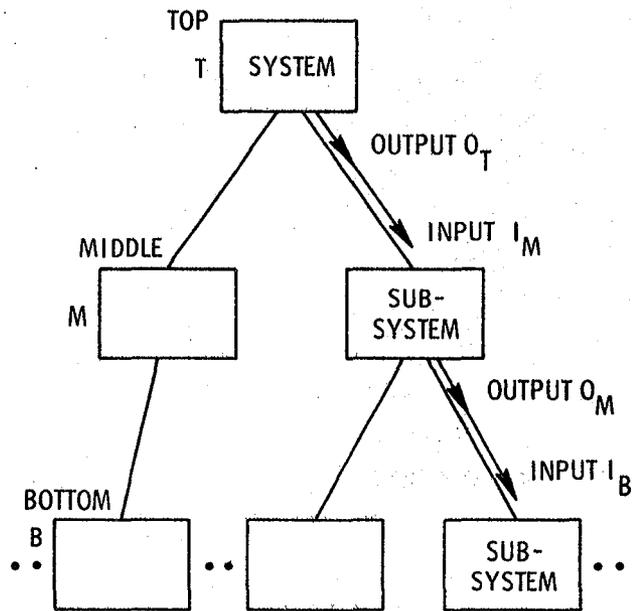
FIGURE 7   ANALYSIS
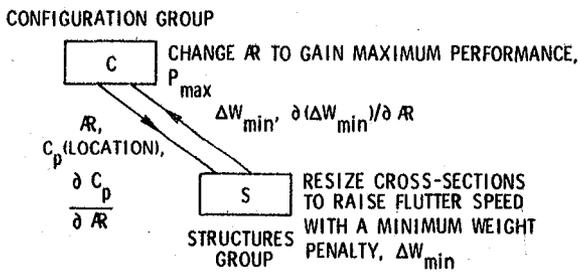
FIGURE 5   TWO ENGINEERING DISCIPLINES
SEARCH FOR A NEW
CONSTRAINED MINIMUM
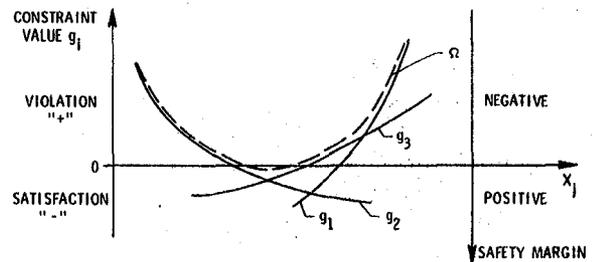
FIGURE 8   CUMULATIVE CONSTRAINT

