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and Space Administration

Scientific and Technical
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SUMMARY

STARS (STructural Analysis RoutineS) is primarily an interactive, graphics-oriented, finite-element computer program for analyzing the static, stability, free-vibration, and dynamic responses of damped and undamped structures, including rotating systems. The element library consists of one-dimensional (1-D) line elements; two-dimensional (2-D) triangular and quadrilateral shell elements; and three-dimensional (3-D) tetrahedral and hexahedral solid elements. These elements enable the solution of structural problems that include truss, beam, space frame, plane, plate, shell, and solid structures, or any combination thereof. Associated algebraic equations are solved by exploiting inherent matrix sparsity. Zero, finite, and interdependent deflection boundary conditions can be implemented by the program. The associated dynamic response analysis capability provides for initial deformation and velocity inputs, whereas the transient excitation may be either forces or accelerations. An effective in-core or out-of-core solution strategy is automatically employed by the program, depending on the size of the problem. Data input may be at random within a data set, and the program offers certain automatic data-generation features. Input data are formatted as an optimal combination of free and fixed formats. Interactive graphics capabilities, using an Evans and Sutherland, Megatek, or any other suitable display terminal, enable convenient display of nodal deformations, mode shapes, and element stresses. The program, developed in modular form for easy modification, is written in FORTRAN for the VAX 11 computer, although earlier development was accomplished using a UNIVAC 1100 computer. Continued development of the program is envisaged, but with care exercised to limit its size (the program now consists of fewer than 12,000 programmed instructions). Applications of the program are anticipated in the fields of aerospace, mechanical, and civil engineering, among others.

1. INTRODUCTION

The general-purpose digital computer program, STARS, has been designed as an efficient tool for analyzing practical structures, as well as for supporting relevant research and development activities; it has also proved to be an effective teaching aid. All such activities are mutually enhancing and interrelated (fig. 1). The current version of the program, capable of solving linear elastic structural problems, will be continuously updated to include other forms of analysis.

In an effort to optimize the program layout, the various subroutines have been grouped into three links. Interaction between the user and the program is effected through a display terminal with or without graphics capabilities; however, a graphics terminal is useful in the accurate preparation of data input and in visualizing structural geometry and analysis results. Thus with reference to figure 2, Link 1 relates to the input phase of the program. Once the data have been entered into the system, the user may create an image of the model on the terminal display screen.

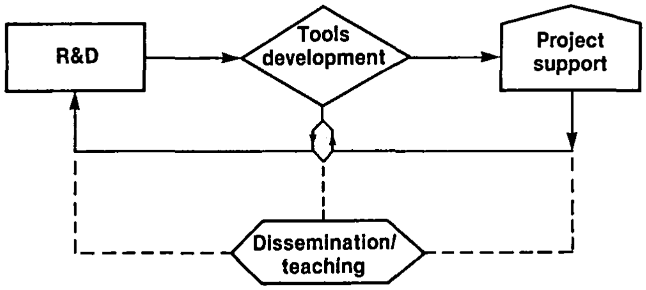


Figure 1. Structural synthesis.

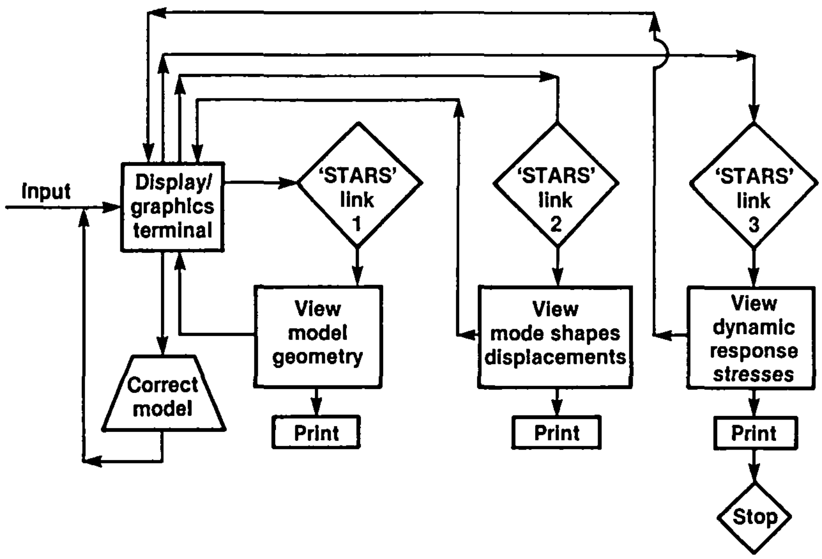


Figure 2. STARS overview.

Subsequent correction or modification of the model may be easily implemented on an interactive basis. Once satisfied with the model format, the user may simply proceed to run Link 2 of the program, which involves major numerical manipulation of input data relative to static, stability, and free-vibration analysis of the structural model. Nodal displacements caused by static loading and the structural mode shapes pertaining to the stability and free-vibration problem may then be displayed using the graphics terminal. Link 3 of the program, the response link, enables computation of structural displacements caused by dynamic loading, as well as element stresses resulting from static and dynamic loads input.

The program can solve static and dynamic problems of nonrotating and rotating structures of general configurations with arbitrary displacement boundary conditions. For static problems, a multiple set of input data is permissible; for dynamic response problems, a single set of force or acceleration data is the usual input. The structural material may be isotropic, orthotropic, or anisotropic. Both viscous and structural damping occurring in practice may be included in the dynamic analysis. A bandwidth minimization option is available, its utilization being highly desirable to ensure economical solution of associated problems.

The free-vibration and dynamic response analysis of structural systems rotating along an arbitrary axis is a useful feature of the STARS program. Such a structure may have a combination of nonrotating and rotating parts, and each part may have a different spin rate. Both rigid body and elastic modes may be computed by the program and the dynamic response analysis is formulated accordingly. The VAX 11 version of the program performs computations in single or double precision, using either real or complex arithmetic operations.

Section 2 provides a concise description of the program, as well as highlights of some of its important features, and section 3 depicts the STARS data input procedure. Section 4 provides summaries of input data and analysis results for a number of sample test cases. A description of the program system is given in section 5.

2. PROGRAM DESCRIPTION

The structure to be analyzed by STARS may be composed of any suitable combination of one-, two-, and three-dimensional elements. The general features of STARS include the following:

1. A general-purpose, compact, finite-element program

2. Elements: bars, beams, triangular and quadrilateral shells, tetrahedral and hexahedral solids

3. Geometry: any relevant structure formed by a suitable combination of the elements in (2)

4. Analysis: natural frequencies and mode shapes of usual and rotating structures with or without structural damping, viscous damping, or both, including initial load (pre-stress) effect; stability (buckling) analysis; dynamic response analysis of usual and rotating structures; and static analysis for thermal and multiple sets of mechanical loading

Special features of the STARS program include the following:

1. Random data input

2. Matrix bandwidth minimizer

3. Automatic node and element generation

4. General nodal deflection boundary conditions

5. Multiple sets of static load input

6. Pre- and post-processor

7. Plot of initial geometry

8. Plots of mode shapes, nodal deformations, and element stresses as functions of time, as required

Structural geometry is described in terms of the global coordinate system (GCS) having a right-handed Cartesian set of X-, Y-, and Z-coordinate axes. Each structural node is assumed to have six degrees of freedom (DOF) consisting of three translations, UX, UY, UZ, and three rotations, UXR, UYR, UZR, which are the undetermined quantities in the associated solution process. Details of some important features of the program are summarized below:

2.1 Nodal and Element Data Generation

The STARS program provides simple linear interpolation schemes that enable automatic generation of nodal and element data. Generation of nodal data is dependent on the occurrence of such features as nodes lying on straight lines and common nodal displacement boundary conditions, but such a generation of element data is possible if the finite-element mesh is repetitive in nature with elements possessing common basic elemental properties. A separate pre-processor called PRESTARS has been developed for automated generation of nodal and element input data for any continuum.

2.2 Matrix Bandwidth Minimization

This feature enables effective bandwidth minimization of the stiffness, inertia, and all other relevant system matrices by reordering input nodal numbers, taking into consideration first-order, as well as second-order, nodal connectivity conditions. Thus with reference to figure 3, the existing nodal numbering may be modified (ref. 1) to minimize the bandwidth of associated matrices. Therefore, any node with minimum first-order connectivity may be chosen as the starting node. Accordingly, any one of nodes 1, 4, 7, 10, 13, and 16, all of which have a minimum first-order nodal connectivity of 2, may be selected as the first node to start the nodal numbering scheme. However, nodes 1, 4, 10, and 13 possess a higher second-order connectivity condition than do nodes 7 and 16. For example, nodes connected to node 1, namely, nodes 2 and 18, are in turn connected to a total of seven nodes, whereas such a connectivity number for either node 7 or 16 happens to be only 6. As such, either node 7 or node 16 may be chosen as the starting node for the renumbering scheme. A revised nodal numbering that minimizes matrix bandwidth is shown in parentheses in figure 3. The present minimization scheme also takes into consideration the presence of nodal interdependent displacement boundary conditions.

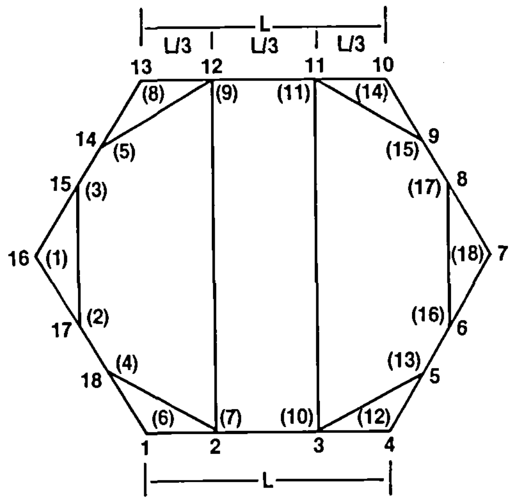


Figure 3. Bandwidth minimization scheme.

2.3 Deflection Boundary Conditions

The nodal displacement relationships may be classified as zero, finite, and interdependent deflection boundary conditions (ZDBC, FDBC, and IDBC). Details of such a formulation are provided in section 3.4. Thus in addition to prescribed zero and finite displacements, the motion of any node in a particular degree of freedom can be related in any desired manner to the motion of the same or any other node in any specified direction.

2.4 Prescribed Loads

A structure may be subjected to any suitable combination of mechanical and thermal loadings. The loads in the mechanical category may be either concentrated at nodes or distributed. Thus uniform pressure may be applied along the length of line elements acting in the direction of the local y- and z-axes. Such uniform surface loads are assumed to act in the direction of the local z-axis of the shell and solid elements, acting respectively on the shell and solid base surfaces.

The effect of thermal loading can be incorporated by the appropriate input of data pertaining to uniform element temperature increases, as well as thermal gradients.

2.5 Static Analysis

Static analysis, performed by setting IPROB = 8 in the input data, is effected by solving the set of linear simultaneous equations:

$$KU = P \tag{1}$$

where

- K = system elastic stiffness matrix
- U = nodal displacement vector
- P = external nodal load vector
- IPROB = integer designating problem type (defined in sec. 3.1)

A multiple set of load vectors is represented by the matrix P incorporating effects of both mechanical and thermal loading. The equations are solved once, initially by Guassian elimination, and solutions pertaining to multiple nodal load cases are obtained by simple back substitution.

2.6 Elastic Buckling Analysis

A buckling analysis is performed by solving the eigenvalue problem,

$$\left(K_E + \gamma K_G\right)U = 0 \tag{2}$$

in which K_E and K_G are elastic stiffness and geometric stiffness matrices, respectively; U is the buckled mode shapes; and γ is the buckling load.

2.7 Free Vibration Analysis

The matrix equation of free vibration for the general case of a spinning structure with viscous and structural damping is expressed (ref. 2) as

$$\left[K_E\left(1 + i^*g\right) + K_G + K'\right]U + \left(C_C + C_D\right)\dot{U} + M\ddot{U} = 0 \tag{3}$$

in which the previously undefined terms are described below and in which a dot indicates differentiation with respect to time

- K' = centrifugal force matrix
- C_C = Coriolis matrix
- C_D = viscous damping matrix
- M = inertia matrix
- g = structural damping parameter
- i^* = imaginary number, $\sqrt{-1}$

Such a structure may have individual nonrotating and also rotating components spinning with different spin rates.

Various reduced sets of equations representing the equation of free vibration pertaining to specific cases are given as follows.

1. Free undamped vibration of nonrotating structures (IPROB = 1):

$$K_E U + M\ddot{U} = 0 \tag{4}$$

2. Free undamped vibration of spinning structures (IPROB = 2):

$$K_E U + C_C\dot{U} + M\ddot{U} = 0 \tag{5}$$

3. Free damped vibration of spinning structures (IPROB = 4,5): defined by equation (3)

4. Free damped vibration of nonspinning structures (IPROB = 6,7):

$$K_E(1 + i^*g)U + C_D\dot{U} + M\ddot{U} = 0 \tag{6}$$

The eigenvalue problem pertaining to the IPROB = 1 and 9 cases is real in nature, but the rest of the above problems involve complex-conjugate roots and vectors. In the special case of a prestressed structure the matrix K_G is automatically included in Equation (6).

In addition, STARS solves the quadratic matrix eigenvalue problem (IPROB = 3) associated with a dynamic element formulation (ref. 3),

$$\left[K_E - \lambda^2 M - \lambda^4\left(M_2 - K_4\right)\right]U = 0 \tag{7}$$

which is in the form of quadratic matrix eigenvalue problem in terms of the eigenvalues $\chi = \lambda^2$ and where both M_2 and K_4 are the higher-order dynamic correction matrices, λ being the natural frequencies. This option is being updated, and a new complete version will be made available shortly.

Pre-stressed structures caused by initial loads may also be analyzed, in which case the relevant eigenvalue problem has the form

$$\left(K_E + K_G - \lambda^2 M\right)U = 0 \tag{8}$$

in which the geometrical stiffness matrix K_G is a function of initial stresses.

2.8 Dynamic Response Analysis

The modal superposition method is employed for the dynamic response analysis, following the computation of structural frequencies and modes. As an example, for a nonrotating, undamped structure, the associated eigenvalue problem of equation (4) is first solved to obtain the first few eigenvectors ϕ and also the eigenvalues. The vectors may consist of a set of rigid body modes ϕ_0 and a number of elastic modes ϕ_e which are next mass-orthonormalized so that the matrix product,

$$\phi^T M \phi = [I] \tag{9}$$

is a unit matrix. A transformation relationship,

$$U = \phi\eta \tag{10}$$

is substituted in the dynamic equation,

$$M\ddot{U} + KU = P(t) \tag{11}$$

and when premultiplied by ϕ^T, yields a set of uncoupled equations,

$$\ddot{\eta}_0 = \phi_0^T P(t) \tag{12}$$

and

$$\ddot{\eta}_e + \Omega^2\eta_e = \phi_e^T P(t) \tag{13}$$

incorporating rigid-body and elastic mode effects, respectively; P(t) is the externally applied, time-dependent forcing function, and Ω^2 is a diagonal matrix, with the terms ω_i^2, ω_i being the natural frequencies. Solutions of equations (12) and (13) can be expressed in terms of Duhamel's integrals, which in turn may be evaluated by standard procedures (ref. 4). In the present analysis, the externally applied time-dependent forcing function must be applied to the structure in appropriate small incremental steps of rectangular pulses. The forcing function may be either load or acceleration vectors; the program also allows application of initial displacement and velocity vectors to the structure. For spinning, as well as damped, structures identified as IPROB = 2, 4, 5, 6, and 7, ϕ^T is replaced by its tranjugate $\bar{\phi}^T$ in the relevant dynamic response formulation.

2.9 Shift Synthesis

The program provides special eigenvalue shifting provisions in the analysis to ensure numerical stability. Such a problem may be encountered in the analysis of aerospace structures, which are designed to be strong and lightweight. For example, the elements of the mass matrix of equation (4) may have numerical values much smaller than those of the stiffness matrix. In such cases, the effect of the mass matrix in the $K - \lambda^2 M$ formulation may be insignificant. Such a problem also occurs in the presence of rigid-body modes characterized by "zero" frequencies. An eigenvalue shift strategy has been developed to accommodate such situations.

