NASA TECHNICAL MEMORANDUM NASA TM-77593

WAVE MULTIPLE SCATTERING BY A FINITE NUMBER OF UNCLOSED CIRCULAR CYLINDERS

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(NASA-TM-77593) WAVE MULTIPLE SCATTERING BY A FINITE NUMBER OF UNCLOSED CIRCULAR CYLINDERS (National Aeronautics and Space Administration) 35 p HC A03/MF A01 CSCL 20N N85-10232
Unclas
G3/32 24171

Translation of "Difraktsiya Voln na Konechnom Chisle Nezamknutykh Tsilindricheskikh Ekranov," Academy of Sciences Ukrainian SSR, Institute of Radiophysics and Electronics, Kharkov, Preprint No. 163, 1980, pp. 1-41

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 AUGUST 1984

ORIGINAL PAGE IS
OF POOR QUALITY

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-77593		

4. Title and Subtitle WAVE MULTIPLE SCATTERING BY A FINITE NUMBER OF UNCLOSED CIRCULAR CYLINDERS

5. Report Date August 1984

6. Performing Organization Code

7. Author(s) E.I. Veliyev and V.V. Veremey

8. Performing Organization Report No.

10. Work Unit No.

9. Performing Organization Name and Address
Leo Kanner Associates
Redwood City, California 94063

11. Contract or Grant No.
NASw-3541

13. Type of Report and Period Covered
Translation

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration, Washington, D.C. 20546

14. Sponsoring Agency Code

15. Supplementary Notes
Translation of "Difraktsiya Voln na Konechnom Chisle Nezamknutykh Tsilindricheskikh Ekranov," Academy of Sciences Ukrainian SSR, Institute of Radiophysics and Electronics, Kharkov, Preprint No. 163, 1980, pp. 1-41

16. Abstract
The boundary-value problem of plane H-polarized electromagnetic wave multiple scattering by a finite number of unclosed circular cylinders is solved. The solution is obtained by two different methods: the method of successive scattering and the method of partial matrix inversion for simultaneous dual equations. The advantages of the successive scattering method are shown. Computer calculations of the surface currents and the total cross section are presented for the structure of two screens.

17. Key Words (Selected by Author(s))

18. Distribution Statement
Unlimited-Unclassified

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22.
Unclassified	Unclassified	35	

WAVE MULTIPLE SCATTERING BY A FINITE NUMBER OF UNCLOSED CIRCULAR CYLINDERS

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Introduction

Study of the problem of wave multiple scattering by structures which consist of a finite number of screens is of interest in the resolution of a number of practical questions in UHF electronics and antenna and measurement technology. /3*

The solution of this problem involves complex numerical computer calculations.

A large number of studies [1-9] have dealt with the problem of wave multiple scattering by a finite number of smooth scatterers. The problem is solved by both rigorous and approximate methods in these studies. The simplest approximate method of solution of the problem of multiple scattering by N smooth scatterers is to disregard the effect of the screens of the structure on each other. It is sufficient for the solution to know the solution of the problem of multiple scattering by each of the scatterers separately. As experiments show however, this method leads to very significant errors even when the scatterers are at a considerable distance from each other. All the remaining approximate methods are reduced to more or less complete accounting for the reciprocal influence of the screens [7, 8, 23].

The problem of multiple scattering of a plane electromagnetic wave by a finite number of flat strips is discussed in [10]. When all the strips are in one plane, the problem is reduced to the solution of one integral Fredholm equation of the second kind. Smooth scatterers and flat tape do not have distinct resonance properties. /4

*Numbers in the margin indicate pagination in the foreign text.

Study of the properties of structures formed of high quality screens capable of intensely scattering an electromagnetic field therefore arouses special interest.

As was shown in [14], an infinite periodic array which consists of unclosed circular cylinders has qualitatively new resonance properties which are of great practical interest. These resonances are observed upon excitation of the natural oscillations of the array by the incident wave [14].

Any actual structure cannot consist of an infinite number of elements. Solution of the problem of wave multiple scattering by a finite number of unclosed screens in the form of cylinders with longitudinal slits is therefore of special importance.

We discuss two rigorous methods of solution of such problems. Following [3], we call the first rigorous method, which is conventionally used in study of the properties of N bodies [12], <u>self consistent</u>. The excitation of each cylinder by the incident wave and unknown fields of adjacent scatterers is analyzed here. This leads to the solution of systems of simultaneous equations or to systems of integral equations. Such systems usually are solved by computer, and the basic complexity is to obtain a system of equations which is convenient for numerical solution.

We call the second method, first used by Schwarzchild [11] in studies of wave multiple scattering by slits, <u>iteration</u> [1]. This method is directly associated with the first one. This method essentially consists of successive study of the stages of wave scattering by the elements of the structure. It is assumed that the solution of the problem of wave multiple scattering by a single element is known. The scattered field of the system is presented in the form of superposition of the scattered fields of the individual elements /5

$$\psi = \sum_{s=1}^{N} \psi_s ,$$

and the scattered field of an arbitrary S-th element in the form of the sum of an infinite number of scattering orders

$$\Psi_s = \sum_{m=1}^{\infty} {}^s\Psi^m_.$$

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A scattered field of the first order is the scattered field of a single S-th scatterer. The scattered field of the second order

is the response of the S-th scatterer to superposition of the scattered fields of the first order of the remaining scatterers, etc. It is easy to note that the boundary conditions of the total field are fulfilled on the surface of each scatterer in this formulation of the problem. The convergence of the series by scattering orders follows from physical considerations and can be proved mathematically.

These two rigorous methods, developed for structures made of smooth scatterers, can also be applied to study of resonance systems of a finite number of unclosed circular cylinders. The interaction between these screens is of special importance in study of the electrodynamic properties of such structures. The use of the iteration method makes it possible to estimate the extent of this interaction. Other advantages of this method over the extremely frequently used [16] self consistent method also are discussed in the work.

1. Solution of Problem of Wave Multiple Scattering by Finite Number of Scatterers by Successive Scattering Method

A plane H polarized wave strikes a structure which consists of a finite number (N) of scatterers in the form of unclosed circular cylindrical screens at angle ϕ_1 to the OY axis (see Fig. 1). /6

The surface of the cylinders are infinitely thin and ideally conducting, cylinder radii are ρ_S, the angular dimensions of the slits are $2\theta_S$ and the orientation angles are ξ_S (S=1, 2, . . ., N). The generatrices of the cylinders are parallel to each other. It is required to determine the field scattered by the structure.

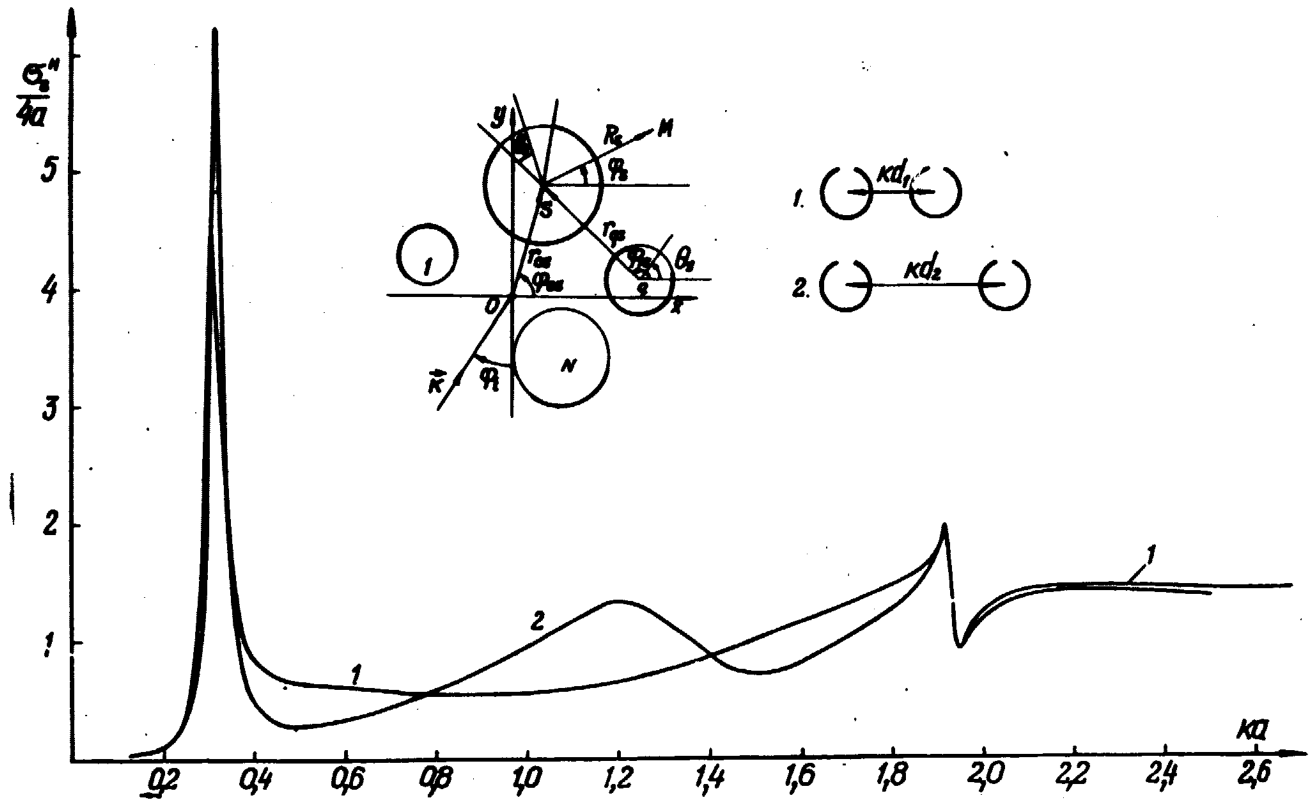

Fig. 1. Total scattering cross section vs. wave dimensions of structure, $\phi_i=0$: 1. $kd_1=4.0$ ka, $\theta_1=\theta_2=1°$; 2. $kd_2=7.0$ ka, $\theta_1=\theta_2=1°$.

