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AN ALTERNATIVE TO REDUCTION OF SURFACE PRESSURE TO SEA LEVEL

J. W. Deardorff
Department of Atmospheric Sciences
Oregon State University
Corvallis, Oregon 97331

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ABSTRACT

The pitfalls of the present method of reducing surface pressure to sea level are reviewed, and an alternative, adjusted pressure, P, is proposed. P is obtained from solution of a Poisson equation over a continental region, using the simplest boundary condition along the perimeter or coastline where P equals the sea level pressure. The use of P would avoid the empiricisms and disadvantages of pressure reduction to sea level, and would produce surface pressure charts which depict the true geostrophic wind at the surface.

1. Introduction

The need for standardizing surface pressures measured at elevated land stations, so as to essentially eliminate purely hydrostatic variatons, is well known (Hewson and Longley, 1944; Saucier, 1955; Wallace and Hobbs, 1977). If some such procedure were not performed, the surface pressure map would merely resemble an inverted topographic map, due to the approximate 12 mb decrease in pressure per 100 m increase in elevation. Synoptic weather influences would be largely masked.

The method presently employed in the US to standardize surface pressures can be summarized as follows (Saucier, 1955; see also Reichelderfer, 1963, Chap. 7 and Appendix 7.2) If p_o is the sea-level reduced pressure to be plotted, and p_s is the actual pressure measured at station elevation z_s above mean level, the starting point for p_o is

$$p_o = p_s \exp(gz_s/R_d\overline{T}^*) + 0.21(mb) \cdot z_s(km) \cdot (\overline{T}^* - \overline{T}^*_n)(C) \qquad (1)$$

where g is gravity, R_d the gas constant for dry air, $\overline{T}^*$ a mean virtual temperature, and $\overline{T}^*_n$ an annual normal value of $\overline{T}^*$. The first term on the right of (1) is the hydrostatically correct sea-level pressure that would obtain, for z_s not too large, if there were an air column extending below the surface down to sea level with a mean virtual temperature $\overline{T}^*$. The second term is the "plateau" correction designed to alleviate inconsistencies noted, on a climatologic basis, if only the first term is used.

In (1), $\overline{T}^*$ is, in its simplest specification, given by

$$\overline{T}^* = \overline{T}^*_s + \tfrac{1}{2}\gamma z_s \qquad (2)$$

where $\overline{T}^*_s$ is the average of the present and 12-hr previous virtual temperature at shelter height, and γ is one-half the dry adiabatic lapse rate g/c_p, where c_p is the specific heat at constant pressure.

In seeking a rational scheme for surface-pressure standardization, however, this method leaves very much to be desired:

1) No air column exists from z_s down to sea level in general; the quantity $\overline{T}^*$ is fictitious.

2) Above an elevation of 305m, γ is taken to be an empirical function of surface temperature, involving a diagram for its evaluation. Moreover, this variation in γ is made a subjective function of geographic location (e.g., Pacific-slope, middle plateau or eastern-slope regions).

3) The definition of $\overline{T}^*_s$ in (2) filters out the diurnal influence of boundary-layer warming and cooling upon the reduced pressure, as pointed out by Sangster (1960). In many applications (e.g., diagnosis of upslope winds or sea breezes) it is necessary to retain the diurnal influence.

4) In the plateau correction term, z_s is not in all cases the true elevation. For a station whose elevation differs "greatly" from that of surrounding stations, z_s is the average elevation of the neighboring stations.

5) To determine precisely the ingredients by which personnel at any par-ticular elevated station reduce their own surface pressure to sea level requires additional information to be found only in non-standard literature references which are generally unavailable on short notice.

6) During conditions of strong temperature contrast over elevated or sloping terrain, gradients of sea-level reduced pressure strongly misrepresent the actual horizontal pressure gradient at the surface. The reduced isobars incorrectly resemble the isotherms then (Sangster, 1960).

7) Despite the complexity of the method presently in use, discrepancies in reduced sea-level pressure for neighboring high stations can still be as large as 10 mb in some situations, according to Saucier.

An alternative to sea-level reduction of pressure was proposed by Sangster (1960); in place of the surface pressure chart one would utilize the geostrophic stream function at the surface, and perhaps also the geostrophic potential function. The contour shapes and spacings of the former closely resemble those of p_o when z_s is small, and otherwise the disadvantages of 1) through 7) would be eliminated. However, Sangster's proposal never became operational, perhaps because the traditional use of pressure, with its familiar units, would have to be abandoned, even over the oceans where there is no problem.

The present motivations for seeking an alternative to sea-level reduction of surface pressure duplicate those of Sangster. However, the alternative proposed here produces the end result desired by the early architects of the sea-level reduction method: standardized pressures whose horizontal gradients yield the true horizontal pressure gradients which depict the geostrophic wind at the surface.

