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(NASA-CR-173917) A STUDY OF METHODS TO PREDICT AND MEASURE THE TRANSMISSION OF SOUND THROUGH THE WALLS OF LIGHT AIRCRAFT. A SURVEY OF TECHNIQUES FOR VISUALIZATION OF NOISE FIELDS (Purdue Univ.) 38 p

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A STUDY OF METHODS TO PREDICT AND MEASURE THE TRANSMISSION OF SOUND THROUGH THE WALLS OF LIGHT AIRCRAFT

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A SURVEY OF TECHNIQUES FOR VISUALIZATION OF NOISE FIELDS

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CONTENTS

1. INTRODUCTION

Noise source identification is a crucial but often difficult task. The noise control engineer often resorts to an iterative investigation to identify the source of the noise. Furthermore, it is not always enough to identify the source, but to also understand the interaction of the source with the noise environment. The noise environment is inevitably complex and invisible to the human eye. Thus, the necessity for the noise control engineer to have access to a tool for visual perception of noise mechanisms is evident.

Several acoustic visualization techniques have evolved and are in use today. In this chapter, a survey of the most widely used methods for visualizing acoustic phenomena is presented. Some of these methods see greatest application to problems unrelated to noise control. Emphasis, however, will be with respect to acoustic processes in the audible frequencies.

With the advent of the digital computer and the advancement of computer graphics, many visual problems are being analyzed on computer graphic systems. Included in this paper is a brief description of the current technology in computer graphics. The experience acquired from the visualization technique survey will serve as basis for recommending an optimum scheme for displaying acoustic fields on computer graphic systems.

2. OPTICAL METHODS OF VISUALIZATION

The oldest and most fundamental techniques for visualizing acoustic phenomena involve optical principles. Optical methods of visualization require an experimental configuration in which the disturbance of a light path by a fluid medium can be observed or recorded on film. These techniques were originally developed in the mid 1800's for visualizing flow fields of compressible fluids [1]. It was known that a density gradient in a fluid medium causes refraction in light passing through the medium. Optical principles were not applied to acoustic visualization until much later, however.

Whilst an abundance of technical information exists concerning optical visualization of ultrasonics [2], few cases of noise field (long-wavelength) visualization by optical methods appear in the literature. Due to physical limitations of experimental configurations (i.e. physical dimensions of optical sources and receivers), the optical methods have proven somewhat ineffective for describing long-wavelength acoustic information. The more common optical methods of visualization are presented in this section, and experimental limitations are discussed.

2.1 Transmission Imaging (Shadowgraph)

The simplest of all optical visualization procedures, the shadow method, is attributed to Dvorak (1880) [3]. A system schematic for this procedure is shown in Figure 1. A spherical mirror (lens), the only optical apparatus, serves to direct parallel light paths from a source. The individual light rays pass through the test medium and are refracted from their original paths. The light then falls onto a screen, and an image of the source is formed. The distance of the image screen from the test medium as well as the size of the light source are chosen such that diffraction effects are detectable but not so great as to lack sharpness.

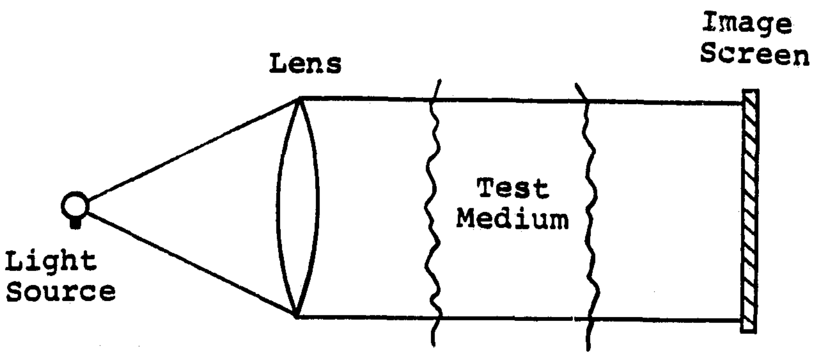

Figure 1. System Schematic for Producing a Shadowgraph

The shadow method visualizes only fields in which a density gradient occurs. Shock waves and supersonic jet flow represent drastic density gradients and have been studied extensively in the aerodynamic disciplines via shadowgraphs [4,5]. Acoustic waves of reasonable amplitude levels do not create a sufficient density gradient and, therefore, cannot be visualized by the shadow method. [Light refraction through a density gradient produced by acoustic emissions in a liquid has been detected by optical methods more sensitive then the shadow method as will be discussed later.]

Williams and Lighthill recognized that a density gradient usually occurs at the interface of two fluid media [6]. A light source and an image screen were positioned below and above a water tank, respectively. Acoustic wave phenomena was simulated by water surface waves on a shadowgraph. Williams and Lighthill were able to visualize wave propagation from simple sources, traversing objects, and jet-stream flow. A shadowgraph of a dipole source appears in Figure 2.

The process of placing a translucent medium in the field between a diffuse light source and an image screen is sometimes referred to as a shadow method. The translucent medium filters portions of the light and appears on the screen as a shadow image. Baumeister and Rice injected color dies into the orifice of a Helmholtz resonator and

