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J Flores
J Barton
T Holst
T Pulliam, Ames Research Center, Moffett Field, California

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Ames Research Center
Moffett Field California 94035

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COMPARISON OF THE FULL-POTENTIAL AND EULER FORMULATIONS FOR COMPUTING TRANSONIC AIRFOIL FLOWS

J. Flores, J. Barton, T. Holst, and T. Pulliam
NASA Ames Research Center
Moffett Field, CA 94035 USA

I. INTRODUCTION

Recently, much attention has been directed toward developing the Euler formulation for various applications in transonic aerodynamics. However, little effort has been made to compare the speed, accuracy, and robustness of these new Euler codes with the full-potential (FP) formulation. The purpose of this paper is to make such a quantitative comparison using a number of transonic airfoil cases.

The computed results are from four transonic airfoil computer codes: (1) TAIR [1,2]; (2) FLO36 [3]; (3) ARC2D [4,5], and (4) FLO52R [6]. Codes (1) and (2) are FP codes, and codes (3) and (4) are Euler codes. The FP codes (TAIR and FLO36) use fully implicit iteration algorithms (AF2 and ADI, respectively); the convergence speed of FLO36 is further enhanced by a multigrid convergence acceleration process. The first Euler code (ARC2D) uses a fully implicit ADI iteration scheme; the second (FLO52R) uses an explicit Runge-Kutta time-stepping algorithm, which is enhanced by a multigrid convergence acceleration scheme.

The TAIR and ARC2D codes were each run using two types of grids. One grid was generated numerically, using an elliptic (Laplacian) solver [2], and the second was generated from an algebraic routine [7]. The FLO36 and FLO52R codes were run using an internally generated grid of the circle plane mapping variety.

The comments and conclusions reached in this study will be expressed generally, that is, in terms of FP versus Euler. The reader should bear in mind that these conclusions have been reached using the four specific codes mentioned above. We expect the results presented herein to be typical, but other codes that use different spatial or iteration algorithms may produce somewhat different results.

II. RESULTS

Figure 1 is a plot of lift coefficient versus the average mesh spacing on the airfoil. The airfoil is a NACA 0012, and the flow conditions are $M_\infty = 0.63$ and $\alpha = 2.0°$. As the grid was refined, the ratio of the number of grid points along the airfoil to the number of grid points away from the airfoil was held fixed. The outer boundary was placed at 12 chords from the airfoil. A study, which consisted of plotting the lift versus distance to the outer boundary, was conducted; it verified that this distance was sufficient to remove outer boundary effects. The TAIR and FLO36 codes produce lift asymptotes of 0.3326 and 0.3333, respectively, and the ARC2D and FLO52R codes produce asymptotic values of 0.3357 and 0.3342, respectively.

Theoretically, all results for the two formulations should reach the same asymptotic value for a subcritical case. The Lock solution (obtained through the Hodograph method and considered "exact") [8] yields 0.335 as the value of the lift. However, Lock extends the NACA 0012 airfoil to a sharp trailing edge at $x/c = 1.0089$, but does not normalize to unit length. In the present results, the NACA 0012 airfoil is both extended and renormalized to unit length. If the Lock result is renormalized, consistent with the present results, the lift coefficient would become 0.3321. This tends to suggest that the FP codes are in better agreement with the "exact" solution for this subcritical case.

Figure 2 is a plot of percent error in the lift coefficient versus CPU time in seconds on the Cray XMP computer for the conditions of Fig. 1 (NACA 0012, $M_\infty = 63$, $\alpha = 2.0°$). The timings from all codes are based on converging the lift to an accuracy of 10^{-4} (four decimal digits). The time-step and convergence acceleration parameters from all codes (in general) have been set at default values; that is, a minimal amount of "tuning" has been included. Thus, the convergence rates are not optimal, but are representative of the convergence rates that would be found in practical applications. Startup times, including initialization and grid generation, have been subtracted from each timing. The error is computed by first constructing the asymptotic values of the lift coefficient (as done in Fig. 1). Then the error is simply the absolute value of the difference between the asymptotic value and the value of the converged lift at a specific level of grid refinement. From Fig. 2 (also Fig. 1), it can be observed that the FP formulations are slightly more accurate than the Euler formulations, especially for the coarser grids. On the coarse grids, the Euler codes are more expensive than the FP codes by an average factor of about 17, based on CPU time. For the finer grids, this factor decreases to about 11.

