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EFFECTS OF THE OCEANS
ON POLAR MOTION:
CONTINUED INVESTIGATIONS

Semi-annual Status Report

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SEMI-ANNUAL REPORT

This progress report for grant NAG 5-145/Supplement 2 covers the period January through June 1984. Most of the research discussed here concerns the pole tide, the oceanic response to the Chandler wobble. The reader should consult earlier progress reports for relevant background material.

During the beginning of the reporting period my efforts were devoted to completing the major project of Supplement 2, a re-examination of Wunsch's (1974) North Sea pole tide theory. Wunsch had proposed that observed pole tide enhancements there (i.e. larger amplitudes than a static tide would possess) resulted from bottom friction, with drag coefficient R, in combination with the depth h of the North Sea decreasing southward. However, his analysis was marred by errors (Wunsch, 1975) and improper approximations. As described in the previous status report, our re-analysis yielded one case where the equations appeared capable of satisfying all necessary boundary conditions: this was a situation in which the North Sea depth was linear in northward distance y, i.e. $h = h_o \alpha y$, and where the drag was inversely proportional to h(y); Wunsch had modeled the depth as proportional to $1+\alpha y$ and treated R as constant. We found the tide's stream function Ψ to be given by

$$\Psi = \sqrt{\frac{2}{\pi}}\, e^{\varepsilon x/2} \sum_{n=1}^{\infty} A_n \frac{1}{\sqrt{\lambda_n}} \sin\left(\frac{n\pi x}{b}\right)\left[y \cosh(\lambda_n y) - \frac{1}{\lambda_n} \sinh(\lambda_n y)\right]$$

where x is eastward distance,

$$\varepsilon = f h_o \alpha / R h \quad , \quad \lambda_n^2 = (\varepsilon/2)^2 + (n\pi/b)^2$$

and f is the Coriolis parameter. This stream function satisfies 3 of Wunsch's boundary conditions, $\Psi = 0$ (i.e. no flow) at the North Sea boundaries x=0, x=b, y=0. The unknown coefficients A_n would be determined from the fourth boundary condition, at the open northern edge of the Sea. Wunsch specified this condition, involving the departure T' of the pole tide height from the static value, as

$$T' = \mathbf{B}_0 \exp(i p_0 x) \qquad \text{at } y=a \quad ,$$

where p_o and $\mathbf{B}_0$ are constants.

At the end of the previous reporting period, I had derived a quite complicated expression for T'(x,y) based on a rigorous general relation between T' and Ψ; determination of the A_n from application of the boundary condition was sure to be troublesome. At the beginning of the present reporting period, I discovered a "shortcut" method of determining $T'(x,y)$; the result was much simpler, namely

$$T' = \frac{f}{gh_0\alpha}\sqrt{\frac{2}{\pi}}\; e^{\varepsilon x/2} \sum_{n=1}^{\infty} A_n \frac{1}{\sqrt{\lambda_n}} \left\{ \sin\left(\frac{n\pi x}{b}\right)\left[\cosh(\lambda_n y) - \frac{\sinh(\lambda_n y)}{2\lambda_n y}\right] + \cos\left(\frac{n\pi x}{b}\right)\left[-\frac{n\pi}{\varepsilon b}\,\frac{\sinh(\lambda_n y)}{\lambda_n y}\right] \right\}.$$

Unfortunately, determination of the A_n revealed fundamental difficulties with the theory. In typical Fourier analysis problems, the sine and cosine coefficients are independent unknowns, determined separately by sine and cosine orthogonality. Our solution for T', however, is in terms of sine and cosine coefficients which are <u>related</u> (and the relation is different for each n!); sine and cosine orthogonality will thus yield different values for An. We found

$$A_n = (-1)^n \frac{gh_0\alpha\,\varepsilon}{f\sqrt{2\pi}}\,\frac{\lambda_n^{3/2}\,a}{n\sinh(\lambda_n a)} \left\{ j\,\frac{(p_0 - \frac{n\pi}{b})B_I - \frac{\varepsilon}{2}B_R}{L_n^2} + j'\,\frac{(p_0 - \frac{n\pi}{b})B_R + \frac{\varepsilon}{2}B_I}{L_n^2} \right\}$$