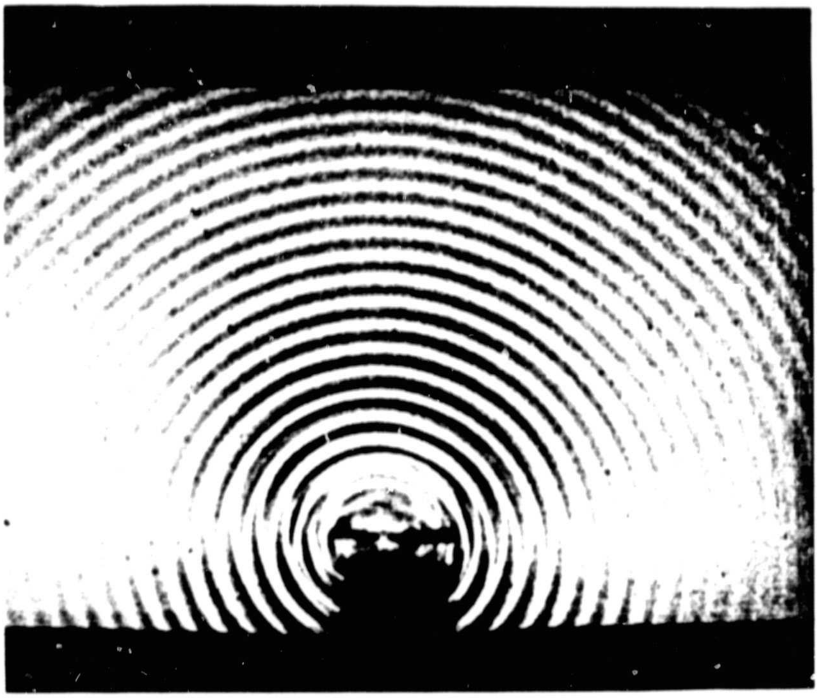

Figure 2. Shadowgraph of a Dipole in a Ripple Tank (printed with permission of Ency. Brit.)

Figure 3. Microphone-Light Sweep in a Reverberant Room (printed with author's permission)

the grazing flow field in order to visualize oscillating flow in a resonator cavity [7]. A high speed camera served as the image screen and recorded the motion as a function of time.

A color photographic technique for visualizing sound fields in both reverberant and free-field spaces is described by Rainey and Neville [8]. A microphone, with a number of colored bulbs affixed closely to it, was traversed through the sound field. The microphone signal was used to switch the lights on and off at voltage levels corresponding to specific sound pressure levels. A camera with long-time exposure was used to record the color patterns on film. Thus, the resulting photograph is built up of a number of microphone traverse sweeps in a manner that is analogous to the method in which a color image is formed on a television picture tube. A black and white photographic reproduction of the color photograph of a reverberant room excited by a 200 Hz sine wave is shown in Figure 3.

2.2 Schlieren Method

Superceding the shadow method of optical visualization is the similar but more sensitive Schlieren method. The principle of the method was developed more than a century ago and is attributed to Foucault (1859) and to Toepler (1864). Foucault applied the principle in order to detect aberrations in optical elements. Toepler, however, recognized and described in his book the wide applicability of the method [9]. The name, "Toepler Method", has therefore become common in some applications. Toepler carried out the first utilization of the Schlieren method to visualize acoustic waves when he observed sound waves in air emitted by an electric spark.

The fundamental system schematic for the Schlieren method is sketched in Figure 4 [10]. The lens, L_1, causes light rays from the source to pass parallel through the test medium. The second lens, L_2, forms an image of the light source in the plane of the knife edge positioned to cut off half the image of the source. An evenly illuminated screen indicates a constant refractive index through the test field. A refraction away from the knife edge will produce a bright patch on the screen; light refracted toward the knife edge will be blocked, and a dark patch will result on the screen. It is often desirable to investigate test media of large area. Since large aberration free lenses are expensive, mirrors are often used in a Z-configuration such as shown in Figure 5.

Since Toepler's time, extensive research has been done to develop the Schlieren technique for the visualization of ultrasonic waves. The method has found widespread use in the study of ultrasonic phenomena in liquids, wherein it is relatively easy to generate ultrasonic, monochromatic