Thus the eigenvalue problem pertaining to equation (4) representing the problem defined as IPROB = 1 may be written as

$$\left(K - \lambda^2 M\right) y = 0 \tag{14}$$

in which λ is the natural frequency of free vibration, y being the eigenvector. The stiffness and mass matrices must be suitably perturbed to handle rigid-body modes and also to maintain numerical stability by negating effects of rounding error. Thus equation (14) is rearranged as

$$\left[K + 4\hat{M} - \left(\tilde{\lambda} + 4\right)\hat{M}\right] y = 0 \tag{15}$$

or,

$$\left(\hat{K} - \hat{\lambda}\hat{M}\right) y = 0 \tag{16}$$

in which

$$\hat{K} = K + 4\hat{M} \tag{17}$$

$$\hat{M} = FM \tag{18}$$

$$\tilde{\lambda} = \lambda^2/F \tag{19}$$

$$\hat{\lambda} = \lambda^2/F + 4 \tag{20}$$

$$F = \max\left(|K_{i,i}|/|M_{i,i}|\right)/10^7 \tag{21}$$

where $|K_{i,i}|$ and $|M_{i,i}|$ typically denote the norms of the diagonal elements and the number 10^7 relates to the computational accuracy of the VAX 11 computer. Once the eigenvalue problem defined by equation (16) is solved, the natural frequencies are simply obtained as

$$\lambda = \sqrt{(\hat{\lambda} - 4)}\sqrt{F} \tag{22}$$

A similar procedure is adopted for the analysis of free-vibration problems defined by IPROB = 6 and 7, as well as for the buckling analysis (IPROB = 9).

In the case of spinning structures, a somewhat similar strategy is used in perturbing appropriate matrices to ensure effective computation of rigid-body modes, as well as numerical stability.

2.10 Formulation for Nodal Centrifugal Forces in Finite Elements

STARS can perform dynamic analyses of structures with nonrotating and rotating parts having different spin rates. Thus with reference to figure 4, a typical element defined by vertices I, J, and K is assumed to rotate around an arbitrary axis in a radial direction with spin rate Ω_R, having components Ω_X, Ω_Y, and Ω_Z in the global X-, Y-, and Z-directions, respectively. Assuming a plane element, the finite-element relationship may be expressed as

$$\mathbf{u} = \mathbf{a}\mathbf{U} \tag{23}$$

with

$$\mathbf{a} = RQ^{-1} \tag{24}$$

in which

- $\mathbf{u}$ = displacement vector at a typical point L within the element in local coordinate system (LCS)
- $\mathbf{U}$ = nodal displacement vector in LCS
- $\mathbf{a}$ = shape function
- $\mathbf{R}$ = portion of shape function matrix, function of coordinates x, y
- $\mathbf{Q}$ = portion of shape function matrix, function of element nodal coordinates in LCS

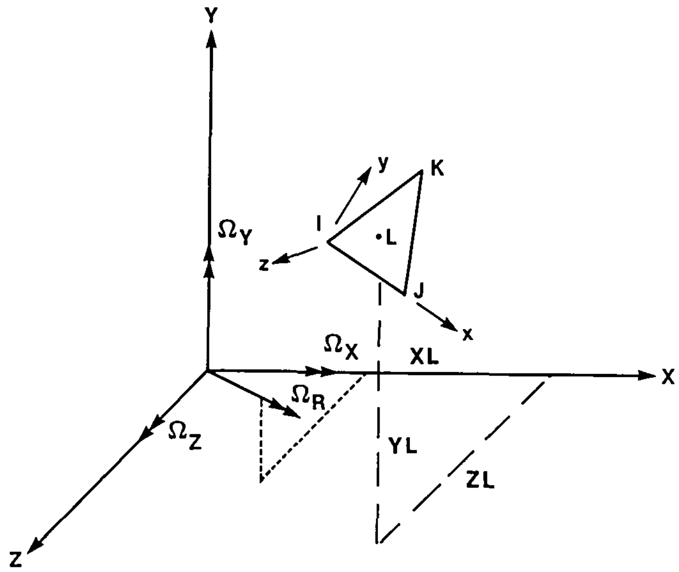


Figure 4. Triangular plane finite element rotating around an arbitrary axis.

Furthermore, by defining the element nodal forces vector in the local coordinate system (x,y,z) as

$$\mathbf{f} = \left(f_x^1,\ f_y^1,\ f_z^1,\ \ldots,\ f_x^3,\ f_y^3,\ f_z^3\right)^T \tag{25}$$

expressions for such nodal centrifugal forces in the planar x- and y-directions owing to spin rates along the global X-, Y-, and Z-directions are derived as follows:

X-axis $\left(\Omega_X\right)$

$$\mathbf{f} = \rho\Omega_X^2 t \left[Q^{-1}\right]^T \int_0^{yk}\int_{lx}^{hx} R^T [\mathbf{XM}]\,dxdy \tag{26}$$

Y-axis $\left(\Omega_Y\right)$

$$\mathbf{f} = \rho\Omega_Y^2 t \left[Q^{-1}\right]^T \int_0^{yk}\int_{lx}^{hx} R^T [\mathbf{YM}]\,dxdy \tag{27}$$

Z-axis $\left(\Omega_Z\right)$

$$\mathbf{f} = \rho\Omega_Z^2 t \left[Q^{-1}\right]^T \int_0^{yk}\int_{lx}^{hx} R^T [\mathbf{ZM}]\,dxdy \tag{28}$$

where ρ is the mass density.

The total element nodal force is obtained as $\mathbf{f} = f(X) + f(Y) + f(Z)$, and in which

$$[\mathbf{XM}] = \begin{bmatrix} M_x\left(M_x x + M_y y + YI\right) + N_x\left(N_x x + N_y y + ZI\right) \\ M_y\left(M_x x + M_y y + YI\right) + N_y\left(N_x x + N_y y + ZI\right) \end{bmatrix} \tag{29}$$

$$[\mathbf{YM}] = \begin{bmatrix} L_x\left(L_x x + L_y y + XI\right) + N_x\left(N_x x + N_y y + ZI\right) \\ L_y\left(L_x x + L_y y + XI\right) + N_y\left(N_x x + N_y y + ZI\right) \end{bmatrix} \tag{30}$$

$$[\mathbf{ZM}] = \begin{bmatrix} L_x\left(L_x x + L_y y + XI\right) + M_x\left(M_x x + M_y y + YI\right) \\ L_y\left(L_x x + L_y y + XI\right) + M_y\left(M_x x + M_y y + YI\right) \end{bmatrix} \tag{31}$$

$[\mathbf{DIR}]$ = direction cosine matrix

$$= \begin{bmatrix} L_x & M_x & N_x \\ L_y & M_y & N_y \\ L_z & M_z & N_z \end{bmatrix} \tag{32}$$

where

XI, YI, ZI = coordinates of node I in the GCS

$lx = (xk)y/yk$, $hx = xj - (xj - xk)y/yk$

t = element thickness

in which xj, xk, yk denote appropriate x and y coordinates of nodes J and K expressed in LCS. These element nodal forces may then be transformed into the GCS as

$$\mathbf{F} = [\mathbf{DIR}]^T\mathbf{f} \tag{33}$$

The above expressions for element nodal centrifugal forces in the LCS owing to arbitrary spin rates as defined by equations (26) to (28) are general in nature for any triangular planar element. Similar expressions are derived for quadrilateral elements by suitably combining such effects for the four constituent triangular elements. For line elements with equivalent nodal lumped masses, the centrifugal forces at a typical node expressed in the GCS have the following form:

$$F_X = m\Omega_Y^2 X + m\Omega_Z^2 X$$

$$F_Y = m\Omega_X^2 Y + m\Omega_Z^2 Y$$

$$F_Z = m\Omega_X^2 Z + m\Omega_Y^2 Z \tag{34}$$

with m being the lumped mass at the node under consideration. In the particular case in which a node is connected to a number of elements with different spin rates, an average spin rate value is assigned to the node.

Once the nodal centrifugal forces have been derived as above and stored in array P, the element stresses in the structure caused by these forces are simply obtained by solving equation (1) (repeated here for convenience),

$$KU = P$$

The stresses are next utilized to derive the structural geometrical stiffness matrix K_G required for solving the free-vibration problems defined in section 1.

2.11 Material Properties

The structural material may be general in nature. Thus the finite-element material properties may be isotropic, orthotropic, or anisotropic. In the most general case of solid elements having anisotropic material properties, defined as material type 3, the stress-strain matrix is expressed as

$$\delta = E\varepsilon \tag{35}$$

with $E_{i,j}$ being elements of the general material matrix of order 6 × 6, defining the relationship between the stress vector δ and the strain vector ε. The elements of the upper symmetric half of the E matrix, as well as coefficients of thermal expansion and material density consisting of 28 coefficients, are the required data input for the pertinent material type. In this connection, it may be noted that the material data input is designed in such a way as to be quite general; the user may easily incorporate effects of various related features, such as varying material axes orientation, by appropriately calculating the elements of the material matrix. If the material is orthotropic, the input scheme remains the same as for the anisotropic case.

Material type 2 pertains to thin, shell elements displaying anisotropic or orthotropic material properties; it requires an input of 13 coefficients. For isotropic material classified as material type 1, only four coefficients constitute the required input data.

2.12 Output of Analysis Results

A dynamic response analysis, in general, yields an output of nodal deformations and element stresses as appropriate functions of time. For line elements, member end-loads and moments constitute the usual output of results. In the case of thin, shell elements, the stresses σ_{xx}, σ_{yy}, and σ_{xy} are calculated at the centroid of the element and at both its top and bottom surfaces. For solid elements, all six components of stresses (σ_{xx}, σ_{yy}, σ_{zz}, σ_{xy}, σ_{yz}, and σ_{zx} are computed at the center of volume of the element. Since free-vibration analysis constitutes a vital preliminary for the dynamic response analysis, the natural frequencies and associated modes are computed by the program and printed out, as desired. Similar results are obtained for elastic buckling analysis. For static problems, the nodal displacements and element stresses are computed for multiple-load cases.

Special printout options make possible a selective output of analysis results. Thus such computed data as stiffness and inertia matrices may be printed out, as desired. Initially, the program automatically prints out the generated nodal coordinates, element data, and other relevant input data.

2.13 Discussions

Additional analysis features such as finite, dynamic element discretizations, improved dynamic analysis capabilities, and various efficient numerical techniques are currently being implemented in the program; the program will be continually updated in the future. A nonlinear analysis capability will also be developed in parallel. Improved pre- and post-processing of data, using an Evans and Sutherland PS 300, Megatek, or other graphics terminals, is being used to permit efficient modeling and analysis, as well as display, of the results pertaining to practical structural problems.

3. DATA INPUT PROCEDURE

3.1 Basic Data

3.1.1 JOB TITLE
Format (13A6)

3.1.2 NN, NEL, NMAT, NMECN, NEP, NET, NTMP, NPR, NBUN
Format (FREE)

1. Description: Basic data parameters.

2. Notes:

NN = total number of nodes

NEL = total number of elements

NMAT = total number of element material types

NMECN = number of material elastic constants, a maximum of numbers, as below

= 4, for isotropic material

= 13, for orthotropic-anisotropic material for 2-D elements (shell, types 2 and 3)

= 28, for orthotropic-anisotropic material for 3-D elements (solid, types 4 and 5)

NEP = total number of line element property types (element type 1)

NET = total number of shell element thickness types (element types 2 and 3)

NTMP = total number of element temperature types

NPR = total number of element pressure types

NBUN = total number of interdependent and finite nodal connectivity conditions (includes IDBC and FDBC in section 2.3)

3.1.3 IPROB, IBAN, NPREC, NC, IDRS, IPLOT, IEIG
Format (FREE)

1. Description: Data defining nature of required solution.

2. Notes:

IPROB = index for problem type, to be set as follows
= 1, undamped free-vibration analysis of nonspinning structures
= 2, undamped, free-vibration analysis of spinning structures
= 3, quadratic matrix eigenproblem option for DEM (dynamic element method) analysis
= 4, free-vibration analysis of spinning structures with diagonal viscous damping matrix
= 5, as for IPROB = 4 with structural damping
= 6, free-vibration analysis of nonspinning structures with general viscous damping
= 7, as for IPROB = 6 with structural damping
= 8, static analysis of structures with thermal and multiple mechanical load cases
= 9, elastic buckling analysis

IBAN = bandwidth minimization option
= 0, performs minimization
= 1, minimization not required

NPREC = specification for solution precision
= 1, real single precision (IPROB = 1, 3, 8, 9)
= 2, real double precision (IPROB = 1, 3, 8, 9)
= 3, complex single precision (IPROB = 2, 4, 5, 6, 7)
= 4, complex double precision (IPROB = 2, 4, 5, 6, 7)

NC = number of sets of nodal loads for IPROB = 8
= 0, for IPROB = 1 through 7
= 1, for IPROB = 9

IDRS = index for dynamic response analysis
= 0, no response analysis required
= 1, performs response analysis

IPLOT = index for graphics display
= 0, no plotting needed
= 1, performs display of input geometry; if satisfactory a restart option enables continuation of current analysis

IEIG = Integer defines eigenproblem solution type
= 0, for solution based on a modified combined Sturm sequence and inverse iteration method
= 1, for an alternative solution technique based on a Lanczos procedure

A dynamic response analysis is achieved by specifying appropriate values for IPROB, IDRS, and IEIG. At end of problem solution, extensive options are available for plotting nodal deformations, mode shapes, and element stresses by utilizing the post-processor program POSTPLOT.

3.1.4 IPLUMP, IMLUMP, NSPIN, IPRINT
Format (FREE)

1. Description: Additional basic data.

2. Notes:

IPLUMP = index for nodal external loads
= 0, no load input
= 1, concentrated nodal load input for IPROB = 8 and 9, as well as for IPROB = 1 through 7 for prestressed structures

IMLUMP = index for nodal lumped mass
= 0, no lumped mass
= 1, lumped nodal mass input (IPROB = 1 through 7)

NSPIN = total number of different element spin types

IPRINT = output print option
= 0, prints final results output only
= 1, prints global stiffness (**K**), mass (**M**), damping or Coriolis (**C**) matrices and values of roots at various stages of convergence
= 2, prints output as in IPRINT = 1, but omits **K**, **M**, and **C** matrices

Mass matrix: Nodal lumped mass matrix is added to consistent mass matrix to evolve the final mass matrix.

Initial load (prestress) effect: To include effect of initial load for the free-vibration problems defined by IPROB = 1 through 7, the intial nodal load is read by setting IPLUMP = 1; also in the presence of lumped mass the user may set IMLUMP = 1.

3.1.5 INDEX, NR, INORM, PU, PL, INDATA (Required if IPROB ≠ 8)
Format (FREE)

1. Description: Data specifications for eigenproblem solution and matrix data input.

2. Notes:

INDEX = indicator for number of eigenvalues and vectors to be computed
= 1, computes NR smallest roots (and vectors) lying within bounds PU, PL
= 2, computes all roots (and vectors) lying within bounds PU, PL

NR = number of roots to be computed (needs no input for INDEX = 2)

INORM = index for vector normalization
= 0, normalizes with respect to a scalar of displacement vector **Y** having largest modulus
= -1, normalizes with respect to a scalar of **Y** or **YD** (velocity) vector having largest modulus

PU = upper bound of roots

PL = lower bound of roots

INDATA = input data option
= 0, basic matrices are automatically computed
= 1, to read basic matrices **K**, **M**, and **C** from user input files

3.1.6 IUV, IDDI, NTTS, NDELT (Required if IDRS = 1)
Format (FREE)

1. Description: Data related to dynamic response analysis.

2. Notes:

IUV = index for initial displacement (U) and velocity (V) input
= 0, no initial data
= 1, either initial displacement or velocity or both are nonzero vectors

IDDI = index for dynamic data input
= 1, nodal load input
= 2, nodal acceleration input

NTTS = total number of sets of load or acceleration data input

NDELT = number of sets of uniform time-increments for response calculation

3.1.7 G
Format (FREE)

1. Description: Structural damping formulation $[K = K(1 + i^{*}g)]$.

2. Notes:

g = structural damping parameter

i^{*} = imaginary number, $\sqrt{-1}$

K = system stiffness matrix

General note:

Each set of data input in succeeding sections is preceded with a relevant comment statement having a dollar sign at the first column, followed by optional descriptive words.

3.2 Nodal Data

3.2.1 $ NODAL DATA

3.2.2 IN, X, Y, Z, UX, UY, UZ, UXR, UYR, UZR, IINC
Format (I5,3E10.4,7I5)

1. Description: NN sets of nodal data input in GCS, at random; table 1 provides a description of the input data.

TABLE 1. - ARRANGEMENT OF NODAL DATA INPUT

Node number	Nodal coordinates			Nodal zero displacement boundary conditions(ZDBC)						Increment
(IN)	(X)	(Y)	(Z)	(UX)	(UY)	(UZ)	(UXR)	(UYR)	(UZR)	(IINC)
				1	2	3	4	5	6	
*	*	*	*	*	*	*	*	*	*	*

2. Notes:

a. A right-handed Cartesian coordinate system (X, Y, Z) is to be chosen to define the global coordinate system (GCS)

b. The asterisk (*) indicates required data input in GCS

c. Each structural node is assumed to have six degrees of freedom (DOF) consisting of three translations, UX, UY, UZ, and three rotations, UXR, UYR, UZR, usually labeled as displacement degrees of freedom 1, 2, 3, and 4, 5, 6, respectively

d. For nodal zero displacement boundary conditions, set value to
= 0, for free motion
= 1, for constrained motion

e. For node generation by increment, set IINC
= 0, for no incrementation
= I, to increment node number of previous input by I until current node number is attained

f. In automatic node generation (note (e)), the imposed displacement boundary conditions of generated intermediate nodes pertain to that of the last data set of the sequence

g. Third point nodes for line elements are assumed to lie on element local x-y plane, and may be chosen as any existing active node or dummy nodes (not connected to any structural member) with UX through UZR set to 1

h. Final data are automatically formed in increasing sequence of node numbers

3.3. Element Data

General note: Element data input may be at random within each data group.