11
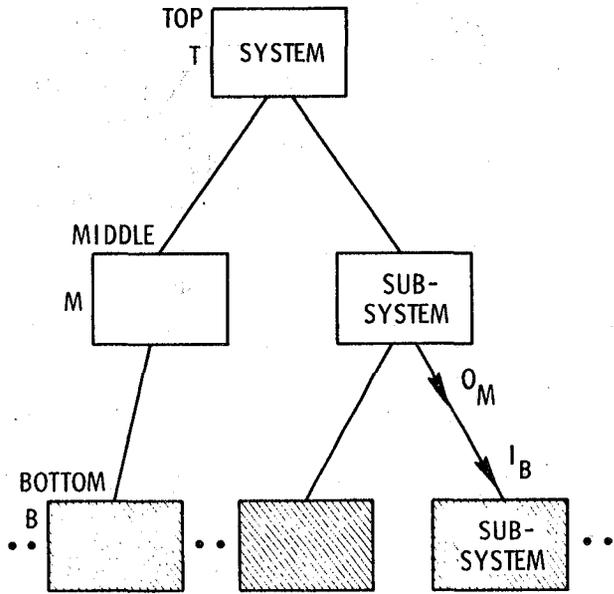
FIGURE 9 OPTIMIZATION: BOTTOM LEVEL

FIGURE 10 OPTIMIZATION: MIDDLE LEVEL
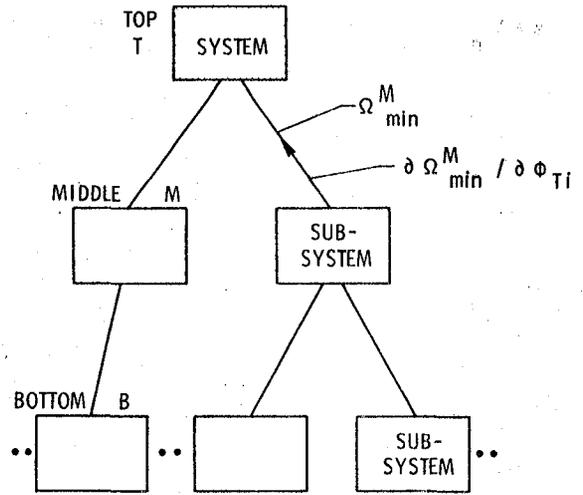
FIGURE 11 OPTIMIZATION: TOP LEVEL

• HEURISTIC: BY EXAMINATION OF THE SYSTEM PHYSICAL MAKE-UP.
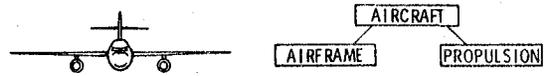
• FORMAL: BY INSPECTION OF THE FUNCTIONAL RELATIONS THAT GOVERN THE PROBLEM
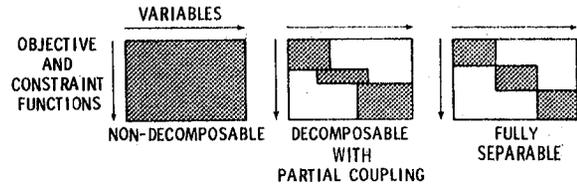
FIGURE 12 MANY WAYS TO DECOMPOSE A SYSTEM
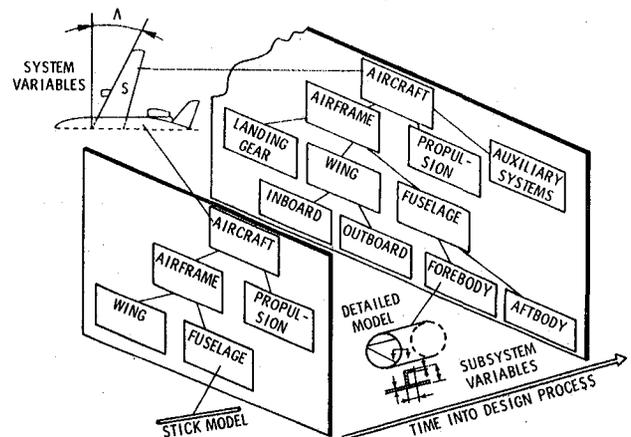
FIGURE 13 DECOMPOSITION ADAPTS TO DESIGN STAGE

FIND A's, I's TO MINIMIZE STRUCTURAL WEIGHT SUBJECT TO CONSTRAINTS ON DEFLECTION AND THE BEAM CONSTRAINTS

CUMULATIVE CONTRAINT Ω AND ITS DERIVATIVES

FIND DIMENSIONS OF THE BEAM CROSS-SECTION TO IMPROVE THE CONSTRAINT SATISFACTION AS MUCH AS POSSIBLE WITHIN THE GIVEN PARAMETERS. THE CONSTRAINTS INCLUDE: STRESS, LOCAL BUCKLING, ETC.
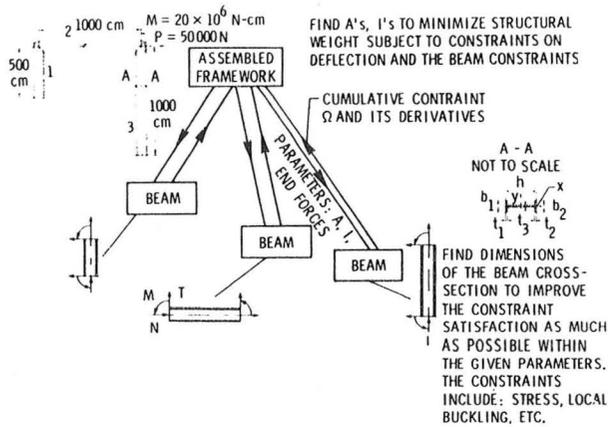
FIGURE 14   TWO-LEVEL FRAMEWORK TEST
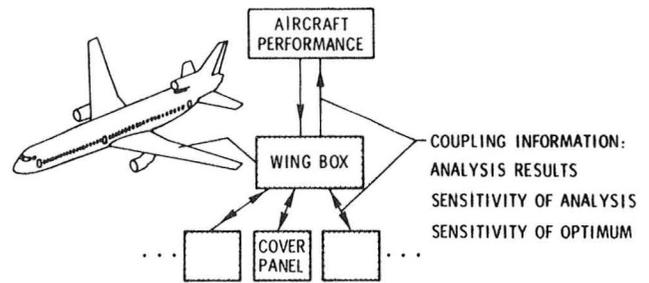
FIGURE 17   DECOMPOSITION FOR WING OPTIMIZATION

COUPLING INFORMATION:
ANALYSIS RESULTS
SENSITIVITY OF ANALYSIS
SENSITIVITY OF OPTIMUM
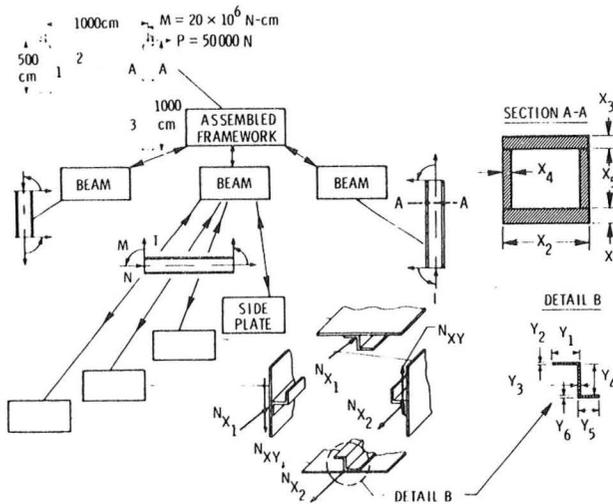
FIGURE 15   THREE-LEVEL FRAMEWORK TEST