using cosine orthogonality, or

$$A_n = (-1)^n \frac{\sqrt{2\pi}\, gh_0\alpha}{fb}\,\frac{-\sqrt{\lambda_n}}{2\cosh(\lambda_n a) - \frac{\sinh(\lambda_n a)}{\lambda_n a}} \left\{ j'\,\frac{\frac{\varepsilon}{2}B_R - (p_0 - \frac{n\pi}{b})B_I}{L_n^2} + j\,\frac{\frac{\varepsilon}{2}B_I + (p_0 - \frac{n\pi}{b})B_R}{L_n^2} \right\}$$

using sine orthogonality, where $B_r = \mathrm{Real}\{B_0\}$, $B_i = \mathrm{Imag}\{B_0\}$, and

$$L_n^2 = \lambda_n^2 + p_0^2 - 2p_0\frac{n\pi}{b}$$

$$j = (e^{-\varepsilon b/2} - e^{\varepsilon b/2})\cos p_0 b \qquad j' = (e^{-\varepsilon b/2} + e^{\varepsilon b/2})\sin p_0 b \quad .$$

These expressions are <u>equal</u> if $\tan^{-1}(B_i/B_r) \equiv \arg\{B_0\} = -\delta_n - \alpha_n$, where

$$\tan\delta_n = \frac{\frac{\varepsilon}{2}j' + (p_0 - \frac{n\pi}{b})j}{\frac{\varepsilon}{2}j - (p_0 - \frac{n\pi}{b})j'} \quad \text{and} \quad \tan\alpha_n = \frac{\varepsilon/2}{n\pi/b}\left[1 - 2\lambda_n a\,\frac{\cosh(\lambda_n a)}{\sinh(\lambda_n a)}\right].$$

This may explain why Wunsch (1974) chose a complex boundary condition at y=a, even though the rest of his equations were strictly real-valued: the non-uniqueness is <u>apparently</u> eliminated if the phase (arg) of B_0 is pre-set. Unfortunately, this approach is inconsistent because $\arg\{B_0\}$ must be a function of n--i.e., for each n a different phase must be selected for B_0.

Such difficulties would not be eliminated if a boundary condition of the form

$$T' = B_0 q(x) \quad \text{at } y=a \ ,$$

for any function q(x), were specified, because after q(x) were expanded in a Fourier series the same ambiquity would appear--unless the Fourier expansion of q(x) happened coincidentally to possess the arg = $-\delta_n - \alpha_n$ type of phases. Since the governing equation for Ψ is an elliptical type of differential equation, its solution requires four boundary conditions [see Carrier & Pearson, 1978], so we cannot simply omit the troublesome boundary condition at y=a (we'd still have to determine the A_n). We must instead conclude that either Wunsch's theory is ill-posed, e.g. the original momentum equations must include other terms such as $\partial/\partial t$ terms or diffusive friction or ..., or else additional theory external to the North Sea is required in order to delineate the proper y=a boundary condition (this may be quite similar to Stoke's Paradox! --see Tritton, 1977, pp. 92-93).

Computer programs were written to calculate the solutions for A_n and thus T' for 5 situations: I) A_n determined from cosine orthogonality, with $B_i = 0$ artifically; II) A_n determined as above with $B_i/B_r = -\tan(\delta_n + \alpha_n)$ illegally; III) A_n determined from cosine orthogonality when the boundary condition is

$$T' = B_r \cos(p_0 x) \quad \text{at } y=a \ ;$$

IV) same as III but with

$$B_R = \sqrt{2}\left(1 + \bar{\Omega}/\sigma\right)(M'-M)\left\{ -\frac{(1+k-h)\bar{\Omega}^2 A^2}{g}\sqrt{\frac{2\pi}{15}}\left[\sqrt{\frac{15}{8\pi}}\sin\theta_a \cos\theta_a\right]\right\}$$

$$p_0 = 1/A\sin\theta_a$$

where θ_a is the colatitude of the North Sea at y=a and all other symbols are from Dickman (1983); and V) same as IV but with B_r larger by a factor of $\sqrt{2}$. In the third situation the A_n are given by

$$A_n = (-1)^n \frac{g h_0 \alpha \varepsilon}{2f\sqrt{2\pi}} \frac{\lambda_n^{3/2} a}{n \sinh(\lambda_n a)} B_R \left\{ \frac{j'(p_0 - \frac{n\pi}{b}) - j\frac{\varepsilon}{2}}{L_n^2} + \frac{j'(p_0 + \frac{n\pi}{b}) - j\frac{\varepsilon}{2}}{\lambda_n^2 + p_0^2 + 2p_0\frac{n\pi}{b}} \right\} .$$

The fourth and fifth situations correspond to forcing the North Sea pole tide by relative motion between the wobbling solid earth and wobbling oceanic body, as in Dickman (1983); this was the goal of the project! In IV time-averaged wobbles are used, while in V maximum wobble amplitudes are considered.

