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SYMBOLS

c	speed of sound in flow
F	source term in acoustic equation
f	source term in Helmholtz equation
$G(\vec{r}, t; \vec{r}_s, t_s)$	Green's function for acoustic wave equation in uniform subsonic flow
$\tilde{G}(\tilde{\vec{r}}, \tilde{t}; \tilde{\vec{r}}_s, \tilde{t}_s)$	Green's function for acoustic wave equation without flow
$G_h(\vec{r}; \vec{r}_s)$	Green's function for subsonic convected Helmholtz equation
$G_{2d}(x, y; x_s, y_s)$	Green's function for subsonic convected Helmholtz equation in two dimensions
$H_0^1(x)$	Hankel function, $J_0(x) + \imath Y_0(x)$
$\imath$	$\sqrt{-1}$
$J_n(x)$	Bessel function of the first kind of order n
k	wave number
M	wind tunnel Mach number, U_∞/c
m	harmonic number of acoustic frequency
N	number of blades
n	normal to duct surface, positive toward duct center
n	harmonic number of loading or thickness function
P	acoustic pressure
p	coefficient of Fourier component of acoustic pressure
R	radial distance
r_t	rotor radius
$\vec{r}, \vec{s}$	position
$\vec{r}_s$	source position

S	duct surface
t	time
t_s	source time
U_∞	velocity of uniform flow in duct
V	duct volume
x, y, z	coordinates in fixed system
$\tilde{x}, \tilde{y}, \tilde{z}$	coordinates in moving system
$Y_n(x)$	Bessel function of the second kind of order n
α	boundary condition coefficient
β	boundary condition coefficient
δ	Dirac delta function
δ_3	three-dimensional Dirac delta function
θ	altitude angle of observer to rotor plane
ξ	distance along blade chord
ρ_o	average density in flow field
ψ	azimuth angle of observer
ω	acoustic frequency
Ω	rotational frequency of rotor

THE INFLUENCE OF A WIND TUNNEL ON HELICOPTER ROTATIONAL NOISE: FORMULATION OF ANALYSIS

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SUMMARY

This paper describes an analytical model that can be used to examine the effects of wind-tunnel walls on helicopter rotational noise. First, a complete physical model of an acoustic source in a wind tunnel is described and a simplified version is then developed. This simplified model retains the important physical processes involved, yet it is more amenable to analysis. Second, the simplified physical model is modeled as a mathematical problem. An inhomogeneous partial differential equation with mixed boundary conditions is set up and then transformed into an integral equation. Details of generating a suitable Green's function and integral equation are given in appendixes. Last, the equation is discussed; it is also given for a two-dimensional case.

INTRODUCTION

It has become important to know the noise levels that a new helicopter will produce before it is put into production. Rules of the International Civil Aviation Organization (ICAO) and possible future rules of the Federal Aviation Administration (FAA) place limits on the allowable noise, measured in terms of effective perceived noise level (EPNL), that a helicopter may produce. To meet these rules, both EPNL and more detailed noise measurements of a helicopter need to be known before manufacturers produce new or modified helicopters. The current analytic prediction methods for the noise levels are completely inadequate for newly designed helicopters. The totally theoretical methods available do not give predictions that correlate well with measurements (for example see the comparisons in refs. 1 and 2). Using measured air loads in the formulations improves the comparisons, but does not yield satisfactory results either (refs. 3 through 5). Empirical noise prediction methods are used to give better correlation with measured noise levels (ref. 6) . However, an empirical method will only yield good results for helicopters that are very similar to those used to produce the empirical predictions. A newly designed helicopter will produce noise different from that of earlier helicopters; moreover, a new helicopter will have to be quieter than many existing helicopters in order to meet proposed noise rules of the FAA and ICAO and community noise standards.

Measuring the noise levels produced by small-scale and prototype models of a new-design helicopter is another way to estimate the noise The noise levels of interest are those produced during flyover, takeoff, and landing. Measurements made in any wind-tunnel test must be translated to the correct reference frame for a flyover measurement Measurements from a full-scale prototype must be corrected for the reverberant effects of the wind-tunnel walls, for there are no anechoic wind tunnels large enough to test a full-size helicopter Measurements from a small-scale model must be corrected for scale effects and may have to be corrected for reverberant wind-tunnel effects as well.

PHYSICAL MODEL OF PROBLEM

The problem of predicting the acoustic field of aeroacoustic sources in a wind tunnel is complicated A model that incorporates all of the physical phenomena is very elaborate. First, the source and its acoustic radiation must be modeled in a flow field The drive system of the wind tunnel also introduces a noise source. The presence of the wind-tunnel walls may have an effect on the source radiation (or strength). The walls produce a complicated semi-reverberant sound field by reflecting the sound. The boundary layer on the wall may alter the acoustic reflection. Also, wind-tunnel walls are a thin shell, therefore, an acoustic field inside the wind tunnel is coupled to structural modes of the wind tunnel which are also coupled to sound radiation from the wind-tunnel walls to the outside. Finally, there is the geometry of the wind tunnel to consider The mean flow may vary because of changes in the cross-sectional area, thus complicating the acoustic field. Obstacles, such as vanes, screens, and drive fans, as well as changes in cross-sectional area, reflect acoustic energy in the wind tunnel. If the tunnel is an open circuit configuration, acoustic pressure at the ends of the duct couples to the acoustic field outside the tunnel radiating away from the wind tunnel.

Modeling all of these phenomena requires a complicated system of coupled partial differential equations with each equation modeling a different aspect of the problem. The following describes the complexity of this type of model. The Ffowcs-Williams and Hawkings equation with a moving observer models the aeroacoustic source and its radiation to a surface just outside the boundary layer of the wind-tunnel wall. Acoustic pressure and velocity couple at the boundary layer to another equation describing acoustic transmission through the boundary layer. At the wall, the acoustic pressure and velocity couple to the wall velocity and acceleration. A fourth-order structural equation governs the vibration of the walls. Velocity and acceleration on the outside of the wall couple to the acoustic field outside. If the wind tunnel is an open circuit, the acoustic pressure field at the inlet and exit of the tunnel couple to the outside acoustic field. Outside the tunnel, the acoustic field can be modeled as a simple potential field with a radiation condition in the far field. At present the above model is impossible to solve, except for possibly the most elementary case involving very simple sources and tunnel geometries.