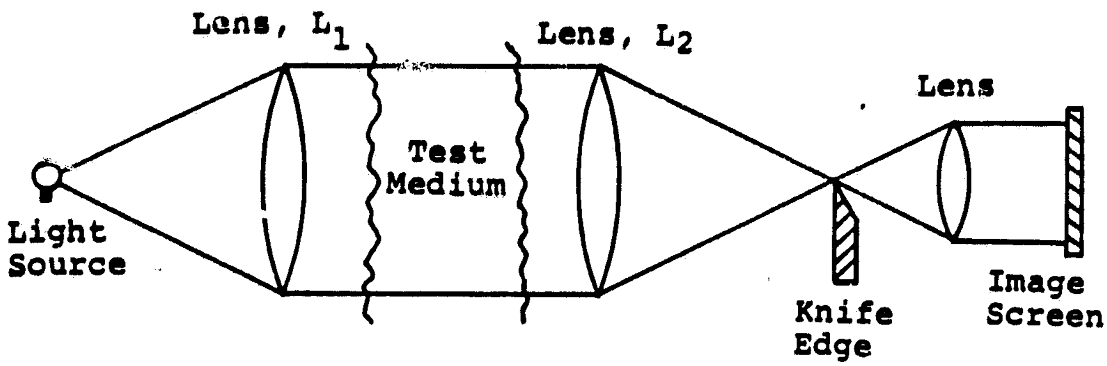

Figure 4. System Schematic for the Schlieren Method

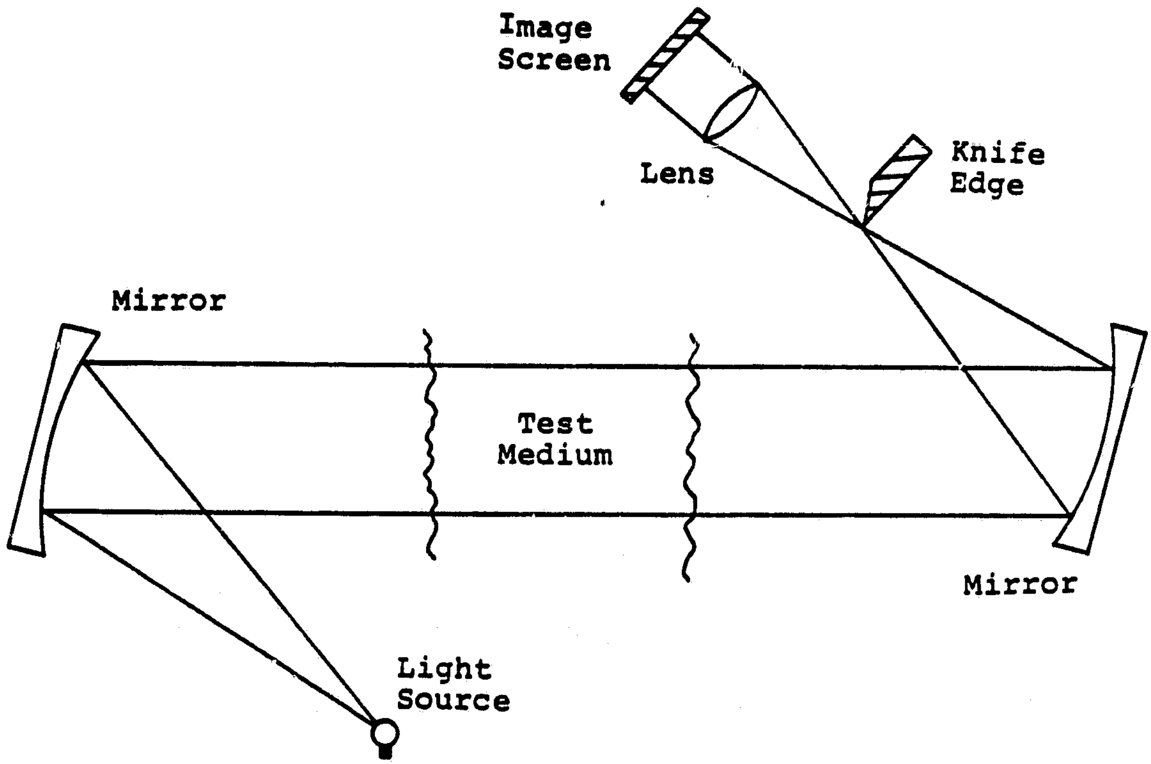

Figure 5. Z-Configuration Schlieren Method Schematic

pressure waves. Klein and Cook have shown that plane, progressive sound waves propagating in a transparent liquid normal to the incident axis of a single wavelength component light will produce a grating effect under proper conditions of wavelength and sound pressure level [11]. If the frequency of the sound waves drops below 20 kHz, however, the optical gratings produce only a decreasing modulation of the phase of light passing through the grate. In addition, the index of refraction becomes small and difficult to detect.

When the Schlieren method is applied to visualization of sound waves in air, a similar limitation results. Bucaro and Dardy demonstrated that in order to detect light diffraction from sound waves below 20 kHz in air, acoustic pressure levels must exceed 180 dB (Re: 1 μ Pa) [12].

Clever techniques have been devised, however, for visualizing sound generation from aerodynamic flows. Kadlec and Davis arranged a self-synchronizing stroboscopic Schlieren system for visualizing quasiperiodic flows [13]. They were able to synchronize a mercury flash lamp to the signal from a condenser microphone positioned in the sound field induced by the interaction of shear layers with a rigid surface. The result was the visualization of edge tone flow fields as shown in the photograph of Figure 6.

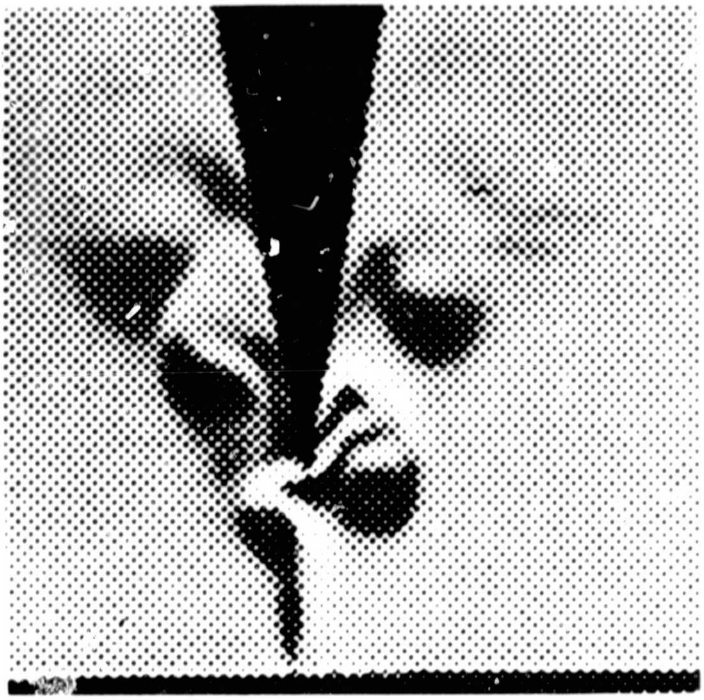

Figure 6. Schlieren Photograph of Edge-Tone Generation
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3. HOLOGRAPHY

The term, 'holography', although originally referring to a document handwritten by the purported author, has been extended to describe a variety of recording techniques. Modern holography was invented by Gabor (1948) as a means of avoiding aberrations in optical elements [14]. Gabor was able to create an interference pattern that was recorded and then reilluminated to reveal a three dimensional image of an object.

The principle of holography can be explained by a mathematical analogy. In order to determine the wave system within a certain sector of space, it is sufficient to know the amplitude and phase distribution at some distinct surface. The hologram is the equivalent of a given set of boundary values, and the reconstruction of the waves corresponds to the solution of the boundary value problem.

3.1 Optical Holography

Optical holography is a method of photographic image recording in which both phase and amplitude of the wavefield are encoded onto a planar photographic transparency. Since photographic emulsions are only senitive to the intensity or energy density of a light wave, phase information of the wave cannot be directly recorded. The problem can be solved, however, by applying the principle of interference. If it is desired to record the total information of an object wave, a reference wave is mixed with the object wave, and the intensity summation and phase differential are recorded in a holographic plane. The hologram is therefore a pattern of interference fringes. One condition for the recording of a hologram is that the wave system be characterized by a unique wavelength and a constant phase relationship (a coherent light source such as a laser-light). An experimental configuration for producing a hologram is sketched in Figure 7.

For the purpose of reconstruction, the hologram is illuminated with a wave which is of the same geometry as the reference wave. When illuminated the hologram serves as a diffraction grating, and three beams emerge, an undeflected beam and two diffracted beams. The diffracted beams produce the real and virtual images of the object field. It is the virtual image that is of interest in practical applications of holography whereas the real image is depth-inverted in appearance. With the use of the light in the image, photographs can be made to interpret the hologram characteristics.