Results are as follows:

I) $A_1 = -2.0\text{x}10^{-4}$, ... , and -0.29 cm $\leq T' \leq 13.1$ cm along the southern coast of the North Sea;

II) $A_1 = 5.2\text{x}10^{-5}$, ... , and -0.07 cm $\leq -T' \leq 3.7$ cm along that coast;

III) $A_1 = -3.1\text{x}10^{-4}$, ... , and -0.41 cm $\leq T' \leq 20.7$ cm along that coast;

IV) $A_1 = 4.7\text{x}10^{-5}$, ... , and -0.06 cm $\leq -T' \leq 3.2$ cm along that coast if ocean-solid earth coupling is 99.9% effective ($M'-M = 0.001 \times M$); and

V) $A_1 = 6.6\text{x}10^{-5}$, ... , and -0.09 cm $\leq -T' \leq 4.5$ cm under the same conditions as in IV. We can conclude, firstly, that (because I-III results are so different) the ill-posed nature of the theory is a serious deficiency; and, secondly, that forcing by ocean-solid earth wobbling at the edge of the Sea may indeed explain North Sea pole tide observations (the enhancements T' in IV have precisely the correct magnitude range!).

Dissipation of energy by North Sea pole tide currents was also computed; preliminary results indicate that such dissipation may explain a significant fraction of Chandler wobble energy loss.

I was scheduled to devote four months to substantiating one aspect of earlier NASA-grant work. Dickman (1983) had found an explanation for the Markowitz wobble, that it is a natural wobble of the coupled rotating ocean-solid earth system, subject to the assumption that ocean - solid earth coupling is primarily non-dissipative and of magnitude $\sim 10^{24}$ N-m. The four-month project was to determine, using fluid dynamics, the topographic drag exerted on a continent by pole tide - type currents. This was, however, to be a simplified analysis, since that torque could also be computed indirectly as a byproduct of the next year's NASA-grant research (but that work, determining the actual wobble-induced flow in realistic oceans, is extremely difficult and could require more than a year's effort...). Thus, the model considered here consisted of a single, vertical-sided continent in oceans of uniform density ρ and depth h overlying a spherical earth (radius = R); the flow was taken to be steady and unforced, and--since we're looking for non-dissipative coupling--the oceans were treated as inviscid.

Prior to the analysis, fluid dynamics texts were consulted (Pedlosky, 1979, chap. 1-3, 6, in which most of the concepts discussed here can be found; Tritton, 1977, chap. 1-15; portions of Lamb 1932, Batchelor 1968, Gill 1983, and a Phillips 1963 review article). The first step in the analysis was to scale the equations. Dimensional analysis quickly revealed that, because of the very slow long-period currents and global length scale characterizing pole tide-type flows, the situation being considered here corresponded almost exactly to <u>geostrophy</u>, where the dominant forces acting on the fluid are the Coriolis and pressure forces. I briefly investigated geostrophy on a sphere--a topic infrequently found in the literature due to the popularity of the beta-plane method. With no radial variations in any quantities and no radial flow, as appropriate for thin oceans, a geostrophic force balance allows only uniform zonal flow. If a continent is added to the oceans, then the presence of such a barrier creates a "Taylor band" of stagnant water in the same latitudes as the continent (this is the spherical equivalent of the Taylor column found in flat geometry).

In order to treat realistic flow problems, then, either radial motion or departures from geostrophy, or both, must be allowed; for our situation the 'force' most likely to break the geostrophic balance is advection, creating an inertial boundary layer near the continent. The quasi-geostrophic continuity, momentum, and vorticity equations are, in vector form,

$$\vec{\nabla}\cdot\vec{u} = 0$$
$$2\rho\vec{\Omega}\times\vec{u} + \rho\vec{u}\cdot\vec{\nabla}\vec{u} = -\vec{\nabla}p$$
$$\vec{u}\cdot\vec{\nabla}\vec{\omega} = (\vec{\omega}+2\vec{\Omega})\cdot\vec{\nabla}\vec{u}$$

where $\vec{u} = (u_r, u_\theta, u_\lambda) \approx (0, u_\theta, u_\lambda)$ is the velocity and $\vec{\omega} = \vec{\nabla}\times\vec{u}$ is its vorticity; and $\vec{\Omega}$ is earth's angular velocity.