2. The proposed alternative

The two components of the geostrophic wind, $\underset{\sim}{V}_g$, if evaluated at the surface, are

$$u_g = -\frac{1}{\rho_s f}\left(\partial p/\partial y\right)_s$$

$$v_g = \frac{1}{\rho_s f}\left(\partial p/\partial x\right)_s \tag{3}$$

where ρ is density, f is the Coriolis parameter, and subscript s refers to evaluation at or very near the surface. Note that, with ∇_H the horizontal gradient operator,

$$\left(\nabla_H p\right)_s \neq \nabla_H p_s$$

over variable terrain, so that a map of p_s alone would not permit $\underset{\sim}{V}_g$ to be obtained. That is, $\left(\nabla_H p\right)_s$ is $\nabla_H p$ evaluated at the surface, whereas $\nabla_H p_s$ is

the horizontal gradient of the surface pressure. Therefore we define a horizontally adjusted pressure, $\mathbf{P}(x,y,)$, by

$$\nabla \mathbf{P} \equiv (\nabla_H p)_s \tag{4}$$

Using the hydrostatic assumption, the well known relation (Hess, 1959, Eq. (12.9)) between $(\nabla_H p)_s$ in (4) and the directly observable quantity ∇p_s is

$$\nabla \mathbf{P} = (\nabla_H p)_s = \nabla p_s + \rho_s g \nabla z_s \tag{5}$$

Eq. (5) may be re-expressed as

$$\nabla \mathbf{P} \equiv (\nabla_H p)_s = \nabla(p_s + \rho_s g z_s) - z_s \nabla(\rho_s g) \tag{6}$$

or, using (4) and (6),

$$\nabla[\mathbf{P} - (p_s + \rho_s g z_s)] = -z_s \nabla(\rho_s g) \tag{7}$$

The divergence of (7) produces the mathematical equation from which $\mathbf{P}$ is to be solved:

$$\nabla^2 S \equiv \nabla^2[\mathbf{P} - (p_s + \rho_s g z_s)] = -\nabla \cdot [z_s \nabla(\rho_s g)] \tag{8}$$

Upon denoting the solution to this Poisson equation by $S(x,y)$, (8) becomes

$$\nabla^2 S = -\nabla \cdot [z_s \nabla(\rho_s g)] \tag{9}$$

with the result

$$\mathbf{P} = p_s + \rho_s g z_s + S \tag{10}$$

Over the oceans, $z_s = 0$ (since pressure observed at ships' bridges is reduced to sea level). Then (9) or (7) indicates that $S = 0$ upon choosing the constants of integration to be zero. Thus, (9) can be solved more simply by standard techniques over some region whose borders lie at or close to sea level so that

the Dirichlet boundary condition

$$S = 0 \quad \text{(at perimeter and over oceans)} \qquad (11)$$

applies. That is, where $z_s = 0$ (10) indicates that for $S = 0$ we have $P = p_s$ as desired. This property is missing from Sangster's proposal -- that a standardized surface pressure should retain the dimensions of pressure and become synonomous with observed pressure over the oceans.

Having utilized (6) instead of (5), it may be noticed that S will often be a relatively small addition to $p_s g z_s$ which resembles a sea-level reduction term (but using p_s instead of a somewhat larger "sub-surface" density). In the next section we show that for a horizontally homogeneous atmosphere S is positive (in the neighborhood of 6 mb for $z_s = 1$km). Only for these reasons does the fictitious procedure of reducing pressure to sea level work as well as it does.

An alternative to boundary-condition (11) comes from (7):

$$\nabla S = -z_s \nabla(p_s g) \qquad (12)$$

which is a gradient condition. Use of (12) permits the technique to be used over any section of terrain, even mesoscale, provided P (or S) is known at one point, at least, along the perimeter or in the interior.

3. Solutions for a horizontally homogeneous atmosphere

In this particular case $p_s = p_s(z_s)$ so that

$$\nabla p_s = (\partial p_s / \partial z_s)\nabla z_s = (\partial p / \partial z)\nabla z_s . \qquad (13)$$

Then (7) becomes

$$\nabla[P - (p_s + p_s g z_s)] = -g(\partial p / \partial z)\nabla(z_s^2 / 2) . \qquad (14)$$

Upon approximating $\partial\rho/\partial z$ with its average value, indicated by overbar, over the terrain heights involved, (14) becomes

$$\nabla[P - (p_s + \rho_s \, gz_s) + g(\overline{\partial\rho/\partial z})z_s^2/2] = 0$$

which yields

$$P = p_s + \rho_s gz_s + g(\overline{\rho}/\overline{T}^*)(g/R_d - \overline{\gamma})z_s^2/2 \qquad (15)$$

upon replacing the density gradient with its hydrostatic value for a mean lapse rate, $\overline{\gamma}$, and upon noting that the constant of integration is zero. Comparison of (15) with (10) indicates that the third term on the right of (15), which is positive, represents S. In this special case no further integration in the horizontal is required to obtain P.