Figure 3 displays a plot of lift coefficient versus the average mesh spacing for a transonic case with a moderate strength shock, NACA 0012, $M_\infty = 0.75$, and $\alpha = 1.0°$. No attempt was made to construct a lift error versus CPU time, as was done in Fig. 2, since, as can be seen in Fig. 3, some of the curves turn over on themselves, making the error measure potentially misleading. We point out here that the asymptotic characteristics of both the FP and Euler formulations are grid-dependent (also apparent in Fig. 1). The algebraic and Laplacian curves for both the FP and Euler formulations show different trends and levels of accuracy. The TAIR (algebraic) and TAIR (Laplacian) results approach their limits from different directions. The level of accuracy for the Euler results is typically less for the algebraic grids, whereas the reverse is true for the FP results. The FP results all approach the same asymptotic limit to within an error of about 1%. The Euler results also approach an asymptotic limit, but the error is significantly less. Another observation from Fig. 3 is that the level of accuracy owing to grid effects can be of the order of the differences in equation formulations (FP versus Euler) for these cases in which the FP is valid.

Utilizing the nonisentropic full-potential formulation [9] in TAIR yields the middle set of curves in Fig. 3. By adding entropy effects to FP formulation, the solutions were improved to within about 4% of the Euler formulation, which it is agreed is the more valid formulation for supercritical cases.

The CPU time at convergence versus the average surface mesh spacing is plotted in Fig. 4 for the conditions shown in Fig. 3 (NACA 0012, $M_\infty = 0.75$, $\alpha = 1°$). This yields a rough estimate of the cost of running each code for different grid sizes, without providing definitive information on the cost to obtain a desired level of accuracy. In general, the Euler codes are more expensive than the FP codes — by a factor of 10 based on CPU time and twice that based on operation count. An interesting observation is that both the ARC2D (Euler) and TAIR (FP) codes converge faster on the Laplacian grid than on the algebraic grid. In fact, the difference between TAIR (algebraic) and TAIR (Laplacian) convergence times is quite large (as much as a factor of 4). The cause for this behavior is not known for certain, but it may be that the stretching is too rapid in the algebraic grids. Because the FP formulation is based on a second-order PDE, it is more likely to be adversely affected by a grid that is nonsmooth or rapidly stretched.

Figure 5 illustrates the asymptotic lift behavior for a strong shock case (RAE 2822, $M_\infty = 0.75$, $\alpha = 3.0°$). Note that these conditions are considered to be beyond the valid range of the full-potential formulation, and only the TAIR and ARC2D codes were run for this case. The FLO codes were not used, a result of the difficulty of the case and the lack of user experience. It can be seen that the results for the TAIR code (algebraic and Laplacian grids) both reach the same asymptotic value of lift. The value obtained is about 1.69, which is grossly in error relative to the Euler results. Thus, the FP formulation is unacceptable for this calculation. The asymptotic values for the ARC2D code (algebraic and Laplacian grids) are in good agreement producing an asymptotic value of lift coefficient near 1.12. The effect of the FP entropy correction is seen to make a major difference in the FP solution, producing errors of a level comparable to those in the previously discussed case (NACA 0012, $M_\infty = 0.75$, $\alpha = 1°$). This improvement in lift is also reflected in a comparable improvement in the surface-pressure distribution, for the nonisentropic FP pressure distribution is in good agreement with the Euler pressure distribution.

Figure 6 presents a comparison of CPU time versus grid refinement for the RAE case. Again we note about an order of magnitude difference in CPU time for FP over Euler. For this case, which is admittedly difficult for isentropic FP, the convergence rates are strongly affected by the different grids. Again, the nonisentropic formulation helped improve the convergence speed of TAIR (Laplacian).

Figure 7 presents a plot of the convergence speed ratio (Euler to FP) versus the average surface mesh spacing for the NACA 0012, $M_\infty = 0.75$, $\alpha = 1°$ case. The convergence speed ratio is plotted based on two criteria: (1) CPU time, and (2) total operation count. Each data point plotted in Fig. 7 is obtained by means of a simple arithmetic average of the results for each formulation, three Euler and five FP (see Fig. 4). Although not monotonic, useful information can be obtained from these curves. The average convergence ratio based on total operations fluctuates from about 9 to 16, and based on CPU time the fluctuation is 4 to 8. The reason for the difference in average convergence speed ratio based on CPU time relative to total operation count is associated with vectorization efficiency. That is, the Euler codes are highly vectorized on the Cray XMP, but the FP codes are not. The Euler-to-FP speed ratio, based on CPU time, could be higher if the FP codes were more efficiently vectorized. However, the possible improvement in FP vectorization efficiency is difficult to estimate, since the AF2 algorithm in two dimensions cannot be vectorized as efficiently as the classical ADI-like implicit schemes or explicit methods. (Note that the AF2 algorithm in three dimensions does not have this disadvantage.)