These equations were also scaled for thin oceans and then reduced accordingly. Functional analysis demonstrated that the only near-geostrophic solutions away from the continent were trivial ones, if the ocean depth h was not a function of longitude λ. For the two-dimensional flow on a sphere, an obvious stream function Ψ could be defined $\left(\frac{\partial\Psi}{\partial\lambda} = h u_\theta \sin\theta \quad , \quad \frac{\partial\Psi}{\partial\theta} = -h u_\lambda\right)$ and it was also shown that the radial vorticity ω_r could be written as

$$\omega_r = -\frac{R}{h}\nabla^2\Psi + \frac{R}{h^2}\vec{\nabla}h\cdot\vec{\nabla}\Psi$$

this expression reveals two regimes for the flow, an "interior" nearly geostrophic flow (since $\omega_r \sim \nabla^2\Psi$) and a boundary flow (near the boundary $\vec{\nabla}h$ will be huge). For the model being considered, h is essentially constant within the oceans, yielding apparently an entirely geostrophic flow except for an infinite, infinitely thin inertial boundary layer.

These conclusions were unchanged even after the initial equations were re-scaled to allow explicitly for the possibility of a boundary layer of intermediate characteristic length. In a last attempt to uncover worthwhile 'spherical' solutions, perturbation methods were tried. For the pole tide the Rossby number "ε" is tiny, so all variables in the 'spherical' equations were expanded in powers of ε and then matched up. Unlike the flat earth case, the combination of continuity and momentum equations here is not "geostrophically degenerate" and instead led to

$$u_\theta \equiv 0 \quad , \quad u_\lambda \neq u_\lambda(\lambda) \quad , \quad \eta \neq \eta(\lambda)$$

where η is the sea-level height; then, with a continental barrier present, the equations once again implied the existence of a Taylor band at the same latitudes as the continent.

It therefore appeared necessary to resort to the well-known beta-plane approach if this brief research project were to be fruitful. On the beta-plane (centered at colatitude $\theta = \theta_0$), the scaled quasi-geostrophic equations were reduced for thin oceans. With L representing the horizontal length scale of the flow, then for the case where L/R is of order ε the governing equation for the flow is (Pedlosky, §6.3)

$$J(p_0\,,\nabla^2 p_0 - \phi p_0 + \beta y) = 0$$

where J is the Jacobian operator, p_0 is the lowest order non-hydrostatic pressure, ϕ is a small parameter, $\beta \equiv \tan\theta_0$, and $y = (\theta_0 - \theta)R$ is the northward variable ($x = \lambda R \sin\theta_0$ is eastward).

Four situations were considered, in order to build gradually to the project goal (this Jacobian equation is highly non-linear!). In all cases the topographic drag force $\vec{D}$ and drag torque $\vec{\mathcal{T}}$ on the continent were calculated as

$$\vec{\mathcal{D}} = \hat{x}\left[-h\int p_0 \cos\nu \, d\ell\right] + \hat{y}\left[-h\int p_0 \sin\nu \, d\ell\right]$$
$$\vec{\mathcal{T}} = \hat{x}\left[hR\int p_0 \sin\nu \, d\ell\right] - \hat{y}\left[hR\int p_0 \cos\nu \, d\ell\right]$$

integrated over the coastline ℓ, which makes an angle ν with the eastward direction.

I) uniform zonal flow $u_x = -U$ from infinity, blocked by an infinite north-south continent (Pedlosky's example of inertial boundary layer). Results are

$$p_0 = \rho f_0 U y \left(1 - e^{-\sqrt{\beta}\, x/L}\right)$$

where $f_0 = 2\Omega\cos\theta_0$ is the Coriolis parameter; and

$$\vec{\mathcal{D}} = 0 \qquad\qquad \vec{\mathcal{T}} = 0$$

(the drag is necessarily zero since the coastline is infinite).