An expansion of (1) for this special case can easily be shown to give essentially the same result as (15), if the plateau correction term is omitted. Only for this case, then, does the present procedure of reduction to sea level make physical sense. In the actual atmosphere, and especially when horizontal temperature gradients and terrain heights are large, a properly adjusted surface pressure requires solution of an elliptic (Poisson) equation. Surface pressure at any given point cannot, in general, be properly standardized independently of the surface pressures (and ρ_s, z_s values) at neighboring points.

4. Discussion

It might be wondered how it is possible to adjust surface pressures everywhere over terrain of whatever irregularity, so that the derivative of P at any point yields the true horizontal pressure gradient, and also maintain the correct boundary condition $P = p_s = p_0$ along the perimeter at sea level (S = 0). The answer to this question becomes one of showing that the area average of (9) is compatible with boundary condition (11). From the divergence theorem in two dimensions, the area integral of $\nabla\cdot\nabla S$ on the left of (9) is the line integral

of ∇S along the perimeter. The latter is zero since S is zero there. The area average of the right-hand side of (9) is zero by the same theorem, since the line integral of $z_s \nabla(\rho_s g)$ is zero along the perimeter. Hence, compatibility is assured, and the average of the forcing function for $\nabla^2 S$ is zero.

The solution for S and P through use of second-order finite differences is not without error, however. To examine its extent, tests were made with a hypothetical continent having square sides of length 4400 m and a 1-km plateau in the central 15% of the region. The grid interval was 200 km. A horizontally homogeneous atmosphere was prescribed for this and lesser heights, so that the true solution was known to be merely the pressure field imposed hydrostatically from greater heights. When a uniform potential temperature, θ, was prescribed for the lowest kilometer, the maximum error in P, utilizing (9)-(11), was 0.06 mb while that in p_0 was 2.6 mb (due perhaps to an inappropriate choice of T^*_n in (1)). When a vertical discontinuity in θ of 12C was prescribed to occur at z = 500 m along the maximum slope of the plateau (1.6×10^{-3}), the error in P increased to 0.46 mb. Since this is a rather severe test case, the tentative conclusion is that errors of solution for P, apart from uncertainties in interpolating the station network data onto the computational grid, will not usually exceed ½ mb and will be up to an order of magnitude smaller than the errors in p_0. An additional error of up to a few hundredths of a millibar in P was noted to occur at sea level along the coast adjacent to the sloping terrain, when the $P = p_0$ boundary condition was applied somewhat farther out to sea.

To obtain the surface geostrophic wind requires knowledge of the surface density. Even if p_s is not available to the user, ρ_s can be estimated to within ½% accuracy knowing P, T^*_s and z_s, through use of (10), ignoring S and utilizing the equation of state. One then finds

$$\rho_s \approx \frac{P}{R_d T^*_s} \bigg/ \left(T + \frac{g z_s}{R_d T^*_s}\right)$$

for use in (3), which becomes

$$u_g = -(\rho_s f)^{-1} \partial P/\partial y$$

$$\text{(15)}$$

$$v_g = (\rho_s f)^{-1} \partial P/\partial x$$

An interesting question arises whether z_s on the right-hand side of (9) should be smoothed before solving for S. This procedure was not done in the example of Sec. 4, and does not seem to be necessary since the solution of a Poisson equation is much smoother than its forcing function. However, Sangster did apply such smoothing.

Another question that arises is how small a height above the surface subscript s should refer to in all the preceeding equations. Instrument shelter height would seem to be satisfactory, through a somewhat greater height within the surface layer might be preferable for better representativeness.

The proposed alternative to sea-level reduction in surface pressure could be implemented much more easily now by the National Weather Service than just a few years ago. Thei. recent development of objectively analyzed pressures on some of the 3-hourly surface pressure charts would permit the inclusion of a solution of the Poisson equation for P with minimal further effort for an area encompassing North America. Presumably, it would be impractical at present to substitute P for p_0 in the one-hourly weather reports, since P cannot be obtained accurately without the foregoing analysis.

The use of P instead of p_0 might well encourage comparisons between numerical forecasts of surface geostrophic winds and their observations. At present there is little incentive to make such comparisons over elevated terrain, using the sea-level reduced pressures for verification purposes, since the latter contain fictitious elements and since the numerical model may not use the same algorithm to reduce its calculated pressures to sea level as do the individual

reporting stations.

5. Summary

A horizontally adjusted surface pressure, P, has been proposed to replace sea-level reduced pressures (p_0) presently in use on the 3-hourly synoptic charts. Unlike p_0, P has a firm, non-empirical physical basis and is uniquely defined. The contours of P show, at a glance, the true direction of the geostrophic wind at the surface even over highly elevated terrain.

Achnowledgment

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