JOINT INSTITUTE FOR ADVANCEMENT OF FLIGHT SCIENCES

A RESEARCH PROGRAM IN ACTIVE CONTROL/AEROELASTICITY

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The program objectives are fully defined in the original proposal entitled "A Research Program in Active Control/Aeroelasticity in the JIAFS at the NASA Langley Research Center" dated August 1, 1981.

Research Conducted during this report period is described below:

Development of Synthesis Methodology for Multi-functional Active Control System.

INTRODUCTION

In the last progress report, formulation of return difference matrix singular values and their gradients with respect to control law parameters, at the plant output was reported. A software for numerical computation was developed. This and previously developed Linear Quadratic Gaussian (LQG) analysis software are presently being used to generate an optimal design scheme. The design objective is to minimize the system root mean square (RMS) response and control activity and also enhance the system stability robustness by increasing the minimum singular value of the return difference matrix at input and/or output.

Since the system response and stability robustness are contradictory in nature, the constrained optimization approach is expected to arrive at an optimal compromise solution.

Optimization Scheme

The system state-space equations and block diagram of the system are shown in Figure 1. The external inputs u_{com}, w and v_{com} may be interpreted as white noise processes in an LQG analysis.

Performance Index (PI)

The performance index to be minimized is defined as the expected value

$$J = E[y^{1T} Q_1 y^1 + u^{1T} Q_2 u^1]_{t \to \infty} \qquad (1)$$

at steady state condition. The output y^1 may be replaced by the design output

y_D if desired. Here y^1 is used to interpret the PI J in terms of singular values to be described later.

Response Constraints (RC)

The response constraints are defined as expected steady state value of the design responses due to stochastic input (mean square responses).

$$g_i \underset{\leq 0}{\triangleq} \frac{E[y_{D_i}]^2_{ss}}{[y_{D_{RMS}}]^2} - 1 \qquad i=1 \cdots N_D \quad (2)$$

Equation (2) may include RMS sensor outputs, control surface deflection and rate, modal deflection and rate, structural bending moment, shearforce, torque, etc. The design response constraints and their gradients with respect to the controller design parameters are computed by solving a set of Lyapunov equations.

Stability Margin Constraints (SMC)

The stability margin constraints at the plant input is defined as

$$g_{n_D+1} \underset{\leq 0}{\triangleq} \sum_{i=1}^{N} [\mathrm{Max}\{0, [\underline{\sigma}_D - \underline{\sigma}(I + KG(s))]\}]^2_{s=jw_i} /N \quad (3)$$

The $\underline{\sigma}_D$ is the desired minimum singluar value of the return difference matrix at plant input (see position (1) in figure 1) and is directly related to the simultaneous gain and phase margins defined in Publication 1. See Publication 2 for detailed explanation of equation (3).

The stability margin constraint at plant output is defined similarly as

$$g_{N_D+2} \underset{\leq 0}{\triangleq} \sum_{i=1}^{N} [\mathrm{Max}\{0, [\underline{\sigma}_D - \underline{\sigma}(I + GK(s))]\}]^2_{s=jw_i} /N \quad (4)$$

The constraints (3) and (4) and their gradients with respect to the controller design parameters are obtained over appropriate frequency range.

The optimization starts at a stable design point where any or all of the constraints may be violated. The designer can choose any set of constraints from equations (2), (3) and (4), to be satisfied. The program computes the gradients of PI and the active and violated constraints. The usable-feasible direction method of CONMIN optimization program is employed to arrive at a feasible design point.

The optimization scheme is presently being evaluated for a drone lateral attitude control system and will later be applied to a large space structure.

Relation between Performance Index and Singular Values

To aid choice of performance index weighting matrices Q_1 and Q_2 and the noise intensity matrices R_u, R_w, and R_v of the white noise processes u_{com} (fictitious), w and v, the performance index J in equation (1) is expressed in terms of singular values of various transfer function matrices and is shown in Figure 2.

To understand the significance of each of the six terms, the general input-output transfer-function matrix relations are shown in Figure 3.