II) same as I but with a square, north-south/east-west continent whose sides are of length L. Results are

$$p_0 = \rho f_0 U y + \rho f_0 U L \sum_{m=0}^{\infty} \frac{16}{\pi^3 (2m+1)^3} \exp\left[-\sqrt{\beta + (\tfrac{\pi}{2L})^2 (2m+1)^2}\, \frac{x}{L}\right] \cos\left[\tfrac{\pi}{2}(2m+1)\frac{y}{L}\right]$$

$$\vec{\mathcal{D}} = -\rho f_0 U h L^2 (\hat{x} + \hat{y}) \qquad\qquad \vec{\mathcal{T}} = \rho f_0 U h R L^2 (\hat{x} - \hat{y}).$$

III) same as II but with flow at infinity similar to the pole tide. True pole tide currents on a spherical earth in global oceans--the "infinity" of the beta plane--would in beta-plane terms approximate to

$$u_x = \tau \frac{\cos 2\theta_0}{\cos\theta_0} \cos\left[\frac{x}{R\sin\theta_0}\right] \qquad\qquad u_y = -\tau \sin\left[\frac{x}{R\sin\theta_0}\right]$$

from Dickman (1984), where $\tau = \sqrt{\frac{15}{8\pi}} \frac{g}{2\Omega R}\left\{T_2^{-1} + \sqrt{\frac{8\pi}{15}}\, \Omega^2 R^2 M_p / g\right\}$, T_2^{-1} being the major spherical harmonic component of the pole tide in response to a wobble of amplitude M_p. However, the closest approximation to these u_x, u_y <u>which is consistent with the beta-plane</u> equations was merely

$$u_x = \tfrac{1}{2}\beta\tau (y/L)^2 \qquad\qquad u_y = -\tau \;,$$

with $U = \tau$. Even with this velocity at infinity, barely more complicated than that in I and II, the shortcut method of solution employed by Pedlosky (see § 3.13) could not be used here. Postulating that the solution includes exponential decay eastward, we eventually found

$$p_o = -\rho f_o U x - \tfrac{1}{6}\beta\rho f_o U L (y/L)^3 + \rho f_o U L \sum_m e^{-2m\pi x/L}\left[A_m \sin(\tfrac{2m\pi}{L}y) + B_m \cos(\tfrac{2m\pi}{L}y)\right].$$

Conceivably the coefficients are given roughly by $A_m = 2\beta(L/2m\pi)^3$, $B_m = \frac{L}{2\pi \ln 2}\frac{(-1)^m}{m^2}$. The net drag is

$$\vec{D} = \rho f_o U h L^2 \{\hat{x}[\beta/12 - 1] + \hat{y}[2\sum B_m - \beta/6]\}$$

$$\vec{T} = \rho f_o U h R L^2 \{\hat{x}[\beta/6 - 2\sum B_m] + \hat{y}[\beta/12 - 1]\}$$

IV) same as III but with a circular continent of radius L. Now the nonlinear Jacobian equation is complicated by <u>highly non-linear boundary conditions</u>. Although an analytic solution for the A_m, B_m as above is clearly beyond the scope of this project, so that p_o remains unknown, it could be proved in this case that the net drag is independent of the Am, Bm. We found

$$\vec{D} = \rho f_o U h L^2 \{\pi\hat{x} + \tfrac{1}{4}\beta\pi\hat{y}\}$$

$$\vec{T} = \rho f_o U h R L^2 \{\pi\hat{y} - \tfrac{1}{4}\beta\pi\hat{x}\}.$$

For flow with the characteristics of the pole tide, $U = \tau \sim 1.6\times10^{-4}$ cm/sec; if the beta plane is centered at $\theta_o \simeq 45°$ and $L \sim 10^8$ cm, then

$$\vec{D} \sim (2.3\hat{x} + 0.6\hat{y}) \times 10^{14} \text{ dyne}$$

$$\vec{T} \sim (1.5\hat{y} - 0.4\hat{x}) \times 10^{23} \text{ dyne-cm}.$$

With 7 such continents but slightly larger in size, $L \sim 5\times10^8$ cm (if the beta plane approach is still valid for such L/R), the net torque would be $\sim 3\times10^{25}$ dyne-cm assuming the results for IV still apply; this is still several orders of magnitude smaller than required by Dickman (1983).

It remains to be seen, when the goal of the next year's project (Supplement 3) is achieved, whether the actual oceans--with irregular continental coastlines, bottom topography, and unsteady turbulent flow--would exert couples on the solid earth $\sim 10^5$ times greater....