3.3.1 $ ELEMENT CONNECTIVITY

3.3.2 IET, IEN, ND1, ND2, ND3, ND4, ND5, ND6, ND7, ND8, IMPP, IEPP/ITHTH, ITMPP, IPRR, IST, INC
Format (16I5)

1. Description: NEL sets of element data input; definition of input data is given in table 2.

TABLE 2. - ELEMENT DATA LAYOUT

Element type (IET)	Element number (IEN)	Node number for vertices 1 (ND1)	2 (ND2)	3 (ND3)	4 (ND4)	5 (ND5)	6 (ND6)	7 (ND7)	8 (ND8)	IMPP	IEPP/ ITHTH	ITMPP	IPRR	IST	INC
Line 1	*	*	*	**	IEC1	IEC2				*	x	*	*	*	*
Shell quadrilateral 2	*	*	*	*	*					*	†	*	*	*	*
Shell triangle 3	*	*	*	*						*	†	*	*	*	*
Solid hexahedron 4	*	*	*	*	*	*	*	*	*	*		*	*	*	*
Solid tetrahedron 5	*	*	*	*	*					*		*	*	*	*

2. Notes:

*	= data as defined
**	= third point node for element type 1
IECI	= integer defining line element end condition pertaining to end I = 0, rigid-ended = 1, pin-ended
IMPP	= integer defining material number
IEPP(x)	= integer defining line element property type
ITHTH(†)	= integer defining shell element thickness type

ITMPP = integer defining element temperature type

IPRR = integer defining element pressure type

IST = integer defining element spin type

INC = integer for element generation by increment
= 0, no increment
= J, increments node numbers of previous elements by J until current element nodal numbers are reached

In automatic element generation (see INC, above), the generated intermediate elements acquire properties the same as the last element in current sequence. Also, a special option enables repetitive use of an element with an input format (I3, I2, 15I5); the integer IET is then replaced by NELNO and IET, where NELNO is the total number of similar elements connecting the specified nodes.

3. Element Description:

The various elements (fig. 5) and associated degrees of freedom are depicted below. X, Y, Z represents global coordinate system (GCS), whereas x, y, z relates to local coordinate system (LCS).

4. Notes:

a. A right-handed Cartesian coordinate system (x, y, z) is to be chosen to define any element local coordinate system (LCS)

b. Any node may be chosen as the first vertex of an element, the local x-axis being along the line connecting vertices 1 and 2

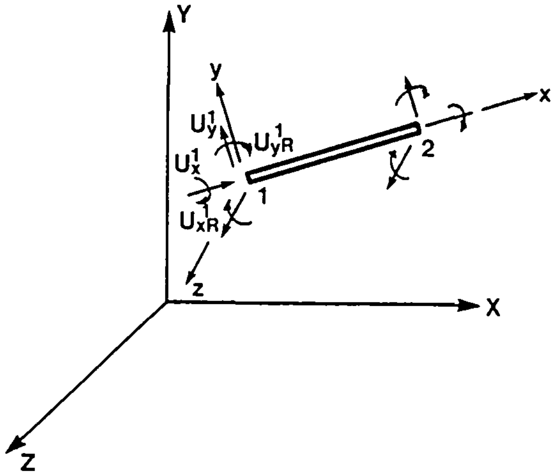
c. For line elements, the local x-y plane is defined as the plane contained by vertices 1, 2, and the specified third point node

d. For thin shell elements, the y-axis lies in the plane of the elements, the z-axis being perpendicular to the x-y plane

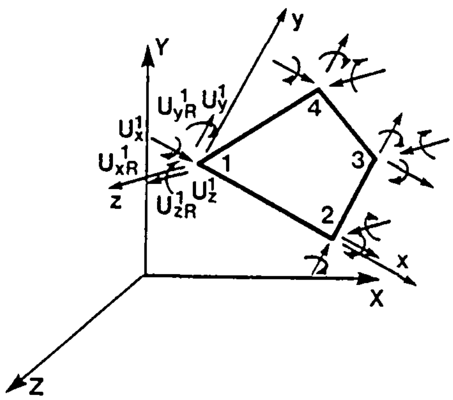
e. The vertices of the shell elements are usually numbered in a counterclockwise sequence when observed from any point along the local positive z-axis

f. For solid elements the y-axis lies in the plane formed by vertices 1-2-3 and 1-2-3-4 for the tetrahedral and hexahedral elements, respectively; the z-axis is perpendicular to the x-y plane, heading toward the 4th node for the tetrahedron and the plane containing the other four nodes for the hexahedral element

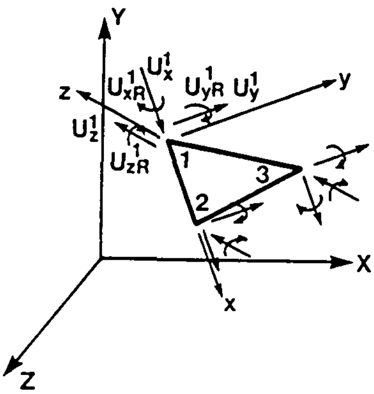
g. The vertices of the solid elements are also numbered in a counterclockwise sequence when viewed from any point on the positive z-axis, lying above the plane under consideration; the fifth vertex of the hexahedron is to be chosen as the node directly above vertex 1



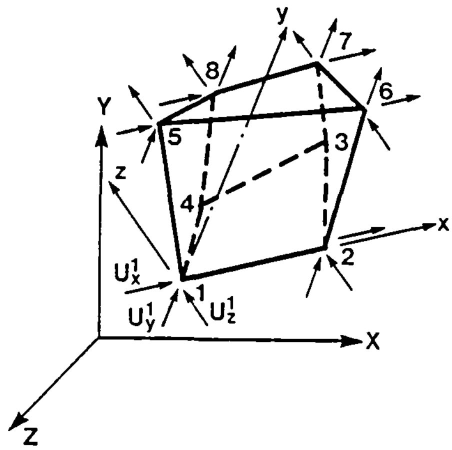
(a) Line element.



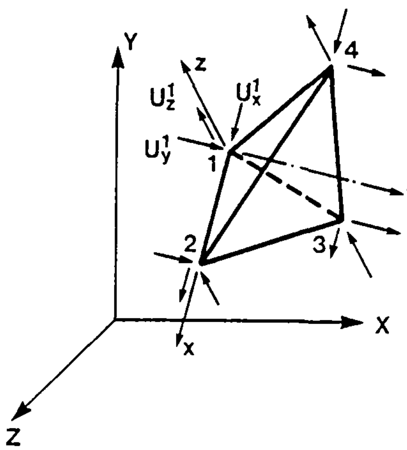
(b) Quadrilateral shell element.



(c) Triangular shell element.



(d) Hexahedral solid element.



(e) Tetrahedral solid element.

Figure 5. Description of finite elements.

5. Structural Modeling:

Since each node is assumed to possess six displacement degrees of freedom, any individual structural form may be simply represented by suppressing appropriate displacement terms. The following rules may be adopted:

Truss structures: to allow only two nodal translational deformations in the plane of the structure; to use line elements

Plane frame: all three in-plane displacements, namely, two translations and one rotation are retained in the formulation; to use line elements

Plane stress/strain: displacement boundary conditions are similar to truss structures; to use shell elements

Plate bending: only the three out-of-plane displacements consisting of one translation and two rotations are considered for the analysis; to use shell elements

Solid structures: the three translational degrees of freedom are retained in the analysis; to use solid elements

Shell, space frame: all six degrees of freedom are to be retained in the solution process; to use shell and line elements, respectively

Suppression of derived nodal motion may be achieved by using zero and interdependent displacement boundary conditions (ZDBC, IDBC), defined in sections 3.2 and 3.4, respectively.

3.3.3 $ LINE ELEMENT BASIC PROPERTIES (Required for line elements only)

3.3.4 IEPP, A, JX, IY, IZ
Format (I5,4E10.4)

1. Description: NEP sets of line element basic property data in element local coordinate system (LCS).

2. Notes:

IEPP = integer denoting line element property type

A = area of cross section

JX = torsional area moment of inertia, about element x-axis

IY = area moment of inertia about element y-axis

IZ = area moment of inertia about element z-axis

3.3.5 $ SHELL ELEMENT THICKNESS (Required for shell elements only)

3.3.6 ITHTH, T
Format (I5,E10.4)

1. Description: NET sets of element thickness data.

2. Notes:

ITHTH = element thickness type

T = thickness

3.3.7 $ ELEMENT MATERIAL PROPERTIES

3.3.8 IMPP, MT
Format (2I5)

3.3.9 E, MU, ALP, RHO (material type 1); or

E11, E12, E14, E22, E24, E44, E55,
E56, E66, ALPX, ALPY, ALPXY, RHO (material type 2); or

E11, E12, E13, E14, E15, E16, E22,
E23, E24, E25, E26, E33, E34, E35,
E36, E44, E45, E46, E55, E56, E66,
ALP1, ALP2, ALP3, ALP4, ALP5, ALP6, RHO (material type 3)
Format (4(7E10.4))

1. Description: NMAT sets of element material property data; the individual material matrices are derived from the 6 × 6, symmetric matrix for general solid material.

2. Notes:

IMPP = material number

MT = material type
= 1, isotropic
= 2, orthotropic-anisotropic, shell elements
= 3, orthotropic-anisotropic, solid elements

E = Young's modulus

$E_{i,j}$ = elements of material stress-strain matrix (i = 1, 6; j = 1, 6)

MU = Poisson's ratio

ALP = coefficient of thermal expansion for isotropic material

ALPX, ALPY, ALPXY = coefficients of thermal expansion, shell elements

ALP1 through
ALP6 = coefficients of thermal expansion, solid elements

RHO = mass per unit volume

3.3.9 $ ELEMENT TEMPERATURE DATA (Required if NTMP ≠ 0)

3.3.10 ITMPP, T, DTDY, DTDZ
Format (2(I5,3E10.4))

1. Description: NTMP number of element temperature types; table 3 shows compatible input data.

TABLE 3. - ELEMENT TEMPERATURE DATA INPUT

Element type	T	DTDY	DTDZ
1	*	*	*
2, 3	*		*
4, 5	*		

2. Notes:

ITMPP = element temperature increase type

T = uniform temperature increase; relates to all elements

DTDY = temperature gradient along element local y-axis; relates to line elements only

DTDZ = temperature gradient along element local z-axis; relates to line and shell elements

* = compatible input data

3.3.11 $ ELEMENT PRESSURE DATA (Required if NPR ≠ 0)

3.3.12 IPRR, PR
Format (5(I5,E10.4))

1. Description: NPR sets of element pressure data

2. Notes:

IPRR = element pressure type

PR = uniform pressure

Pressure directions for line elements: uniform pressure allowed in local y- and z-direction only and the program calculates as input both end loads and moments; while pressure corresponding to a first nodal input pertains to y-direction, a subsequent input for the same node signifies pressure acting in the z-direction

Pressure directions for shell elements: uniform pressure allowed in local z-direction only, program computes nodal load input

Pressure directions for solid elements: uniform pressure allowed on base surface defined by nodes 1-2-3-4 and 1-2-3 for hexahedral and tetrahedral elements, respectively, acting in local z-direction; program computes nodal load input data

3.4 Data in Global Coordinate System

General note: Data input in global coordinate system may be at random within each data group.

3.4.1 $ ELEMENT SPIN RATE DATA (Required if NSPIN ≠ 0).

3.4.2 IST, SPX, SPY, SPZ
Format (I5,3E10.4)

1. Description: NSPIN sets of spin data.

2. Notes:

IST = spin type

SPX, SPY, SPZ = components of element spin rate in global X-, Y-, and Z-directions, respectively

3.4.3 $ DEFLECTION BOUNDARY CONDITION DATA (Required if NBUN ≠ 0)

3.4.4 INI, IDOFJ, INIP, IDOFJP, CONFCT
Format (4(I4,I1,I4,I1,E10.4))

1. Description: NBUN sets of nodal deflection boundary condition data.

2. Notes:

INI = node number I

IDOFJ = Jth DOF associated with node I

INIP = node number I'

IDOFJP = J'th DOF associated with node J'

CONFCT = connectivity factor

J and J' vary between 1 and 6

3. Additional Notes:

The nodal displacement boundary conditions relationship is expressed as

$$U_{i,j} = a_{m,n} U_{m,n}$$

$$= a_{i,j} U_{i,j} + a_{i',j'} U_{i',j'} + \cdots$$

The input scheme is shown in table 4.

TABLE 4. - DATA LAYOUT FOR DISPLACEMENT BOUNDARY CONDITIONS

Node 1	DOF	Node 2	DOF	Connectivity coefficient	Terminology
i	j	i'	j'	$a_{i',j'}$	IDBC
i	j	i	j	$a_{i,j}$	FDBC
i	j	i	j	0.	ZDBC

in which

i,i' = node numbers

j,j' = degrees of freedom

$a_{i,j}$, $a_{i',j'}$ = connectivity coefficients

IDBC, FDBC, and ZDBC are, respectively, the interdependent, finite, and zero displacement boundary conditions. The ZDBC may also be conveniently implemented by following the rules given in table 1, which is generally recommended for such cases.

3.4.5 $ NODAL LOAD DATA (Required if IPLUMP ≠ 0)

3.4.6 IN, IDOF, P, IDOFE
Format (2I5,E10.4,I5)

1. Description: NC sets of nodal force data.

2. Notes:

IN = node number

IDOF and IDOFE are, respectively, the start and end degrees of freedom assigned with the same P value; default value for IDOFE is IDOF

P = nodal load

Each data set is to be terminated by setting a neagtive value for IN.

3.4.7 $ NODAL MASS DATA (Required if IMLUMP ≠ 0)

3.4.8 IN, IDOF, M, IDOFE
Format (2I5,E10.4,I5)

1. Description: Nodal lumped mass data.

2. Notes:

M = nodal mass

Other definitions are as in section 3.4.6.

3.4.9 $ NODAL INITIAL DISPLACEMENT AND VELOCITY DATA (Required if IUV = IDRS = 1)

3.4.10 IN, IDOF, UI, VI
Format (2I5,2E15.5)

1. Description: Initial displacements and velocities data.

2. Notes:

IN = node number

IDOF = degree of freedom

UI = initial displacement value

VI = initial velocity value

Data set is terminated if IN is read as -1.

3.4.11 $ NODAL FORCE ACCELERATION DATA (Required if NTTS ≠ 0 and IDRS = 1)

3.4.12 TZ
Format (E15.5)

3.4.13 IN, IDOF, PZ
Format (2I5,E15.5)

1. Description: NTTS sets of dynamic nodal load (IDDI = 1) or acceleration (IDDI = 2) input data.

2. Notes:

TZ = time-duration of load application

PZ = nodal force or acceleration data

Each data set is terminated by setting IN value to -1. Other definitions are as given in section 3.4.6.

3.4.14 $ INCREMENTAL TIME DATA FOR RESPONSE CALCULATION (Required if NDELT ≠ 0 and IDRS = 1)

3.4.15 DELT, IDELT
Format (E15.5,I5)

1. Description: NDELT sets of uniform incremental time input data for dynamic response calculation.

2. Notes:

DELT = uniform incremental time-step

IDELT = total number of uniform time-steps in the data set

3.5 Additional Basic Data

3.5.1 $ VISCOUS DAMPING DATA (Required if IPROB = 4 or 5)

3.5.2 (C(I,1),I = 1,N)
Format (6E10.4)

1. Description: User input of diagonal viscous damping matrix.

2. Notes:

C = diagonal viscous damping matrix

N = order of matrix

3.5.3 $ COEFFICIENTS FOR PROPORTIONAL VISCOUS DAMPING (Required if IPROB = 6 or 7)

3.5.4 ALPHA, BETA (Required if IPROB = 6 or 7)
Format (2E10.4)

1. Description: Proportional viscous damping formulation **C** = ALPHA***K** + BETA***M**.

2. Notes:

ALPHA and BETA are damping parameters.

K and **M** are system stiffness and mass matrices.

3.5.5 $ USER INPUT OPTION FOR VISCOUS DAMPING MATRIX (Required if IPROB = 6 or 7 and ALPHA and BETA set to 0)

3.5.6 ((C(I,J),J = 1,M11),I = 1,6)
Format (6E10.4)

1. Description: NN sets of user input of banded viscous damping matrix C(N,M11) in blocks of six rows of bandwidth M11, one row at a time (N = 6*NN).

2. Notes:

Data file must conform to IDBC, FDBC, and ZDBC, inherent in the problem.