Visualization by optical holography of acoustic fields in the audible range is governed by the same limitations that exist for the previous optical methods. The conditions on wavelength and amplitude have rendered this method

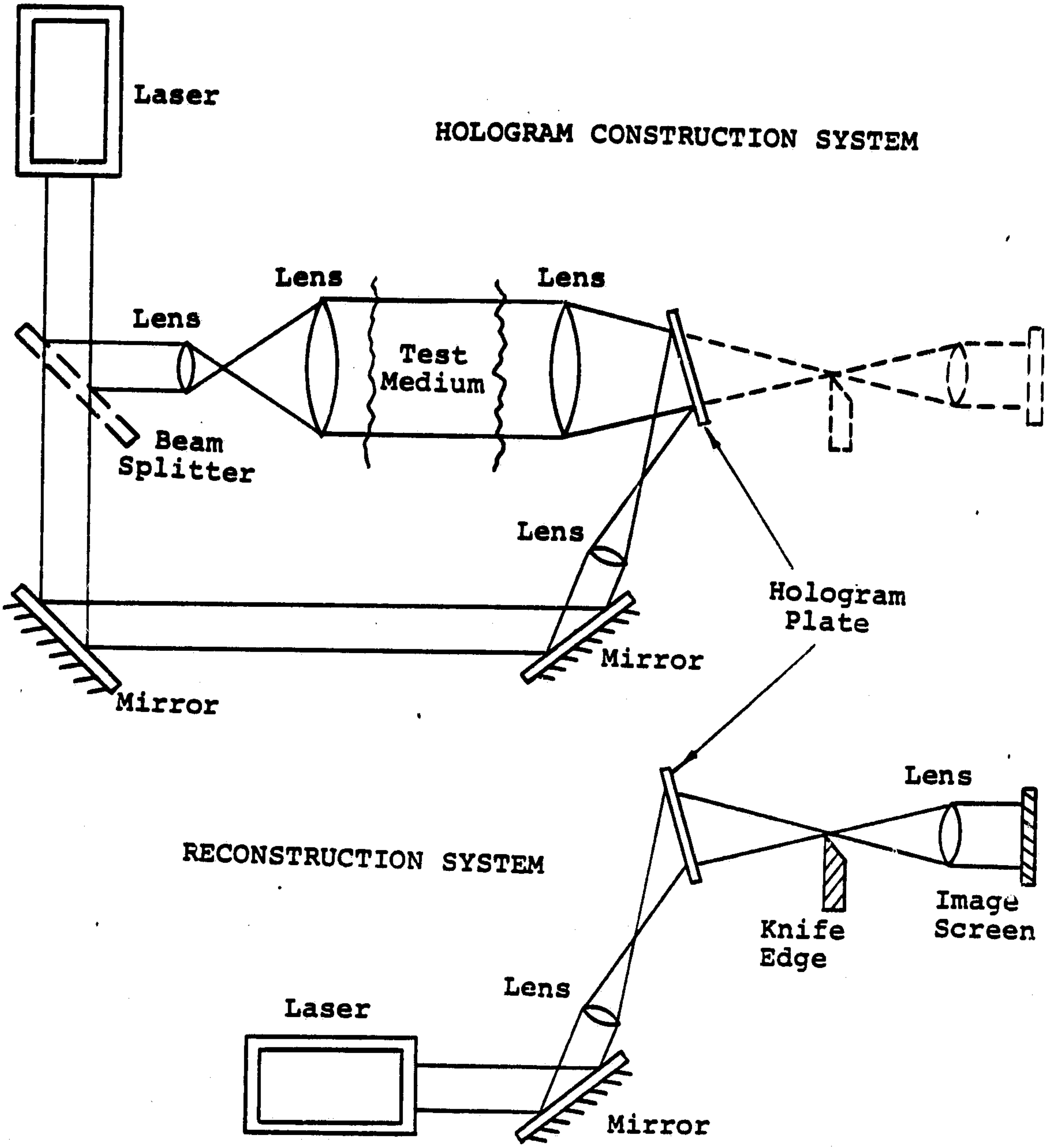

Figure 7. System Schematic for Producing a Hologram

ineffective for noise field visualization. A common application of laser holography, however, is the visualization of vibrating structures [15,16]. Vibration patterns of a plate appear in Figure 8. Although the understanding of structural vibration is of benefit to noise control, this topic will not be addressed here.

J. A. Clark devised an optical holographic technique by which a tone burst below 20 kHz in water could be visualized [17]. Two optical beams were used for this method. One hologram was recorded while the acoustic medium was at rest, and a second while the medium was dynamically loaded. The two holograms were superimposed and optically processed to reconstruct an image of the sound field. The image was found to contain patterns of optical interference fringes. If the acoustic pressure was constant in the direction of the optical system axes, these patterns could be directly interpreted as contours of equal acoustic pressure. A reconstructed hologram of the density field of a 16 kHz tone burst propagating in water is shown in Figure 9.

3.2 Acoustical Holography

The principle that makes holography possible is based upon the wave properties of electromagnetic radiation. Since wave phenomena is fundamental to acoustic radiation, and sound waves obey the same equations as light waves, the holographic technique can be applied to an acoustic domain.