This paper presents an investigation of the sound field inside the wind tunnel relatively close to the source, so a simpler model, which retains the essential physical character of the acoustic field inside a wind tunnel, can be used. The model consists of a known harmonic acoustic source of finite dimension inside an infinite duct of constant cross-sectional area. The duct is assumed to contain a uniform subsonic flow. The wall boundary will be represented by a simple impedance condition.

This simpler model assumes that several physical effects are negligible. First, no wind tunnel is an infinitely long uniform duct; all contain drive fans and vanes, changes in area and shape, and are closed loops or open loops. All of these factors will affect the transmission of acoustic energy in the wind tunnel. Often, these changes are far from the test section which contains the noise source and the acoustic field of interest; therefore, it can be assumed that the changes have a negligible effect on the sound field in the test section. Vanes will have a small amount of acoustic backscatter. The drive fan will be far from the test section so its presence as an extra noise source and its influence in scattering sound should be small. Most changes in the cross section of a wind tunnel are gradual so they will not produce a strong acoustic reflection. The topological departure from an infinite duct is manifested by changes in the geometry far from the test section. The fundamental acoustic mode of the wind tunnel will be a very low frequency since the wavelength will be of the order of the length of the tunnel in a closed circuit tunnel, or twice the length of the tunnel in an open tunnel. High-frequency structual modes of the tunnel, where the acoustic source frequencies occur, should not strongly couple into the problem.

Next, the boundary conditions at the wall are not exact It is assumed that the influence of the boundary layer is small and can be incorporated into a point impedance condition. This local impedance condition also precludes any coupling between the acoustic field and structural modes of the wind tunnel.

Last are the assumptions involving the source This model assumes that the source radiation is known and that the amplitude is low enough for the linearized acoustic equation to apply For convenience, it is also assumed that the source can be decomposed into its separate frequency components, implying that the source is both periodic and that the linear equation holds Finally, it is assumed that the presence of the walls does not affect the acoustic radiation from the source. This is reasonable, because the acoustic source is not typically close to any wall; as a result the pressure and velocity induced by the presence of the wall are small compared to the pressure and velocity at the source produced by the noise sources on a helicopter These assumptions are consistent with modeling the source as a distribution of rotating acoustic monopoles, dipoles, and quadrupoles

MATHEMATICAL FORMULATION OF PROBLEM

An inhomogeneous, hyperbolic partial differential equation with mixed boundary conditions on the walls models the general problem of an acoustic source inside an infinite duct with uniform subsonic flow No analytic solution is known for this problem with arbitrary sources, arbitrary duct shape, and arbitrary wall impedance In this section, the problem will be formulated as a differential equation with boundary conditions and transformed into an integral equation.

The acoustic problem is modeled with linearized potential flow. Acoustic pressure P will be the variable used. A complex pressure is used to keep track of the phase information The physical acoustic pressure is represented by the real part of P.

The inhomogeneous convected wave equation with appropriate boundary conditions describes this acoustic flow. The wave equation comes from considering a small perturbation to the Euler equations describing flow in an inviscid, homogeneous fluid which does not conduct heat. A combination of the equations describing conservation of mass and momentum will produce the wave equation, as in reference 7 For a uniform mean flow $\vec{U}_{\infty}$ with acoustic source term F the governing equation is

$$\nabla^2 \mathrm{P}(\vec{r}, t) - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + \vec{U}_{\infty} \cdot \nabla\right)^2 \mathrm{P}(\vec{r}, t) = \mathrm{F}(\vec{r}, t) \tag{1}$$

The boundary condition on the walls of the duct relates the acoustic pressure to the normal gradient of pressure:

$$\alpha \mathrm{P} - \beta \nabla \mathrm{P} \cdot \vec{n} = 0 \tag{2}$$

The coefficient combination β/α has the dimensions of pressure/velocity and is equivalent to specifying the impedance without flow The normal to the duct wall is n (positive inward) Also, a radiation condition must be enforced in the duct far away from all sources The radiation condition used will be explained later when the integral relation is developed for the acoustic potential in a duct.

The coordinate system, as shown in figure 1, is oriented such that the only mean flow component is in the positive x-direction. No loss of generality results, because the flow is assumed to be uniform in a duct of constant cross section. The governing equation becomes

$$\nabla^2 \mathrm{P}(\vec{r}, t) - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + U_{\infty} \cdot \frac{\partial}{\partial x}\right)^2 \mathrm{P}(\vec{r}, t) = \mathrm{F}(\vec{r}, t) \tag{3}$$

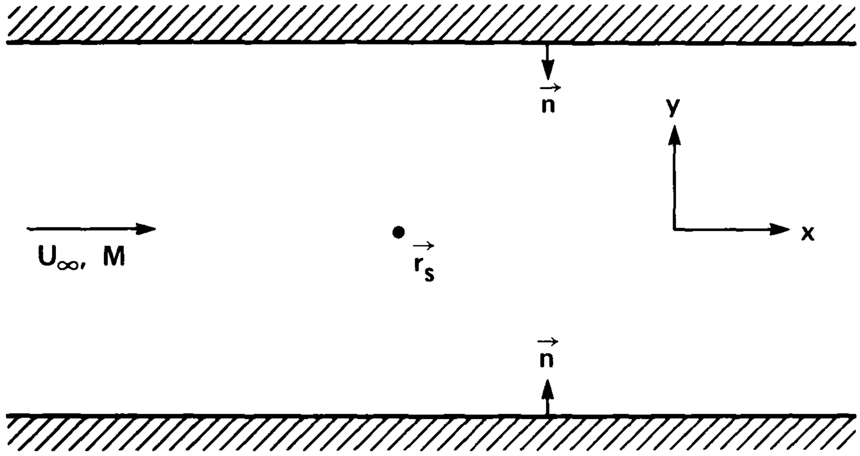

Figure 1. – Fixed reference frame

The acoustic sources considered and, therefore, the resulting acoustic field, will be periodic. A Fourier series decomposition will be used so each frequency component can be considered separately. The acoustic pressure and source will now have the forms

$$P(\vec{r}, t) = \sum_{\omega} a(\omega) e^{-i\omega t} p(\vec{r}) \tag{4}$$

and

$$F(\vec{r}, t) = \sum_{\omega} b(\omega) e^{-i\omega t} f(\vec{r}) \tag{5}$$

With this transformation there is a change to the variable $k = \omega/c$. The equation becomes

$$\nabla^2 p(\vec{r}) - M^2 \frac{\partial^2 p(\vec{r})}{\partial x^2} + 2iMk \frac{\partial p(\vec{r})}{\partial x} + k^2 p(\vec{r}) = f(\vec{r}) \tag{6}$$