4. SAMPLE PROBLEMS

This section provides the input data, as well as relevant outputs, of 11 typical test cases involving static, stability, free-vibration, and dynamic response analyses of representative structures. The input data are prepared in accordance with the procedures described in section 3; the required run-stream is given in section 5. Details of such analyses are in the descriptions that follow in which each structural geometry is described in a right-handed, rectangular coordinate system and the associated input data are defined in consistent unit form.

4.1 Space Truss: Static Analysis

The static analysis of the space truss depicted in figure 6 (ref. 5) was performed to yield nodal deformations and element forces. A load of 300 lb acts at node 7 along the axial direction of the member connecting nodes 7 and 9; another load of 500 lb is applied at node 10 in the direction of the structural base centerline. Also, the three members in the upper tier of the structure are subjected to an uniform temperature increase of 100°.

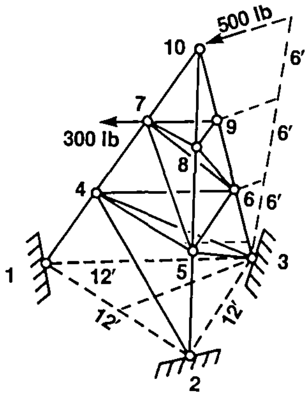


Figure 6. Space truss.

Important data parameters —

Young's modulus, E	=	1.0×10^7
Poisson's ratio, μ	=	0.3
Coefficient of thermal expansion, α	=	12.5×10^{-6}

STARS input data —

```
SPACE TRUSS - MECHANICAL AND THERMAL LOADING
11,21,1,4,1,Ø,1,Ø,Ø
8,1,2,1,Ø,Ø,Ø
1,Ø,Ø,2
$ NODAL DATA
    1       Ø.Ø       6.Ø      Ø.Ø    1    1    1    Ø    Ø    Ø
    2       Ø.Ø      -6.Ø      Ø.Ø    1    1    1    Ø    Ø    Ø
    3     1Ø.39       Ø.Ø      Ø.Ø    1    1    1    Ø    Ø    Ø
    4     1.155       4.Ø      6.Ø
    5     1.155      -4.Ø      6.Ø
    6     8.Ø81       Ø.Ø      6.Ø
    7     2.3Ø9       2.Ø     12.Ø
    8     2.3Ø9      -2.Ø     12.Ø
    9     5.773       Ø.Ø     12.Ø
   1Ø     3.464       Ø.Ø     18.Ø
   11      12.Ø       3.Ø      Ø.Ø
$ ELEMENT CONNECTIVITY
    1    1    1    4   11    1    1    Ø    Ø    Ø    1    1
    1    2    2    4   11    1    1                   1    1
    1    3    2    5   11    1    1                   1    1
    1    4    3    5   11    1    1                   1    1
    1    5    3    6   11    1    1                   1    1
    1    6    3    4   11    1    1                   1    1
    1    7    4    5   11    1    1                   1    1
    1    8    5    6   11    1    1                   1    1
    1    9    6    4   11    1    1                   1    1
    1   1Ø    4    7   11    1    1                   1    1
    1   11    5    7   11    1    1                   1    1
    1   12    5    8   11    1    1                   1    1
    1   13    6    8   11    1    1                   1    1
    1   14    6    9   11    1    1                   1    1
    1   15    6    7   11    1    1                   1    1
    1   16    7    8   11    1    1                   1    1
    1   17    8    9   11    1    1                   1    1
    1   18    9    7   11    1    1                   1    1
    1   19    7   1Ø   11    1    1                   1    1    1
    1   2Ø    8   1Ø   11    1    1                   1    1    1
    1   21    9   1Ø   11    1    1                   1    1    1
$ LINE ELEMENT BASIC PROPERTIES
    1   Ø.Ø1389
$ ELEMENT MATERIAL PROPERTIES
    1    1
    1Ø.ØE6        Ø.3  12.5E-Ø6
$ ELEMENT TEMPERATURE DATA
    1    1ØØ.Ø
$ NODAL LOAD DATA
   1Ø    1    -5ØØ.Ø
    7    1    -259.8
    7    2     15Ø.Ø
   -1
```

STARS analysis results: nodal deformations and element stresses -

LOAD CASE NO. 1

NODE	X-DISPL.	Y-DISPL.	Z-DISPL.	X-ROTN.	Y-ROTN.	Z-ROTN.
1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
2	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
3	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
4	-Ø.251729E-Ø1	Ø.194816E-Ø1	-Ø.276224E-Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
5	-Ø.245333E-Ø1	Ø.19478ØE-Ø1	-Ø.24789ØE-Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
6	-Ø.298415E-Ø1	Ø.286691E-Ø1	Ø.4651Ø5E-Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
7	-Ø.1343Ø2E+ØØ	Ø.479256E-Ø1	-Ø.321279E-Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
8	-Ø.124453E+ØØ	Ø.479262E-Ø1	-Ø.38Ø713E-Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
9	-Ø.12938ØE+ØØ	Ø.564544E-Ø1	Ø.542351E-Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
1Ø	-Ø.4Ø3689E+ØØ	Ø.418542E-Ø1	Ø.33Ø398E-Ø2	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
11	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ

ELEMENT STRESSES

ELEMENT NO.	END1 END5	END2 END6	END3 END7	END4 END8	PX1/PX2 SXT SXX	PY1/PY2 SYT SYY	PZ1/PZ2 SXYT SZZ	MX1/MX2 SXB SXY	MY1/MY2 SYB SYZ	MZ1/MZ2 SXYB SZX
1	1	4			Ø.785575E+Ø3 -Ø.785575E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
2	2	4			-Ø.7153ØØE-Ø2 Ø.7153ØØE-Ø2	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
3	2	5			Ø.464121E+Ø3 -Ø.464121E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
4	3	5			Ø.817221E-Ø1 -Ø.817221E-Ø1	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
5	3	6			-Ø.116939E+Ø4 Ø.116939E+Ø4	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
6	3	4			-Ø.146365E+Ø3 Ø.146365E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
7	4	5			-Ø.632629E-Ø1 Ø.632629E-Ø1	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
8	5	6			-Ø.32Ø435E-Ø3 Ø.32Ø435E-Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
9	6	4			Ø.15ØØØ7E+Ø3 -Ø.15ØØØ7E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
1Ø	4	7			Ø.7Ø524ØE+Ø3 -Ø.7Ø524ØE+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
11	5	7			Ø.4618Ø7E-Ø1 -Ø.4618Ø7E-Ø1	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE-ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
12	5	8			Ø.464118E+Ø3 -Ø.464118E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
13	6	8			-Ø.177856E+ØØ Ø.177856E+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
14	6	9			-Ø.927915E+Ø3 Ø.927915E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
15	6	7			-Ø.321363E+Ø3 Ø.321363E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
16	7	8			Ø.4187Ø1E-Ø1 -Ø.4187Ø1E-Ø1	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
17	8	9			Ø.83ØØ78E-Ø1 -Ø.83ØØ78E-Ø1	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
18	9	7			Ø.825195E-Ø1 -Ø.825195E-Ø1	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
19	7	1Ø			Ø.29Ø373E+Ø3 -Ø.29Ø373E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
2Ø	8	1Ø			Ø.29Ø373E+Ø3 -Ø.29Ø373E+Ø3	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ
21	9	1Ø			-Ø.11Ø159E+Ø4 Ø.11Ø159E+Ø4	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ Ø.ØØØØØØE+ØØ

4.2 Space Frame: Static Analysis

A space frame with rigid connections, shown in figure 7, (ref. 6) is subjected to nodal forces and moments. Results of such analysis are presented below.

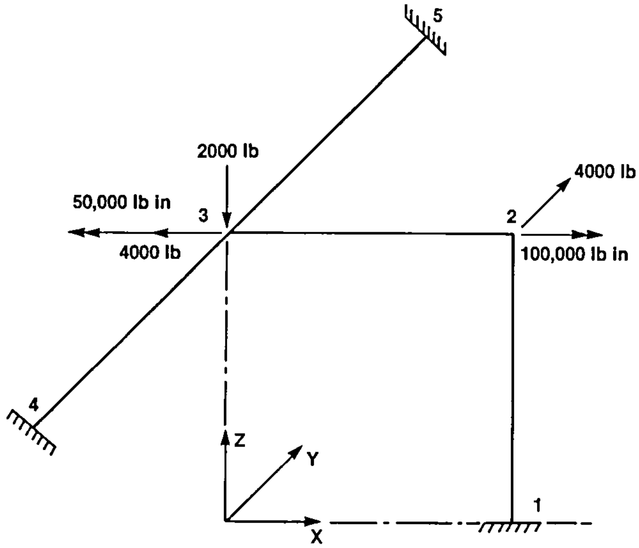


Figure 7. Space frame structure.

Important data parameters —

Young's modulus, E	=	30.24×10^6
Poisson's ratio, μ	=	0.2273
Cross-section area, A	=	25.13
Member length, ℓ	=	120

STARS input data —

```
SPACE FRAME CASE
6,4,1,4,1,Ø,Ø,Ø,Ø
8,1,2,1,Ø,Ø,Ø
1,Ø,Ø,2
$ NODAL DATA
    1      12Ø.Ø        Ø.Ø        Ø.Ø     1     1     1     1     1     1
    2      12Ø.Ø        Ø.Ø      12Ø.Ø     Ø     Ø     Ø     Ø     Ø     Ø
    3        Ø.Ø        Ø.Ø      12Ø.Ø     Ø     Ø     Ø     Ø     Ø     Ø
    4        Ø.Ø     -12Ø.Ø      12Ø.Ø     1     1     1     1     1     1
    5        Ø.Ø      12Ø.Ø      12Ø.Ø     1     1     1     1     1     1
    6       1Ø.Ø       1Ø.Ø        Ø.Ø     1     1     1     1     1     1
$ ELEMENT CONNECTIVITY
    1    1    1    2    6    Ø    Ø                    1     1
    1    2    2    3    6    Ø    Ø                    1     1
    1    3    3    4    6    Ø    Ø                    1     1
    1    4    3    5    6    Ø    Ø                    1     1
$ LINE ELEMENT BASIC PROPERTIES
    1     25.13      125.7     62.83      62.83
$ ELEMENT MATERIAL PROPERTIES
    1    1
  3Ø.24EØ6     Ø.2273
$ NODAL LOAD DATA
    2    4   1ØØØØØ.Ø
    2    2     4ØØØ.Ø
    3    1    -4ØØØ.Ø
    3    3    -2ØØØ.Ø
    3    4   -5ØØØØ.Ø
   -1
```

STARS analysis results —

LOAD CASE NO. 1

NODE	X-DISPL.	Y-DISPL.	Z-DISPL.	X-ROTN.	Y-ROTN.	Z-ROTN.
1	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.125288E+00	0.347953E+00	0.196027E-04	-0.239969E-02	-0.121545E-02	0.323397E-02
3	-0.125397E+00	0.103330E-03	-0.804946E-01	-0.580122E-03	-0.283265E-03	0.910380E-03
4	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
5	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
6	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

ELEMENT STRESSES

ELEMENT NO.	END1 / END5	END2 / END6	END3 / END7	END4 / END8	PX1/PX2 SXT SXX	PY1/PY2 SYT SYY	PZ1/PZ2 SXYT SZZ	MX1/MX2 SXB SXY	MY1/MY2 SYB SYZ	MZ1/MZ2 SXYB SZX
1	1	2			-0.124139E+03	-0.931688E+03	0.261767E+04	-0.417341E+05	-0.193156E+06	-0.785065E+05
					0.124139E+03	0.331688E+03	-0.261767E+04	0.417341E+05	-0.120964E+06	-0.332961E+05
2	2	3			-0.690813E+03	0.232395E+03	-0.129390E+04	0.234814E+05	0.397457E+05	0.255969E+05
					0.690813E+03	-0.232395E+03	0.129390E+04	-0.234814E+05	0.115522E+06	0.229051E+04
3	3	4			-0.654366E+03	0.523180E+03	-0.980653E+03	0.365551E+04	0.437120E+05	0.234344E+05
					0.654366E+03	-0.523180E+03	0.980653E+03	-0.365551E+04	0.739664E+05	0.393472E+05
4	3	5			0.654366E+03	0.131882E+04	0.249337E+04	-0.365551E+04	-0.164730E+06	0.870856E+05
					-0.654366E+03	-0.131882E+04	-0.249337E+04	0.365551E+04	-0.134475E+06	0.711728E+05

4.3 Plane Stress: Static Analysis

Figure 8 (ref. 7) depicts the finite-element model of the symmetric half of a deep beam subjected to a concentrated load, as shown.

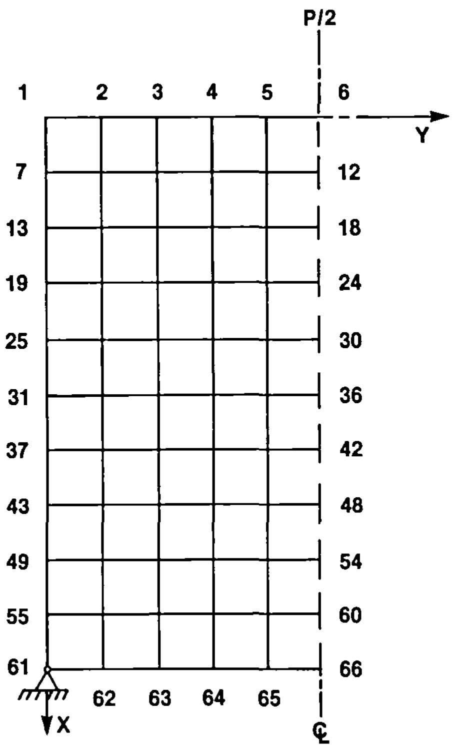


Figure 8. Deep beam example.

Important data parameters –

Young's modulus, E	= 30 × 10^6
Poisson's ratio, μ	= 0.2
Nodal load, P	= 10,000
Beam side length, ℓ	= 20

STARS input data —

```
PLANE STRESS CASE - TEZCAN
66,5Ø,1,4,Ø,1,Ø,Ø,Ø
8,1,2,1,Ø,Ø,Ø
1,Ø,Ø,2
$ NODAL DATA
    1       Ø.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
   55      18.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø    6
   61      2Ø.Ø        Ø.Ø        Ø.Ø    1    1    1    1    1    1
    2       Ø.Ø        2.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
   62      2Ø.Ø        2.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø    6
    3       Ø.Ø        4.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
   63      2Ø.Ø        4.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø    6
    4       Ø.Ø        6.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
   64      2Ø.Ø        6.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø    6
    5       Ø.Ø        8.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
   65      2Ø.Ø        8.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø    6
    6       Ø.Ø       1Ø.Ø        Ø.Ø    Ø    1    1    1    1    1
   66      2Ø.Ø       1Ø.Ø        Ø.Ø    Ø    1    1    1    1    1    6
$ ELEMENT CONNECTIVITY
    2    1    1    7    8    2    Ø    Ø    Ø    Ø    1    1
    2   1Ø   55   61   62   56                        1    1    Ø    Ø    Ø    6
    2   11    2    8    9    3                        1    1
    2   2Ø   56   62   63   57                        1    1                   6
    2   21    3    9   1Ø    4                        1    1
    2   3Ø   57   63   64   58                        1    1                   6
    2   31    4   1Ø   11    5                        1    1
    2   4Ø   58   64   65   59                        1    1                   6
    2   41    5   11   12    6                        1    1
    2   5Ø   59   65   66   6Ø                        1    1                   6
$ SHELL ELEMENT THICKNESSES
    1       Ø.1
$ ELEMENT MATERIAL PROPERTIES
    1    1
   3Ø.ØE6        Ø.2
$ NODAL LOAD DATA
    6    1    5ØØØ.Ø
   -1
```