$TSKIN = C_0 + C_1 (1-\beta) + C_2 (1-\beta)^2$

• SYMMETRIC UPPER AND LOWER WING BOX COVER SKINS
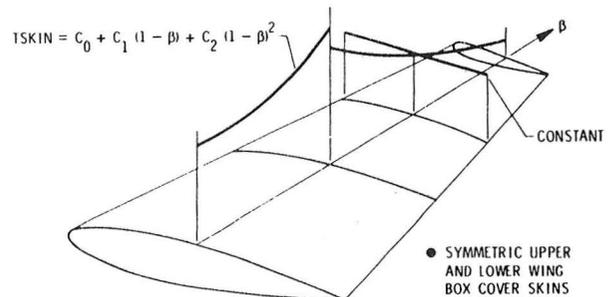
FIGURE 18   SKIN THICKNESS DISTRIBUTION, SIMPLIFIED FOR INITIAL TESTING

OBJECTIVE
MINIMUM OF BLOCK FUEL CONSUMPTION

CONSTRAINTS ON
PERFORMANCE
AERODYNAMICS
STRUCTURE

DESIGN VARIABLES:
h/c, Λ, S, R, C_t/C_r
AND
STRUCTURAL CROSS-SECTIONS
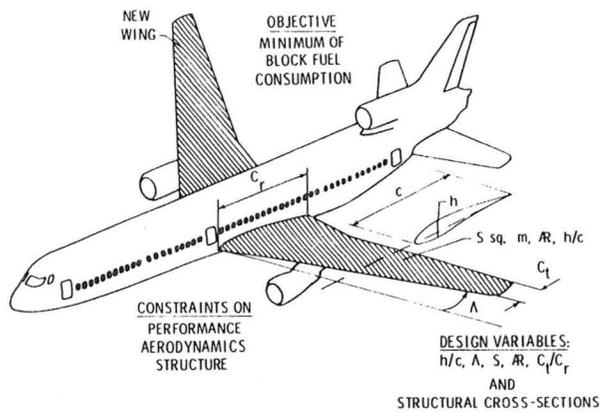
FIGURE 16   MULTILEVEL OPTIMIZATION OF A WING
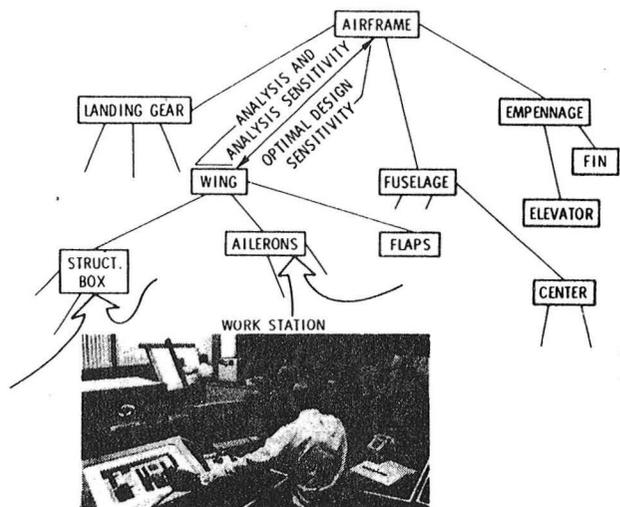
FIGURE 19   DESIGN OFFICE ORGANIZATION

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4. Title and Subtitle AEROSPACE ENGINEERING DESIGN BY SYSTEMATIC DECOMPOSITION AND MULTILEVEL OPTIMIZATION		5. Report Date June 1984
		6. Performing Organization Code 505-33-53-12
7. Author(s) Jaroslaw Sobieszczanski-Sobieski,* Jean-Francois M. Barthelemy,** and Gary L. Giles*		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Virginia 23665		11. Contract or Grant No.
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16. Abstract

This paper describes a method for systematic analysis and optimization of large engineering systems, by decomposition of a large task into a set of smaller subtasks that can be solved concurrently. The subtasks may be arranged in hierarchical levels. Analyses are carried out in each subtask using inputs received from other subtasks, and are followed by optimizations carried out from the bottom up. Each optimization at the lower levels is augmented by analysis of its sensitivity to the inputs received from other subtasks to account for the couplings among the subtasks in a formal manner. The analysis and optimization operations alternate iteratively until they converge to a system design whose performance is maximized with all constraints satisfied. The method, which is still under development, is tentatively validated by test cases in structural applications and an aircraft configuration optimization.

17. Key Words (Suggested by Author(s)) systematic analysis, optimization, decomposition, sensitivity, subtasks	18. Distribution Statement Unclassified - Unlimited Star Category - 05

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