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SEMI-ANNUAL STATUS REPORT

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1. Introduction

It is highly desirable to construct a reliable, shock capturing finite difference method to solve the Euler equations for inviscid, supersonic flow past fighter and missile type of configurations. The numerical method must have a firm theoretical foundation and must be robust and efficient. It should be able to treat subsonic pockets in a predominantly supersonic flow. The method must also be easily applicable to the complex topologies of the aerodynamic configuration under consideration. In the rest of this report, we briefly describe our ongoing approach to this task, We first present in Section 2, a background sketch on a scheme developed by Osher and his co-workers. This scheme and analogous extensions to steady supersonic flows is the basic numerical method. Some results of the first year's effort are presented in Section 3. Results obtained during the second year's effort are presented in Section 4. Results obtained during the third year's effort are presented in Section 5. Results obtained during this year's effort are presented in Section 6. A list of references is complied in Section 7. A list of papers written during this period is attached.

2. The Osher Scheme: A background

Engquist and Osher [1,2,3] developed circa 1980 a monotone, upwind scheme for scalar conservation laws and applied it to the small disturbance equation of transonic flow. This scheme is now incorporated in production computer programs at both NASA's Ames Research Center and Langley Research Center and has been found to be much more robust than the conventional Murman-Cole algorithm. The Engquist-Osher algorithm was also more reliable in the expansion shocks were excluded and steady flows were computed more quickly. Osher then developed a generalization of the scalar algorithm for hyperbolic systems of conservation laws [4] which was further studied by Osher and Solomon [5].

The Osher scheme for systems is an upwind shock "capturing" algorithm applied to the unsteady Euler equations. It resembles several others, such as Godunov's [6,7], Roe's [8] and Steger and Warmings's [9]. It can be applied to essentially all hyperbolic systems of conservation laws arising in physics, but becomes relatively simple for Euler's equations in general geometries using body fitted coordinates. The latter extension resulted from a collaboration between Osher and Chakravarthy [10,11]. The Osher scheme is based on a Riemann solver, as in Godunov's but compression waves are used to approximate shocks. This leads to a smoother and simpler algorithm. The numerical flux functions are written in closed form and include various switches which make them upwind. Osher's algorithm reaches steady shock solutions exactly (for constant states on either side of the shock) on the grid with a one or two point monotone transition (the two point transition property carries over for problems with nonzero gradients on either side of the shock).

The Osher algorithm for the unsteady Euler equation is currently first order accurate and explicit in time. An extension to second order accuracy and implicit temporal differencing is possible. While the Osher methodology has so far been applied extensively to the unsteady Euler equations, the scheme was actually developed for general systems of conservation laws. Thus an algorithm may also be developed for steady supersonic flows where the marching direction would be a spatial coordinate. These extensions are now incorporated into the proposed task outline.

3. Results of the First Year's Effort

So far we have extended the basic algorithm to be second order accurate. This involves using and modifying flux limiting techniques first developed by Van Leer [12], cf. also Harten [13]. The technique fits very well into our basic algorithm because of its inherent use of a nonlinear field by field decomposition.

The resulting algorithm is now at least second order accurate and variation diminishing, hence no overshoot is possible. It still possesses steady, discrete shock profiles having a one or two point transition. Moreover, the entropy condition, which ensures that only physically correct limit solutions occur, is still valid. The full scheme involves, at most, a five point discretization.

Explicit calculations were implemented through Richtmyer's version of the Lax-Wendroff time discretization. The boundary treatment involves using the first order scheme one point from the boundary, and using a natural boundary condition Riemann problem solver at the physical and far field boundaries.

4. Some Results Obtained During the Second Year's Effort

During this period, the details of the first order Osher algorithm were worked out for hyperbolic space marching for Euler's equations in body-fitted coordinates. Further work was done to make it second order accurate and "variation diminishing." Numerical comparisons with Roe's algorithm were performed for this problem.

Implicit time marching algorithms were devised for the first and second order unsteady algorithms. Using the upwind nature of the space differences, it was found that extremely fast convergence to steady state was possible. This was done using techniques borrowed from elliptic equations to invert the implicit scheme for very large time steps. No approximate factorization was used.

Algorithms for the unsteady problem based on triangulation of the space region were devised. The control volume approximations use the basic first order Osher algorithm together with multidimensional, coordinate free, flux limiters. A very flexible and powerful algorithm is being developed. Implicit calculations, using the algorithm withour approximate factorization, can lead to rapid convergence to steady state in very general geometries.

Finally, schemes based on Riemann solvers are known to have an "entropy glitch," i.e. an $O(\Delta x)$ expansion shock at sonic expansions. For the Osher scheme, we have removed this by adding a simple multiplier to our usual viscosity in the appropriate field. Besides cosmetics, this also helps the robustness of implicit calculations.

5. Some Results Obtained During the Third Year's Effort

During this period we devised MUSCL type high resolution algorithms [17]. These are similar (5 point, TVD, entropy condition satisfying) to those we devised earlier [18], but are somewhat more flexible, and simpler to implement. The original MUSCL idea is due to van Leer [15].

We also constructed high resolution supersonic space marching algorithms for Euler's equations.

We began to extend our techniques to laminar compressible Navier-Stokes flow, avoiding spurious boundary layrers using results of [16], [19].

The efficient implementation of these algorithms is proceeding quite smoothly.

6. Some Results Obtained During the Present Period

During this period we made our high resolution algorithms even more accurate. In [20], we gave a systematic procedure for constructing semi-discrete approximations to scalar conservation laws. Except for isolated critical points, these schemes will have $2m - 1$ order accuracy, $2m$ order dissipation, and a bandwidth using $2m + 1$ points, for m any integer between two and eight. They are in conservation form and TVD.

In [21] and [22] we included the simple 5 point, thirde order accurate, TVD method into our Euler and Navier-Stokes codes, and much improvement was found. Shocks were very crisply resolved, and phenomena such as boundary larger separation were computed accurately.

There are improvements in time discretization to be obtained. It would also be good to remove the degeneracy to first order accuracy which is a problem in the efficient use of the higher than third order methods. Work is proceeding in this direction. The goal of a very efficient compressible Navier-Stokes code is within reach.

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8. Papers Written During the Present Period

[1] Sukumar Chakravarthy and Stanley Osher, "Computing with High Resolution Upwind Schemes for Hyperbolic Equations," to appear in Proceedings of AMS - SIAM Summer Seminar, La Jolla, CA (1983)

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