The free-field Green's function for the convected Helmholtz equation will be used. The Green's function, $G_h(\vec{r}; \vec{r}_s)$, describes the acoustic response at the location $\vec{r}$ to a harmonic source of frequency ω at the location $\vec{r}_s$ in unbounded space For a uniform mean flow in the positive x-direction, $G_h(\vec{r}, \vec{r}_s)$ satisfies the following partial differential equation with a radiation condition at infinity:

$$\nabla^2 G_h(\vec{r};\vec{r}_s) - M^2 \frac{\partial^2 G_h(\vec{r};\vec{r}_s)}{\partial x^2} + 2iMk\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x} + k^2 G_h(\vec{r};\vec{r}_s) = \delta_3(\vec{r}-\vec{r}_s) \quad (7)$$

The solution (derived in appendix A) is

$$G_h(\vec{r};\vec{r}_s) = \frac{-e^{ik\frac{[R-M(x-x_s)]}{(1-M^2)}}}{4\pi R} \quad (8)$$

with

$$R = \sqrt{(x-x_s)^2 + (1-M^2)[(y-y_s)^2 + (z-z_s)^2]} \quad (9)$$

In unbounded space, this Green's function gives the solution to equation (6) by evaluating the integral

$$p_f(\vec{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_h(\vec{r};\vec{r}_s) f(\vec{r}_s) d\vec{r}_s \quad (10)$$

The subscript f on the pressure coefficient will be used to designate this pressure as the pressure in unbounded or free space. Next, an integral relation is formed for $p(\vec{r})$ on and inside the duct by use of Green's second formula The relations are

$$\frac{p(\vec{r})}{2} = \int\int (-p(\vec{r})[\nabla G_h(\vec{r};\vec{r}_s) \cdot \vec{n}] + G_h(\vec{r},\vec{r}_s)[\nabla p(\vec{r}) \cdot \vec{n}])dS + \quad (11a)$$

$$\int\int\int G_h(\vec{r};\vec{r}_s) f(\vec{r}) dV$$

on the duct surface S, and

$$p(\vec{r}) = \int\int (-p(\vec{r})[\nabla G_h(\vec{r};\vec{r}_s) \cdot \vec{n}] + G_h(\vec{r},\vec{r}_s)[\nabla p(\vec{r}) \cdot \vec{n}])dS + \quad (11b)$$

$$\int\int\int G_h(\vec{r};\vec{r}_s) f(\vec{r}) dV$$

in the duct volume V. The derivation of this relation is given in appendix B. The corresponding equations for the two-dimensional case are given in appendix C.

On the surface of the duct S, the integral relation and the boundary condition combine to give

$$\frac{p(\vec{r})}{2} = \int\int p(\vec{r})\left[-\nabla G_h(\vec{r};\vec{r}_s)\cdot\vec{n} + G_h(\vec{r};\vec{r}_s)\frac{\alpha}{\beta}\right]dS + \tag{12}$$

$$\int\int\int G_h(\vec{r};\vec{r}_s)f(\vec{r})dV$$

when $\beta \neq 0$. The restriction that $\beta \neq 0$ is applicable to any solid or porous surface since $\beta = 0$ implies a vacuum. This is an exact equation for the acoustic pressure $p(\vec{r})$ on the surface of the duct. All quantities except $p(\vec{r})$ are known. In general, this equation cannot be solved analytically; however, it can be solved numerically. From the solution of $p(\vec{r})$ on the surface of the duct, the value of $p(\vec{r})$ anywhere inside the duct, and thus the acoustic pressure in the field, can be found from

$$p(\vec{r}) = \int\int p(\vec{r})\left[-\nabla G_h(\vec{r};\vec{r}_s)\ \ \vec{n} + G_h(\vec{r};\vec{r}_s)\frac{\alpha}{\beta}\right]dS + \tag{13}$$

$$\int\int\int G_h(\vec{r};\vec{r}_s)f(\vec{r})dV$$

In equations (12) and (13), the term $\int\int\int G_h(\vec{r},\vec{r}_s)f(\vec{r})dV$ is equal to $p_f(\vec{r})$, the acoustic pressure which would exist with the same source, $f(\vec{r})$, in uniform flow and no walls. This term can be calculated in many ways. One method is described in the following paragraph.

It is convenient to calculate the acoustic pressure field in the frequency domain since the reverberant field will be calculated in the frequency domain. The following formulas give the acoustic pressure spectrum for the rotor mth blade passage harmonic, $\omega = mN\Omega$. The formulas were taken from reference 8; the observer is in motion to give the wind-tunnel reference frame. Subcript h refers to the hub:

$$P_f(x,y,z) = \frac{mN^2\Omega}{4\pi R[1 - M\cos(\delta_r)]} e^{\frac{imN\Omega[R - M(x - x_h)]}{(1-M^2)c}} \tag{14}$$

$$\sum_{n=-\infty}^{\infty} e^{i(mN-n)(\psi+\frac{\pi}{2})} \int_o^{r_t} J_{(mN-n)}\left(\frac{mN\Omega r\cos(\theta)}{c[1 - M\cos(\delta_r)]}\right)$$

$$\left[m\Omega\rho_0(mN - n)t_{(mN-n)}V_n + \frac{i\sin(\theta)}{c}l_{(mN-n)}\right]dr$$