STARS analysis results —

NODE	X-DISPL.	Y-DISPL.	Z-DISPL.	X-ROTN.	Y-ROTN.	Z-ROTN.
1	0.643004E-02	0.784168E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
2	0.686331E-02	0.806537E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
3	0.736951E-02	0.882648E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
4	0.814826E-02	0.991451E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
5	0.942354E-02	0.984274E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
6	0.135581E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
7	0.645125E-02	0.339509E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
8	0.685836E-02	0.327585E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
9	0.735772E-02	0.281376E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
10	0.810701E-02	0.135669E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
11	0.944763E-02	-0.202705E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
12	0.106295E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
13	0.650795E-02	-0.353817E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
14	0.683854E-02	-0.546589E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
15	0.730059E-02	-0.111159E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
16	0.797687E-02	-0.215918E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
17	0.877384E-02	-0.212967E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
18	0.919536E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
19	0.654701E-02	-0.242513E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
20	0.675883E-02	-0.242862E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
21	0.714184E-02	-0.252697E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
22	0.764636E-02	-0.249769E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
23	0.812575E-02	-0.171392E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
24	0.833486E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
25	0.649748E-02	-0.274906E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
26	0.659352E-02	-0.248456E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
27	0.688814E-02	-0.219499E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
28	0.726972E-02	-0.181145E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
29	0.759755E-02	-0.108600E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
30	0.772861E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
31	0.631290E-02	-0.176818E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
32	0.633246E-02	-0.125004E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
33	0.658259E-02	-0.845376E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
34	0.691461E-02	-0.587820E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
35	0.718585E-02	-0.332733E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
36	0.728973E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
37	0.596216E-02	-0.453016E-06	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
38	0.596853E-02	0.743781E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
39	0.625301E-02	0.104449E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
40	0.661288E-02	0.854564E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
41	0.688862E-02	0.441688E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
42	0.699000E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
43	0.539932E-02	0.206086E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
44	0.548947E-02	0.303468E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
45	0.592653E-02	0.294909E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
46	0.639034E-02	0.206152E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
47	0.670621E-02	0.983889E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
48	0.681496E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
49	0.451550E-02	0.394097E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
50	0.489274E-02	0.491623E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
51	0.565929E-02	0.394860E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
52	0.627523E-02	0.214191E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
53	0.662510E-02	0.879983E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
54	0.673927E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
55	0.300094E-02	0.378987E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
56	0.427165E-02	0.504907E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
57	0.557725E-02	0.139993E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
58	0.624712E-02	-0.457609E-06	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
59	0.660310E-02	-0.241771E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
60	0.671494E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
61	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
62	0.430391E-02	-0.765803E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
63	0.554128E-02	-0.654390E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
64	0.623057E-02	-0.445990E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
65	0.657645E-02	-0.226519E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
66	0.668556E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

ELEMENT STRESSES

ELEMENT NO.	END1 END5	END2 END6	END3 END7	END4 END8	PX1/PX2 SXT SXX	PY1/PY2 SYT SYY	PZ1/PZ2 SXYT SZZ	MX1/MX2 SXB SXY	MY1/MY2 SYB SYZ	MZ1/MZ2 SXYB SZX
1	1	7	8	2	0.143384E+03	0.107016E+03	-0.260104E+03	0.143384E+03	0.107016E+03	-0.260104E+03
2	7	13	14	8	0.239304E+03	-0.186143E+03	-0.607161E+02	0.239304E+03	-0.186143E+03	-0.607161E+02
3	13	19	20	14	-0.348182E+03	-0.216837E+03	0.459592E+03	-0.348182E+03	-0.216837E+03	0.459592E+03
4	19	25	26	20	-0.163769E+04	-0.131780E+03	0.843353E+03	-0.163769E+04	-0.131780E+03	0.843353E+03
5	25	31	32	26	-0.335924E+04	-0.848659E+02	0.105358E+04	-0.335924E+04	-0.848659E+02	0.105358E+04
6	31	37	38	32	-0.538550E+04	-0.127262E+03	0.125524E+04	-0.538550E+04	-0.127262E+03	0.125524E+04
7	37	43	44	38	-0.787078E+04	-0.282555E+03	0.166297E+04	-0.787078E+04	-0.282555E+03	0.166297E+04
8	43	49	50	44	-0.112623E+05	-0.790640E+03	0.263614E+04	-0.112623E+05	-0.790640E+03	0.263614E+04
9	49	55	56	50	-0.163355E+05	-0.159126E+04	0.514415E+04	-0.163355E+05	-0.159126E+04	0.514415E+04
10	55	61	62	56	-0.241927E+05	-0.963766E+04	0.122654E+05	-0.241927E+05	-0.963766E+04	0.122654E+05
11	2	8	9	3	-0.840132E+02	0.207459E+03	-0.233322E+03	-0.840132E+02	0.207459E+03	-0.233322E+03
12	8	14	15	9	-0.761663E+03	-0.922655E+03	0.583228E+03	-0.761663E+03	-0.922655E+03	0.583228E+03
13	14	20	21	15	-0.196668E+04	-0.890848E+03	0.161039E+04	-0.196668E+04	-0.890848E+03	0.161039E+04
14	20	26	27	21	-0.324355E+04	-0.505299E+03	0.220386E+04	-0.324355E+04	-0.505299E+03	0.220386E+04
15	26	32	33	27	-0.431822E+04	-0.342976E+03	0.250988E+04	-0.431822E+04	-0.342976E+03	0.250988E+04
16	32	38	39	33	-0.530784E+04	-0.532538E+03	0.288429E+04	-0.530784E+04	-0.532538E+03	0.288429E+04
17	38	44	45	39	-0.625966E+04	-0.109059E+04	0.356589E+04	-0.625966E+04	-0.109059E+04	0.356589E+04
18	44	50	51	45	-0.691433E+04	-0.217278E+04	0.466162E+04	-0.691433E+04	-0.217278E+04	0.466162E+04
19	50	56	57	51	-0.621450E+04	-0.470548E+04	0.572053E+04	-0.621450E+04	-0.470548E+04	0.572053E+04
20	56	62	63	57	-0.425137E+03	-0.198628E+04	0.149334E+04	-0.425137E+03	-0.198628E+04	0.149334E+04
21	3	9	10	4	-0.472056E+03	-0.371183E+03	0.221827E+03	-0.472056E+03	-0.371183E+03	0.221827E+03
22	9	15	16	10	-0.185435E+04	-0.224936E+04	0.212951E+04	-0.185435E+04	-0.224936E+04	0.212951E+04
23	15	21	22	16	-0.398151E+04	-0.156003E+04	0.314191E+04	-0.398151E+04	-0.156003E+04	0.314191E+04
24	21	27	28	22	-0.485999E+04	-0.662376E+03	0.308726E+04	-0.485999E+04	-0.662376E+03	0.308726E+04
25	27	33	34	28	-0.506124E+04	-0.531420E+03	0.303415E+04	-0.506124E+04	-0.531420E+03	0.303415E+04
26	33	39	40	34	-0.492160E+04	-0.933600E+03	0.320350E+04	-0.492160E+04	-0.933600E+03	0.320350E+04
27	39	45	46	40	-0.445751E+04	-0.169963E+04	0.354638E+04	-0.445751E+04	-0.169963E+04	0.354638E+04
28	45	51	52	46	-0.340813E+04	-0.270233E+04	0.371169E+04	-0.340813E+04	-0.270233E+04	0.371169E+04
29	51	57	58	52	-0.136223E+04	-0.268085E+04	0.255089E+04	-0.136223E+04	-0.268085E+04	0.255089E+04
30	57	63	64	58	-0.304189E+03	0.448787E+03	0.372647E+03	-0.304189E+03	0.448787E+03	0.372647E+03
31	4	10	11	5	-0.673933E+03	-0.272642E+04	0.179107E+04	-0.673933E+03	-0.272642E+04	0.179107E+04
32	10	16	17	11	-0.680483E+04	-0.387664E+04	0.554921E+04	-0.680483E+04	-0.387664E+04	0.554921E+04
33	16	22	23	17	-0.751821E+04	-0.893684E+03	0.401277E+04	-0.751821E+04	-0.893684E+03	0.401277E+04
34	22	28	29	23	-0.683325E+04	-0.234737E+03	0.293327E+04	-0.683325E+04	-0.234737E+03	0.293327E+04
35	28	34	35	29	-0.583750E+04	-0.432099E+03	0.248987E+04	-0.583750E+04	-0.432099E+03	0.248987E+04
36	34	40	41	35	-0.470407E+04	-0.105916E+04	0.240206E+04	-0.470407E+04	-0.105916E+04	0.240206E+04
37	40	46	47	41	-0.339649E+04	-0.179718E+04	0.239540E+04	-0.339649E+04	-0.179718E+04	0.239540E+04
38	46	52	53	47	-0.189863E+04	-0.213440E+04	0.207309E+04	-0.189863E+04	-0.213440E+04	0.207309E+04
39	52	58	59	53	-0.625687E+03	-0.124948E+04	0.118445E+04	-0.625687E+03	-0.124948E+04	0.118445E+04
40	58	64	65	59	-0.316193E+02	0.146181E+04	0.168679E+03	-0.316193E+02	0.146181E+04	0.168679E+03
41	5	11	12	6	-0.239134E+05	-0.106444E+05	0.129045E+05	-0.239134E+05	-0.106444E+05	0.129045E+05
42	11	17	18	12	-0.158184E+05	-0.461348E+02	0.497836E+04	-0.158184E+05	-0.461348E+02	0.497836E+04
43	17	23	24	18	-0.111853E+05	0.645630E+03	0.210061E+04	-0.111853E+05	0.645630E+03	0.210061E+04
44	23	29	30	24	-0.842533E+04	0.414873E+03	0.125923E+04	-0.842533E+04	0.414873E+03	0.125923E+04
45	29	35	36	30	-0.642358E+04	-0.220669E+03	0.969579E+03	-0.642358E+04	-0.220669E+03	0.969579E+03
46	35	41	42	36	-0.468073E+04	-0.101786E+04	0.883437E+03	-0.468073E+04	-0.101786E+04	0.883437E+03
47	41	47	48	42	-0.301527E+04	-0.167224E+04	0.826096E+03	-0.301527E+04	-0.167224E+04	0.826096E+03
48	47	53	54	48	-0.151628E+04	-0.170116E+04	0.664179E+03	-0.151628E+04	-0.170116E+04	0.664179E+03
49	53	59	60	54	-0.461707E+03	-0.571001E+03	0.355761E+03	-0.461707E+03	-0.571001E+03	0.355761E+03
50	59	65	66	60	-0.459945E+02	0.187102E+04	0.581728E+02	-0.459945E+02	0.187102E+04	0.581728E+02

4.4 Plate Bending: Vibration Anaylsis

A square cantilever plate was analyzed to yield the natural frequencies and associated mode shapes. Figure 9 depicts the plate with a 4 × 4 finite-element mesh, the bottom edge along the x-axis being clamped.

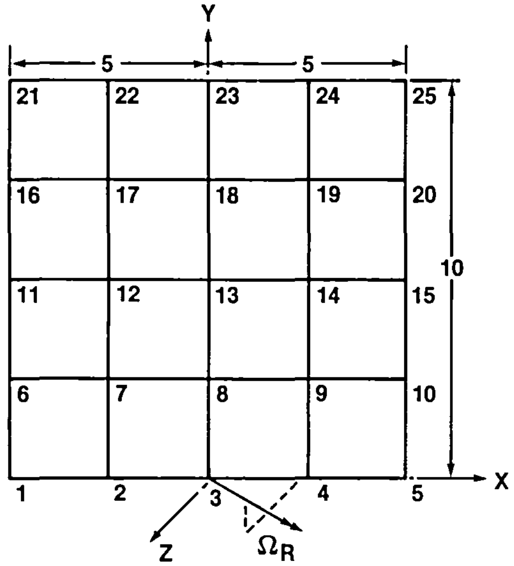


Figure 9. Square cantilever plate.

Important data parameters -

Young's modulus, E	$= 10 \times 10^6$
Side length, ℓ	$= 10$
Plate thickness, t	$= 0.1$
Poisson's ratio, μ	$= 0.3$
Mass density, ρ	$= 0.259 \times 10^{-3}$

STARS input data —

```
SQUARE 4-BY-4 PLATE     NON-SPINNING STRUCTURE
25,16,1,4,Ø,1,Ø,Ø,Ø
1,1,2,1,Ø,Ø,Ø
Ø,Ø,Ø,2
1,6,Ø,6ØØØ.Ø,Ø.Ø,Ø
$ NODAL DATA
    1       -5.Ø        Ø.Ø         Ø.Ø    1    1    1    1    1    1    Ø
    5        5.Ø        Ø.Ø         Ø.Ø    1    1    1    1    1    1    1
    6       -5.Ø        2.5         Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø    Ø
   1Ø        5.Ø        2.5         Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø    1
   11       -5.Ø        5.Ø         Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø    Ø
   15        5.Ø        5.Ø         Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø    1
   16       -5.Ø        7.5         Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø    Ø
   2Ø        5.Ø        7.5         Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø    1
   21       -5.Ø       1Ø.Ø         Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø    Ø
   25        5.Ø       1Ø.Ø         Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø    1
$ ELEMENT CONNECTIVITY
    2    1    1    2    7    6    Ø    Ø    Ø    Ø    1    1    Ø    Ø    Ø
    2    4    4    5   1Ø    9    Ø    Ø    Ø    Ø    1    1    Ø    Ø    Ø    1
    2    5    6    7   12   11    Ø    Ø    Ø    Ø    1    1    Ø    Ø    Ø
    2    8    9   1Ø   15   14    Ø    Ø    Ø    Ø    1    1    Ø    Ø    Ø    1
    2    9   11   12   17   16    Ø    Ø    Ø    Ø    1    1    Ø    Ø    Ø
    2   12   14   15   2Ø   19    Ø    Ø    Ø    Ø    1    1    Ø    Ø    Ø    1
    2   13   16   17   22   21    Ø    Ø    Ø    Ø    1    1    Ø    Ø    Ø
    2   16   19   2Ø   25   24    Ø    Ø    Ø    Ø    1    1    Ø    Ø    Ø    1
$ SHELL ELEMENT THICKNESSES
    1      Ø.1
$ ELEMENT MATERIAL PROPERTIES
    1    1
  1.ØE+Ø7       Ø.3Ø        Ø.Ø  Ø.259E-3
```

STARS output summary — The output summary is presented in table 5.

TABLE 5. - NATURAL FREQUENCIES OF A SQUARE CANTILEVER PLATE

Mode number	Natural frequency ω, rad/sec	Nondimensional parameter, $\gamma = \omega \ell^2 \sqrt{\rho t/D}$
1	213.47	3.59
2	528.06	8.88
3	1217.10	20.47
4	1559.30	26.22
5	1806.30	30.38

Note: D = plate flexural rigidity
$= Et^3/12(1-\mu^2)$

4.5 General Shell: Vibration Analysis

A circular cylindrical shell is shown in figure 10 in which quadrilateral shell elements are used for structural discretization to perform a free-vibration analysis.

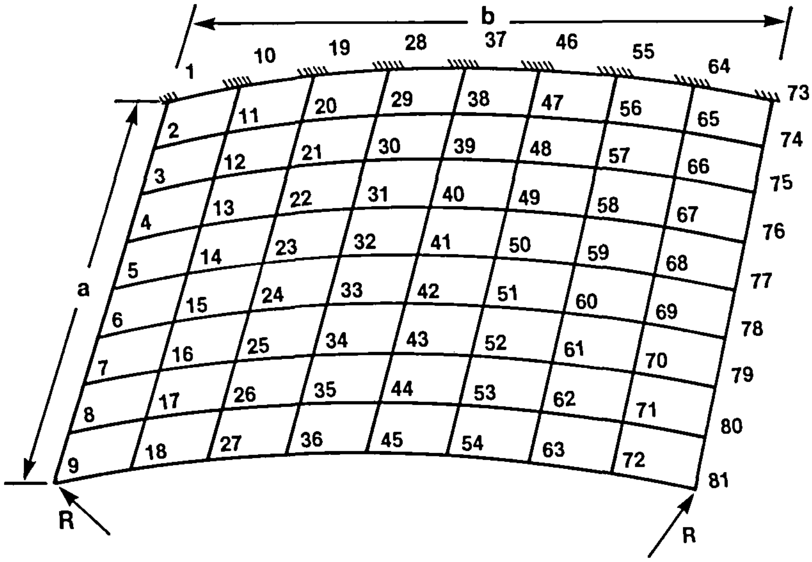


Figure 10. Finite element model of a cylindrical shell.