We consider the case of nonoverlapping screens. We will find the solution of the problem by the successive scattering method proposed in [1, 11], which has a number of advantages because of the clear physical interpretation.

We write the total field of the structure in the form of superposition of the incident field and the scattered fields sought from each unclosed cylinder:

$$H_z^{tot} = H_z^i + H_z^{scat} = H_z^i + \sum_{S=1}^{N_s} H_z^{scat}. \quad (1)$$

Field H_z^{wave} everywhere outside the elements of the structure satisfies the Helmholtz equation and, on the surfaces of the cylinders, the boundary condition of Neumann, the condition of radiation to infinity and the condition of finite energy in any bounded region of space.

Following [1], we present the scattered field of the S-th cylinder in the form of an infinite sum over the so called orders of scattering:

$$ {}^{s}H_z^{\text{scat}} = \sum_{m=1}^{\infty} {}^{s}H_z^{m}. \tag{2} $$

We define the field of the first order of scattering as the response of a single S-th cylindrical screen to an incident plane wave, i.e., the Helmholtz equation, the radiation condition and the Neumann boundary condition of the surface of the S-th cylinder are satisfied by superposition of fields $H_z^i + {}^S H_z^1$. The scattered field of the first order can consequently be written in the form [17] /8

$$ {}^{s}H_z^1 = \sum_{(m)} {}^{s}\mu_m^1 \, J_m'(\kappa\rho_s) \cdot H_m^{(1)}(\kappa R_s)\, e^{im\varphi_s} \quad , R_s > \rho_s . \tag{3} $$

where (R_S, ϕ_S) are polar coordinates centered on the axis of the S-th cylinder, J'_m, $H_m^{(1)}$ are Bessel functions, and ${}^S\mu_m^1$ are the Fourier coefficients of the current density function of the first order on the surface of the S-th cylinder.

For determination of coefficients ${}^S\mu_m^1$, we subordinate total field $H_z^i + {}^S H_z^1$ to the Neumann boundary condition on the surface of the S-th cylinder

$$ \frac{\partial}{\partial R_s}\left(H_z^i + {}^{s}H_z^1\right)\bigg|_{R_s=\rho_s} = 0 \ , \quad \xi_s + \theta_s < \varphi_s < 2\pi + \xi_s - \theta_s . \tag{3'} $$

In the system of functional equations of the first order obtained in this manner, inversion of the static portion of the matrix can be carried out by the method of the Riemann-Gilbert problem, and a system of linear algebraic equations of the second kind can be obtained [19]:

$$ {}^{s}\mu_n^1 = \sum_{(m)} C_{nm}^{s}\, {}^{s}\mu_m^1 + {}^{s}b_n^1 , \tag{4} $$

where

$$ C_{nm}^{s} = \begin{cases} \frac{(-1)^n}{n} e^{-in\xi_s} i\pi(\kappa\rho_s)^2 J_0'(\kappa\rho_s) \cdot H_0^{(1)}(\kappa\rho_s) \cdot V_{n-1}^{-1}, & m=0, n\neq 0 \\ (-1)^{m+n} \frac{|m|}{n} \cdot e^{-i(m-n)\xi_s} \delta_m^s V_{n-1}^{m-1}, & m\neq 0, n\neq 0 \\ (-1)^{m+1} |m| \cdot e^{im\xi_s} \delta_m^s W_m, & m\neq 0, n=0 \\ -i\pi(\kappa\rho_s)^2 \cdot J_0'(\kappa\rho_s) \cdot H_0^{(1)\prime}(\kappa\rho_s) \cdot W_0, & m=0, n=0 ; \end{cases} \tag{5} $$

$$ {}^{s}b_n^1 = \begin{cases} \frac{(-1)^n}{n} e^{-in\xi_s} i\pi(\kappa\rho_s)^2 A_s^0 \sum_{(m)} J_{-m}'(\kappa\rho_s) e^{im(\xi_s+\varphi_i)} V_{n-1}^{m-1}, & n\neq 0 \\ -i\pi(\kappa\rho_s)^2 A_s^0 \sum_{(m)} J_{-m}'(\kappa\rho_s) e^{im(\xi_s+\varphi_i)} W_m, & n=0 ; \end{cases} \tag{6} $$

$$ A_s^0 = e^{i\kappa r_{js} \sin(\varphi_i + \varphi_{js})} , $$

j is a random number from 1 to N. /9

System of linear algebraic equations of the second kind (4) is Fredholmian [19] and can be solved by the reduction method. Its solution can consequently be obtained with any preassigned accuracy. System (4) can be solved by the successive approximation method in a number of cases [17], which substantially simplifies the calculations.

By summing the scattered first order fields of each cylinder, we obtain the solution of the multiple scattering problem in the single scatterer approximation. As has been stated, this approximation gives a strongly distorted concept of the field scattered by the system. The degree of accuracy of the results can be estimated by means of accounting for the subsequent scattering orders.

We will call the response of the S-th cylinder to superposition of the first order scattering field the second order scattering field of this cylinder, i.e., we assume that superposition of the scattered first order fields from the cylinders with numbers $i \neq S$ $(i=1, 2, \ldots, N)$ and the scattered second order field of the S-th cylinder satisfies the Helmholtz equation, the boundary conditions on the surface of the cylinder and the radiation condition. We write this total field in the form

$$\sum_{q=1}^{N}{}' {}^{q}H_z^1 + {}^{S}H_z^2 = {}^{S}H_z^{n2},$$

where ${}^{S}H_z^{2}$ is the second order scattering field of the S-th cylinder, /10
and the Σ' symbol specifies that the value of summation notation q=S is excluded.

The second order field which satisfies the wave equation and the radiation condition outside cylinder S (like the first order field) can be written in the form

$$ {}^{S}H_z^2(R_S, \varphi_S) = \sum_{(m)} {}^{S}\mu_m^2 \, J_m'(\kappa\rho_S) H_m^{(1)}(\kappa R_S) e^{im\varphi_S}. \tag{7}$$

With Eq. (3) and (7) taken into account, we write field ${}^{S}H_z^{n2}$

$$ {}^{S}H_z^{n2} = \sum_{q=1}^{N}{}' \sum_{(m)} {}^{q}\mu_m^1 \, J_m'(\kappa\rho_q) \cdot H_m^{(1)}(\kappa R_q) e^{im\varphi_q} + \sum_{(m)} {}^{S}\mu_m^2 \, J_m'(\kappa\rho_S) \cdot H_m^{(1)}(\kappa R_S) e^{im\varphi_S}. \tag{8}$$

For determination of ${}^{S}\mu_m^{2}$, we subordinate field ${}^{S}H_z^{n2}$ to the Neumann boundary condition on the surface of the S-th cylinder, i.e.,

$$\begin{cases} \dfrac{\partial}{\partial R_S} {}^{S}H_z^{n2} \Big|_{R_S=\rho_S} = 0, & \xi_S + \theta_S < \varphi_S < 2\pi + \xi_S - \theta_S, \\ {}^{S}\mu^2 = 0, & \xi_S - \theta_S < \varphi_S < \xi_S + \theta_S. \end{cases} \tag{9}$$

In subordination of the total field to the boundary conditions, a change must be made to the (R_S, ϕ_S) coordinate system connected with the S-th cylinder. The summation theorem for Bessel functions must be used for this

$$H_m^{(1)}(\kappa R_q)\cdot e^{im(\varphi_q-\varphi_{qs})}=\sum_{(p)} J_p(\kappa R_s)\cdot H_{p+m}^{(1)}(\kappa r_{qs})\cdot e^{ip(\pi-\varphi_s+\varphi_{qs})} \quad (10)$$

Series (10) converges with $R_S<r_{qs}$, ϕ_{qs} is the characteristic position angle of the q-th cylinder relative to the S-th cylinder and r_{qs} is the distance between these cylinders.