The coefficient of the thickness term is

$$t_n = \int_{\xi_{le}}^{\xi_{te}} t(\xi, r)e^{\frac{in\xi}{r}} d\xi \tag{15}$$

The coefficient of the loading term is

$$l_n = \int_{\xi_{le}}^{\xi_{te}} l(\xi, r)e^{\frac{in\xi}{r}} d\xi \tag{16}$$

The other variables are.

$$R = \sqrt{(x - x_h)^2 + (1 - M^2)[(y - y_h)^2 + (z - z_h)^2]} \tag{17a}$$

$$1 - M\cos(\delta_r) = \frac{(1 - M^2)R}{R - M(x - x_s)} \tag{17b}$$

and

$$V_0 = 1, V_{\pm 1} = \pm\frac{iMc\cos(\alpha)}{2\Omega r} \tag{17c}$$

The acoustic pressure in the wind tunnel (duct) is found by solving the equations in several steps. First, the free-field pressure p_f is found on the inside surface of the duct. Evaluation of equation (14) is one example of how this term may be found. Substituting $p_f = \int\int\int G_h(\vec{r};\vec{r}_s)f(\vec{r})dV$ into equation (12) yields an integral equation for the acoustic pressure on the inside duct surface. Solution of the integral equation will yield the acoustic pressure on the duct surface. Then, the acoustic pressure anywhere inside the duct may be found by substituting the surface pressure into the integral in equation (13) and evaluating the integrals.

APPENDIX A

DERIVATION OF THE FREE-FIELD GREEN'S FUNCTION FOR THE SUBSONIC CONVECTED ACOUSTIC EQUATION

The following partial differential equation (A1) and boundary condition (A2) for $G(\vec{r}, t; \vec{r}_s, t_s)$ describe the effect of an acoustic source at the location $\vec{r}_s$ with time dependence $f(t_s)$ on a receiver at the location $\vec{r}$ and time t in unbounded space A uniform subsonic flow with Mach number M in the positive x-direction is modeled.

$$\left(\nabla^2 - M^2 \frac{\partial^2}{\partial x^2} - \frac{2M}{c} \frac{\partial^2}{\partial x \partial t} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) G(\vec{r}, t, \vec{r}_s, t_s) = f(t_s) \delta_3(\vec{r} - \vec{r}_s) \quad (A1)$$

The boundary condition at infinity requires a radiation condition at infinity of

$$\lim_{\vec{r} \to \infty} \left[(\vec{r} - \vec{r}_s)(\vec{r} \quad \nabla G_h(\vec{r}; \vec{r}_s) + \frac{\partial G_h(\vec{r}; \vec{r}_s)}{c \partial t}) \right] = 0 \quad (A2)$$

This ensures that waves are outgoing at large distances and that contributions to surface integrals at infinity are zero

In the original coordinates (fig 1) the observer and source, $\vec{r}$ and $\vec{r}_s$, are fixed in space and the fluid is moving in the positive x-direction with Mach number M. A solution can be found with a Galilean transformation to the reference frame where the fluid has no time-averaged velocity (M = 0) and the source and observer are moving uniformly in the negative x-direction with Mach number M. The transformation shown in figure 2 is as follows:

$$\vec{r} = (x, y, z) \quad (A3a)$$

$$\tilde{\vec{r}} = \vec{r} - \vec{M} c t \quad (A3b)$$

$$\tilde{x} = x - Mct \quad (A3c)$$

$$\tilde{y} = y \quad (A3d)$$

$$\tilde{z} = z \quad (A3e)$$

$$\tilde{t} = t \tag{A3f}$$

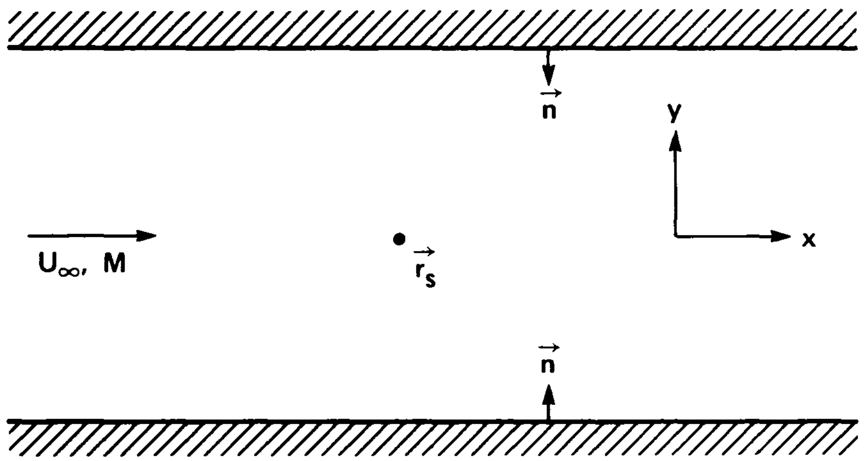

Figure 1. – Fixed reference frame.

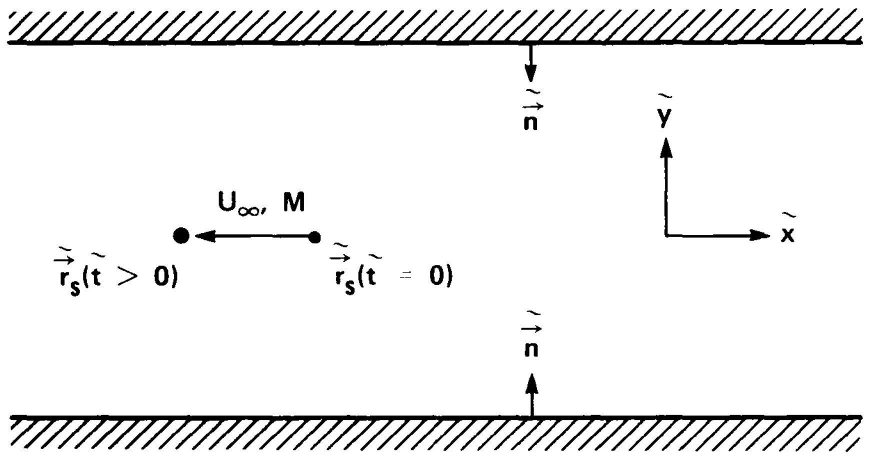

Figure 2. – Moving reference frame.