From equations (5) and (7) it is easy to see that if $Q_1=R_v=R_w=[0]$, $Q_2=R_u=I$ then minimizing J would improve stability margin at plant input. On the other hand, if $Q_2=R_u=R_w=[0]$, $Q_1=R_v=I$ then minimizing J would improve stability margin at plant output. Each term in equation (5) can be interpreted by examining the input-output transfer function relations in equation (6).

CONCISE STATEMENT OF RESEARCH ACCOMPLISHED

A control law synthesis methodology for multifunctional active control system to satisfy RMS load and response constraints as well as to meet stability robustness requirements at plant input and output has been developed. Modern control theory, singular value analysis and optimization techniques have been utilized. All stability and response derivative expressions were derived analytically for sensitivity study. The software is incorporated as an update to the AB/LAD general control design software package PADLOCS (see publication no. 3).

PUBLICATIONS

1. Mukhopadhyay, V. and Newsom, J. R., "A Multiloop System Stability Margin Study Using Matrix Singular Values," Journal of Guidance Control and Dynamics, Vol. 7, No. 5, Sep.-Oct. 1984, pp. 582-587.

2. Newsom, J. R. and Mukhopadhyay, V., "A Multiloop Robust Controller Design Study Using Singular Value Gradients," Journal of Guidance Control and Dynamics (to be published).

3. Newsom, J. R., Adams, W. M., Mukhopadhyay, V., Abel, I. and Tiffani, S. H., "Active Controls: A Look at Analytical Methods," Proceedings of 14th Congress of the International Council of the Aeronatuical Sciences, Paper No. ICAS-84-4.2.3., Vol. 1, pp. 230-242, September 1984.

PLANT

$$\dot{x}_s = F x_s + G_u u + G_w W \qquad x \rightarrow N_S$$
$$y = H x_s + v_{COM} \qquad u \rightarrow N_C$$
$$y_D = H_D x_s \qquad y \rightarrow N_O$$
$$y_D \rightarrow N_D$$

CONTROLLER

$$\dot{x}_c = A x_c + B y \qquad x_c \rightarrow M$$
$$u = C x_c + D y + u_{COM} \qquad D v = [0]$$