Important data parameters —

Side length, a, b	= 10
Radius, r	= 20
Thickness, t	= 0.1
Young's modulus, E	= 29.5×10^6
Poisson's ratio, μ	= 0.3
Mass density, ρ	= 0.733×10^{-3}

STARS input data —

```
     SHELL ELEM         8 -BY- 8    CURVED SHELL     FREE-VIB
81,64,1,4,0,1,0,0,0
1,1,2,1,0,0,0
0,0,0,0
1,6,0,90000.0,0.0,0
$ NODAL DATA
    1        0.0        0.0        0.0    1    1    1    1    1    1    0
    2       1.25        0.0        0.0    0    0    0    0    0    0    0
    9       10.0        0.0        0.0    0    0    0    0    0    0    1
   10        0.0       1.25  0.2803754    1    1    1    1    1    1    0
   11       1.25       1.25  0.2803754    0    0    0    0    0    0    0
   18       10.0       1.25  0.2803754    0    0    0    0    0    0    1
   19        0.0        2.5   0.478218    1    1    1    1    1    1    0
   20       1.25        2.5   0.478218    0    0    0    0    0    0    0
   27       10.0        2.5   0.478218    0    0    0    0    0    0    1
   28        0.0       3.75  0.5959826    1    1    1    1    1    1    0
   29       1.25       3.75  0.5959826    0    0    0    0    0    0    0
   36       10.0       3.75  0.5959826    0    0    0    0    0    0    1
   37        0.0        5.0  0.6350833    1    1    1    1    1    1    0
   38       1.25        5.0  0.6350833    0    0    0    0    0    0    0
   45       10.0        5.0  0.6350833    0    0    0    0    0    0    1
   46        0.0       6.25  0.5959826    1    1    1    1    1    1    0
   47       1.25       6.25  0.5959826    0    0    0    0    0    0    0
   54       10.0       6.25  0.5959826    0    0    0    0    0    0    1
   55        0.0        7.5   0.478218    1    1    1    1    1    1    0
   56       1.25        7.5   0.478218    0    0    0    0    0    0    0
   63       10.0        7.5   0.478218    0    0    0    0    0    0    1
   64        0.0       8.75  0.2803754    1    1    1    1    1    1    0
   65       1.25       8.75  0.2803754    0    0    0    0    0    0    0
   72       10.0       8.75  0.2803754    0    0    0    0    0    0    1
   73        0.0       10.0        0.0    1    1    1    1    1    1    0
   74       1.25       10.0        0.0    0    0    0    0    0    0    0
   81       10.0       10.0        0.0    0    0    0    0    0    0    1
$ ELEMENT CONNECTIVITY
    2    1    1    2   11   10    0    0    0    0    1    1
    2    8    8    9   18   17    0    0    0    0    1    1    0    0    0    1
    2    9   10   11   20   19    0    0    0    0    1    1
    2   16   17   18   27   26    0    0    0    0    1    1    0    0    0    1
    2   17   19   20   29   28    0    0    0    0    1    1
    2   24   26   27   36   35    0    0    0    0    1    1    0    0    0    1
    2   25   28   29   38   37    0    0    0    0    1    1
    2   32   35   36   45   44    0    0    0    0    1    1    0    0    0    1
    2   33   37   38   47   46    0    0    0    0    1    1
    2   40   44   45   54   53    0    0    0    0    1    1    0    0    0    1
    2   41   46   47   56   55    0    0    0    0    1    1
    2   48   53   54   63   62    0    0    0    0    1    1    0    0    0    1
    2   49   55   56   65   64    0    0    0    0    1    1
    2   56   62   63   72   71    0    0    0    0    1    1    0    0    0    1
    2   57   64   65   74   73    0    0    0    0    1    1
    2   64   71   72   81   80    0    0    0    0    1    1    0    0    0    1
$ SHELL ELEMENT THICKNESSES
    1        0.1
$ ELEMENT MATERIAL PROPERTIES
    1    1
0.2950E+080.3000E+000.0000E+000.7332E-03
```

STARS output summary — The output summary is presented in table 6.

TABLE 6. - NATURAL FREQUENCIES OF A CYLINDRICAL CANTILEVER SHELL

Mode number	Natural frequencies ω, rad/sec	Nondimensional parameter, $\gamma = \omega a^2\sqrt{\rho t/D}$
1	702.1363	10.60
2	1117.6793	16.99
3	1936.3007	30.65
4	2765.2993	42.23
5	3045.5002	47.68
6	3901.2104	65.45

4.6 General Solid: Vibration Analysis

A cube idealized by hexahedral solid elements is shown in figure 11. The nodes lying in the X-Y plane are assumed to be fixed. Details of the natural frequency analysis of the cube are presented herein.

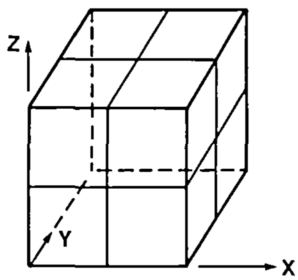


Figure 11. Cube discretized by hexahedral elements.

Important data highlights —

Side length, ℓ = 10
Young's modulus, E = 10×10^6
Poisson's ratio, μ = 0.3
Mass density, ρ = 2.349×10^{-4}

STARS input data —

```
HEXAHEDRON CASE - 2 BY 2
27,8,1,4,Ø,Ø,Ø,Ø,Ø
1,1,2,1,Ø,Ø,Ø
Ø,Ø,Ø,2
1,6,Ø,15ØØØØ.Ø,Ø.Ø,Ø
$ NODAL DATA
    1      ØØ.Ø      ØØ.Ø      ØØ.Ø    1    1    1    1    1    1
    2       5.Ø      ØØ.Ø      ØØ.Ø    1    1    1    1    1    1
    3      1Ø.Ø      ØØ.Ø      ØØ.Ø    1    1    1    1    1    1
    4      1Ø.Ø       5.Ø      ØØ.Ø    1    1    1    1    1    1
    5      1Ø.Ø      1Ø.Ø      ØØ.Ø    1    1    1    1    1    1
    6       5.Ø      1Ø.Ø      ØØ.Ø    1    1    1    1    1    1
    7      ØØ.Ø      1Ø.Ø      ØØ.Ø    1    1    1    1    1    1
    8      ØØ.Ø       5.Ø      ØØ.Ø    1    1    1    1    1    1
    9       5.Ø       5.Ø      ØØ.Ø    1    1    1    1    1    1
   1Ø      ØØ.Ø      ØØ.Ø       5.Ø                   1    1    1
   11       5.Ø      ØØ.Ø       5.Ø                   1    1    1
   12      1Ø.Ø      ØØ.Ø       5.Ø                   1    1    1
   13      1Ø.Ø       5.Ø       5.Ø                   1    1    1
   14      1Ø.Ø      1Ø.Ø       5.Ø                   1    1    1
   15       5.Ø      1Ø.Ø       5.Ø                   1    1    1
   16      ØØ.Ø      1Ø.Ø       5.Ø                   1    1    1
   17      ØØ.Ø       5.Ø       5.Ø                   1    1    1
   18       5.Ø       5.Ø       5.Ø                   1    1    1
   19      ØØ.Ø      ØØ.Ø      1Ø.Ø                   1    1    1
   2Ø       5.Ø      ØØ.Ø      1Ø.Ø                   1    1    1
   21      1Ø.Ø      ØØ.Ø      1Ø.Ø                   1    1    1
   22      1Ø.Ø       5.Ø      1Ø.Ø                   1    1    1
   23      1Ø.Ø      1Ø.Ø      1Ø.Ø                   1    1    1
   24       5.Ø      1Ø.Ø      1Ø.Ø                   1    1    1
   25      ØØ.Ø      1Ø.Ø      1Ø.Ø                   1    1    1
   26      ØØ.Ø       5.Ø      1Ø.Ø                   1    1    1
   27       5.Ø       5.Ø      1Ø.Ø                   1    1    1
$ ELEMENT CONNECTIVITY
    4    1    1    2    9    8   1Ø   11   18   17    1
    4    2    2    3    4    9   11   12   13   18    1
    4    3    9    4    5    6   18   13   14   15    1
    4    4    8    9    6    7   17   18   15   16    1
    4    5   1Ø   11   18   17   19   2Ø   27   26    1
    4    6   11   12   13   18   2Ø   21   22   27    1
    4    7   18   13   14   15   27   22   23   24    1
    4    8   17   18   15   16   26   27   24   25    1
$ ELEMENT MATERIAL PROPERTIES
    1    1
    1.ØE+7        Ø.3        Ø.Ø   2.349E-4
```

STARS output summary - The output summary is presented in table 7.

TABLE 7. - NATURAL FREQUENCIES OF A SOLID CUBE (2 × 2 MESH)

Mode number	Natural frequency parameter $\hat{\omega} = \omega/(E/\rho)^{1/2}$, rad/sec	Exact solution $\hat{\omega}$
1	0.07378	0.06801
2	0.07378	0.06801
3	0.09994	0.09288
4	0.1695	0.1611
5	0.21001	0.1819
6	0.21001	0.1819

4.7 Spinning Cantilever Beam: Vibration Analysis

A cantilever beam spinning about the Y-axis is shown in figure 12.

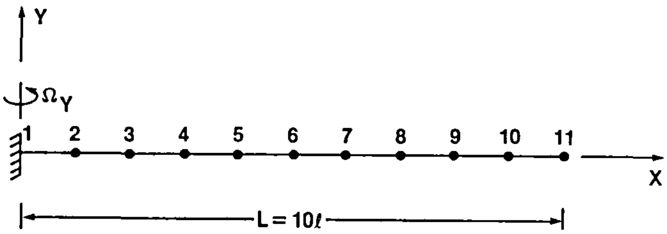


Figure 12. Spinning cantilever beam.

Important data — The structure is assumed to possess both viscous and structural damping.

Young's modulus, E	= 30×10^6
Cross-sectional area, A	= 1.0
Moment of inertia:	
About Y-axis	= 1/12
About Z-axis	= 1/24
Element length, ℓ	= 6
Nodal translational mass	= 1
Nodal mass moment of inertia	= 1/35
Scalar viscous damping	= 0.628318
Structural damping coefficient	= 0.01
Spin rate, Hz	= 0.1

STARS input data —

```
SPINNING CANTILEVER BEAM - 1Ø ELEMENT IDEALIZATION - VISC AND STRUCT DAMPING
12,1Ø,1,4,1,Ø,Ø,Ø,Ø
5,1,4,1,Ø,Ø,Ø
Ø,1,1,2
1,6,Ø,5ØØ.,Ø.,Ø
Ø.Ø1
$ NODAL DATA
    1       Ø.Ø       Ø.Ø       Ø.Ø    1    1    1    1    1    1
    2       6.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
    3      12.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
    4      18.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
    5      24.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
    6      3Ø.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
    7      36.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
    8      42.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
    9      48.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
   1Ø      54.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
   11      6Ø.Ø       Ø.Ø       Ø.Ø    Ø    Ø    Ø    Ø    Ø    Ø
   12      25.Ø      15.Ø       Ø.Ø    1    1    1    1    1    1
$ ELEMENT CONNECTIVITY
    1    1    1    2   12    Ø    Ø    Ø    Ø    Ø    1    1    Ø    Ø    1    Ø
    1   1Ø   1Ø   11   12    Ø    Ø    Ø    Ø    Ø    1    1    Ø    Ø    1    1
$ LINE ELEMENT BASIC PROPERTIES
    1       1.Ø    Ø.125ØØØ.Ø83333330.Ø4166667
$ ELEMENT MATERIAL PROPERTIES
    1    1
  3Ø.ØE+Ø6       Ø.3Ø
$ ELEMENT SPIN RATE DATA
    1       Ø.Ø   Ø.628318        Ø.Ø
$ NODAL MASS DATA
    2    1         1.Ø    3
    3    1         1.Ø    3
    4    1         1.Ø    3
    5    1         1.Ø    3
    6    1         1.Ø    3
    7    1         1.Ø    3
    8    1         1.Ø    3
    9    1         1.Ø    3
   1Ø    1         1.Ø    3
   11    1         1.Ø    3
    2    4 Ø.Ø285714    6
    3    4 Ø.Ø285714    6
    4    4 Ø.Ø285714    6
    5    4 Ø.Ø285714    6
    6    4 Ø.Ø285714    6
    7    4 Ø.Ø285714    6
    8    4 Ø.Ø285714    6
    9    4 Ø.Ø285714    6
   1Ø    4 Ø.Ø285714    6
   11    4 Ø.Ø285714    6
   -1
$ VISCOUS DAMPING DATA
Ø.ØØØØE+ØØØ.ØØØØE+ØØØ.ØØØØE+ØØØ.ØØØØE+ØØØ.ØØØØE+ØØØ.ØØØØE+ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØØØ.628318ØØ
Ø.ØØØØE+ØØØ.ØØØØE+ØØØ.ØØØØE+ØØØ.ØØØØE+ØØØ.ØØØØE+ØØØ.ØØØØE+ØØ
```

STARS output summary — The output summary is presented in table 8.

TABLE 8. - SPINNING CANTILEVER BEAM

Mode	Structure without damping (IPROB = 2)	Structure with viscous damping (IPROB = 4)	Structure with viscous and structural damping (IPROB = 5)
1	2.4319	$-0.3093 \pm 2.3919i^*$	$-0.3194 \pm 2.3868i^*$
2	3.4379	$-0.3119 \pm 3.4093i^*$	$-0.3277 \pm 3.4041i^*$
3	15.3113	$-0.3167 \pm 15.3048i^*$	$-0.3918 \pm 15.2935i^*$
4	21.6549	$-0.3166 \pm 21.6502i^*$	$-0.4246 \pm 21.6438i^*$
5	43.0587	$-0.3202 \pm 43.0563i^*$	$-0.5322 \pm 43.0820i^*$
6	60.8850	$-0.3202 \pm 60.8833i^*$	$-0.6243 \pm 60.8726i^*$

Note: Natural frequencies for various problem types for a spin rate Ω = 0.1 Hz (0.6283 rad/sec) $\left(i^* = \sqrt{-1}\right)$

4.8 Spinning Cantilever Plate: Vibration Analysis

The cantilever plate model described in section 4.4 is chosen for this sample problem. The plate is spun along the Z-axis with a uniform spin rate $\Omega_Z = 0.8*\omega_N^1$, ω_N^1 being the first natural frequency of vibration of the nonrotating plate. Table 9 provides the first few natural frequencies of the plate in nondimensional form, ω being the natural frequencies. Also presented in the table are the results of the free-vibration analysis of the plate rotating along an arbitrary axis, the spin rate being $\Omega_R \approx 0.8*\omega_N^1$, with components $\Omega_X = \Omega_Y = \Omega_Z \approx 0.8\omega_N^1/\sqrt{3}$.

STARS input data —

```
SQUARE 4-BY-4 PLATE    SPINNING STRUCTURE
25,16,1,4,Ø,1,Ø,Ø,Ø
2,1,4,1,Ø,Ø,Ø
Ø,Ø,1,2
1,6,Ø,6ØØØ.Ø,Ø.Ø,Ø
$ NODAL DATA
     1     -5.Ø      Ø.Ø      Ø.Ø   1   1   1   1   1   1   Ø
     5      5.Ø      Ø.Ø      Ø.Ø   1   1   1   1   1   1   1
     6     -5.Ø      2.5      Ø.Ø   Ø   Ø   Ø   Ø   Ø   Ø   Ø
    1Ø      5.Ø      2.5      Ø.Ø   Ø   Ø   Ø   Ø   Ø   Ø   1
    11     -5.Ø      5.Ø      Ø.Ø   Ø   Ø   Ø   Ø   Ø   Ø   Ø
    15      5.Ø      5.Ø      Ø.Ø   Ø   Ø   Ø   Ø   Ø   Ø   1
    16     -5.Ø      7.5      Ø.Ø   Ø   Ø   Ø   Ø   Ø   Ø   Ø
    2Ø      5.Ø      7.5      Ø.Ø   Ø   Ø   Ø   Ø   Ø   Ø   1
    21     -5.Ø     1Ø.Ø      Ø.Ø   Ø   Ø   Ø   Ø   Ø   Ø   Ø
    25      5.Ø     1Ø.Ø      Ø.Ø   Ø   Ø   Ø   Ø   Ø   Ø   1
$ ELEMENT CONNECTIVITY
    2    1    1    2    7    6   Ø   Ø   Ø   Ø   1   1   Ø   Ø   1
    2    4    4    5   1Ø    9   Ø   Ø   Ø   Ø   1   1   Ø   Ø   1
    2    5    6    7   12   11   Ø   Ø   Ø   Ø   1   1   Ø   Ø   1
    2    8    9   1Ø   15   14   Ø   Ø   Ø   Ø   1   1   Ø   Ø   1
    2    9   11   12   17   16   Ø   Ø   Ø   Ø   1   1   Ø   Ø   1
    2   12   14   15   2Ø   19   Ø   Ø   Ø   Ø   1   1   Ø   Ø   1
    2   13   16   17   22   21   Ø   Ø   Ø   Ø   1   1   Ø   Ø   1
    2   16   19   2Ø   25   24   Ø   Ø   Ø   Ø   1   1   Ø   Ø   1
$ SHELL ELEMENT THICKNESSES
    1      Ø.1
$ ELEMENT MATERIAL PROPERTIES
    1    1
   1.ØE+Ø7      Ø.3Ø        Ø.Ø   Ø.259E-3
$ ELEMENT SPIN RATE DATA
    1       Ø.Ø        Ø.Ø      17Ø.86
```

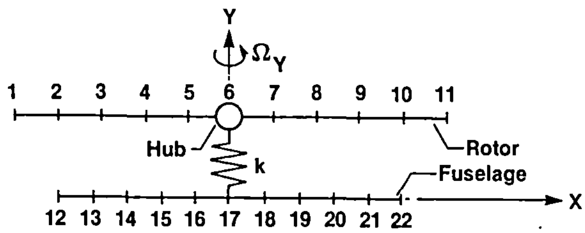
STARS output summary — The output summary is presented in table 9.

