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APPROXIMATION METHODS FOR INVERSE PROBLEMS INVOLVING THE VIBRATION OF BEAMS WITH TIP BODIES

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APPROXIMATION METHODS FOR INVERSE PROBLEMS

INVOLVING THE VIBRATION OF BEAMS WITH TIP BODIES*

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Abstract

We outline two cubic spline based approximation schemes for the estimation of structural parameters associated with the transverse vibration of flexible beams with tip appendages. The identification problem is formulated as a least squares fit to data subject to the system dynamics which are given by a hybrid system of coupled ordinary and partial differential equations. The first approximation scheme is based upon an abstract semigroup formulation of the state equation while a weak/variational form is the basis for the second. Cubic spline based subspaces together with a Rayleigh-Ritz-Galerkin approach was used to construct sequences of easily solved finite dimensional approximating identification problems. Convergence results are briefly discussed and a numerical example demonstrating the feasibility of the schemes and exhibiting their relative performance for purposes of comparison is provided.

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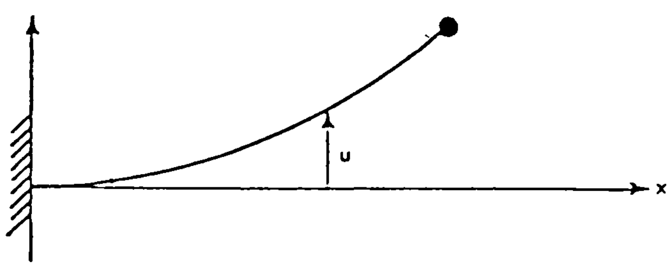
In this short paper we briefly outline two cubic spline based approximation schemes for the solution of inverse problems involving the vibration of flexible beams with attached tip bodies. The identification problem is formulated as the least squares fit to data of a hybrid system of coupled partial and ordinary differential equations describing the dynamics of the beam and tip bodies. The resulting optimization problem is infinite dimensional and as such, necessitates the use of some form of approximation. The schemes we have developed are based upon the construction of a sequence of approximating identification problems in which the underlying constraining state equations are semi-discrete finite dimensional approximations to the infinite dimensional distributed system which governs the original identification problem. Our study includes both theoretical convergence results and numerical testing.



Although our general approach applies to a broad class of problems, to illustrate the two methods, we consider the problem of identifying the spatially invariant flexural stiffness q_1 and linear mass density q_2 for a beam of length ℓ clamped at one end and cantilevered at the other with a tip (point) mass of magnitude q_3 which is also to be identified. Using the Euler-Bernoulli theory and elementary Newtonian mechanics, the system of

$$q_2 u_{tt}(t,x) = -q_1 u_{xxxx}(t,x) + f(t,x) \qquad x \in (0,\ell),\ t \in (0,T) \tag{1}$$

$$q_3 u_{tt}(t,\ell) = q_1 u_{xxx}(t,\ell) + g(t) \qquad t \in (0,T) \tag{2}$$

describing the transverse deflection of the beam and tip mass is obtained where f and g denote externally applied lateral loads (see [2]). The boundary conditions are given by

$$u(t,0) = u_x(t,0) = u_{xx}(t,\ell) = 0 \qquad t \in (0,T). \tag{3}$$

The initial conditions are of the form

$$u(0,x) = \phi(x), \quad u_t(0,x) = \psi(x) \qquad x \in [0,\ell]. \tag{4}$$

The identification problem is stated formally as

(ID): Find $q = (q_1,q_2,q_3)^T \in Q$, Q a compact subset of R^{+3}, which minimizes

$$J\big(q;u(\cdot,\cdot;q)\big) = \sum_{i=1}^{\nu} \sum_{j=1}^{\mu} |u(t_i,x_j;q) - \bar{u}(t_i,x_j)|^2$$

subject to $u(\cdot,\cdot;q)$ being a solution to (1) – (4) where $\{\bar{u}(t_i,x_j)\}_{\substack{i=1,\ldots,\nu \\ j=1,\ldots,\mu}}$ denote a set of given displacement observations.