In Fig. 8, an attempt is made to shed some light on an interesting controversy in which the Euler and FP formulations are involved: the proper level of solution convergence. Because of the differencing of the dependent variable ϕ to obtain the pressure distribution, truncation error is added to any FP solution. Since this error adds to the lack-of-convergence error (theoretically), the FP solution must be converged more tightly than the Euler solution for the same level of accuracy in the lift calculation. Figure 8 shows a plot of error in lift versus rms error in the dependent variable (E_{rms}), pressure for the Euler formulation, and ϕ for the FP formulation. The exact definitions for these two different types of error are displayed in Fig. 8. The two curves shown in Fig. 8 were produced from the NACA 0012, $M_\infty = 0.75$, $\alpha = 1°$ case. Initially, the test case was run until tight convergence was obtained. Then, the converged dependent variables and converged lift coefficient were saved and the case was rerun. The curves shown in Fig. 8 were obtained by plotting the lift error versus the rms dependent-variable error every 50 iterations. Convergence in this case for FP and Euler solutions were about 300 and 1600 iterations, respectively. This explains the difference in number of data points plotted for each code. For this case, the FP solution does need to be converged more tightly for the same error in lift. For a lift error of about 10^{-4}, the FP solution needs to be dropped about an order more in rms error.

III. CONCLUSIONS

A study involving four transonic airfoil computer codes, two FP and two Euler, has been performed. The major conclusions of the study are as follows: (1) the FP codes are faster than the Euler codes by about an order of magnitude based on CPU time on the Cray XMP; (2) the FP formulation loses accuracy as transonic flow develops, but entropy corrections yield FP solutions comparable to those of the Euler; (3) grid coarseness and type can be significant in affecting both accuracy and convergence characteristics; (4) the FP formulation must be more tightly converged than the Euler formulation for comparable levels of accuracy in the lift coefficient; and (5) in general, good accuracy for adequate meshes can be obtained with both formulations, irrespective of the solution method.

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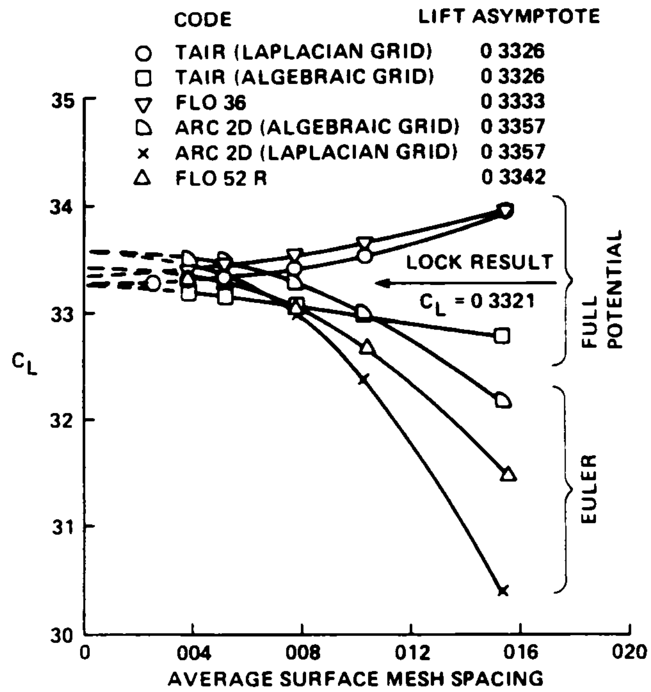

Fig. 1 Lift versus grid refinement: NACA 0012, $M_\infty = 0.63$, $\alpha = 2°$.