By using Eq. (10), we obtain from Eq. (9) the system of functional equations for determination of ${}^S\mu_m^{\,2}$

$$\begin{cases} \sum_{(m)} {}^s\mu_m^2\, J_m'(\kappa\rho_s)\, H_m^{(1)\prime}(\kappa\rho_s) e^{im\varphi_s} = \\ \qquad = -\sum_{(m)} J_m'(\kappa\rho_s)\, {}^s\Omega_m^1\, e^{im\varphi_s}, \quad \xi_s+\theta_s<\varphi_s<2\pi+\xi_s-\theta_s, \\ \sum_{(m)} {}^s\mu_m^2\, e^{im\varphi_s} = 0, \quad \xi_s-\theta_s<\varphi_s<\xi_s+\theta_s, \end{cases} \quad (11)$$

where
$${}^s\Omega_m^1 = \sum_{q=1}^{N}{}' \sum_{(p)} {}^q\mu_p^1\, J_p'(\kappa\rho_q)\, e^{i(p-m)\varphi_{qs}} H_{p-m}^{(1)}(\kappa r_{qs}).$$ /11

The Fourier coefficients of the first order surface current density function ${}^q\mu_m^{\,1}$ are determined from the solution of system (4). System of Eq. (11) is similar to the system of functional equations obtained by solution of the problem of excitation of a single cylinder by a plane wave, i.e., by determination of the first order scattering field. These systems only differ on the right sides, which characterize the exciting field. This is completely understandable, since both the first and second order scattering fields are defined as the response of the same cylinder to different perturbing fields. This is the incident plane wave in the first case and the first order scattering field of the remaining cylinders in the second.

By applying the method of the Riemann-Gilbert problem to system

(11), we obtain a system of linear algebraic equations of the second kind for ${}^S\mu_m{}^2$, the matrix of which equals the matrix of system of Eq. (4)

$$
{}^s\mu_n^2 = \sum_{(m)} C_{nm}^s\, {}^s\mu_m^2 + {}^s b_n^2 ,
$$

$$
{}^s b_n^2 = \begin{cases} i\pi(\kappa\rho_s)^2 \dfrac{(-1)^m}{m} e^{-im\xi_s} \sum\limits_n {}^s\Delta_n^2 e^{in(\xi_s+\pi)} V_{m-1}^{n-1}, & m \neq 0 \\ -i\pi(\kappa\rho_s)^2 \sum\limits_{(n)} {}^s\Delta_n^2 e^{in(\xi_s+\pi)} W_n, & m = 0, \end{cases} \qquad (12)
$$

where $\quad {}^s\Delta_m^2 = J_m'(\kappa\rho_s)\, {}^s\Omega_m^1 .$ /12

System (12) is analagous to system (4). It can also be solved by the reduction method and by the successive approximations method in particular cases.

We will call the response of the S-th cylinder to the second order scattered field of the remaining cylinders the third order scattered field

$$
\sum_{q=1}^{N}{}' \, {}^q H_z^2 + {}^s H_z^3 = {}^s H_z^{sn},
$$

in which $\quad \dfrac{\partial}{\partial R_s} {}^s H_z^{ms} \Big|_{R_s=\rho_s} = 0 \quad$ with $\quad \xi_s + \theta_s < \varphi_s < 2\pi + \xi_s - \theta_s .$

The third order scattered field is found just like the second order field was found.

In a similar manner, it is easy to determine the fields of all the remaining orders of scattering. By then summing the scattering orders, we obtain the field scattered by the structure

$$
H_z^{scat} = \sum_{q=1}^{N}\sum_{n=1}^{\infty} {}^q H_z^n = \sum_{q=1}^{N}\sum_{(m)} \mu_m^q J_m'(\kappa\rho_q) H_m^{(1)}(\kappa R_q) e^{im\varphi_q} \qquad (13)
$$

where $\quad \mu_m^q = \sum\limits_{n=1}^{\infty} {}^q\mu_m^n , \ (R_q > \rho_q,\ q = 1,2,\ldots,N).$

${}^{s}\mu_m^n$ are determined for random parameters of the structure by solution of the system of linear algebraic equations of the second kind

$$ {}^{s}\mu_m^n = \sum_{(p)} C_{mp}^{s}\, {}^{s}\mu_p^n + {}^{s}b_m^n . \tag{14}$$

where

$$ {}^{s}b_m^n = \begin{cases} i\pi(k\rho_s)^2 A_s^{n-1} \frac{(-1)^m}{m} e^{-im\xi_s} \sum\limits_{(p)} {}^{s}\Delta_p^n e^{ip(\xi_s+\alpha)} V_{m-1}^{p-1}, & m \neq 0 \\ -i\pi(k\rho_s)^2 A_s^{n-1} \sum\limits_{(p)} {}^{s}\Delta_p^n e^{ip(\xi_s+\alpha)} W_p, & m = 0, \end{cases}$$

$$ {}^{s}\Delta_p^m = J_p'(k\rho_s)\, {}^{s}\Omega_p^{m-1}, \qquad A_s^i = \begin{cases} A_s^0, & i = 0 \\ 1, & i \neq 0, \end{cases}$$

$$ {}^{s}\Omega_m^i = \begin{cases} e^{im\varphi_i}, & i = 0 \\ \sum\limits_{q=1}^{N}{}' \sum\limits_{(p)} {}^{q}\mu_p^i\, J_p'(k\rho_q)\, e^{i(p-m)\varphi_{qs}} H_{p-m}^{(1)}(kr_{qs}), & i \neq 0 . \end{cases}$$

The resulting solution of the multiple scattering problem can be called rigorous if, by assigning the order of iterations, i.e., by taking account of a specific number of scattering orders, the solution of the problem can be obtained with any preassigned accuracy. /13

We note that the matrix of the system of equations is defined only by the cylinder number and does not depend on the scattering order. This substantially speeds up the calculation process and simplifies its management.

More than that, in the particular case when the cylinders have narrow slits ($\theta_S \to 0$, $S=1, \ldots, N$), system (14) permits analytical solution. Analysis of the behavior of the coefficients of the matrix of the system of equations shows that, with increase of indices n and m and when θ_S (angular dimension of the slit)$\to 0$, C_{nm}^{S} with $m \neq 0$, $n \neq 0$, $m \neq n$ are very small, and they can be disregarded in solution of the system of equations, i.e.,

$$ {}^{s}\mu_0^n = C_{00}^{s}\, {}^{s}\mu_0^n + \sum_{m \neq 0} C_{0m}^{s}\, {}^{s}\mu_m^n + {}^{s}b_0^n ,$$

$$ m \neq 0, \quad {}^{s}\mu_m^n = C_{m0}^{s}\, {}^{s}\mu_0^n + C_{mm}^{s}\, {}^{s}\mu_m^n + {}^{s}b_m^n .$$

In this manner, by leaving the diagonal elements and the elements of the zero column and zero line in the matrix of the system of equations, the solution can be written in the form

$$\begin{cases} {}^{s}\mu_0^n = \dfrac{\sum\limits_{m\neq 0} C_{0m}^{s}\,{}^{s}b_m^n/(1-C_{mm}^{s}) + {}^{s}b_0^n}{1 - C_{00}^{s} - \sum\limits_{m\neq 0} C_{0m}^{s}\cdot C_{m0}^{s}/(1-C_{mm}^{s})} \\ {}^{s}\mu_m^n = (\mu_0^n\cdot C_{m0}^{s} + {}^{s}b_m^n)/(1-C_{mm}^{s}),\ m\neq 0. \end{cases} \qquad (15)$$

A numerical test of Eq. (15) was carried out. The results were compared with the rigorous solution of the problem of wave multiple scattering by a single unclosed cylinder. With $\theta<10°$, the error with $ka<5.0$ is from 0.01% to 1%. At angles $\theta \lesssim 1°$, Eq. (15) gives an accurate result (to within 1%) over a significantly wider range of change of ka (see Table 1). /14