The first approximation scheme, which is described in detail in [6], is based upon the recasting of (1) - (4) as an abstract evolution equation set in an infinite dimensional Hilbert space. Let $H = R\times H^0(0,\ell)$ with inner product $\langle(\eta,\phi),(\zeta,\psi)\rangle_H = \eta\zeta + \langle\phi,\psi\rangle_0$ where $\{H^k,\langle\cdot,\cdot\rangle_k\}$ denote the usual Sobolev spaces and Sobolev inner products. Define the operators $M_0(q):H\to H$ and $A_0(q):D\subset H\to H$ by $M_0(q)(\eta,\phi) = (q_3\eta,q_2\phi)$ and $A_0(q)\big(\phi(\ell),\phi\big) = \big(-q_1\phi'''(\ell),q_1\phi''''\big)$ respectively where $D = \{(\eta,\phi)\varepsilon H:\phi\varepsilon H^4(0,\ell),\ \phi(0) = \phi'(0) = 0,\ \phi''(\ell) = 0,\ \eta = \phi(\ell)\}$. Assume $f \in L_2([0,T],H^0(0,\ell))$, $g \in L_2(0,T)$, $\phi \in H^2(0,\ell)$ and $\psi \in H^0(0,\ell)$ and let $F_0(t) = \big(g(t),f(t,\cdot)\big)$, $\hat{\phi} = \big(\phi(\ell),\phi\big)$ and $\hat{\psi} = \big(\psi(\ell),\psi\big)$ where $\psi(\ell) \in R$ is specified if it is not well defined. We then rewrite (1) - (4) as an abstract second-order system in H;

$$M_0(q)\, D_t^2\hat{u}(t) + A_0(q)\hat{u}(t) = F_0(t), \qquad t \in (0,T) \tag{5}$$

$$\hat{u}(0) = \hat{\phi}, \qquad D_t\hat{u}(0) = \hat{\psi} \tag{6}$$

where $\hat{u}(t) = \big(u(t,\ell),u(t,\cdot)\big) \in H$ and $D_t = \frac{d}{dt}$. Let $V = \{(\eta,\phi)\varepsilon H:\phi\varepsilon H^2(0,\ell),\phi(0) = \phi'(0) = 0,\eta = \phi(\ell)\}$ and define $L:V\subset H\to H$ by $L\big(\phi(\ell),\phi\big) = (0,\phi'')$. The operator $A_0(q)$ can then be written in factored form as $A_0(q) = q_1L^*L$ where $L^*:Dom(L^*)\subset H\to H$ is given by $L^*(\eta,\phi) = (-\phi'(\ell),\phi'')$ for $(\eta,\phi)\varepsilon Dom(L^*) = \{(\eta,\phi)\varepsilon H : \phi\varepsilon H^2(0,\ell),\ \phi(\ell) = 0\}$.

Let $Z = H \times H$ with inner product $\langle z,w\rangle_q = \langle q_1z_1,w_1\rangle_H + \langle M_0(q)z_2,w_2\rangle_H$. Choosing our state as $z(t) = \big(L\hat{u}(t),D_t\hat{u}(t)\big)$ we rewrite (5), (6) as the first-order system in Z given by

$$\dot{z}(t) = A(q)z(t) + F(t;q), \quad z(0) = z_0 \tag{7}$$

where $A(q):\mathrm{Dom}(L^*)\times V \subset Z \to Z$ is given in matrix form by

$$A(q) = \begin{bmatrix} 0 & L \\ -q_1 M_0(q)^{-1} L^* & 0 \end{bmatrix},$$

$F(t;q) = \left(0, M_0(q)^{-1} F_0(t)\right)$ and $z_0 = (L\hat{\phi}, \hat{\psi})$ ($\hat{\phi}$ is assumed to be an element in V). The operator $A(q)$ is densely defined and skew self adjoint. It therefore generates (see [8]) a C_0 group of unitary operators $\{S(t;q): -\infty < t < \infty\}$ on Z. The mild solution (strong or classical if F and z_0 are sufficiently regular) to (7) is then given by (see [5])

$$z(t) = S(t;q)z_0 + \int_0^t S(t-\tau;q)F(\tau;q)d\tau. \tag{8}$$

Problem (ID) can then be written as

(IDA): Find $q \varepsilon Q$ which minimizes $J\left(q; C_1(\cdot)z(\cdot;q)\right)$ subject to $z(\cdot;q)$ being given by (8) where the operators $C_1(x):Z \to R$ are defined by

$$C_1(x)\left((\eta,\phi),(\zeta,\psi)\right) = \int_0^x \int_0^\tau \phi(\sigma)d\sigma d\tau \quad \text{for} \quad x \ \varepsilon \ [0,\ell].$$