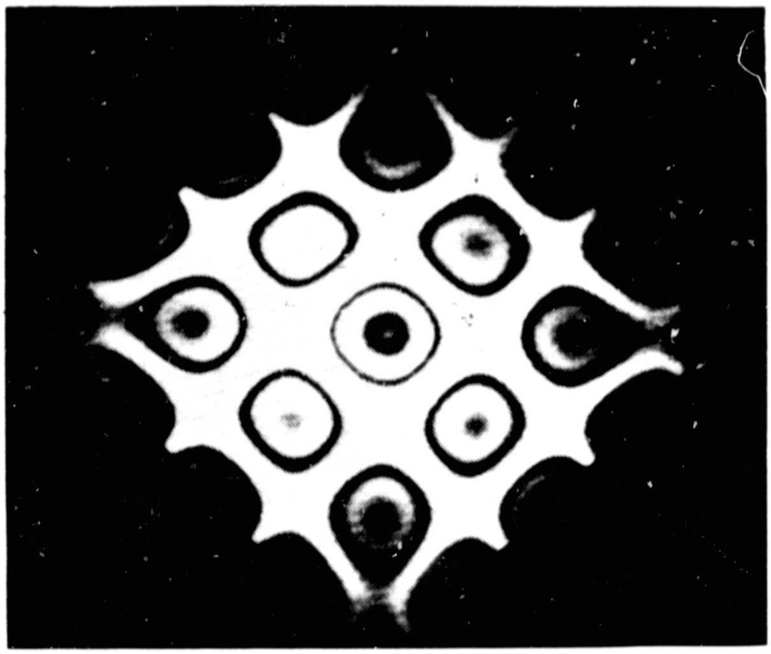

Figure 8. Hologram of Vibration Patterns on a Plate (printed with permission of ASME Mag.)

Figure 9. Hologram of a 16 kHz Tone-Burst in Water (printed with author's permission)

In place of a laser light, a monochromatic acoustic wave serves as the reference wave for creating the interferogram. If a laser beam is used to reconstruct the image, however, the additional procedure of photo-reducing the hologram is necessary. [The ratio of the wavelength recorded to the wavelength of the laser light must approach one in order to reduce distortion.]

In acoustic holography the difficulty is in generating the acoustical hologram. Conventional photographic procedures have proven inadequate for recording the acoustic information. In the study of ultrasonics, several schemes exist for recording holograms with sound-sensitive apertures and reproducing the hologram in real-time or with the various acoustic image conversion methods [18,19]. Iizuka developed a technique by which acoustic fields could be mapped by means of standard photographic film [20]. Color film containing a layer of developing reagent was subjected to a uniform light field to begin the developing process. The film was then placed in an acoustic field. The regions of high sound intensity caused an increase in the rate of diffusion of the developing reagent to the grain sites. The result was a visible pattern corresponding to the field's intensity distribution. Although more sensitive to ultrasonic waves, an acoustic resonator driven by a speaker at 315 Hz was successfully mapped. It is believed that the film negative could be useful for preparing acoustic holograms.

A method for detecting acoustic sound sources using long-wavelength acoustical holography was developed by Watson [21]. In order to avoid the complexities of capturing acoustic energy photographically, Watson attached diode lights (LED's) to four microphones situated on a scanner frame. A wave oscillator served to create the reference wave, which was an electronically synthesized plane wave. The radiated sound waves from the source are summed with the reference wave and used to light the LED's attached to the respective microphones of the scanner. A camera with an open shutter records the scanning of the acoustic field, and the resultant negative is the hologram.

Williams, Maynard, and Skudrzyk developed a holographic technique which can produce high resolution images of sound sources regardless of the wavelength [22]. They constructed a plane array of 256 microphones and were able to directly record the sound pressure level and phase via digital computer. No reference wave is required, thus, the technique avoids interferograms and the physical means of reconstruction, which had been the source of the resolution limitation. The sound sources are reconstructed digitally by the computer, and the results are displayed with computer graphics. The computer-graphics images could be rotatable 3-D projections, four-color maps, or stereographic projections for true 3-D viewing. The reconstruction of sound radiation from a rectangular plate excited at a resonant mode appears in Figure 10.

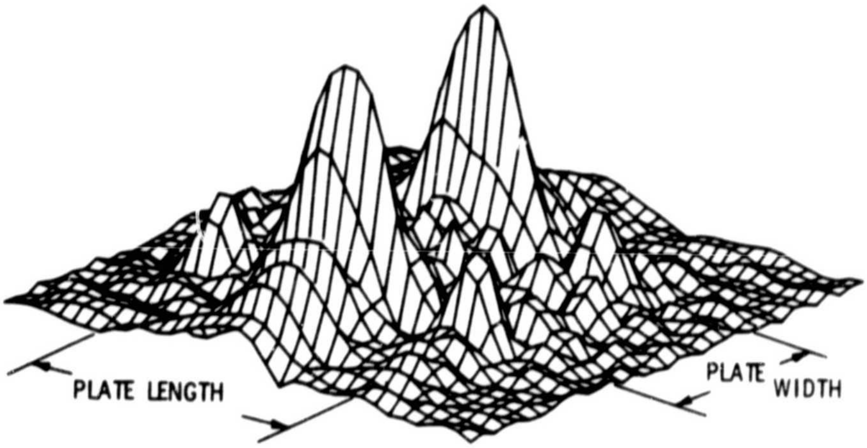

Figure 10. Hologram Image of Radiation from a Plate
(printed with author's permission)

4. ANIMATION OF ACOUSTICAL PHENOMENA

A new dimension can be added to acoustic visualization by the use of animation. Acoustics is a dynamic process of fluid mechanics. Yet, all the visualization methods aforementioned have resulted in a static representation of a dynamic process; time being held constant or averaged. The capability to visualize an acoustic process in the time domain greatly enhances the conceptual understanding of the phenomena. Certainly, transient behavior is not fully represented by a static model. Several persons have devised methods for the purpose of research and education by which an acoustic process can be visualized through animation. Three animation techniques are outlined in this section.

4.1 Baumeister and Rice

As previously described, Baumeister and Rice in order to investigate flow oscillation in Helmholtz resonators constructed a shadowgraph configuration. The frequency of the oscillation, however, was too high to be detected by human eye. Therefore, colored dies were injected into the flow field, and a high speed camera served as the image screen of the shadowgraph. The oscillating motion at the mouth of the resonator was recorded at high speed and then reviewed at a slower speed. The oscillating motion could be easily followed and evaluated visually.

4.2 Williams and Lighthill

In their study of aerodynamic generation of sound, Williams and Lighthill have demonstrated a method for animation of acoustic sources. The shadowgraph configuration consisted of a light source and an image screen positioned below and above a water tank, respectively. The shadows cast onto the screen were recorded on moving film. The depth of the water was chosen to represent sound in the sense that the waves in the ripple tank travelled at a constant speed. Waves were induced by the mass fluctuation of a foreign body being pushed in and out of the water. By the use of the oscillating plungers, the radiation from monopoles, dipoles, and quadrapoles could be compared and evaluated. Effects of wave cancellation, jet flow, and traversing objects were animated in the ripple tank.