Because the solution of the multiple scattering problem as $\theta_S \to 0$ of an unclosed cylinder is written in explicit form, a closed expression can be obtained for the field scattered by the structure by introducing some designations (similar equations were obtained in [1], which dealt with study of the scattering properties of circular cylinders)

$$H_z^{\text{pac}} = \sum_{s=1}^{N}\sum_{m=-\infty}^{\infty} J_m'(\kappa\rho_s)\cdot H_m^{(1)}(\kappa R_s)\, e^{im\varphi_s}\sum_{n=1}^{\infty}{}^{s}\rho_m^n ,$$

where

$${}^{s}\rho_m^n = \prod_{\mu=(1)}^{(n-1)}\sum_{s^\mu}{}'\sum_{m^\mu} J_{m^\mu}'(\kappa\rho_{s^\mu})\,{}^{s^\mu}R_{m^{\mu}}^{m^{\mu-1}}(\varphi_{s^\mu s^{\mu-1}}, \kappa r_{s^\mu s^{\mu-1}})\cdot$$

$$\cdot A_{s^{n-1}}^{0} \sum_{(p)} e^{ip\varphi_i}\,{}^{s^{n-1}}F_{m^{n-1}}^{p} ,$$

in which

$${}^{s}R_m^{m'}(\varphi_{s's}, \kappa r_{s's}) = \sum_{(p)} e^{i(p-m')\varphi_{s's}} H_{p-m'}^{(1)}(\kappa r_{s's})\,{}^{s}F_m^p ,$$

$${}^{s}F_m^p = \begin{cases} \dfrac{C_{m0}^{s}}{D_m^{s} D_0^{s}}\left(\sum\limits_{t\neq 0} \dfrac{C_{0t}^{s}}{1-C_{tt}^{s}}\,{}^{s}N_t^p + {}^{s}N_0^p\right) + \dfrac{1}{D_m^{s}}\,{}^{s}N_m^p ,\ m\neq 0 \\ \dfrac{1}{D_0^{s}}\left(\sum \dfrac{C_{0t}^{s}}{1-C_{tt}^{s}}\,{}^{s}N_t^p + {}^{s}N_0^p\right),\quad m=0, \end{cases}$$

where /17

$$
{}^{s}N_m^p = \begin{cases} i a(\kappa\rho_s)^2 \frac{(-1)^m}{m} e^{-im\xi_s} J_p'(\kappa\rho_s) e^{ip\xi_s} V_{m-1}^{p-1}, & m \neq 0 \\ -ia(\kappa\rho_s)^2 J_p'(\kappa\rho_s) e^{ip\xi_s} W_p, & m = 0 \end{cases}
$$

$$
\mathcal{D}_m^S = \begin{cases} 1 - C_{mm}^S, & m \neq 0 \\ 1 - C_{00}^S - \sum_{m' \neq 0} \frac{C_{om'}^S C_{m'0}^S}{1 - C_{m'm'}^S}, & m = 0. \end{cases}
$$

In order to better explain the iteration procedure of obtaining the solution of the problem of multiple scattering by a finite number of scatterers, we repeat the stages of calculation in order to show that the boundary conditions are fulfilled on the surface of each unclosed cylinder.

We show that

$$
\frac{\partial}{\partial R_s} H_z^{tot} \Big/_{R_s = \rho_s} = 0,
$$

with $\xi_s + \theta_s < \varphi_s < 2\pi + \xi_s - \theta_s$ (we will subsequently imply this condition).
$s = 1, 2, \ldots, N$

In this manner

$$
\frac{\partial}{\partial R_s} H_z^{tot} = \frac{\partial}{\partial R_s} H_z^i + \frac{\partial}{\partial R_s} H_z^{scat} = \frac{\partial}{\partial R_s} H_z^i + \frac{\partial}{\partial R_s} H_z^s + \sum_{s'}^{N}{}' \frac{\partial}{\partial R_s} H_z^{s'} =
$$

$$
= \left[\frac{\partial}{\partial R_s} H_z^i + \frac{\partial}{\partial R_s} H_z^s\right] + \sum_{s'}{}' \frac{\partial}{\partial R_s} H_z^{s'}, \text{ with } R_s = \rho_s.
$$

That is, the total field consists of the superposition of the incident wave, the wave scattered by the S-th cylinder and the waves scattered by the remaining cylinders. By presenting the scattered fields in the form of an infinite sum of the scattering orders, we write

$$
\frac{\partial}{\partial R_s} H_z^{tot} = \left(\frac{\partial}{\partial R_s} H_z^i + \frac{\partial}{\partial R_s} {}^sH_z^1\right) + \frac{\partial}{\partial R_s} \sum_{m=2}^{\infty} {}^sH_z^m + \frac{\partial}{\partial R_s} \sum_{s'}{}' \sum_{m=1}^{\infty} {}^{s'}H_z^m
$$

with $R_s = \rho_s$.

TABLE 1.

			μ_0	μ_1	μ_2	μ_3	μ_4
$ka=0{,}375,\ \theta=5°$	строгое a	Re	0.3745	0.1849	0.0418	-0.0528	-0.0203
		Im	1.5938	-0.1199	-0.1012	-0.0286	0.0394
	приближ. b	Re	0.3745	0.1849	0.0418	-0.0528	-0.0203
		Im	1.5938	-0.1199	-0.101[illegible]	-0.0286	0.0394
$ka=1{,}0,\ \theta=5°$	строгое a	Re	-0.4274	0.2821	0.1186	0.0151	-0.0381
		Im	-1.0010	-0.8787	-0.3621	-0.1158	-0.0185
	приближ. b	Re	-0.4274	0.2821	0.1186	0.0151	-0.0381
		Im	-1.0010	-0.8787	-0.3621	-0.1158	-0.0186
$ka=2{,}0,\ \theta=5°$	строгое a	Re	-1.5530	1.0493	0.4876	0.379[illegible]	0.[illegible]508
		Im	-0.2515	-1.5575	-1.3550	-0.8032	-0.39[illegible]4
	приближ. b	Re	-1.5530	1.0497	0.4876	0.3796	0.0507
		Im	-0.2516	-1.5587	-1.3548	-0.8034	-0.3924
$ka=0.375,\ \theta=10°$	строгое a	Re	-2.2217	-0.0418	-0.1718	0.0312	0.0764
		Im	0.3349	-0.6081	0.0283	0.1070	-0.0221
	приближ. b	Re	-2.2221	-0.0424	-0.1718	0.0312	0.0765
		Im	0.3354	-0.6078	0.0284	0.1070	-0.0221
$ka=1{,}0,\ \theta=10°$	строгое a	Re	-0.3221	0.2313	0.2045	0.0131	-0.0684
		Im	-1.0646	-0.7286	-0.3581	-0.1633	-0.0166
	приближ. b	Re	-0.3218	0.2293	0.2037	0.1298	-0.0686
		Im	-1.0659	-0.7207	-0.3527	-0.1617	0.0156
$ka=2{,}0,\ \theta=10°$	строгое a	Re	-1.6206	-0.5715	1.5045	0.2148	-0.3552
		Im	-0.6306	-4.7018	-1.3099	-1.4946	-0.2277
	приближ. b	Re	-1.6601	-0.8767	1.4465	0.1380	-0.3180
		Im	-0.6244	-4.4720	-1.4337	-1.4337	-0.1891

Key: a. Rigorous b. Approximate

Because of equality (3'), the sum of the first two terms equals zero /18

$$\frac{\partial}{\partial R_s}\psi = \left(\frac{\partial}{\partial R_s}{}^{s}H_z^{s} + \sum_{s'}{}'\frac{\partial}{\partial R_s}{}^{s'}H_z^{1}\right) + \frac{\partial}{\partial R_s}\sum_{m=1}^{\infty}{}^{s}H_z^{m} + \frac{\partial}{\partial R_s}\sum_{s'}{}'\sum_{m=1}^{\infty}{}^{s'}H_z^{m}, \quad \text{with} \quad R_s = \rho_s.$$

It follows from equality (9) that we can drop out the first two terms and in the same manner for all scattering orders, since the scattered fields from the remaining cylinders constitute the excitation of any individual cylinder and as a response to which the field of the next scattering order is formed.

Consequently, the n-th order of excitation

$$\frac{\partial}{\partial R_s}{}^{s}H_z^{n+1} + \frac{\partial}{\partial R_s}\sum_{s'}{}^{s'}H_z^{n} = 0\Big/_{R_s=\rho_s},$$

and hence

$$\frac{\partial}{\partial R_s}\psi\Big/_{R_s=\rho_s} = 0 \qquad \text{(in metal)}.$$

The physical principles which are the basis of the iteration algorithm of construction of the solution of the problem of multiple scattering by a finite number of scatterers give the right to speak of the convergence of the iteration process, i.e., the series by scattering orders converges $\sum_{m=1}^{\infty}{}^{s}H_z^{m}$. This also can be proved mathematically.

It is completely evident that the convergence rate of the series depends on the incident wavelength as well as on the geometric dimensions of the structure (primarily on the distance between scatterers). The rate of convergence of this series can be estimated numerically (see Section III). Estimates show that, even when the cylinders are located quite close to each other (there is one diameter between cylinders), the scattering order series converges quite rapidly (eight members of the series is sufficient to obtain results which are accurate to within 0.1%).

The availability of clearly defined stages of construction of the solution of the problem and the algorithmic nature of the solu- /19 tion makes the iteration method very convenient for computer programming of the problems. Compared with direct self consistent method (described in Section II), there are substantial savings in working storage here, which makes rigorous solution of such problems possible even on a computer of medium capacity. A great advantage of the iteration method is the physical foundations of presentation of the scattered field in the form of superposition of the scattering orders. This method makes it possible to estimate the extent of interaction between structural elements and to show the region to which a given approximation can be restricted and the frequency region and structures for which single scattering theory can be used, i.e., to generally disregard the interaction.