For each $N = 1,2\ldots$, let $S_3(\Delta^N)$ denote the space of cubic spline functions on the interval $[0,\ell]$ corresponding to the uniform partition $\Delta^N = \{0, \frac{\ell}{N}, \frac{2\ell}{N}, \cdots, \ell\}$ (see [7]). Let $W^N = \{(\eta,\phi)\varepsilon H: \phi \varepsilon S_3(\Delta^N), \phi(\ell) = 0\}$, $V^N = \{(\eta,\phi)\varepsilon H: \phi\varepsilon S_3(\Delta^N), \phi(0) = \phi'(0) = 0, \eta = \phi(\ell)\}$ and $Z^N = W^N \times V^N$. Then

Z^N is a 2N+4 dimensional subspace of Z which satisfies $Z^N \subset \mathrm{Dom}(A(q)) = \mathrm{Dom}(L^*)\times V$ for all $q\varepsilon Q$. Let P_q^N denote the orthogonal projection of Z onto Z^N with respect to the $\langle\cdot,\cdot\rangle_q$ inner product. Define $A^N(q):Z^N \to Z^N$ by $A^N(q) = P_q^N A(q)$ and noting that $A^N(q)$ is a linear operator defined on a finite dimensional space (and is therefore bounded), consider the initial value problem

$$\dot{z}^N(t) = A^N(q)z^N(t) + P_q^N(t)F(t;q), \qquad t \; \varepsilon \; (0,T)$$

$$z^N(0) = P_q^N z_0,$$

and its solution

$$z^N(t) = \exp\big(A^N(q)t\big)P_q^N z_0$$

$$+ \int_0^t \exp\big(A^N(q)(t-\tau)\big)P_q^N F(\tau;q)d\tau \qquad t \; \varepsilon \; [0,T]. \tag{9}$$

The approximating identification problems for the first method take the form

(IDN_1): Find $q\varepsilon Q$ which minimizes $J\big(q;C_1(\cdot)z^N(\cdot;q)\big)$ subject to $z^N(\cdot;q)$ being given by (9).

Under rather mild assumptions it can be shown that each of the problems (IDN_1) admits a solution $\bar{q}^N$ and that the sequence $\{\bar{q}^N\}$ contains a convergent subsequence $\{\bar{q}^{N_k}\}$, $\bar{q}^{N_k} \to \bar{q} \; \varepsilon \; Q$ as $k \to \infty$. Using standard approximation results from linear semigroup theory (see [5]) and the properties of spline functions (see [7]) to establish convergence of the state approximations, it can then be argued that $\overline{q}$ is a solution to problem (ID).

The second approximation scheme involves the rewriting of (1) - (4) in an equivalent weak/variational form (see [4]). Let H and V be as they have been defined previously. Define the inner product on V, $\langle\cdot,\cdot\rangle_V$ by $\langle\hat{\phi},\hat{\psi}\rangle_V = \langle L\hat{\phi},L\hat{\psi}\rangle_H$. We have the usual dense embeddings $V \subset H \subset V'$ where V' is the dual of V. We consider the weak form of (1) - (4)

$$\langle M_0(q)D_t^2\hat{u}(t),\hat{\theta}\rangle_H + a\left(\hat{u}(t),\hat{\theta};q\right) = \langle F_0(t),\hat{\theta}\rangle_H, \quad \hat{\theta} \in V,\ t \in (0,T) \tag{10}$$

$$\hat{u}(0) = \hat{\phi}, \quad D_t\hat{u}(0) = \hat{\psi} \tag{11}$$

where the bilinear form $a(\cdot,\cdot;q)$ on $V\times V$ is defined by $a(\hat{\phi},\hat{\psi};q) = q_1\langle L\hat{\phi},L\hat{\psi}\rangle_H = \langle A_0(q)\hat{\phi},\hat{\psi}\rangle_H$. The derivatives in the definition of the operator $A_0(q)$ are interpreted in the distributional sense and $A_0(q)$ is considered to be an element in $\mathscr{L}(V,V')$, the space of continuous linear operators from V into V'.