My grant-related activities during the reporting period also included presenting a talk on the self-consistent dynamic pole tide at the Spring 1984 AGU meetings in Cincinnati (a copy of the abstract is attached); and the supervision of two graduate students (D. Steinberg, C. Ammon). Steinberg has been working for over 1 1/2 years on the consequences of Dahlen's (1976; see also Merriam 1973) self-gravitating, loading static pole tide model. Previously, the FORTRAN programs he had written to implement Dahlen's theory had yielded tide characteristics that did not agree with Dahlen's published values. During the reporting period, Steinberg discovered that the published values for the tide's effect on the Chandler wobble frequency implicitly included the effect of the mantle's response to the pole tide load. This had not been clearly stated in the literature and explained why a tide supposedly enhanced (through self-gravitation and loading) compared to earlier non-loading static tide models would affect the wobble period <u>less</u>. Steinberg's programs were able to separate the tide's own effect from that of the loaded mantle, and he showed that the loading, gravitating tide does indeed affect the wobble period more than the non-loading tide [however, the mantle response to loading reverses the net effect!]. It then became clear that the loading, gravitating tide <u>is</u> enhanced in magnitude, compared to non-loading pole tide models, at least according to the tide's spherical harmonic 2, -1 coefficient (which determines the tide's effect on wobble period). The next step--the final one in Steinberg's M.A. thesis research--will be to construct the loading, gravitating tide at various ports, and compare it with the actual pole tide; such a comparison (never before published) would allow departures of the actual tide from equilibrium to be accurately quantified.

Ammon completed a year-long investigation of techniques for deconvolving Chandler wobble data. It was hoped that, by treating the Markowitz wobble as a second free wobble of the earth, the deconvolution filters of Smylie et al. (1973) could be modified to generate more accurate wobble excitation time series. Unfortunately the project got bogged down in reproducing the original Smylie et al. filters: perhaps that paper was a bit vague in providing details of the filters; in any event, as constructed by us the filters appeared unstable, with undesirable side-effects. Without a much longer and intensive effort, it is not likely that those filters could be "fine-tuned" enough to generate recognizable or reasonable excitation time series.

Following the Spring AGU meeting I reviewed once again the Supplement 1 research upon which the talk was based. The review revealed several errors, primarily involving the normalization used for the complex spherical harmonic functions. After correcting these errors it turned out that my theoretically computed self-consistent dynamic pole tide was, at least in global oceans, very similar to the idealized static pole tide; for example, the effect of the dynamic pole tide on the Chandler wobble period is likely to be within 1 day (i.e. 3%) of the static tide's effect. I completed the manuscript on "the self-consistent dynamic pole tide in global oceans," and submitted it to Geophysical Journal of the Royal Astronomical Society at the end of the reporting period; because of the greatly differing conclusions resulting from the revisions, I also circulated preprints to a number of colleagues. To save on photocopying and mailing costs, I will be mailing reprints to NASA.

The Self-Consistent Dynamic Pole Tide

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It has been known since Newcomb (1892) that the pole tide, the oceanic response to the Chandler wobble, acts--through its mass redistribution (products of inertia)--to lengthen the Chandler wobble period. The extent of lengthening, traditionally calculated by assuming a static response, is ∿ 1 month.

Such estimates, however, fail to treat the pole tide self-consistently. The tide's effect on wobble period, computed according to conservation of angular momentum (Liouville equation), is a function of the tide height; the latter is found rigorously according to conservation of linear momentum (e.g. Laplace tidal eqns) and depends on the wobble period. The two must be determined jointly or the lengthening estimate will be incorrect.

At the Fall 1983 Meeting I reported on a first investigation of the self-consistent pole tide. The oceans were taken as global, and linearized bottom friction of different strengths was considered. In all cases the resulting tide lengthened the wobble period by only ∿ half the static amount.

But a non-static tide must be supported by non-zero currents, and relative angular momentum associated with the currents will also affect the wobble period. Self-consistent theory incorporating such effects has now been developed. Results to date indicate that the angular momentum contribution to wobble period lengthening is surprisingly large: more than three times that of the tide's products of inertia.

Although oceanic non-globality can be expected to modify these results, it appears that the self-consistent and dynamic nature of the pole tide is crucial to the tide's effect on wobble.

1. Spring Meeting
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8. a. see #3 above

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9. C (contributed)