We note in concluding this section that the field within the cylindrical screens can be determined in the following manner:

$$ {}^{s}H_z^{scat} = \sum_m {}^{s}\mu_m J_m(\kappa R_s) H_m^{(1)}(\kappa \rho_s) e^{im\varphi_s} \quad , R_s < \rho_s , $$

where

$$ {}^{s}\mu_m = \sum_{n=1}^{\infty} {}^{s}\mu_m^n . $$

II. Rigorous Solution of Multiple Scattering Problem Based on Self Consistent Method

Following [3], we will call the method described in this section self consistent. The substance of the self consistent method is that the field scattered by some cylinder is found on the assumption that /20 the field scattered by the remaining cylinders is known, i.e., by considering all the scatterers in turn and presenting the exciting field in the form of the superposition of a plane wave and unknown scattered fields of the remaining elements of the structure, after application of the boundary conditions, we arrive at a system of simultaneous equations for the unknown amplitudes of the scattered fields of the structural elements [12].

Quite high efficiency of the self consistent method is achieved in the case in question by partial inversion of the matrix of the system of functional equations of the first kind, similar to the way this was done in study of the unclosed cylinder array [14].

We present the total field in the form of superposition of the incident and scattered fields. The field scattered by the structure is written in the form of superposition of the fields scattered by the individual elements

$$H_z^{\text{total}} = H_z^i + H_z^{\text{scat}} = H_z^i + \sum_{q=1}^{N} {}^q H_z^{\text{scat}}. \tag{16}$$

We will use the (R_S, ϕ_S) coordinate system (S=1, 2, . . ., N) tied to the centers of the corresponding cylinders to record the incident and scattered fields.

By presenting the scattered field in the form of superposition of the bilayer potentials and by using the property of periodicity of the surface current density function on the cylinders with respect to ϕ_S, we obtain [17]

$$H_z^{\text{scat}} = \sum_{q=1}^{N} \sum_{(m)} \mu_m^q J_m'(\kappa\rho_q) \cdot H_m^{(1)}(\kappa R_q) e^{im\varphi_q}, \quad R_q > \rho_q.$$

By applying the summation theorem for cylindrical functions, the field scattered by the structure can be written in the (R_S, ϕ_S) coordinate system of a random cylinder of number S

$$H_z^{\text{scat}} = \sum_{m=-\infty}^{\infty} \mu_m^s J_m'(\kappa\rho_s) \cdot H_m^{(1)}(\kappa R_s) \cdot e^{im\varphi_s} +$$

$$+ \sum_{q=1}^{N} \sum_{(m)} \mu_m^q \cdot J_m'(\kappa\rho_q) \cdot e^{im\varphi_{qs}} \cdot \tag{17}$$

$$\cdot \sum_{(p)} J_p(\kappa R_s) \cdot H_{m+p}(\kappa r_{qs}) \cdot e^{ip(\pi - \varphi_s + \varphi_{qs})}, \quad R_s > \rho_s, \; R_s < r_{qs}.$$

Total field (16) should satisfy the wave equation, radiation condition, the condition of finite energy in any finite space, as well as the boundary conditions on the surface of each scatterer. /21

The Neumann boundary condition applies in our case:

$$\frac{\partial}{\partial R_s} H_z^{tot} \Big|_{R_s=\rho_s} = 0 \qquad \text{with} \quad \xi_s + \theta_s < \varphi_s < 2\pi + \xi_s - \theta_s \quad S = 1,2, \ldots N. \tag{18}$$

The following condition is fulfilled in the remaining portion of the interval of change of ϕ_S:

$$\sum_{p=-\infty}^{\infty} \mu_p^s \cdot e^{ip\varphi_s} = 0 \, , \quad \xi_s - \theta_s < \varphi_s < \xi_s + \theta_s \, . \tag{19}$$

We substitute the expressions for the incident and scattered fields (16), (17) in Eq. (18) and we differentiate. The resulting equation here together with Eq. (19) form a system of functional summator equations with a trigonometric kernel for the unknown μ_p^S:

$$\begin{cases} \sum\limits_{(m)} \mu_m^s \cdot J_m'(\kappa\rho_s) \cdot H_m^{(1)}(\kappa\rho_s) e^{im\varphi_s} = -A_s^0 \sum\limits_{(m)} J_m'(\kappa\rho_s) e^{im(\varphi_s+\varphi_i)} - \\ \qquad - \sum\limits_{(m)} J_m'(\kappa\rho_s) \Omega_m^s \cdot e^{im\varphi_s} , \qquad \xi_s + \theta_s < \varphi_s < 2\pi + \xi_s - \theta_s \\ \sum\limits_{(m)} \mu_m^s e^{im\varphi_s} = 0 \, , \qquad \xi_s - \theta_s < \varphi_s < \xi_s + \theta_s \, , \end{cases} \tag{20}$$

where /22

$$\Omega_m^s = \sum_{\substack{q=1 \\ q \neq s}}^{N} \sum_{(p)} \mu_p^q J_p'(\kappa\rho_q) H_{p-m}^{(1)}(\kappa r_{qs}) e^{i(p-m)\varphi_{qs}} ;$$

$$A_s^0 = e^{i\kappa r_{js} \sin(\varphi_i + \varphi_{js})} \, .$$

A similar system of equations was obtained in [14] in solution of the problem of multiple scattering of a plane wave by an infinite periodic array of unclosed circular cylinders. However, in view of the fact that the surface current densities on the cylinders are different in the problem in question in distinction from [15, 14], the system of functional equations for μ_n^q, with $q=1, 2, \ldots, N$, must be considered jointly. As is well known, systems of the first kind are unsuitable for numerical analysis. We therefore partially invert the matrix which corresponds to the problem of wave multiple diffraction by a single cylinder by the Riemann-Gilbert method. The procedure of expansion of the matrix in the completely continuous and easily inverted portion is discussed in detail in [20].

Thus, by using the Riemann-Gilbert method, we obtain a set of simultaneous linear algebraic equations of the second kind which are /23
convenient for computer utilization:

$$\mu_n^s = \sum_{m=-\infty}^{\infty} C_{nm}^s \mu_m^s + \sum_{q=1}^{N}{}' \sum_{p=-\infty}^{\infty} M_{np}^{qs} \mu_p^q + b_n^s, \quad \begin{matrix} n=\pm 1, \pm 2, \ldots \\ s=1,2,\ldots,N, \end{matrix} \tag{21}$$

where C_{nm}^S and b_n^S were determined in Section I (with $b_n^S = {}^S b_n^1$)

$$M_{np}^{qs} = \begin{cases} i\pi(\kappa\rho_s)^2 \dfrac{(-1)^n}{n} e^{-in\xi_s} J_p'(\kappa\rho_q) \displaystyle\sum_{m=-\infty}^{\infty} J_{-m}'(\kappa\rho_s) \cdot H_{p-m}^{(1)}(\kappa r_{qs}) \cdot \\ \qquad \cdot e^{i(p-m)\varphi_{qs}} e^{im\xi_s} V_{n-1}^{m-1}(-\theta_s), \quad n \neq 0 \\ -i\pi(\kappa\rho_s)^2 J_p'(\kappa\rho_q) \displaystyle\sum_{m=-\infty}^{\infty} J_{-m}'(\kappa\rho_s) \cdot H_{p-m}^{(1)}(\kappa r_{qs}) \cdot \\ \qquad e^{i(p-m)\varphi_{qs}} e^{im\xi_s} W_m(-\theta_s), \quad n = 0. \end{cases}$$

The scattered field should satisfy the condition on the edge, i.e., the condition of limited energy in any finite volume of space. It can be shown that, to satisfy this condition, the Fourier coefficients of the surface current density function must be found in Gilbert space $\tilde{\ell}_2$, i.e., the inequality

$$\sum_{n=-\infty}^{\infty} |\mu_n|^2 |n| < \infty.$$

should be fulfilled.

By using estimates for Bessel functions of higher order, it can be shown that

$$\sum_{n,m=-\infty}^{\infty} |C_{nm}^{s}|^2 |n| < \infty \quad , \quad \sum_{n,m=-\infty}^{\infty} |M_{n,m}^{qs}|^2 |n| < \infty .$$

System of Eq. (21) can be solved by the reduction method with any preassigned accuracy. /24

The matrix of the resulting system of equations is of a distinct cellular nature. In some particular case of mutual location of the cylinders, the matrix of the system of equations can be cellularly Teplitsian, which permits the use of the efficient numerical algorithms developed for the calculation of such systems [18].

We discuss the total scattering cross section of a system of unclosed circular cylinders.