We define a Galerkin approximation $\hat{u}^N$ to $\hat{u}$ as the solution to

$$\langle M_0(q)D_t^2\hat{u}^N(t),\hat{\theta}^N\rangle_H + a\left(\hat{u}^N(t),\hat{\theta}^N;q\right) = \langle F_0(t),\hat{\theta}^N\rangle_H, \quad \hat{\theta}^N \in V^N,\ t \in (0,T) \tag{12}$$

$$\hat{u}^N(0) = P^N\hat{\phi}, \quad D_t\hat{u}^N(0) = P^N\hat{\psi} \tag{13}$$

where V^N is as it was defined above and P^N is now the orthogonal projection of H onto V^N with respect to the $\langle\cdot,\cdot\rangle_H$ inner product.

The approximating identification problems are then given by

(IDN_2): Find $q \varepsilon Q$ which minimizes $J\left(q;C_2(\cdot)\hat{u}^N(\cdot;q)\right)$ subject to $\hat{u}^N(\cdot;q)$ being the solution to (12), (13) where the operators $C_2(x):V \to R$ are defined by $C_2(x)\left(\phi(\ell),\phi\right) = \phi(x), x \varepsilon [0,\ell]$.

Using standard variational arguments of the type found in [3] it can be argued that under sufficient regularity assumptions, for $\{q^N\} \subset Q$ with $q^N \to q^*$ we have $\hat{u}^N(q^N) \to \hat{u}(q^*)$ in V and $D_t\hat{u}^N(q^N) \to D_t\hat{u}(q^*)$ in H as $N \to \infty$ where $\hat{u}^N(q^N)$ is the solution to (12), (13) corresponding to q^N and $\hat{u}(q^*)$ is the solution to (10), (11) corresponding to q^*. This in turn yields a convergence result analogous to the one stated above for the first scheme. A more detailed discussion of these results appears in [1].

We demonstrate the feasibility of our methods with a simple example. We consider a beam of length 1 and use the two schemes described above to estimate its stiffness q_1, its linear mass density q_2 and the magnitude of a tip mass q_3. We assumed that the system was initially at rest $(\phi=\psi=0)$ and then excited via the distributed lateral load $f(t,x) = e^x \sin 2\pi t$ and the point force $g(t) = 2e^{-2t}$ acting on the tip mass. Displacement observations at positions $x_j = .5, .75, 1.0, j=1,2,3$, at times $t_i = .5i, i=1,2,\ldots,10$, were generated using the "true" parameter values $q_1 = 1.0$, $q_2 = 3.0$ and $q_3 = 1.5$, the first two natural modes of the system and a standard Galerkin method. The approximating identification problems (IDN_1) and (IDN_2) were solved using an iterative Levenberg-Marquardt scheme with the approximating state equations solved at each iteration using a variable order Adams Predictor Corrector method. The "start-up" values for the unknown parameters were taken to be $q_1^0 = .7$, $q_2^0 = 2.7$ and $q_3^0 = 1.7$. Our results for methods 1 and 2 are given in Tables 1 and 2 respectively below.

Table 1

N	$\bar{q}_1^N$	$\bar{q}_2^N$	$\bar{q}_3^N$	CPU (m:s)
2	1.00083	2.99817	1.50090	0:08.70
3	1.00072	3.00317	1.49868	0:29.38
4	1.00061	2.99227	1.50141	1:01.19
5	1.00009	2.98711	1.50195	1:35.26
6	1.00061	2.99039	1.50144	2:45.20

Table 2

N	$\bar{q}_1^N$	$\bar{q}_2^N$	$\bar{q}_3^N$	CPU (m:s)
2	1.00057	3.04455	1.48957	0:09.19
3	1.00067	3.01256	1.49707	0:22.10
4	1.00027	3.00922	1.49721	0:57.57
5	1.00016	2.98936	1.50262	1:22.52
6	.99912	2.99720	1.49952	2:52.76

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16. Abstract

We outline two cubic spline based approximation schemes for the estimation of structural parameters associated with the transverse vibration of flexible beams with tip appendages. The identification problem is formulated as a least squares fit to data subject to the system dynamics which are given by a hybrid system of coupled ordinary and partial differential equations. The first approximation scheme is based upon an abstract semigroup formulation of the state equation while a weak/variational form is the basis for the second. Cubic spline based subspaces together with a Rayleigh-Ritz-Galerkin approach was used to construct sequences of easily solved finite dimensional approximating identification problems. Convergence results are briefly discussed and a numerical example demonstrating the feasibility of the schemes and exhibiting their relative performance for purposes of comparison is provided.

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