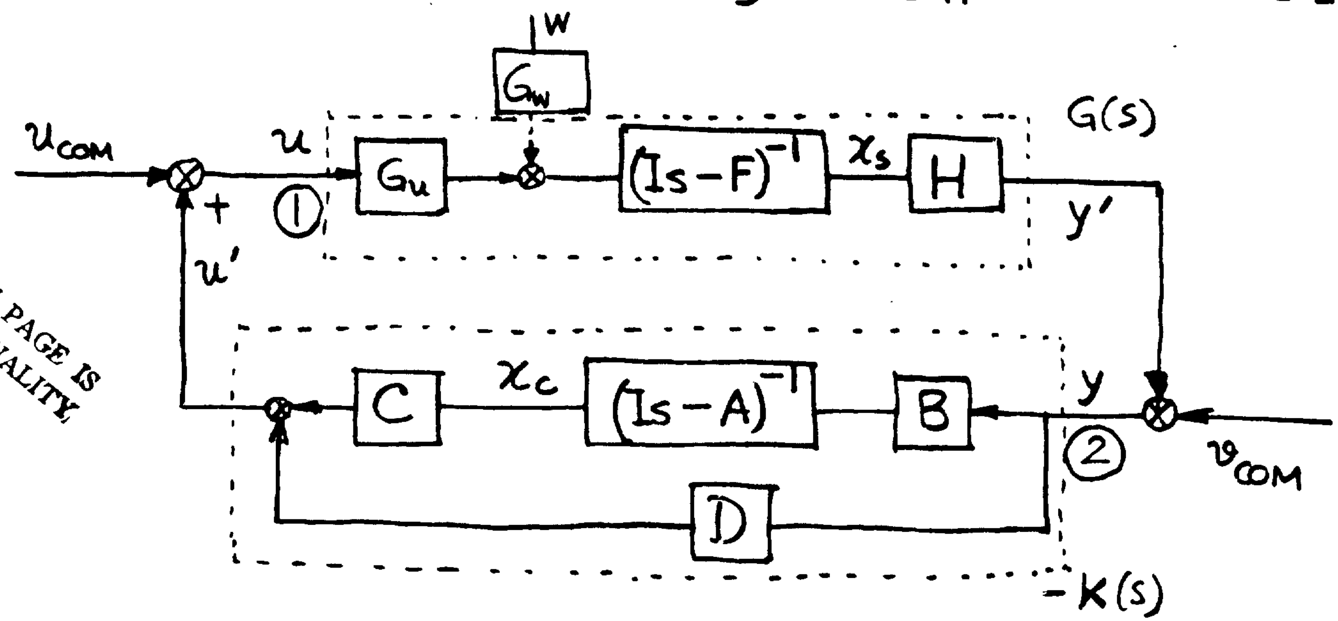

FIGURE -1 BLOCK DIAGRAM

PERFORMANCE INDEX

$$J = E\left[y'^T Q_1 y' + u'^T Q_2 u' \right]_{t\to\infty}$$

$$= tr\left[(H^T Q_1 H) X_s \right] + tr\left[Q_2 U \right]_{t\to\infty}$$

$$= \frac{1}{\pi}\int_0^{\infty} \Big\{ \sum \sigma_i^2 \left[Q_1^{1/2} (I+GK)^{-1} G \quad R_u^{1/2} \right]$$
$$+ \sum \sigma_i^2 \left[Q_1^{1/2} (I+GK)^{-1} G_w(s) R_w^{1/2} \right]$$
$$+ \sum \sigma_i^2 \left[Q_1^{1/2} (I+GK)^{-1} GK \, R_v^{1/2} \right]$$
$$+ \sum \sigma_i^2 \left[Q_2^{1/2} (I+KG)^{-1} KG \quad R_u^{1/2} \right]$$
$$+ \sum \sigma_i^2 \left[Q_2^{1/2} (I+KG)^{-1} KG_w(s) R_w^{1/2} \right]$$
$$+ \sum \sigma_i^2 \left[Q_2^{1/2} (I+KG)^{-1} K \quad R_v^{1/2} \right] \Big\}_{s=j\omega} d\omega \quad \text{-----} \ (5)$$

WHERE

$$G(s) = H(Is-F)^{-1} G_u$$
$$-K(s) = \left[C(Is-A)^{-1} B + D \right]$$
$$G_w(s) = H(Is-F)^{-1} G_w$$

FIGURE-2

GENERAL INPUT-OUTPUT RELATIONS

$$\begin{Bmatrix} y' \\ y \\ u' \\ u \end{Bmatrix} = \begin{bmatrix} (I+GK)^{-1}G & (I+GK)^{-1}G_w(s) & -(I+GK)^{-1}GK \\ (I+GK)^{-1}G & (I+GK)^{-1}G_w(s) & (I+GK)^{-1} \\ -(I+KG)^{-1}KG & -(I+GK)^{-1}KG_w(s) & -(I+KG)^{-1}K \\ (I+KG)^{-1} & -(I+GK)^{-1}KG_w(s) & -(I+KG)^{-1}K \end{bmatrix} \begin{Bmatrix} u_{COM} \\ w \\ v_{COM} \end{Bmatrix} \quad \cdots\cdots (6)$$

NOTE

$$(I+GK)^{-1} + (I+GK)^{-1}GK = I$$
$$(I+KG)^{-1} + (I+KG)^{-1}KG = I \quad \cdots\cdots (7)$$

$$\frac{1}{k}\underline{\sigma}[I+GK] \leq \underline{\sigma}[I+KG] \leq k\,\underline{\sigma}[I+GK]$$

WHERE $k = \min\left[\dfrac{\bar{\sigma}[G]}{\underline{\sigma}[G]}, \dfrac{\bar{\sigma}[K]}{\underline{\sigma}[K]}\right] \geq 1$

FIGURE-3