We write the scattered field in the form

$$H_z^{\text{scat}} = \sum_{m=-\infty}^{\infty} \mu_m^s \cdot J_m'(\kappa\rho_s) \cdot H_m^{(1)}(\kappa R_s) \cdot e^{im\varphi_s} +$$

$$+ \sum_{p=-\infty}^{\infty} H_p^{(1)}(\kappa R_s) \left\{ \sum_{\substack{q \neq s \\ q=1}}^{N} \sum_{m=-\infty}^{\infty} \mu_m^q \cdot J_m'(\kappa\rho_q) \cdot J_{m-p}(\kappa r_{qs}) e^{i(m-p)\psi_{qs}} \right\} e^{ip\varphi_s} =$$

$$= \sum_{m=-\infty}^{\infty} \tilde{\mu}_m^s H_m^{(1)}(\kappa R_s) \cdot e^{im\varphi_s} \qquad \text{with } R_s > r_{qs} > \rho_s ,$$

where

$$\tilde{\mu}_m^s = \mu_m^s \cdot J_m'(\kappa\rho_s) + \sum_{q=1}^{N}{}' \sum_{p=-\infty}^{\infty} \mu_p^q \cdot J_p'(\kappa\rho_q) \cdot J_{p-m}(\kappa r_{qs}) e^{i(p-m)\psi_{qs}} .$$

By determining the flow of the Umov-Poynting vector through a cylindrical surface of infinitely large radius with the center on the axis of one cylinder, we obtain

$$\sigma_s^H = Re \int_0^{2\pi} E_\varphi \cdot H_z^* R_s \cdot d\varphi_s = \frac{4}{\kappa} \sum_{m=-\infty}^{\infty} |\tilde{\mu}_m^s|^2 . \qquad (22)$$

The series found by calculation of $\tilde{\mu}_m^S$ proves to be slowly converging however. Thus, Eq. (22) proves to be unsuitable for the calculations.

By using optical theorem, a convenient equation for calculation of the total scattering cross section was successfully obtained:

$$\sigma_s^N = -\frac{4}{K}\sum_{m=-\infty}^{\infty} Re\left(e^{-im\varphi_i}\tilde{\mu}_m^s\right) =$$

$$= k\epsilon\sum_{q=1}^{N}\sum_{m=-\infty}^{\infty} \mu_m^q J_m'(\kappa\rho_q) \cdot e^{im\varphi_q} e^{ik\rho_q \sin(\varphi_s + \varphi_i)}.$$

Study of the total scattering cross section permits analysis of the scattering power of the structure.

III. Discussion of Calculation Results

Computer calculation of various characteristics of a structure which consists of a finite number of unclosed cylinders was performed by both the self consistent and iteration methods. The advantages of the iteration method was shown in the preceding sections. They are primarily the algorithmic and graphic nature of the method and then the economical use of the working memory of the machine, which makes it possible to calculate multielement structures on medium capacity machines without the use of external memory, and finally the advantage in calculation time.

The question arises however of the convergence rate of the iteration process. It is physically obvious that the convergence is the higher the greater the distance between scatterers.

Numerical computer utilization of the iteration method makes it possible for us to estimate the convergence rate of the successive scattering process.

We limited ourselves to study of structures which consist of two scatterers in the calculations, since the purpose was to test the successive scattering procedure for subsequent application to study of multielement structures.

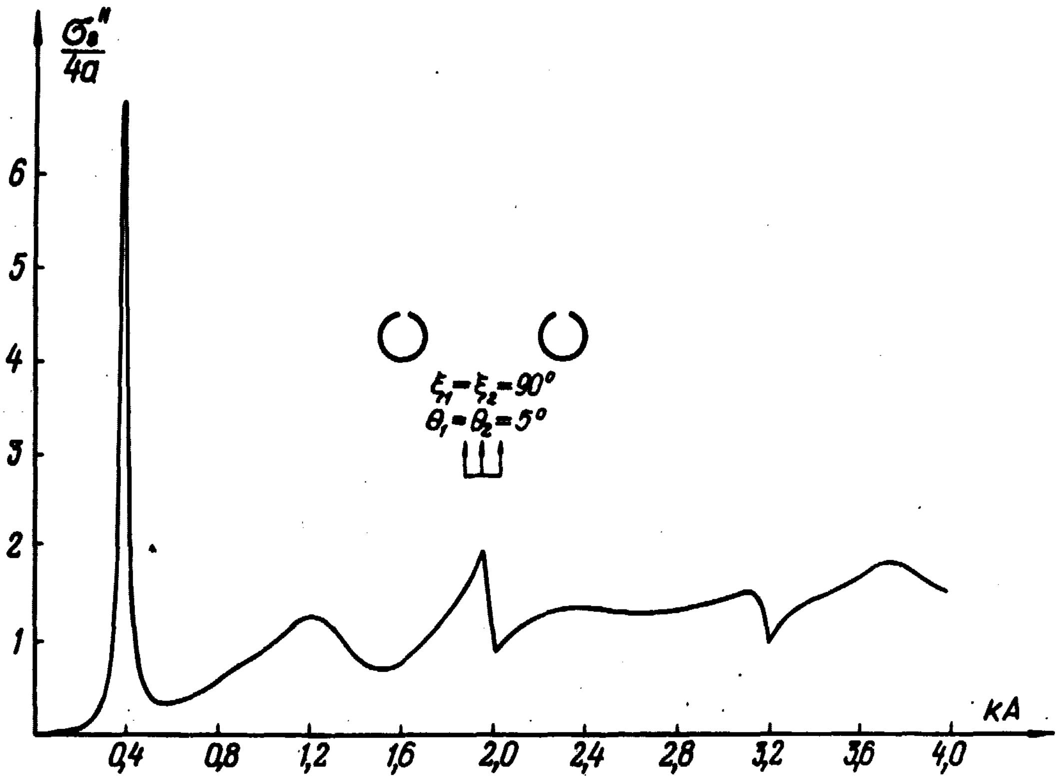

Fig. 2. Total scattering cross section vs. ka: kd=7.0 ka, $\theta_1=\theta_2=5°$.

The calculation results were compared with the results of rigorous calculation of a similar structure by the self consistent method. It turned out that the total scattering cross section of a structure with kd_1=4.0 ka and kd_2=7.0 ka (where d is the distance between scatterers, a is the cylinder radius, k=2 π/λ) with eight successive scatterings taken into account can be calculated to within 0.5% (see Fig. 1-3 and Table 2). As should have been expected, the accuracy of calculations with kd_1 is higher than when kd_2, and 5-6 iterations are sufficient for calculation of such a structure. The accuracy of calculations in the resonance regions is reduced because of amplification of the effect of the elements on each other. The calculation error is less than 0.5% however. /26

The real and imaginary parts of the Fourier coefficients of the surface current density in response to the electromagnetic field next to the scatterers are presented in Tables 3-8. The Tables 3-8 data are accurate to within 10^{-6}. It is easily seen here how, by incorporating the additions which respond in turn to all stages of scattering, i.e., repeated successively scattered waves, we arrive at a more complete accounting of the interaction between the scatterers.

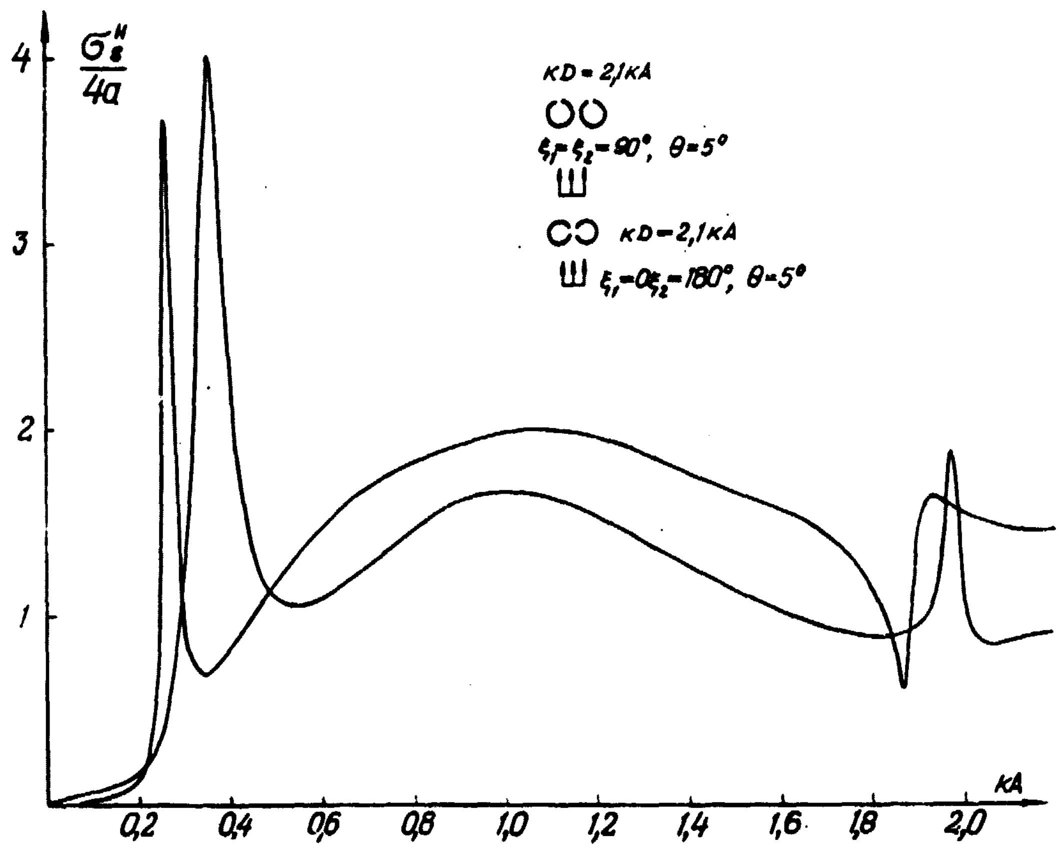

Fig. 3. Total scattering cross section vs. ka: 1. kd=2.1 ka, $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=5°$; 2. kd=2.1 ka, $\xi_1=0°$, $\xi_2=180°$, $\theta_1=\theta_2=5°$.