Equation, A1, transforms to

$$\left[\tilde{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial \tilde{t}^2}\right]\tilde{G}(\tilde{\vec{r}}, \tilde{t}; \tilde{\vec{r}}_s, \tilde{t}_s) = \tilde{f}(\tilde{t}_s)\delta_3(\tilde{\vec{r}} - \tilde{\vec{r}}_s) \qquad (A4)$$

The transformed equation has a solution, which is an integration over time and space,

$$\tilde{G}(\tilde{\vec{r}}, \tilde{t}; \tilde{\vec{r}}_s, \tilde{t}_s) = \qquad (A5)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} -\frac{\tilde{f}(\tilde{t}_s)\delta_3(\tilde{\vec{s}} - \tilde{\vec{r}}_s)\delta\left(\tilde{t}_s - \left(\tilde{t} - \frac{|\tilde{\vec{r}} - \tilde{\vec{s}}|}{c}\right)\right)}{4\pi|\tilde{\vec{r}} - \tilde{\vec{s}}|} d\tilde{t}_s \, d\tilde{\vec{s}}$$

A spatial integration with the three-dimensional delta function leads to a time-integration,

$$\tilde{G}(\tilde{\vec{r}}, \tilde{t}, \tilde{\vec{r}}_s, \tilde{t}_s) = \int_{-\infty}^{\infty} -\frac{\tilde{f}(\tilde{t}_s)\delta\left(\tilde{t}_s - \left(\tilde{t} - \frac{|\tilde{\vec{r}} - \tilde{\vec{r}}_s|}{c}\right)\right)}{4\pi|\tilde{\vec{r}} - \tilde{\vec{r}}_s|} d\tilde{t}_s \qquad (A6)$$

A time-integration with the delta function yields

$$\tilde{G}(\tilde{\vec{r}}, \tilde{t}; \tilde{\vec{r}}_s, \tilde{t}_s) = \left(\frac{-\tilde{f}(\tilde{t}_s)}{4\pi|\tilde{\vec{r}} - \tilde{\vec{r}}_s|\left[\frac{\partial}{\partial \tilde{t}_s}\left(\frac{|\vec{r} - \vec{r}_s|}{c}\right) + 1\right]}\right) \qquad (A7)$$

at $\tilde{t}_s = \tilde{t} - \frac{|\tilde{\vec{r}} - \tilde{\vec{r}}_s|}{c}$.

The moving source and observer have the following forms, a constant plus a component linearly proportional to time:

$$\tilde{\vec{r}}_s = \tilde{\vec{r}}_{so} - \vec{M}c\tilde{t}_s \qquad (A8a)$$

$$\tilde{x}_s = \tilde{x}_{so} - \mathrm{M}c\tilde{t}_s \tag{A8b}$$

$$\tilde{y}_s = \tilde{y}_{so} \tag{A8c}$$

$$\tilde{z}_s = \tilde{z}_{so} \tag{A8d}$$

$$\tilde{\vec{r}} = \tilde{\vec{r}}_o - \vec{\mathrm{M}}c\tilde{t} \tag{A8e}$$

$$\tilde{x} = \tilde{x}_o - \mathrm{M}c\tilde{t} \tag{A8f}$$

$$\tilde{y} = \tilde{y}_o \tag{A8g}$$

$$\tilde{z} = \tilde{z}_o \tag{A8h}$$

$$|\tilde{\vec{r}} - \tilde{\vec{r}}_s| = \sqrt{(\tilde{x}_o - \mathrm{M}c\tilde{t} - \tilde{x}_{so} + \mathrm{M}c\tilde{t}_s)^2 + (\tilde{y}_o - \tilde{y}_{so})^2 + (\tilde{z}_0 - \tilde{z}_{so})^2} \tag{A8ı}$$

A source-time derivative of the distance is

$$\frac{\partial}{\partial \tilde{t}_s}|\tilde{\vec{r}} - \tilde{\vec{r}}_s| = \frac{\mathrm{M}c(\tilde{x}_0 - \mathrm{M}c\tilde{t} - \tilde{x}_{so} + \mathrm{M}c\tilde{t}_s)}{|\tilde{\vec{r}} - \tilde{\vec{r}}_s|} \tag{A9a}$$

or

$$\frac{\partial}{\partial \tilde{t}_s}|\tilde{\vec{r}} - \tilde{\vec{r}}_s| = \frac{\mathrm{M}c(\tilde{x}_o - \tilde{x}_{so})}{|\tilde{\vec{r}} - \tilde{\vec{r}}_s|} - \mathrm{M}^2 c \tag{A9b}$$

Evaluating the derivative gives

$$\tilde{\mathrm{G}}(\tilde{\vec{r}}, \tilde{t}; \tilde{\vec{r}}_s, \tilde{t}_s) = \frac{-\tilde{f}(\tilde{t} - \frac{|\tilde{\vec{r}} - \tilde{\vec{r}}_s|}{c})}{4\pi\left[|\tilde{\vec{r}} - \tilde{\vec{r}}_s|(1 - \mathrm{M}^2) + \mathrm{M}(\tilde{x}_o - \tilde{x}_{so})\right]} \tag{A10}$$

In the moving coordinate system,

$$\tilde{\mathrm{G}}(\tilde{\vec{r}}, \tilde{t}; \tilde{\vec{r}}_s, \tilde{t}_s) = \frac{-\tilde{f}(\tilde{t} - \frac{|\tilde{\vec{r}} - \tilde{\vec{r}}_s|}{c})}{4\pi\left[(1 - \mathrm{M}^2)|\tilde{\vec{r}} - \tilde{\vec{r}}_s| + \mathrm{M}(\tilde{x} - \tilde{x}_s + \mathrm{M}c\tilde{t} - \mathrm{M}c\tilde{t}_s)\right]} \tag{A11}$$

Now the original coordinates from the original reference frame can be substituted into the formula for $G(\vec{r}, t, \vec{r}_s, t_s)$.