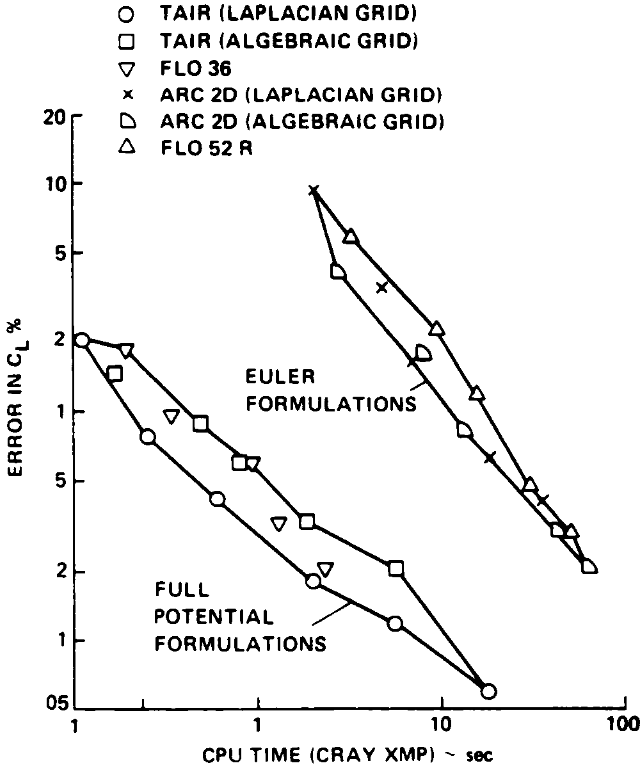

Fig. 2 Lift error versus CPU: NACA 0012, $M_\infty = 0.63$, $\alpha = 2°$.

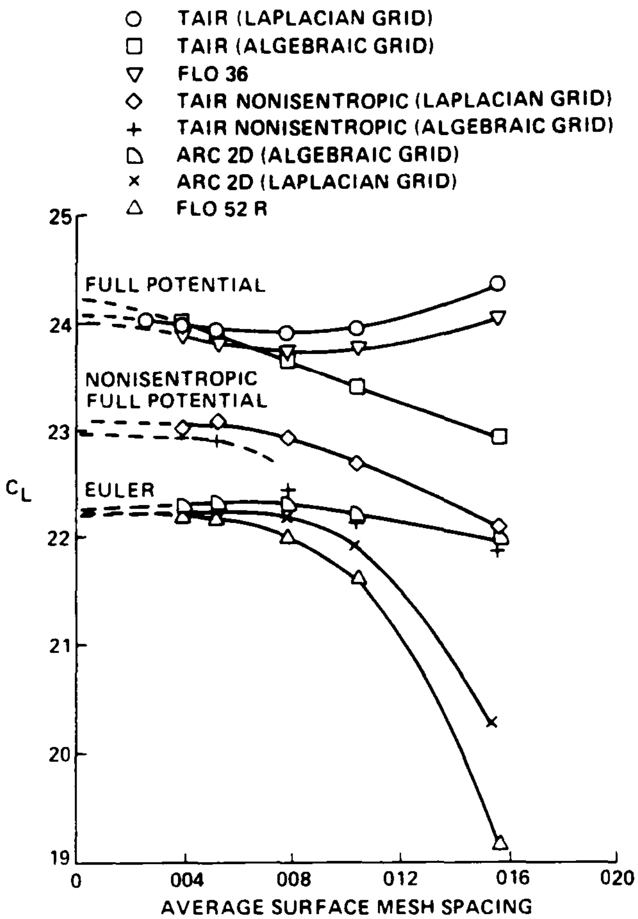

Fig. 3 Lift versus grid refinement: NACA 0012, M_∞ = 0.75, α = 1°.