Thus, to obtain Fourier coefficients with less than 1% error and ka=0.4 and kd_1, it is sufficient to allow for six successive scatterings and, for ka=0.4 and kd_2, only four are sufficient. It cannot be stated here that the elements of the structure in question are at a substantial distance from each other. The distance between cylinders is one diameter with kd_1.

TABLE 2. TOTAL SCATTERING CROSS SECTIONS OBTAINED BY ITERATION AND SELF CONSISTENT METHODS FOR STRUCTURES CONSISTING OF TWO CYLINDERS: $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=1°$

ka	kd = 4,0 ka		kd = 7,0 ka		
ka	$1/4a\ \sigma_s''$ Самосоглас. (a)	$1/4a\ \sigma_s''$ Итерационн. (b)	ka	$1/4a\ \sigma_s''$ Самосоглас. (a)	$1/4a\ \sigma_s''$ Итерационн. (b)
0.2	0.0822719	0.0822625	0.2	[illegible]	0.065309
0.3	3.439394	3.435422	0.3	2.742066	2.743112
0.35	I.657383	I.657577	0.32	6.078781	6.081347
0.45	0.661186	0.681187	0.33	4.120604	4.121048
0.6	0.608246	0.608413	0.35	I.554921	I.56185
0.8	0.531977	0.532443	0.4	0.458834	0.458818
I.0	0.551453	0.561441	0.45	0.305071	0.306863
I.I	0.585818	0.586090	0.5	0.282176	0.282149
I.2	0.643281	0.642972	0.7	0.453291	0.453278
I.4	0.850676	0.850773	0.8	0.596771	0.596738
I.5	0.995453	0.995359	I.0	0.963409	0.963266
I.6	I.14778	I.147332	I.I	I.186883	I.186810
I.7	I.29306	I.292651	I.2	I.321486	I.321451
I.8	I.441822	I.441869	I.28	I.222493	I.222393

Key: a. Self consistent
b. Iteration

We also investigated the case when the cylinders are located very close to each other (but do not overlap). To calculate the total scattering cross section with less than 5% error even in the case kd=2.1 ka, only eight iterations are sufficient (see Table 9). To obtain results with very high accuracy (in calculation of both the near and distant fields), it is sufficient to account for 18-19 successive scatterings. In this case, the correction to the Fourier coefficients in the 19th iteration is less than 10^{-3} (see Table 10), which is less than 0.1%. /37

TABLE 3. REAL PARTS OF FOURIER COEFFICIENTS OF SURFACE CURRENT DENSITY FUNCTIONS OF DIFFERENT SCATTERING ORDERS WITH $ka=0.4$, $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=1°$, $kd=4.0\ ka$; PRECISE VALUES OF COEFFICIENTS:

$$\mathrm{Re}\,'\mu_0 = -0.0866650846,$$
$$\mathrm{Re}\,'\mu_1 = 0.138034414,$$
$$\mathrm{Re}\,'\mu_{-1} = 0.243494507$$

a № итерации (i)	$\mathrm{Re}\,'\mu_0^i$	$\mathrm{Re}\sum_i{}'\mu_0^i$	$\mathrm{Re}\,'\mu_1^i$	$\mathrm{Re}\sum_i{}'\mu_1^i$	$\mathrm{Re}\,'\mu_{-1}^i$	$\mathrm{Re}\sum_i{}'\mu_{-1}^i$
1	-0.3251	-0.3251	-0.1578	-0.1578	0.1578	0.1578
2	0.1589	-0.1661	0.0134	-0.1444	0.0753	0.2331
3	0.0638	-0.1024	0.0056	-0.1486	0.0116	0.2447
4	0.0151	-0.0873	0.0010	-0.1379	0.0001	0.2449
5	0.0017	-0.0856	-0.0000	-0.1379	-0.0009	0.2440
6	-0.0005	-0.0861	-0.0001	-0.1380	-0.0004	0.2436
7	-0.0004	-0.0865	-0.0000	-0.1380	-0.0001	0.2436
8	-0.0001	-0.0866	-0.0001	-0.1380	-0.0000	0.2435

Key: a. Iteration number (i)

Conclusion

By using the method of solution of the problem of multiple scattering by a finite number of scatterers developed for systems which consist of smooth nonresonant components, the problem of multiple scattering of a H polarized wave by N unclosed screens in the form of cylinders with longitudinal slits with distinct resonance properties was solved.

TABLE 4. IMAGINARY PORTIONS OF FOURIER COEFFICIENTS OF SURFACE CURRENT DENSITY FUNCTIONS OF DIFFERENT SCATTERING ORDERS WITH $ka=0.4$, $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=1°$, $kd=4.0\ ka$; PRECISE VALUES OF COEFFICIENTS:

$$Im\,{}^1\mu_0 = 1.41681392,$$
$$Im\,{}^1\mu_1 = 0.151179847,$$
$$Im\,{}^1\mu_{-1} = -0.184379659$$

a (i)	$Im\,{}^1\mu_0^i$	$Im\sum_i {}^1\mu_0^i$	$Im\,{}^1\mu_1^i$	$Im\sum_i {}^1\mu_1^i$	$Im\,{}^1\mu_{-1}^i$	$Im\sum_i {}^1\mu_{-1}^i$
1	1.3203	1.3203	0.1499	0.1499	-0.1499	-0.1499
2	0.1129	1.4532	0.0068	0.1567	-0.0164	-0.1663
3	0.0006	1.4540	-0.0023	0.1533	-0.0128	-0.1781
4	-0.0104	1.4236	-0.0015	0.1518	-0.0060	-0.1831
5	-0.0050	1.4186	-0.0006	0.1512	-0.0012	-0.1843
6	-0.0015	1.4171	-0.0001	0.1511	-0.0001	-0.1845
7	-0.0003	1.4168	-0.0000	0.1511	0.0000	-0.1844
8	0.0000	1.4168	0.0000	0.1511	0.0000	0.1844

Key: a. Iteration number (i)

The problem was solved by two methods: self consistent, which takes full account of the interaction between cylinders, and the iteration, in which the degree of accounting for the interaction is directly proportional to the number of iterations.

It was shown on the basis of numerical calculations that the iteration method of solution gives correct results, even in the case of cylinders located very close to each other (more than 20 iterations must be taken into consideration here).

TABLE 5. REAL PARTS OF FOURIER COEFFICIENTS OF SURFACE CURRENT DENSITY FUNCTIONS OF DIFFERENT SCATTERING ORDERS WITH $ka=0.4$, $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=1°$, $kd=7.0\ ka$; PRECISE VALUES OF COEFFICIENTS:

$$Re\,{}^1\mu_0 = -0.419716396,$$
$$Re\,{}^1\mu_1 = -0.158262608,$$
$$Re\,{}^1\mu_{-1} = 0.196181449$$

a № итерации (i)	$Re\,{}^1\mu_0^i$	$Re\sum_{(i)}{}^1\mu_0^i$	$Re\,{}^1\mu_1^i$	$Re\sum_i{}^1\mu_1^i$	$Re\,{}^1\mu_{-1}^i$	$Re\sum_i{}^1\mu_{-1}^i$
1	-0.3251	-0.3251	-0.1578	-0.1578	0.1578	0.1578
2	-0.0537	-0.3788	0.0015	-0.1564	0.0366	0.1944
3	-0.0294	-0.4082	-0.0010	-0.1574	0.0029	0.1973
4	-0.0089	-0.4171	-0.0006	-0.1580	-0.0005	0.1967
5	-0.0021	-0.4192	-0.0002	-0.15[illegible]	-0.0004	0.1964
6	-0.0004	-0.4196	-0.0000	-0.1582	-0.0001	0.1962
7	-0.0001	-0.4197	-0.0000	-0.1582	-0.0000	0.1962
8	-0.0000	-0.4197	-0.0000	-0.1582	-0.0000	0.1962

Key: a. Iteration number (i)

The basis for solution of the problem of multiple scattering by a finite number of smooth scatterers is solution of the problem of wave multiple scattering by a single element.