TABLE 9. - NATURAL FREQUENCY PARAMETERS OF A SPINNING SQUARE CANTILEVER PLATE

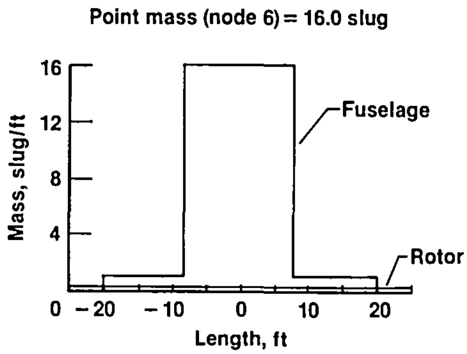
Mode number	Natural frequency of parameter $\gamma = \omega \ell^2 \sqrt{\rho t/D}$	
	$\Omega_Z = 0.8\omega_N^1$, 170.86 rad/sec	Ω_R = 170.86 rad/sec, $\Omega_X = \Omega_Y = \Omega_Z$ = 98.65 rad/sec
1	10.6103	7.4377
2	16.4093	13.4362
3	29.5585	26.4286
4	32.9242	30.3492
5	39.2103	36.1341
6	58.3640	56.2620

4.9 Helicopter Structure: Vibration Analysis

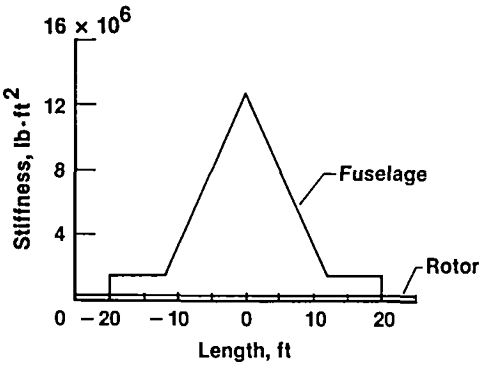
A coupled helicopter rotor-fuselage system is shown in figure 13 (ref. 8), along with relevant stiffness and mass distributions, which are suitably approximated for the discrete-element modeling of the structure. Numerical free-vibration analysis was performed for the structure with the rotor spinning at 10 rad/sec ($\Omega_Y = 10$); such results are presented in table 10, along with the results for the corresponding nonspinning case.



(a) Discrete element model.



(b) Structural mass distribution.



(c) Structural stiffness distributions.

Figure 13. Coupled helicopter rotor-fuselage system.

STARS input data —

```
   HELICOPTER BLADES AND FUSELAGE ANALYSIS TEST
23,21,6,4,2,0,0,0,0
2,0,4,1,0,0,0
0,1,1,2
1,12,0,200.0,0.0,0
$ NODAL DATA
    1      -25.0        1.0        0.0     0     0     1     1     1     0     0
    5       -5.0        1.0        0.0     0     0     1     1     1     0     1
    6        0.0        1.0        0.0     0     0     1     1     1     0     0
   11       25.0        1.0        0.0     0     0     1     1     1     0     1
   12      -20.0        0.0        0.0     0     0     1     1     1     0     0
   22       20.0        0.0        0.0     0     0     1     1     1     0     1
   23       10.0        0.5        0.0     1     1     1     1     1     1     0
$ ELEMENT CONNECTIVITY
    1    1    1    2   23    0    0    0    0    0    1    1    0    0    1    0
    1   10   10   11   23    0    0    0    0    0    1    1    0    0    1    1
    1   11   12   13   23    0    0    0    0    0    2    1    0
    1   12   13   14   23    0    0    0    0    0    2    1    0
    1   13   14   15   23    0    0    0    0    0    3    1    0
    1   14   15   16   23    0    0    0    0    0    4    1    0
    1   15   16   17   23    0    0    0    0    0    5    1    0
    1   16   17   18   23    0    0    0    0    0    5    1    0
    1   17   18   19   23    0    0    0    0    0    4    1    0
    1   18   19   20   23    0    0    0    0    0    3    1    0
    1   19   20   21   23    0    0    0    0    0    2    1    0
    1   20   21   22   23    0    0    0    0    0    2    1    0
    1   21    6   17   23    0    0    0    0    0    6    2    0
$ LINE ELEMENT BASIC PROPERTIES
    1      1.0        1.0        1.0        1.0
    2    100.0        1.0        1.0        1.0
$ ELEMENT MATERIAL PROPERTIES
    1    1
    2.0E05        0.3        0.0        0.3
    2    1
   1.53E06        0.3        0.0       1.23
    3    1
   3.45E06        0.3        0.0       1.23
    4    1
   7.29E06        0.3        0.0       16.0
    5    1
  11.30E06        0.3        0.0       16.0
    6    1
    1.0E08        0.3        0.0        0.0
$ ELEMENT SPIN RATE DATA
    1      0.0       10.0        0.0
$ NODAL MASS DATA
    6    1      16.0
    6    2      16.0
    6    3      16.0
   -1
```

STARS output summary — The output summary is presented in table 10.

TABLE 10. — NATURAL FREQUENCIES OF A HELICOPTER STRUCTURE

Mode number	Natural frequencies, spin rates		Mode shape
	$\Omega_Y = 0$	$\Omega_Y = 10$	
1,2,3	0	0	Rigid body
4	4.645	11.83	Rotor 1st antisymmetric bending
5	5.093	11.90	Rotor 1st symmetric bending
6	23.088	23.19	Fuselage 1st bending
7	27.93	36.98	Rotor 2nd antisymmetric bending
8	28.22	38.16	Rotor 2nd symmetric bending
9	38.40	39.29	Rotor 3rd antisymmetric bending

4.10 Rocket Structure: Dynamic Response Analysis

A rocket is simply idealized by two line elements, as shown in figure 14 (ref. 4), which is subjected to a pulse loading function at the base. Results of the dynamic response analysis follow.

(a) Rocket structure.

(b) Pulse loading.

Figure 14. Rocket subjected to dynamic loading.

Important data parameters — Arbitrary element and material properties data are assumed for the analysis to correlate results with available ones expressed in parametric form.

Young's modulus, E	=	100
Poisson's ratio, μ	=	0.3
Area of cross-section, A	=	1.0
Mass density, ρ	=	1.0
Length of an element, ℓ	=	5.0
Pulse load intensity, P_0	=	10.0
Duration of load, sec	=	1.0
Total time period for response evaluation	=	2.0

STARS input data —

```
DYNAMIC RESPONSE CASE - PRZEMIENIECKI
4,2,1,4,1,Ø,Ø,Ø,Ø
1,1,2,Ø,1,Ø,Ø
Ø,Ø,Ø,2
1,3,Ø,2Ø.Ø,Ø.Ø,Ø
Ø,1,1,2
$ NODAL DATA
    1      Ø.Ø      Ø.Ø      Ø.Ø    Ø    1    1    1    1    1
    2      5.Ø      Ø.Ø      Ø.Ø    Ø    1    1    1    1    1
    3     1Ø.Ø      Ø.Ø      Ø.Ø    Ø    1    1    1    1    1
    4      5.Ø      5.Ø      Ø.Ø    1    1    1    1    1    1
$ ELEMENT CONNECTIVITY
    1    1    1    2    4    Ø    Ø                   1    1
    1    2    2    3    4    Ø    Ø                   1    1
$ LINE ELEMENT BASIC PROPERTIES
    1      1.Ø      Ø.Ø      Ø.Ø      Ø.Ø
$ ELEMENT MATERIAL PROPERTIES
    1    1
     1ØØ.Ø     Ø.3      Ø.Ø      1.Ø
$ DYNAMIC NODAL FORCE DATA
           1.Ø
    1    1      1Ø.Ø
   -1
$ INCREMENTAL TIME DATA FOR DYNAMIC RESPONSE ANALYSIS
          Ø.1Ø    1Ø
          Ø.2Ø     5
```

STARS analysis results at typical time-steps –

DYNAMIC RESPONSE AT TIME =Ø.7ØØØE+ØØ

NODE	X-DISPL.	Y-DISPL.	Z-DISPL.	X-ROTN.	Y-ROTN.	Z-ROTN.
1	Ø.74468ØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
2	Ø.194953E+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
3	-Ø.139465E+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
4	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ

ELEMENT STRESSES

ELEMENT NO.	END1 / END5	END2 / END6	END3 / END7	END4 / END8	PX1/PX2 SXT SXX	PY1/PY2 SYT SYY	PZ1/PZ2 SXYT SZZ	MX1/MX2 SXB SXY	MY1/MY2 SYB SYZ	MZ1/MZ2 SXYB SZX
1	1	2			Ø.811733E+Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
					-Ø.811733E+Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
2	2	3			Ø.498461E+Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
					-Ø.498461E+Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØE+ØØ	Ø.ØØØØØØE+ØØ

DYNAMIC RESPONSE AT TIME =Ø.12ØØE+Ø1

NODE	X-DISPL.	Y-DISPL.	Z-DISPL.	X-ROTN.	Y-ROTN.	Z-ROTN.
1	Ø.1Ø7ØØ8E+Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
2	Ø.664217E+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
3	Ø.414986E+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
4	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ

ELEMENT STRESSES

ELEMENT NO.	END1 / END5	END2 / END6	END3 / END7	END4 / END8	PX1/PX2 SXT SXX	PY1/PY2 SYT SYY	PZ1/PZ2 SXYT SZZ	MX1/MX2 SXB SXY	MY1/MY2 SYB SYZ	MZ1/MZ2 SXYB SZX
1	1	2			Ø.1Ø9945E+Ø2	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
					-Ø.1Ø9945E+Ø2	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
2	2	3			Ø.668836E+Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ
					-Ø.668836E+Ø1	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ	Ø.ØØØØØØE+ØØ

4.11 Plate, Beam, and Truss Structures: Buckling Analysis

A buckling analysis was performed for a simply supported square plate model, described in section 4.4, subjected to a uniform unit stress acting along the two edges parallel to the y-axis; relevant input data and analysis results are as follows.

STARS input data —

```
SQUARE 4-BY-4 PLATE - BUCKLING ANALYSIS
25,16,1,4,Ø,1,Ø,Ø,Ø
9,1,2,1,Ø,Ø,Ø
I,Ø,Ø,2
1,1,Ø,2ØØØØ.Ø,Ø.Ø,Ø
$ NODAL DATA
   1    -5.ØØ     Ø.Ø      Ø.Ø   Ø  1  1  Ø  Ø  1  Ø
   2    -2.5Ø     Ø.Ø      Ø.Ø   1  1  1  Ø  Ø  1
   4     2.5Ø     Ø.Ø      Ø.Ø   1  1  1  Ø  Ø  1  1
   5     5.ØØ     Ø.Ø      Ø.Ø   Ø  1  1  Ø  Ø  1  Ø
   6    -5.ØØ     2.5Ø     Ø.Ø   Ø  1  1  Ø  Ø  1  Ø
   7    -2.5Ø     2.5Ø     Ø.Ø   Ø  Ø  Ø  Ø  Ø  1
   9     2.5Ø     2.5Ø     Ø.Ø   Ø  Ø  Ø  Ø  Ø  1  1
  1Ø     5.ØØ     2.5Ø     Ø.Ø   Ø  1  1  Ø  Ø  1
  11    -5.ØØ     5.ØØ     Ø.Ø   Ø  1  1  Ø  Ø  1  Ø
  12    -2.5Ø     5.ØØ     Ø.Ø   Ø  Ø  Ø  Ø  Ø  1
  14     2.5Ø     5.ØØ     Ø.Ø   Ø  Ø  Ø  Ø  Ø  1  1
  15     5.ØØ     5.ØØ     Ø.Ø   Ø  1  1  Ø  Ø  1
  16    -5.ØØ     7.5Ø     Ø.Ø   Ø  1  1  Ø  Ø  1  Ø
  17    -2.5Ø     7.5Ø     Ø.Ø   Ø  Ø  Ø  Ø  Ø  1
  19     2.5Ø     7.5Ø     Ø.Ø   Ø  Ø  Ø  Ø  Ø  1  1
  2Ø     5.ØØ     7.5Ø     Ø.Ø   Ø  1  1  Ø  Ø  1
  21    -5.ØØ    1Ø.ØØ     Ø.Ø   Ø  1  1  Ø  Ø  1  Ø
  22    -2.5Ø    1Ø.ØØ     Ø.Ø   1  1  1  Ø  Ø  1
  24     2.5Ø    1Ø.ØØ     Ø.Ø   1  1  1  Ø  Ø  1  1
  25     5.ØØ    1Ø.ØØ     Ø.Ø   Ø  1  1  Ø  Ø  1
$ ELEMENT CONNECTIVITY
   2   1   1   2   7   6  Ø  Ø  Ø  Ø  1  1  Ø  Ø  Ø
   2   4   4   5  1Ø   9  Ø  Ø  Ø  Ø  1  1  Ø  Ø  Ø  1
   2   5   6   7  12  11  Ø  Ø  Ø  Ø  1  1  Ø  Ø  Ø
   2   8   9  1Ø  15  14  Ø  Ø  Ø  Ø  1  1  Ø  Ø  Ø  1
   2   9  11  12  17  16  Ø  Ø  Ø  Ø  1  1  Ø  Ø  Ø
   2  12  14  15  2Ø  19  Ø  Ø  Ø  Ø  1  1  Ø  Ø  Ø  1
   2  13  16  17  22  21  Ø  Ø  Ø  Ø  1  1  Ø  Ø  Ø
   2  16  19  2Ø  25  24  Ø  Ø  Ø  Ø  1  1  Ø  Ø  Ø  1
$ SHELL ELEMENT THICKNESSES
   1      Ø.1
$ ELEMENT MATERIAL PROPERTIES
   1   1
  1.ØE+Ø7      Ø.3Ø
$ NODAL LOAD DATA
   1   1     .125
   6   1     .25Ø
  11   1     .25Ø
  16   1     .25Ø
  21   1     .125
   5   1    -.125
  1Ø   1    -.25Ø
  15   1    -.25Ø
  2Ø   1    -.25Ø
  25   1    -.125
  -1
```

STARS analytical results - The analytical results are presented in table 11.

TABLE 11. - CRITICAL LOAD OF A SIMPLY SUPPORTED SQUARE PLATE

Mode number	Buckling load parameter	
	STARS solution	Analytical solution
1	361.6305	361.5240

The cantilever beam defined in section 4.7 is the subject of the buckling analysis; the relevant details are given below.

STARS input data —

```
CANTILEVER BEAM - 1Ø ELEMENT IDEALIZATION - BUCKLING ANALYSIS
12,1Ø,1,4,1,Ø,Ø,Ø,Ø
9,1,2,1,Ø,Ø,Ø
1,Ø,Ø,2
1,1,Ø,12ØØØ.,Ø.,Ø
$ NODAL DATA
    1       Ø.Ø        Ø.Ø        Ø.Ø    1    1    1    1    1    1
    2       6.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
    3      12.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
    4      18.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
    5      24.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
    6      3Ø.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
    7      36.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
    8      42.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
    9      48.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
   1Ø      54.Ø        Ø.Ø        Ø.Ø    Ø    Ø    1    1    1    Ø
   11      6Ø.Ø        Ø.Ø        Ø.Ø    Ø    1    1    1    1    Ø
   12      25.Ø       15.Ø        Ø.Ø    1    1    1    1    1    1
$ ELEMENT CONNECTIVITY
    1    1    1    2   12    Ø    Ø    Ø    Ø    Ø    1    1    Ø    Ø    1    Ø
    1   1Ø   1Ø   11   12    Ø    Ø    Ø    Ø    Ø    1    1    Ø    Ø    1    1
$ LINE ELEMENT BASIC PROPERTIES
    1       1.Ø      Ø.125   Ø.Ø83333   Ø.Ø41667
$ ELEMENT MATERIAL PROPERTIES
    1    1
  3Ø.ØE+Ø6       Ø.3Ø
$ NODAL LOAD DATA
   11    1       -1.Ø
   -1
```

STARS analytical results — The analytical results are presented in table 12.

TABLE 12. - CRITICAL LOAD OF A CANTILEVER BEAM

Mode number	Buckling load parameter	
	STARS solution	Analytical solution
1	7011.2935	7010.4223

A simple truss (fig. 15) (ref. 4) is also analyzed to determine the critical loads. The associated input data and analytical results are given below.

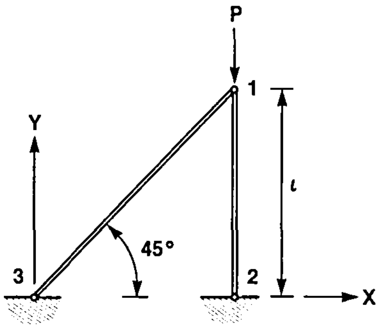


Figure 15. Truss structure.

STARS input data —

```
PRZ - TRUSS BUCKLING ANALYSIS
4,2,1,4,1,Ø,Ø,Ø,Ø
9,1,2,1,Ø,Ø,Ø
1,Ø,Ø,2
1,2,Ø,2ØØØØ.Ø,Ø.Ø,Ø
$ NODAL DATA
    1      1ØØ.Ø       1ØØ.Ø         Ø.Ø    Ø    Ø    1    1    1    1
    2      1ØØ.Ø         Ø.Ø         Ø.Ø    1    1    1    1    1    1
    3        Ø.Ø         Ø.Ø         Ø.Ø    1    1    1    1    1    1
    4        Ø.Ø        5Ø.Ø         Ø.Ø    1    1    1    1    1    1
$ ELEMENT CONNECTIVITY
    1    1    3    1    4    1    1    Ø    Ø    Ø    1    1
    1    2    2    1    4    1    1    Ø    Ø    Ø    1    1
$ LINE ELEMENT BASIC PROPERTIES
    1      Ø.1
$ ELEMENT MATERIAL PROPERTIES
    1    1
   1Ø.ØEØ3       Ø.2
$ NODAL LOAD DATA
    1    2        -1.Ø
   -1
```

STARS analysis results — The analytical results are presented in table 13.