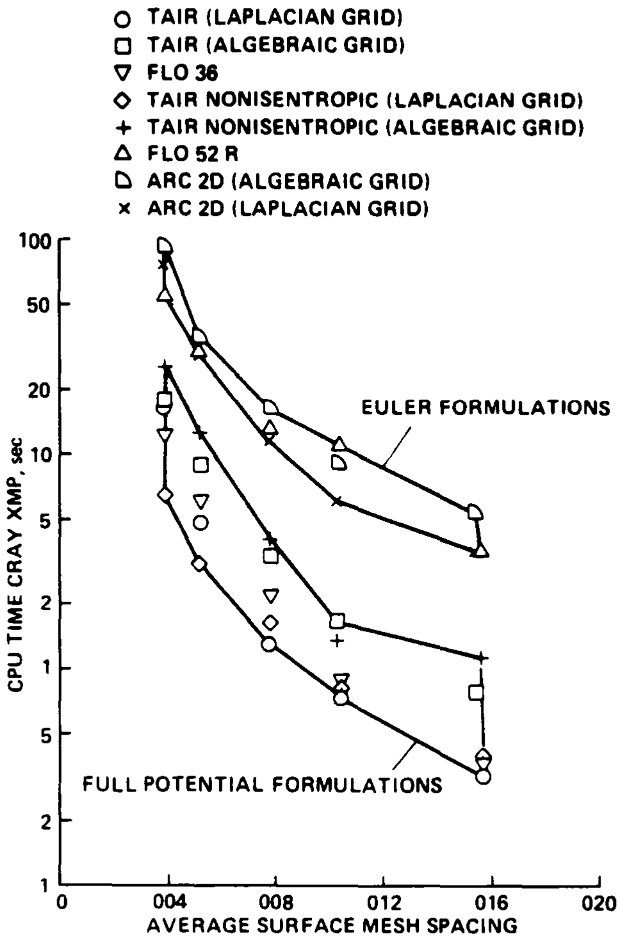

Fig. 4 CPU versus grid refinement: NACA 0012, M_∞ = 0.75, α = 1°.

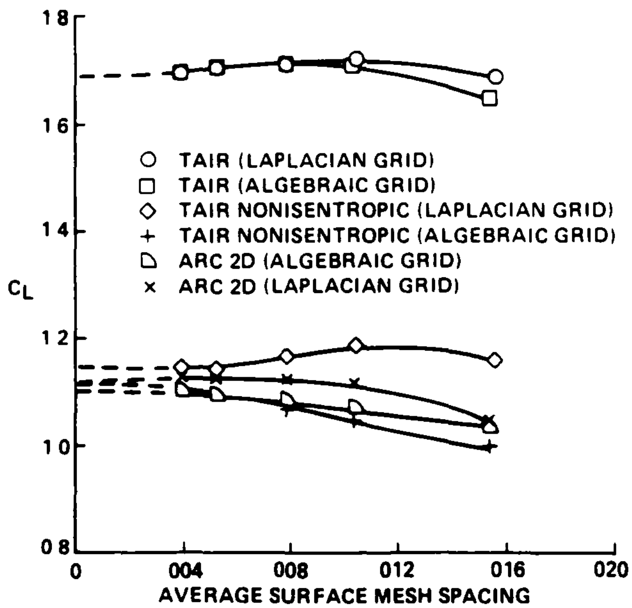

Fig. 5 Lift versus grid refinement: RAE 2822, M_∞ = 0.75, α = 3°.

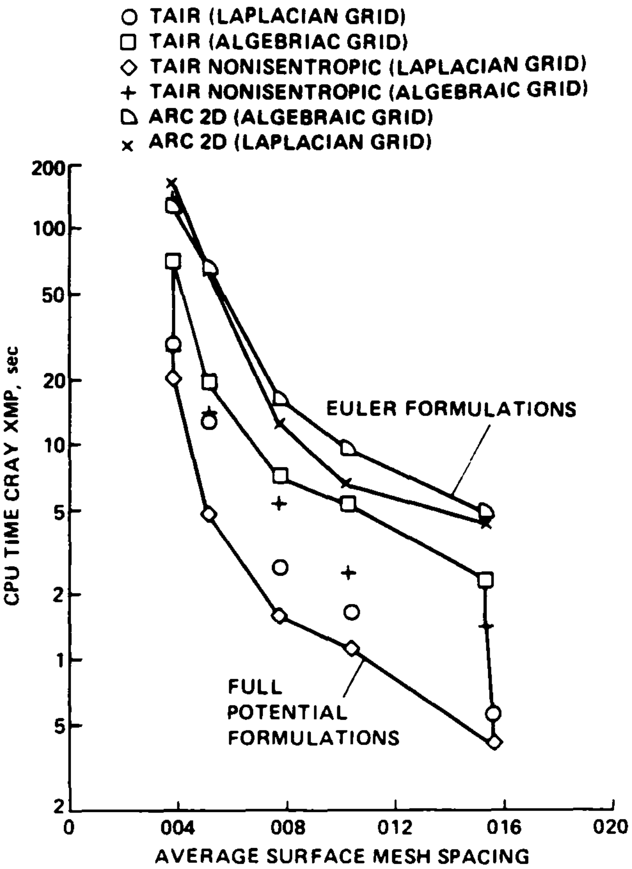

Fig. 6 CPU versus grid refinement: RAE 2822, $M_\infty = 0.75$, $\alpha = 3°$.