Efficient use of the iteration method became possible due to the solution of the problem of a single unclosed screen obtained

TABLE 6. IMAGINARY PORTIONS OF FOURIER CO-
EFFICIENTS OF SURFACE CURRENT DENSITY FUNCTIONS
OF DIFFERENT SCATTERING ORDERS WITH
$ka=0.4$, $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=1°$, $kd=7.0\ ka$;
PRECISE VALUES OF COEFFICIENTS:

$Im\,{}^1\mu_0$ = I.50478019,
$Im\,{}^1\mu_1$ = 0.I67739616,
$Im\,{}^1\mu_{-1}$ = 0.0909355889

a № итерации (i)	$Im\,{}^1\mu_0^i$	$Im\sum_i {}^1\mu_0^i$	$Im\,{}^1\mu_1^i$	$Im\sum_i {}^1\mu_1^i$	$Im\,{}^1\mu_{-1}^i$	$Im\sum_i {}^1\mu_{-1}^i$
I	I.32033	I.0320	0.I499	0.I499	-0.I499	-0.I499
2	0.I55I	I.4754	0.0I359	0.I635	0.0444	-0.I055
3	0.0275I	I.5030	0.0035	0.I670	0.0II3	-0.0942
4	0.0027	I.5056	0.0006	0.I677	0.0027	-0.09I4
5	-0.0004	I.5052I	0.000I	0.I677	0.0005	-0.0909
6	-0.0003	I.5049	0.0000	0.I677	0.000I	-0.0909
7	-0.0000	I.5049	-0.0000	0.I677	0.0000	-0.09088
8	-0.0000	I.5049	-0.0000	0.I677	0.0000	-0.09088

Key: a. Iteration number (i)

in analytical form (in the case of narrow slits, $\theta<10°$), i.e., just as for a smooth scatterer (closed screen).

By using the procedure of inversion of the static portion of the matrix of the system of equations in solution of the problem of multiple scattering by a single cylinder with a longitudinal slit [21], the problem of multiple scattering by N unclosed screens by /39
the self consistent method was reduced to solution of a system of Fredholm equations of the second kind, more precisely, to a set of

TABLE 7. REAL PORTIONS OF FOURIER COEFFICIENTS OF SURFACE CURRENT DENSITY FUNCTIONS OF DIFFERENT SCATTERING ORDERS WITH $ka=1.57$, $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=1°$, $kd=4.0\ ka$; PRECISE VALUES OF COEFFICIENTS:

$Re\,\mu_0 = 1.01885437$,
$Re\,\mu_1 = -0.399932611$,
$Re\,\mu_{-1} = -0.11026788$

a № итерации (i)	$Re\,\mu_0^i$	$\sum_i Re\,\mu_0^i$	$Re\,\mu_1^i$	$Re\sum_i \mu_1^i$	$Re\,\mu_{-1}^i$	$Re\sum_i \mu_{-1}^i$
1	1.1883	1.1883	0.0443	0.0443	-0.0443	-0.0443
2	-0.2602	0.9281	-0.2333	-0.1892	-0.4983	-0.5425
3	0.1325	1.0606	0.1112	-0.0780	0.1994	-0.3432
4	-0.0585	1.0021	-0.0439	-0.1219	-0.0773	-0.4204
5	0.0227	1.0248	0.0154	-0.1066	0.0269	-0.3934
6	-0.0079	1.0169	-0.0047	-0.1113	-0.0083	-0.4018
7	0.0024	1.0193	0.0012	-0.1101	0.0021	-0.3097
8	-0.0006	1.0187	-0.0002	-0.1103	-0.0006	-0.4003

Key: a. Iteration number (i)

N simultaneous systems of linear algebraic equations of the second kind.

The advantages of the iteration method of solution over the self consistent method were shown in the work. In our opinion, the most significant advantage of the iteration method is the physical foundations of its construction. It is evident that the degree of interaction between the structural elements depends on many parameters, primarily on the relationships between the wavelength and dimensions

TABLE 8. IMAGINARY PARTS OF FOURIER COEFFICIENTS OF SURFACE CURRENT DENSITY FUNCTIONS OF DIFFERENT SCATTERING ORDERS WITH $ka=1.57$, $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=1°$, $kd=4.0\ ka$; PRECISE VALUES OF COEFFICIENTS:

$Im\,{}^1\mu_0$ = 0.47I6833I8,
$Im\,{}^1\mu_1$ = I.304718845,
$Im\,{}^1\mu_{-1}$ = -1.76I85287 .

a № итерации (i)	$Im\,{}^1\mu_0^i$	$Im\sum_i {}^1\mu_0^i$	$Im\,{}^1\mu_1^i$	$Im\sum_i {}^1\mu_1^i$	$Im\,{}^1\mu_{-1}^i$	$Im\sum_i {}^1\mu_{-1}^i$
I	0.7863	0.7863	I.4847	I.4847	-I.4847	-I.4847
2	-0.4005	0.3858	-0.2I80	I.2667	-0.3439	-I.8286
3	0.I043	0.490I	0.04I3	I.3080	0.0723	-I.7563
4	-0.0202	0.4699	-0.0008	I.3072	-0.0012	-I.7575
5	0.0000	0.4699	-0.0050	I.3022	-0.0088	-I.7663
6	0.0027	0.4726	0.0036	I.3058	0.0063	-I.7600
7	-0.00I9	0.4706	-0.00I8	I.3040	-0.0032	-I.7632
8	0.0009	0.47I7	0.0008	I.3047	0.00I3	-I.76I8

Key: a. Iteration number (i)

of the screens and the distance between the structural elements. For calculation of a multicomponent structure, it therefore is very useful to indicate zones of strong and weak interaction. The iteration method of solution makes it possible to do this. The iteration method of solution of the multiple scattering problem has still other advantages here. Because of the strict sequence of operations, it is easy to perform it by computer, and it does not require large memory volume. All this permits the solution of such complex problems even on a medium capacity computer.

TABLE 9. TOTAL SCATTERING CROSS SECTION OBTAINED BY ITERATION AND SELF CONSISTENT METHODS FOR STRUCTURES CONSISTING OF TWO CYLINDERS: $\xi_1=\xi_2=90°$, $\theta_1=\theta_2=5°$, kd=2.1 ka

ka	a $\frac{1}{4a}\sigma_s''$ Самосогласованный метод	b $\frac{1}{4a}\sigma_s''$ Итерационный метод
0.2	0.13359	0.13102
0.22	0.20697	0.20292
0.25	0.40982	0.40130
0.3	1.5106	1.4963
0.31	1.9937	1.9904
0.33	3.2112	3.2243
0.36	3.7107	3.6646
0.39	2.4939	2.6185
0.4	2.1628	2.1325
0.45	1.3245	1.3217
0.48	1.1568	1.1549
0.5	1.1040	1.1018
0.55	1.0765	1.0726
0.6	1.1221	1.1177

Key: a. Self consistent method
b. Iteration method

In conclusion, the authors thank V.P. Shestopalov for continual attention to the work and comprehensive discussion of the results.

TABLE 10. ZERO FOURIER COEFFICIENT OF SURFACE CURRENT DENSITY FUNCTION OF DIFFERENT SCATTERING ORDERS FOR STRUCTURE OF CLOSELY SPACED CYLINDERS, ka=4.0, $\theta_1=\theta_2=5°$, $\xi_1=\xi_2=90°$, kd=2.1 ka

Номер итерации a	$Re\,{}^1\mu_0^i$	$Re\sum_i {}^1\mu_0^i$	$Im\,{}^1\mu_0^i$	$Im\sum_i {}^1\mu_0^i$
I	0.44347	0.44347	374469	3.4469
2	0.30470	0.748I7	-2.3I35	I.I334
3	-I.0369	-0.2888	0.84754	I.98094
4	0.75795	0.469I43	-0.06775	I.9I3I8
5	-0.38434	0.084795	-0.25552	I.65766
6	0.0847I	0.I69509	0.24658	I.90424
9	0.05248	0.I9442	0.002346	I.80868
II	0.00382	0.I7473	0.0I748	I.806I2
12	0.00444	0.I79I7	- 0.009536	I.79869
I4	0.00352	0.I7727	0.00060	I.8000I
I5	-0.00I4I	0.I7585	-0.00I58	I.79847
I6	0.000II	0.I7597	0.00I2I	I.79969
I8	-0.00038	0.I7595	0.000I4	I.79923
I9	0.00023	0.I76I9	0.00007	I.79930
20	-0.00008	0.I76I08	-0.000II	I.799I8

Key: a. Iteration number

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