TABLE 13. - CRITICAL LOAD OF A SIMPLE TRUSS

Mode number	Buckling load parameter	
	STARS solution	Analytical solution
1	261.20389	261.20387

5. SYSTEM DESCRIPTION

To log on to the system, the relevant procedure is dependent on the type of computer in which the program is residing. Thus as an example, if the user wishes to activate the program on the HMC (Harvey Mudd College) VAX 11 computing system, the user name KGUPTA and the current password have to be utilized to gain admittance to the main system. Once logged in, the user is placed immediately in the [KGUPTA] directory. At the same time, the file LOGIN.COM is automatically activated, enabling utilization of useful commands such as copying and deleting files and moving from one subdirectory to another. The [KGUPTA] directory lists all STARS subdirectories, the five major ones being MAINIB, EIGSOL, RESPONSE, OBJECT, and EXE. Details of these subdirectories, as well as other relevant information, are given next; the definition of various parameters can be found in section 3.

5.1 MAINIB Subdirectory

The MAINIB subdirectory contains all the subroutines (source codes) that form the input module. A brief description of these subroutines is given here.

ADMAT	adds submatrices to form stiffness matrix of shell elements
ASEMBL	assembles various element matrices in GCS
BANMIN	minimizes bandwidth of system matrices by nodal renumbering
BLOCK1	rewrites system matrices in predetermined block format in real single precision data
BLOCK2	as in BLOCK1 with data in real double precision
BLOCK3	as in BLOCK1 with data in complex single precision
BLOCK4	as in BLOCK1 with data in complex double precision
BLOCKX	calls BLOCK1 through BLOCK4 depending on type of data precision required
CFV	a general routine to compute centrifugal force vector in GCS for various elements owing to spin
COMFILE	common statement which passes files NTK (stiffness matrix), NTMS (inertia matrix), etc.
COMPMIB	command file which compiles an individual subroutine in MAINIB subdirectory
COMRES	common statement which passes IDRS, IUV, IDDI, NTTS, TF, and DELT, etc., pertaining to dynamic response analysis
CORESIZE	sets NAC (available core size) value
CORTS	obtains coordinates of triangular shell element in LCS

DIRCOS	computes direction cosine matrix for any general finite element
EDINPT	reads element input data
ELIM	forms stiffness matrix of a quadrilateral element after eliminating effect of fifth node at centroid
GCINPT	reads input data defined in GCS
GEO	computes basic parameters used by subroutines TPBKG and TPKG
HEXCOR	determines vertices of each of six tetrahedrons forming a hexahedron
INP	reads user data input of system matrices and stores in NFILE in blocks of six rows by M11 (half-bandwidth) columns
ISAMAX	a subroutine used in connection with matrix inversion; finds the index of element having maximum absolute value
LINEK	forms stiffness matrix, pressure, and temperature load matrices in LCS for line elements
LINEK4	forms higher-order stiffness matrix in LCS for line elements
LINEKG	forms geometrical stiffness matrix in LCS for line elements
LINEM	forms mass matrix in LCS for line elements
LINEM2	forms higher-order mass matrix in LCS for line elements
LINEML	forms lumped mass matrix in LCS for line elements (translational mass only in GCS)
LNCCKP	obtains $\mathbf{C_C}$ (Coriolis) and $\mathbf{K}^{*}$ (centrifugal force) matrices in GCS
MAINBN	converts generated system matrices into predetermined block format
MAINI	main input link driver, which calls major subroutines to form system matrices
MAINIB	primary driver program for MAINI and MAINBN
MASEM	combines 6 × 6 and 9 × 9 matrices into 18 × 18 matrix
MMULT	performs matrix multiplication
NASEM	general routine for matrix assembly; places element matrix (**VS**(N1, N1)) of arbitrary dimension into system matrix **V**(6, M11)
NODCON	effects nodal conversion after bandwidth minimization operations
NODCOR	reads nodal coordinate data at random, sets final data in sequence
QCA	generates coordinates of centroid of a quadrilateral element in GCS

QDR	develops quadrilateral element K, K_G, and M matrices
QDRASM	assembles triangular element matrices to form quadrilateral element matrix in LCS
QSHCF	computes centrifugal forces in a thin quadrilateral shell element owing to spin about the X-, Y-, and Z-axes in GCS; called by CFV
SAXPY	a subroutine for matrix inversion; effects constant times a vector plus a vector
SGEDI	a subroutine for matrix inversion; computes the determinant and inverse of a matrix
SGEFA	a subroutine for matrix inversion; factors a real matrix by Gaussian elimination
SPINAV	calculates nodal spin rates by averaging such values of connecting elements
SSCAL	a subroutine for matrix inversion; scales a vector by a constant
SSWAP	a subroutine for matrix inversion; interchanges two vectors
TESM	generates triangular element submatrices containing nodal coordinate data
TETKTP	generates tetrahedron stiffness, temperature, and pressure matrices in LCS
TETM	generates tetrahedron mass matrix in LCS
TMP	forms triple matrix product of order 3
TPBK	generates stiffness matrix for plate bending element
TPBKG	forms geometrical stiffness matrix for triangular plate bending element
TPKG	forms geometrical stiffness matrix for triangular plane-stress/plane-strain element
TPLK	generates stiffness matrix for plane triangular element
TSCTL	calculates X2, X3, and Y3 coordinate data in LCS for triangular shell element
TSHCF	computes centrifugal forces in element LCS in a thin triangular shell element owing to spin in GCS about the X-, Y-, and Z-axes; called by CFV
TSHK	forms triangular-shell element stiffness matrix
TSHM	forms mass matrix for triangular-shell element by appropriately combining corresponding plane-stress and plate-bending elements

TSHTP	generates triangular-shell element temperature and pressure matrices
UNIT	normalizes a vector
VECPRO	obtains a vector cross product
WTBR1	writes on BR (load etc.) matrix in real single precision
WTBR2	writes on BR matrix in real double precision
WTBR3	writes on BR matrix in complex single precision
WTBR4	writes on BR matrix in complex double precision

5.2 EIGSOL Subdirectory

This subdirectory contains all the subroutines (source codes) that form the eigenvalue solution and linear simultaneous equation solver module. These subroutines are described below.

BANMAT	obtains solution of simultaneous equations EX = B, E being either Hermitian or real symmetric; B is a NC set of vectors, X being the corresponding solution stored in B
BISECN	isolates desired first NR roots lying within bound PU, PL
COMDIMV	a common statement passing some basic integer variables (NAC, etc.)
COMEIGBIS	passes integer arrays common to both EIGSOL and BISECN routines
COMFIL	a common statement passing files NTK, NTMS, etc.
COMFILE	as in COMFIL, but passing additional files
COMPEIGS	command file which compiles all subroutines
COMPARM	common statement passing integer variables
CORESIZE	sets NAC value
EIGNV	computes number of roots in system smaller than P
EIGSOL	main driver routine for this link
EIGSS	main subroutine in EIGSOL
INPUT	reads and stores matrix data input in block format
MULT	forms A1 - XMULT*A2; A1 and A2 are matrices, XMULT being a factor
VECMLT	multiplies two vectors
VECTOR	determines eigenvalue and eigenvector using special inverse iteration scheme

VMULT	multiplies a matrix and a vector
VORTHO	orthogonalizes a set of independent vectors belonging to repeated roots

5.3 RESPONSE Subdirectory

The RESPONSE subdirectory contains the subroutines (source code) for the third link of STARS which are listed below.

CDP	reads complex double precision data into BR matrix
COMPRES	command file which compiles all subroutines
CSP	reads complex single-precision data into BR matrix
DYNRES	obtains dynamic response using modal superposition
MSNRM	effects mass orthonormalization of eigenvectors
RDP	reads real double-precision data into BR matrix
RESCDP	reads all response related data (U0, V0, TZ, F, or A) and converts them into complex double precision
RESCSP	as in RESCDP with data in complex single precision
RESPONSE	main driver for this link
RESRDP	as in RESCDP with data in real double precision
RESRSP	as in RESCDP with data in real single precision
RSP	reads real single-precision data into BR matrix
STRESS	calculates element stresses

5.4 OBJECT Subdirectory

The OBJECT subdirectory contains the following object files and object libraries.

MAINIB.OBJ	EIGSOL.OBJ	RESPONSE.OBJ
MAINIB.LRS	EIGSOL.LRS	RESPONSE.LRS
	EIGSOL.LRL	RESPONSE.LRL
	EIGSOL.LCS	RESPONSE.LCS
	EIGSOL.LCM	RESPONSE.LCM

Note: The file specification .OBJ indicates an object (compiled) file of the relevant primary driver routine, and all other file specifications indicate object library files, containing compiled versions of the various subroutines.

LRS	real single-precision (ORS) version
LRL	real double-precision (ORL) version
LCS	complex single-precision (OCS) version
LCM	complex double-precision (OCM) version

The library format allows the user to readily identify the location of an object file. A full library documentation follows.

EIGSOL.LRS:	BANMAT, BISECN, EIGNV, EIGSS, INPUT, MULT, VECMLT, VECTOR, VMULT, VORTHO
EIGSOL.LRL:	contains (ORL) version of the object files in EIGSOL.LRS
EIGSOL.LCS:	contains (OCS) version of the object files in EIGSOL.LRS
EIGSOL.LCM:	contains (OCM) version of the object files in EGISOL.LRS
MAINIB.LRS:	ADMAT, ASEMBL, BANMIN, BLOCK1, BLOCK2, BLOCK3, BLOCK4, BLOCKX, CFV, CORTS, DIRCOS, EDINPT, ELIM, GCINPT, GEO, HEXCOR, INP, ISAMAX, LINEK, LINEK4, LINEKG, LINEM, LINEM2, LINEML, LNCCKP, MAINBN, MAINI, MASEM, MMULT, NASEM, NODCON, NODCOR, QCA, QDR, QDRASM, QSHCF, SAXPY, SGEDI, SGEFA, SPINAV, SSCAL, SSWAP, TESM, TETKTP, TETM, TMP, TPBK, TPBKG, TPKG, TPLK, TSCTL, TSHCF, TSHK, TSHM, TSHTP, UNIT, VECPRO, WTBR1, WTBR2, WTBR3, WTBR4
RESPONSE.LRS:	CDP, CSP, DYNRES, MSNRM, RDP, RESCDP, RESCSP, RESRDP, RESRSP, RSP, STRESS
RESPONSE.LRL:	contains (ORL) version of the object files in RESPONSE.LRS
RESPONSE.LCS:	contains (OCS) version of the object files in RESPONSE.LRS
RESPONSE.LCM:	contains (OCM) version of the object files in RESPONSE.LRS

5.5 EXE Subdirectory

The EXE subdirectory contains the following.

LKEIGSOL.COM	command file that links EIGSOL and creates EIGSOLRS.EXE, EIGSOLRL.EXE, EIGSOLCS.EXE, or EIGSOLCM.EXE, depending on data type
LKMAINIB.COM	command file that links MAINIB and creates MAINIB.EXE
LKRES.COM	command file that links RESPONSE and creates RESPRS.EXE, RESPRL.EXE, RESPCS.EXE, or RESPCM.EXE, depending on data type

The .EXE files created by the above .COM files are stored in the EXE subdirectory.

5.6 Editing, Compiling, Linking, and Executing STARS

To implement desired modifications in a subroutine in the program, any one of the edit modes available on the VAX computer (currently there are three: SOS, EDT, and TECO) may by suitably utilized. Once the editing is complete, all other versions of the file just edited should be deleted to save storage space and to protect the current version from deletion. The modified subroutine file is next compiled suitably.

To compile files resident in the MAINIB subdirectory, the COMPMIB.COM file (resident in the MAINIB subdirectory) is used. For example, to compile MAINI.FOR, the following instruction is used,

```
$ @COMPMIB MAINI
```

In general, any subroutine in the MAINIB subdirectory can be compiled by activating the COMPMIB.COM file as shown above; that is:

```
$ @COMPMIB file name
```

Compilation of subroutines in the EIGSOL subdirectory is done by activating the COMPEIGS.COM file (found in the EIGSOL subdirectory). To compile subroutine VORTHO.FOR, for example, the relevant instruction is

```
$ @COMPEIGS VORTHO ORS, ORL, OCS, or OCM
```

COMPEIGS.COM will copy corresponding procedure files (procedure files contain various parameter cards) depending on whether the object is ORS, ORL, OCS, or OCM. If the same file is compiled more than once, there is no need to recopy all the procedure files. To avoid copying again, the instruction "GO" is used at the end of the @COMP command string.

Compilation of routines resident in the RESPONSE subdirectory is done exactly as in the EGISOL subdirectory except that the COMPRES.COM file is utilized for that purpose.

After compiling the subroutine, the file must be properly linked so that the modifications can be incorporated into the new execution file. To link the MAINIB, EIGSOL, or RESPONSE subroutines, the appropriate command file in the EXECUTN subdirectory is activated. For example, after having compiled MAINI.FOR, a new MAINIB.EXE is created by the command,

```
$ @LKMAINIB
```

To link EIGSOL or RESPONSE, either LKEIGSOL or LKRES is activated while indicating the data type. For instance, for real single-precision data, such commands are as follows:

```
$ @LKEIGSOL ORS or $ @LKRES ORS
```

To execute or run STARS, the following procedure is adopted:

```
$ ASSIGN INPUT.DAT FOR005
$ ASSIGN MAINIB.OUT FOR006
$ RUN EXE:MAINIB
$ ASSIGN EIGSOL.OUT FOR006
$ RUN EXE:EIGSOL(RS, RL, CS, or CM)
$ ASSIGN RESP.OUT FOR006
$ RUN EXE:RESP(RS, RL, CS, or CM)
```

These commands may be entered separately or as part of a single command file.

Ames Research Center
Dryden Flight Research Facility
National Aeronautics and Space Admnistration
Edwards, California, March 26, 1984

REFERENCES

1. Collins, R. J.: Bandwidth Reduction by Automatic Renumbering. Int. J. Numer. Methods Eng., vol. 6, 1973, pp. 345-356.

2. Gupta, K. K.: Development of a Unified Numerical Procedure for Free Vibration Analysis of Structures. Int. J. Numer. Methods Eng., vol. 17, 1981, pp. 187-198.

3. Gupta, K. K.: On a Finite Dynamic Element Method for Free Vibration Analysis of Structures. Comput. Methods Appl. Mech. Eng., vol. 9, 1976, pp. 105-120.

4. Przemieniecke, J. S.: Theory of Matrix Structural Analysis. McGraw-Hill, New York, 1968.

5. Merian, L.: Statics. Second ed., John Wiley and Sons, 1971.

6. Ross, C. T. F.: Computer Methods in Engineering. Elis Howard Publishers, U. K., 1982.

7. Tezcan, S. S.: Application of Matrix Algebra to Problems of Plane Stress, Plane Strain, Bending of Plates and Cylindrical Shells. Wiss. Z. Hochsch. Architek. Bauw, Weimar, 1965.

8. Rutkowski, M. J.: The Vibration Characteristics of a Coupled Helicopter Rotor-Fuselage by a Finite Element Analysis. NASA TP-2118, 1983.

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15. Supplementary Notes

Program tapes are available from the author (Mail Stop D-OFS).

16. Abstract

STARS (STructural Analysis RoutineS) is primarily an interactive, graphics-oriented, finite-element computer program for analyzing the static, stability, free vibration, and dynamic responses of damped and undamped structures, including rotating systems. The element library consists of one-dimensional (1-D) line elements, two-dimensional (2-D) triangular and quadrilateral shell elements, and three-dimensional (3-D) tetrahedral and hexahedral solid elements. These elements enable the solution of structural problems that include truss, beam, space frame, plane, plate, shell, and solid structures, or any combination thereof. Associated algebraic equations are solved by exploiting inherent matrix sparsity. Zero, finite, and interdependent deflection boundary conditions can be implemented by the program. The associated dynamic response analysis capability provides for initial deformation and velocity inputs, whereas the transient excitation may be either forces or accelerations. An effective in-core or out-of-core solution strategy is automatically employed by the program, depending on the size of the problem. Data input may be at random within a data set, and the program offers certain automatic data-generation features. Input data are formatted as an optimal combination of free and fixed formats. Interactive graphics capabilities, using an Evans and Sutherland, Megatek, or any other suitable display terminal, enable convenient display of nodal deformations, mode shapes, and element stresses. The program, developed in modular form for easy modification, is written in FORTRAN for the VAX 11 computer, although earlier development was accomplished using a UNIVAC 1100 computer. Continued development of the program is envisaged, but with care exercised to limit its size (the program now consists of fewer than 12,000 programmed instructions). Applications of the program are anticipated in the fields of aerospace, mechanical, and civil engineering, among others.

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