4.3 Heckl

Heckl in his analytical studies of sound sources has animated wave propagation [22]. Heckl evaluated simple sources, radiation from surfaces, and vibration in solid media by first discretizing the problem domain to represent selected particles. The time variable was then incremented and the progressive motion was photographically recorded and combined to form a moving film. The result was the animation of particle motion in various acoustic configurations. One frame describing radiation from a solid surface is shown in Figure 11. The movie graphically depicts acoustic behavior. Heckl has used two interesting techniques. The motion from many particles in a field much larger than the source is used to show wave propagation. In addition, the analytical technique allows for the evaluation of the source nearfield.

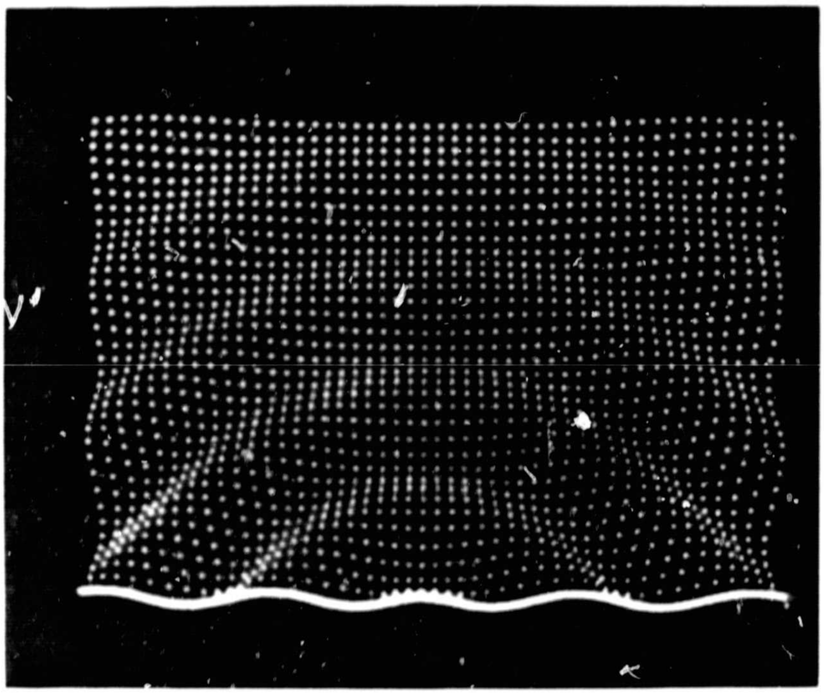

Figure 11. Animation Frame of Radiation from a Surface (printed with author's permission)

5. COMPUTER GRAPHICS

Improvements in experimental techniques and analytical modelling have resulted in the acquisition of large amounts of data. Thus, engineers are confronted with the task of evaluating large amounts of data. Manual inspection of computer printouts or hand plotting of graphs is slow and often ineffective for identifying trends. The digital computer and the technology of computer graphics have made significant advances is the last two decades. With increasing computer technology and decreasing computer costs, the ability of the computer to display information graphically has given the engineer fast and useful access to problem data.

A field representing particle motion is particularly difficult to display by computer graphics. The acoustic fields are three-dimensional. Thus, the two-dimensional display screen must demonstrate a feature for depth perception. In addition, the representation of a vector quantity possesses direction as well as magnitude. Recently, Feldman at Purdue University under research contract from NASA developed a general purpose program for visualization of three-dimensional vector fields [23]. Presented here is a brief description of the graphics program with emphasis on acoustic visualization.

5.1 Three-Dimensional Vector Display

Until recently graphics techniques were limited to two dimensional display. Since these were the only adequate means to visualize data, the engineer was obliged to either collect data in a manner conforming to the display or accept and learn conventions where three-dimensional data is displayed in two dimensions such as contour plots or waterfall plots. The three-dimensionality of many problems was neglected or poorly understood. Feldman's program, spatial data visualizer (SDV), is capable of displaying three-dimensional fields by a wide variety of techniques on the most current computer graphics hardware, and thus, will be used to conduct a survey of the most effective methods for computer graphic visualization of acoustic fields.

Discrete three-dimensional vector fields can be displayed by SDV on an Evans and Sutherland PS300 computer graphics system. In order to display discrete data on the graphics screen, a symbolic representation is used. SDV provides several selective shapes for vector representation with 3-D symmetry and automatic scaling. The feature of SDV that gives the images depth is termed 'depth cueing'. Intensity depth cueing is the ability to gradually fade the brightness of a line drawn on the screen to a level associated with the depth position of the end points. The graphic display of the discrete particle velocities of an acoustic dipole are represented at a distance and close to the source in Figure 12.

Due to available local intelligence, SDV also allows for the real-time translation and rotation of the graphic display. With similarity to observing an object while rotating it by hand, this provides the capability to turn and move the object field in real time as the analyst deciphers the field. A sequence of computer generated displays in Figure 13 demonstrates the translation-rotation capability. The display image represents an acoustic mode shape in an airplane described in terms of particle velocity.

Data reduction techniques are available for analyzing large numbers of vector displays in a 3-D region. Data reduction infers that the total information is reduced by selective viewing of the portions of interest. In addition to rotation and translation, the observer can 'zoom' in and out from either a perspective or orthographic view. The display can be sliced by two planes so as to perform real-time hither and yon clipping. In Figure 14, the particle velocity mode shapes associated with the cavity resonance in a box are displayed from various view points.