$$G(\vec{r}, t; \vec{r}_s, t_s) = \frac{-f\left(t - \frac{|\vec{r} - \vec{M}ct - \vec{r}_s + \vec{M}ct_s|}{c}\right)}{4\pi[(1 - M^2)|\vec{r} - \vec{M}ct - \vec{r}_s + \vec{M}ct_s| + M(x - x_s)]} \tag{A12}$$

$$c(t - t_s) = |\vec{r} - \vec{M}ct - \vec{r}_s + \vec{M}ct_s| \tag{A13}$$

This equation needs to be solved for the source time so that the formula for $G(\vec{r}, t; \vec{r}_s, t_s)$ can be evaluated:

$$c^2(t - t_s)^2 = (x - x_s - Mct + Mct_s)^2 + (y - y_s)^2 + (z - z_s)^2 \tag{A14}$$

$$= (x - x_s)^2 + 2(x - x_s)Mc(t_s - t) + M^2c^2(t - t_s)^2 + (y - y_s)^2 + (z - z_s)^2$$

Collecting terms yields a quadratic equation in $c(t - t_s)$:

$$(1 - M^2)c^2(t - t_s)^2 + 2M(x - x_s)c(t - t_s) - \tag{A15}$$

$$(x - x_s)^2 - (y - y_s)^2 - (z - z_s)^2 = 0$$

with solutions

$$c(t - t_s) = \frac{-M(x - x_s)}{1 - M^2} \tag{A16}$$

$$\pm \frac{\sqrt{M^2(x - x_s)^2 + (1 - M^2)[(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2]}}{1 - M^2}.$$

The positive root is used to ensure causality·

$$R = \frac{-M(x - x_s)}{1 - M^2} + \tag{A17}$$

$$\frac{\sqrt{M^2(x - x_s)^2 + (1 - M^2)[(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2]}}{1 - M^2}$$

Substituting the solved value of t_s gives

$$G(\vec{r}, t; \vec{r}_s, t_s) = \frac{-f(t - \frac{R}{c})}{4\pi[(1 - M^2)R + M(x - x_s)]} \qquad (A18a)$$

or

$$G(\vec{r}, t; \vec{r}_s, t_s) = \qquad (A18b)$$

$$\frac{-f\left(t - \frac{-M(x-x_s)+\sqrt{M^2(x-x_s)^2+(1-M^2)[(x-x_s)^2+(y-y_s)^2+(z-z_s)^2]}}{c(1-M^2)}\right)}{4\pi\sqrt{(x - x_s)^2 + (1 - M^2)[(y - y_s)^2 + (z - z_s)^2]}}$$

A time-dependent harmonic source is substituted into the equation. Let $f(t) = e^{-i\omega t}, k = \frac{\omega}{c}$; then, the partial differential equation becomes

$$(\nabla^2 - M^2 \frac{\partial^2}{\partial x^2} + 2iMk\frac{\partial}{\partial x} + k^2)G_h(\vec{r}, \vec{r}_s) = \delta_3(\vec{r} - \vec{r}_s) \qquad (A19)$$

and the solution becomes

$$G_h(\vec{r}; \vec{r}_s) = \frac{-e^{ik\left(\frac{+\sqrt{(x-x_s)^2+(1-M^2)[(y-y_s)^2+(z-z_s)^2]}-M(x-x_s)}{(1-M^2)}\right)}}{4\pi\sqrt{(x - x_s)^2 + (1 - M^2)[(y - y_s)^2 + (z - z_s)^2]}} \qquad (A20)$$

This solution gives waves that propagate away from the source and decay with distance. Except in the case of $M = 0$, the surfaces of constant phase and constant amplitude do not coincide. The function $G_h(\vec{r}; \vec{r}_s)$ is very well behaved far from the source and has the following limit properties:

$$\lim_{\vec{r} \to \infty} G_h(\vec{r}; \vec{r}_s) = 0 \qquad (A21)$$

$$\lim_{\vec{r} \to \infty} \frac{\partial G_h(\vec{r}; \vec{r}_s)}{\partial s} = 0, \ \vec{s} = a\vec{x} + b\vec{y} + c\vec{z}$$

$$\lim_{x\to\infty} \mathrm{G}_{\boldsymbol{h}}(\vec{r};\vec{r}_s) = 0$$

$$\lim_{x\to\infty} \frac{\partial \mathrm{G}_{\boldsymbol{h}}(\vec{r};\vec{r}_s)}{\partial s} = 0, \;\; \vec{s} = a\vec{x} + b\vec{y} + c\vec{z}$$

APPENDIX B

DERIVATION OF GREEN'S FORMULA FOR THE SUBSONIC CONVECTED HELMHOLTZ EQUATION IN A UNIFORM DUCT

An integral equation can be constructed which describes the harmonic acoustic pressure coefficient p inside an infinite duct with a uniform subsonic flow by utilizing the Green's function for the acoustic pressure and Green's second formula. The subsonic flow in the positive x-direction has Mach number M. The equation for the Green's function in subsonic flow is

$$(\nabla^2 - M^2 \frac{\partial^2}{\partial x^2} + 2ikM\frac{\partial}{\partial x} + k^2)G_h(\vec{r};\vec{r}_s) = \delta_3(\vec{r} - \vec{r}_s) \qquad (B1)$$

And the equation for the acoustic pressure from an arbitrary source is

$$(\nabla^2 - M^2 \frac{\partial^2}{\partial x^2} + 2ikM\frac{\partial}{\partial x} + k^2)p(\vec{r}) = f(\vec{r}) \qquad (B2)$$

By restricting the formulation to cases in which $\alpha/\beta \neq 0$ and $\alpha/\beta \neq \infty$, it can be shown from the differential equation formulation of the problem that the acoustic pressure in the duct will decay with distance away from the source Thus, the far-field pressure has the following form:

$$\lim_{\vec{r} \to \infty} p(\vec{r}) = 0 \qquad (B3a)$$

$$\lim_{\vec{r} \to \infty} \frac{\partial p(\vec{r})}{\partial s} = 0, \quad \vec{s} = a\vec{x} + b\vec{y} + c\vec{z} \qquad (B3b)$$