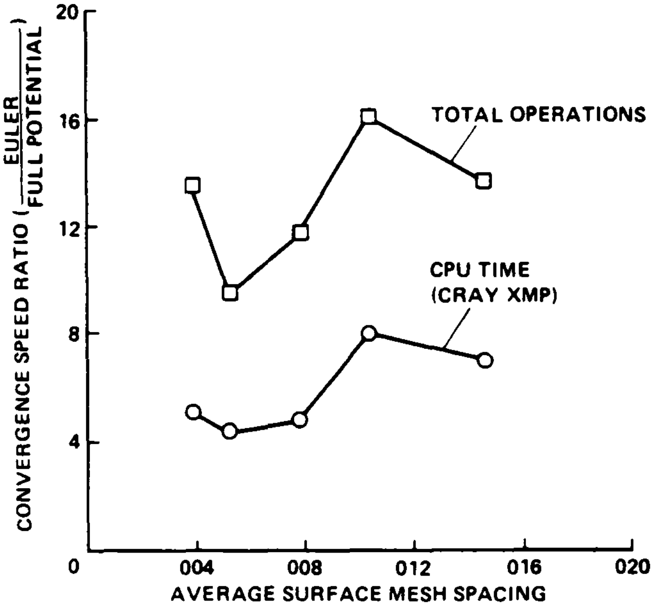

Fig. 7 Convergence speed versus grid refinement: NACA 0012, $M_\infty = 0.75$, $\alpha = 1°$.

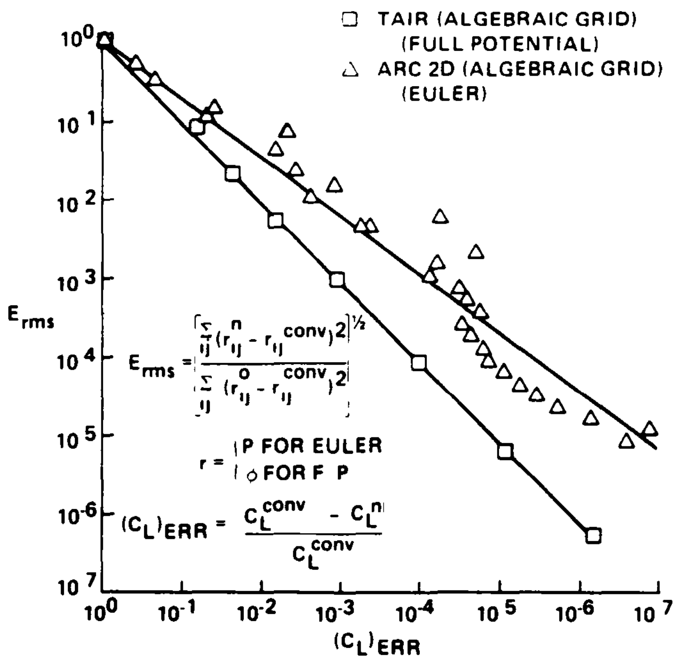

Fig. 8 Root-mean-square error versus lift error: NACA 0012, $M_\infty = 0.75$, $\alpha = 1°$.

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J. Flores, J. Barton, T. Holst, T. Pulliam

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Point of Contact: J. Flores, Ames Research Center, MS 202A-14, Moffett Field, CA 94035
(415) 965-5369 or FTS 448-5369

16. Abstract

The purpose of this paper is to perform a quantitative comparison between the Euler and full potential formulations with respect to speed and accuracy The robustness of the codes used is tested by a number of transonic airfoil cases.

The computed results are from four transonic airfoil computer codes The full potential codes (TAIR and FLO36) use fully implicit iteration algorithms (AF2 and ADI, respectively) with the convergence speed of FLO36 further enhanced by a multigrid convergence acceleration process The first Euler code (ARC2D) uses a fully implicit ADI iteration scheme. The second Euler code (FLO52) uses an explicit Runge-Kutta time-stepping algorithm which is enhanced by a multigrid convergence acceleration scheme

Quantitative comparisons are made using various plots of lift coefficient versus the average mesh spacing along the airfoil. Besides yielding an asymptotic limit to the lift coefficient, these results also demonstrate the truncation error behavior of the various codes. Plots of the lift coefficient error versus the average mesh spacing along the airfoil are also included (The error is computed by the absolute value of the difference between the asymptotic limit of the lift coefficient and the locally computed value) The lift coefficient error is also plotted versus CPU time and/or the number of floating point operation counts required per computation.

From the various plots, quantitative conclusions regarding the full potential and Euler formulations with respect to accuracy, speed, and robustness can be presented.

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