5.2 <u>Color Graphics</u>

Although characterized by qualities such as brightness and purity, the word "color" in common usage typically refers to hue. Hue is a circular function changing from red to orange to yellow to green to blue to magenta, and

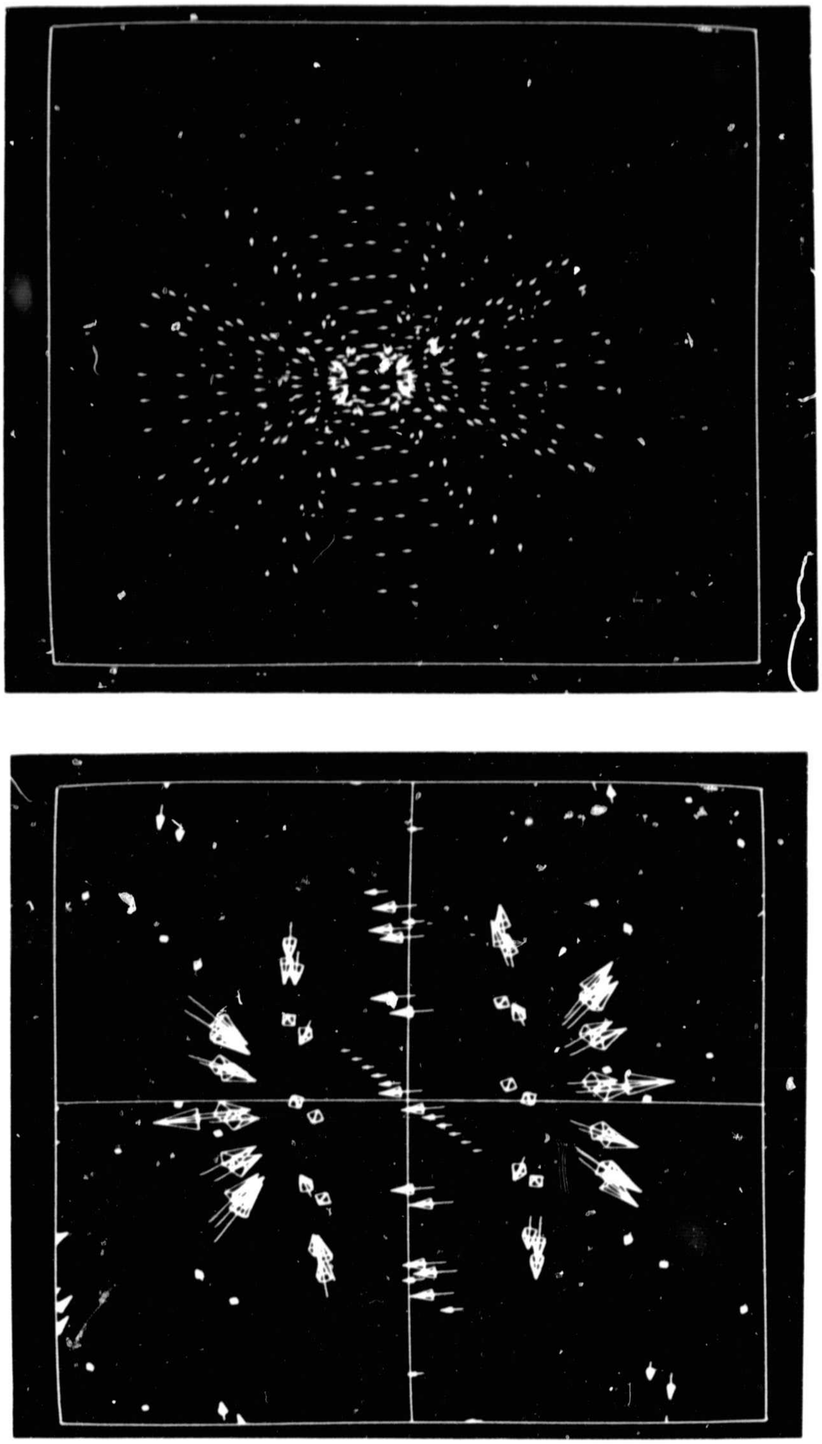

Figure 12. Computer Graphic Display of a 3-D Dipole

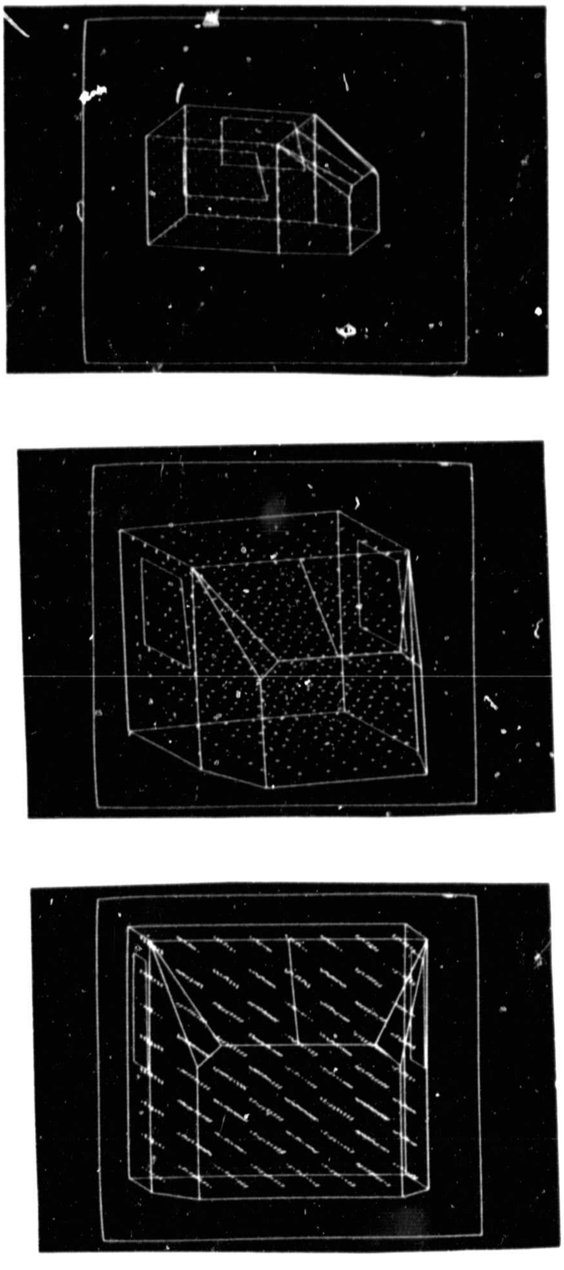

Figure 13. Particle Acceleration Mode in the Airplane

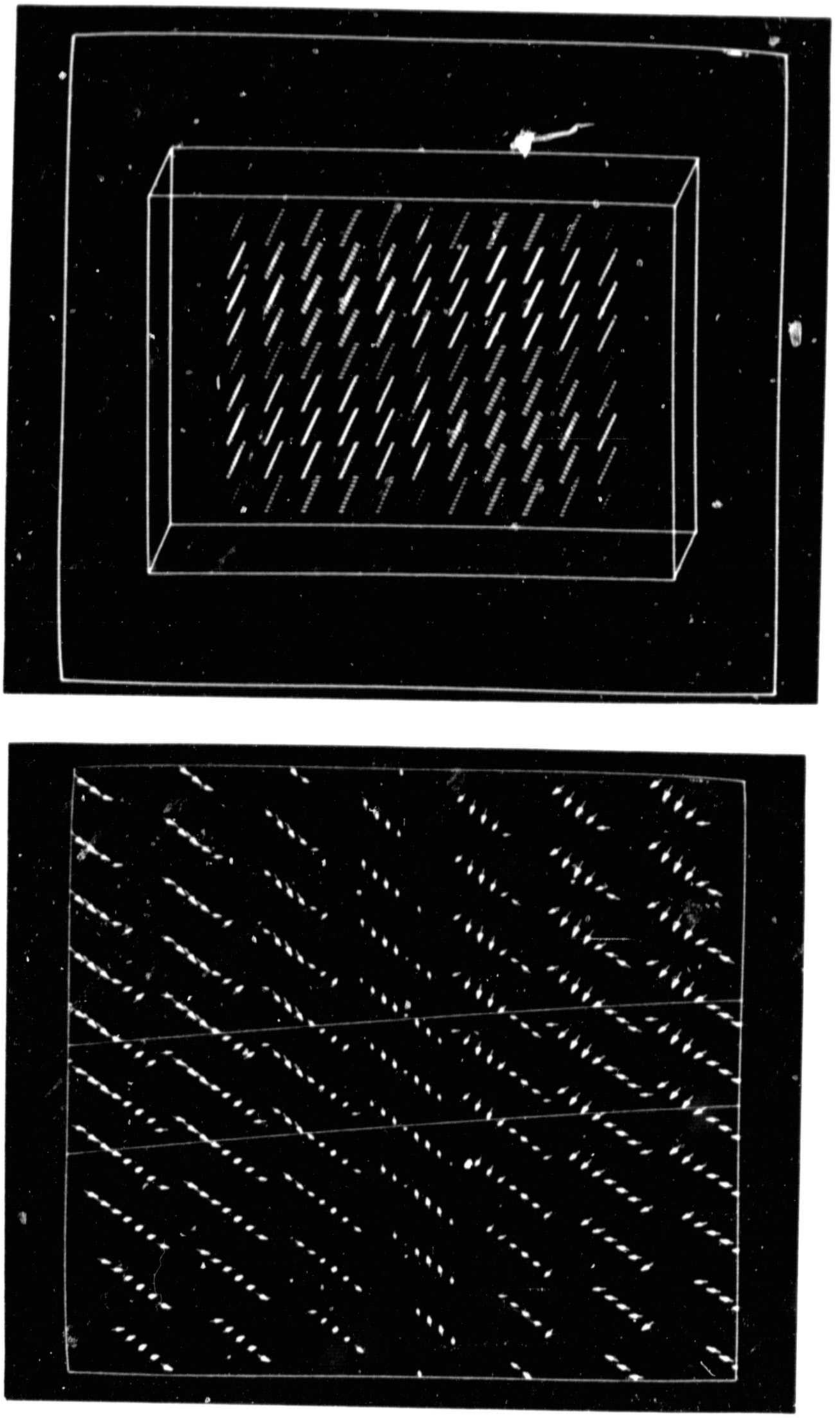

Figure 14. Particle Acceleration Modes in the Box

back to red. Saturation refers to the amount of gray in a color. Intensity describes the lightness or darkness of a color. The human eye is very sensitive to changes in color, consequently, the advantage of displaying data values in color is evident.

Color graphic display is provided by SDV on a Ramtek 9400 raster graphics terminal. SDV provides a choice from three color scales for representing the vector magnitude: red-green-blue, red-green-magenta, and cyan-magenta-yellow. Color shading enhances the three-dimensionality of the symbols. The hue, as well as saturation and intensity, can be manually adjusted. A color legend on each display allows for easy visual scaling of vector magnitudes.

The capability to define an object within the vector field is also provided by SDV. The object is described by polygonal surfaces. Intensity is used to shade the object for 3-D perspective and to create transparency effect.

5.3 Animation by Computer Graphics