Green's second formula is

$$\int\int \left[-p(\vec{r})\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial n} + G_h(\vec{r};\vec{r}_s)\frac{\partial p(\vec{r})}{\partial n}\right]dS = \qquad (B4)$$

$$\iiint \left[p(\vec{r}) \nabla^2 G_h(\vec{r};\vec{r}_s) - G_h(\vec{r};\vec{r}_s) \nabla^2 p(\vec{r}) \right] dV$$

with the surface integral evaluated over the inside surface of the duct and the volume integral evaluated over the interior of the duct. The normal to the surface with positive direction inward is n. Substituting $\nabla^2 G_h(\vec{r};\vec{r}_s)$ and $\nabla^2 p(\vec{r})$ into equation (B4) gives

$$\iint \left[-p(\vec{r}) \frac{\partial G_h(\vec{r},\vec{r}_s)}{\partial n} + G_h(\vec{r};\vec{r}_s) \frac{\partial p(\vec{r})}{\partial n} \right] dS = \tag{B5}$$

$$\iiint p(\vec{r}) \left[M^2 \frac{\partial^2 G_h(\vec{r};\vec{r}_s)}{\partial x^2} + 2ikM \frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x} - k^2 G_h(\vec{r};\vec{r}_s) + \delta_3(\vec{r} - \vec{r}_s) \right] dV -$$

$$\iiint G_h(\vec{r};\vec{r}_s) \left[M^2 \frac{\partial^2 p(\vec{r})}{\partial x^2} + 2ikM \frac{\partial p(\vec{r})}{\partial x} - k^2 p(\vec{r}) + f(\vec{r}) \right] dV$$

Canceling terms gives

$$\iint \left[-p(\vec{r}) \frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial n} + G_h(\vec{r};\vec{r}_s) \frac{\partial p(\vec{r})}{\partial n} \right] dS = \tag{B6}$$

$$\iiint M^2 \left[p(\vec{r}) \frac{\partial^2 G_h(\vec{r};\vec{r}_s)}{\partial x^2} - G_h(\vec{r},\vec{r}_s) \frac{\partial^2 p(\vec{r})}{\partial x^2} \right] dV +$$

$$\iiint 2ikM \left[p(\vec{r}) \frac{\partial G_h(\vec{r},\vec{r}_s)}{\partial x} - G_h(\vec{r},\vec{r}_s) \frac{\partial p(\vec{r})}{\partial x} \right] dV +$$

$$\iiint [p(\vec{r}) \delta_3(\vec{r} - \vec{r}_s) - G_h(\vec{r};\vec{r}_s) f(\vec{r})] dV$$

The terms in the first integral on the right-hand side of equation B6 can be evaluated analytically by integrating by parts for $x = -\infty$ to $+\infty$

$$\iiint M^2 \left[p(\vec{r}) \frac{\partial^2 G_h(\vec{r},\vec{r}_s)}{\partial x^2} - G_h(\vec{r};\vec{r}_s) \frac{\partial^2 p(\vec{r})}{\partial x^2} \right] dV = \tag{B7}$$

$$\int\int M^2\left[p(\vec{r})\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x} - G_h(\vec{r};\vec{r}_s)\frac{\partial p(\vec{r})}{\partial x}\right]_{x=-\infty}^{x=\infty} dA$$

$$-\int\int\int M^2\left[\frac{\partial p(\vec{r})}{\partial x}\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x} - \frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x}\frac{\partial p(\vec{r})}{\partial x}\right]dV = 0$$

with the integration over the cross-sectional area of the duct in the surface integral. Both integrals are identically zero. The terms in the area integral,

$$p(\vec{r})\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x} - G_h(\vec{r};\vec{r}_s)\frac{\partial p(\vec{r})}{\partial x} = 0 \tag{B8}$$

at $x = \pm\infty$, since as $x \rightarrow \infty$, $G_h(\vec{r};\vec{r}_s) = 0$, $\frac{\partial G_h(\vec{r},\vec{r}_s)}{\partial x} = 0$, $p(\vec{r})$ is finite, and $\frac{\partial p(\vec{r})}{\partial x}$ is finite.

The second volume integral in equation B6 is also identically zero. Again integration by parts produces terms that are all zero:

$$\int\int\int\left[p(\vec{r})\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x} - G_h(\vec{r},\vec{r}_s)\frac{\partial p(\vec{r})}{\partial x}\right]dV = \tag{B9}$$

$$\int\int\left[\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x}\int p(\vec{r})dx - \frac{\partial p(\vec{r})}{\partial x}\int G_h(\vec{r},\vec{r}_s)dx\right]_{-\infty}^{\infty} dA -$$

$$\int\int\int_{-\infty}^{\infty}\left[p(\vec{r})\frac{\partial^2 G_h(\vec{r},\vec{r}_s)}{\partial x^2} - G_h(\vec{r};\vec{r}_s)\frac{\partial^2 p(\vec{r})}{\partial x^2}\right]dx dA =$$

$$\int\int\left[\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial x}\int p(\vec{r})dx - \frac{\partial p(\vec{r})}{\partial x}\int G_h(\vec{r};\vec{r}_s)dx\right]_{-\infty}^{\infty} dA = 0$$

Both the functions $G_h(\vec{r};\vec{r}_s)$ and $p(\vec{r})$ are well behaved enough so that at $x = \pm\infty$, $\frac{\partial G_h(\vec{r},\vec{r}_s)}{\partial x} = 0$, $\frac{\partial p(\vec{r})}{\partial x} = 0$, $\int p(\vec{r})dx$ is finite, and $\int G_h(\vec{r};\vec{r}_s)dx$ is finite. Applying these constraints, an integral relation is found for the acoustic pressure $p(\vec{r})$ on the surface,

$$\frac{p(\vec{r})}{2} = \int\int\left[-p(\vec{r})\frac{\partial G_h(\vec{r};\vec{r}_s)}{\partial n} + G_h(\vec{r};\vec{r}_s)\frac{\partial p(\vec{r})}{\partial n}\right]dS + \tag{B10a}$$

$$\int\int\int G_h(\vec{r};\vec{r}_s)f(\vec{r})dV$$

And in the duct volume the equation is

$$p(\vec{r}) = \int\int\left[-p(\vec{r})\frac{\partial G_h(\vec{r},\vec{r}_s)}{\partial n} + G_h(\vec{r};\vec{r}_s)\frac{\partial p(\vec{r})}{\partial n}\right]dS + \qquad (B10b)$$