The visualization of acoustic fields as a function of time greatly enhances the conceptual understanding. SDV includes the feature for animated motion of 3-D vector fields. In order to create an animation, incremental time frames of spatial data are generated. The frames are then sequentially flashed on the screen in a cyclic or tumbling order. The speed of the animation is adjustable. The animated display includes all the features outlined in Section 5.1, including real-time rotation and zoom.

6. CONCLUSION

The first three sections are a survey of the most common methods for visualizing acoustic phenomena. The emphasis of the survey is with respect to acoustic processes in the audible frequencies. It is apparent that some of the visualization techniques are underdeveloped, and additional research must precede the widespread and regular application to noise control. Acoustical holography, however, is a powerful tool and shows great promise for future application.

The survey of three-dimensional vector representations discussed in Section 5 is used as an indicator of the most effective techniques for visualizing acoustic fields with computer graphics. The capability to sense depth is essential and depth cueing on the PS300 is superior to the color shading technique used on the raster graphics screen. Real-time rotation and zoom are also important as these techniques simulate human diagnostic methods. Real-time techniques are also much superior to interrupted frame by frame or iterative techniques. Animation provides a great deal of insight particularly for propagation or transient problems. The color vector displays are somewhat limited and provide only a slight improvement over the classical two-dimensional techniques for acoustic display.

ACKNOWLEDGEMENTS

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