$$-\int\int\int G_h(\vec{r};\vec{r}_s)f(\vec{r})dV$$

APPENDIX C

INTEGRAL EQUATION FOR TWO-DIMENSIONAL DUCTS

The three-dimensional case can easily be reduced to a two-dimensional problem by assuming that all quantities are independent of z. The walls become two parallel plates at $y = y_l$ and $y = y_u$ and the sources become infinite line sources in the z-direction. The resulting acoustic field is also uniform in the z-direction. Putting these restrictions into the three-dimensional equation simplifies the equation so that $G_h(\vec{r};\vec{r}_s)$ is the only quantity that depends on z. The integrals in the surface and volume equations take on the following form with the restrictions that $\alpha/\beta \neq 0$ and $\alpha/\beta \neq \infty$:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x, y_l, z)\left(-\frac{\partial}{\partial y} + \frac{\alpha \imath k c}{\beta}\right) G_h(\vec{r};\vec{r}_s) dz dx + \tag{C1}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x, y_u, z)\left(\frac{\partial}{\partial y} + \frac{\alpha \imath k c}{\beta}\right) G_h(\vec{r};\vec{r}_s) dz dx +$$

$$\int_{y_l}^{y_u}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y, z) G_h(\vec{r}, \vec{r}_s) dz dx dy =$$

$$\int_{-\infty}^{\infty} p(x, y_l)\left(-\frac{\partial}{\partial y} + \frac{\alpha \imath k c}{\beta}\right) \int_{-\infty}^{\infty} G_h(\vec{r};\vec{r}_s) dz dx +$$

$$\int_{-\infty}^{\infty} p(x, y_u)\left[\frac{\partial}{\partial y} + \frac{\alpha \imath k c}{\beta} p(x, y_u)\right] \int_{-\infty}^{\infty} G_h(\vec{r}, \vec{r}_s) dz dx +$$

$$\int_{y_l}^{y_u} \int_{-\infty}^{\infty} f(x,y) \int_{-\infty}^{\infty} \mathrm{G}_h(\vec{r};\vec{r}_s)\mathrm{d}z\mathrm{d}x\mathrm{d}y$$

The integration in the z-direction can be done analytically (ref. 9) for all of the integrals since

$$\mathrm{G}_{2d}(x,y;x_s,y_s) = \int_{-\infty}^{\infty} \mathrm{G}_h(\vec{r};\vec{r}_s)\mathrm{d}z = \tag{C2}$$

$$\int_{-\infty}^{\infty} \frac{-e^{\imath k\left(\frac{+\sqrt{(x-x_s)^2+(1-\mathrm{M}^2)[(y-y_s)^2+(z-z_s)^2]}-\mathrm{M}(x-x_s)}{(1-\mathrm{M}^2)}\right)}}{4\pi\sqrt{(x-x_s)^2+(1-\mathrm{M}^2)[(y-y_s)^2+(z-z_s)^2]}}\mathrm{d}z =$$

$$\frac{\sqrt{1-\mathrm{M}^2}e^{-\imath k\mathrm{M}(x-x_s)}}{4k\imath}\mathrm{H}_0^1\left(\frac{k\sqrt{(x-x_s)^2+(1-\mathrm{M})(y-y_s)^2}}{1-\mathrm{M}}\right)$$

This provides a two-dimensional Green's function for the convected Helmholtz equation which will be called $\mathrm{G}_{2d}(x,y,x_s,y_s)$ The function H_0^1 is the Hankel function of the first kind of order 0. The surface integral equation is now

$$\frac{\mathrm{p}(x,y)}{2} = \tag{C3}$$

$$\int_{-\infty}^{\infty} \mathrm{p}(x,y_l)\left(-\frac{\partial}{\partial y}+\frac{\alpha\imath kc}{\beta}\right)\mathrm{G}_{2d}(x,y_l,x_s,y_s)\mathrm{d}x +$$

$$\int_{-\infty}^{\infty} \mathrm{p}(x,y_u)\left(\frac{\partial}{\partial y}+\frac{\alpha\imath kc}{\beta}\right)\mathrm{G}_{2d}(x,y_u;x_s,y_s)\mathrm{d}x +$$

$$\int_{y_l}^{y_u} \int_{-\infty}^{\infty} f(x,y)\mathrm{G}_{2d}(x,y;x_s,y_s)\mathrm{d}x\mathrm{d}y$$

This is now the equation to be solved for $p(x, y)$. All quantities are known except $p(x, y)$. As before, the last integral represents the complex acoustic pressure that would exist in free space without the duct from the source $f(x, y)$ and can be replaced by $p_f(x, y)$ if the free acoustic field is known. After $p(x, y)$ is solved for on the surfaces y_u and y_l the acoustic pressure and thus all of the acoustic quantities, can be evaluated inside the duct with

$$p(x, y) = \tag{C4}$$

$$\int_{-\infty}^{\infty} p(x, y_l)\left(-\frac{\partial}{\partial y} + \frac{\alpha \imath k c}{\beta}\right) G_{2d}(x, y_l; x_s, y_s) dx +$$

$$\int_{-\infty}^{\infty} p(x, y_u)\left(\frac{\partial}{\partial y} + \frac{\alpha \imath k c}{\beta}\right) G_{2d}(x, y_u; x_s, y_s) dx dy +$$

$$\int_{y_l}^{y_u} \int_{-\infty}^{\infty} f(x, y) G_{2d}(x, y, x_s, y_s) dx dy$$

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16 Abstract

This paper describes an analytical model that can be used to examine the effects of wind-tunnel walls on helicopter rotational noise. First, a complete physical model of an acoustic source in a wind tunnel is described and a simplified version is then developed. This simplified model retains the important physical processes involved, yet it is more amenable to analysis. Second, the simplified physical model is modeled as a mathematical problem. An inhomogeneous partial differential equation with mixed boundary conditions is set up and then transformed into an integral equation. Details of generating a suitable Green's function and integral equation are given in appendixes. Last, the equation is discussed; it is also given for a two-dimensional case.

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