IDENTIFICATION OF DYNAMIC COEFFICIENTS

OF ANNULAR TURBULENT SEALS

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This paper presents an identification procedure to determine dynamic coeffi-
cients of annular turbulent seals in turbopumps. Measurements were carried out at a
built test rig with two symmetrical arranged seals. A rigid rotating shaft is sur-
rounded by an elastically supported housing, which is excited by impact forces. The
relative radial motion between the rotating parts and the housing, respectively bet-
ween the seal surfaces, is measured by displacement pick-ups and from the time sig-
nals complex frequency response functions can be calculated. Finally an analytical
model, depending on the seal parameters, is fitted to the measured data, to find the
dynamic coefficients.

INTRODUCTION

It is well known, that forces in annular pressure seals have a large influence
on the vibrations of turbopump rotors. Therefore the knowledge about this forces is
very important. In order to describe the real behavior of the seals, hydrostatic as
well as hydrodynamic effects have to be considered (Ref. 1,2,3,4). Caused by this
seal forces unstable bending vibrations may occur.

In the design stage a machine designer wants to know, whether a turbopump rotor
will run stable during operation and what size the stability threshold speed will
have. Precalculations should be carried out to evaluate the stability behavior.
For this task in many cases a linear rotor model with inertia-, damping- and stiff-
ness coefficients can be employed. It is no problem to find stiffness and inertia
input data for the shaft. In contrary there is a uncertainty concerning the input
data of seals.

Theoretical models exist to determine the required dynamic seal characteristics
(Ref. 1,2,3,4), but there is a necessity to have more measured data to compare them
with theoretical results and to modify existing seal models. In the case of turbu-
lent seals in pumps only few experimental data are available today (Ref. 2,5,6,7).
Childs and Dressman (Ref. 5) have developed a test facility for dynamic testing of
straight and convergent tapered seals. Radial and tangential force components, re-
sulting from a circular centered orbit, could be measured and the result were com-
pared with various theoretical predictions. The circumferential forces are in a good
correlation with the predictions of a finite length theory derived by Childs (Ref. 4).
On the other hand the radial forces are underpredicted by this theory. Childs et al
(Ref. 6) have designed another test rig for testing the leakage and rotordynamic
characteristics of the interstage seal configurations for the High Pressure Fuel
Turbopump (HPFTP) of the Space Shuttle Main Engine. In this test program the seals
are geometrically similar to the HPFTP seals and are operating at the original high
Reynolds numbers. Again in this test program it is planned to measure the seal for-
ces directly.

In our paper an identification procedure is presented to determine the inertia-, damping- and stiffness coefficients of straight annular turbulent seals. A preliminary test facility has been developed consisting of a very stiff rotating shaft, which is surrounded by an elastically supported housing. Two symmetrical seals are arranged between the shaft and the housing and water flows axially across these two seals. The housing is excited by impact forces, resulting in a relative radial motion between rotor and housing, respectively between the seal surfaces. The measured input and output time signals are transformed to the frequency domain and complex frequency response functions are obtained, expressing the dynamic behavior of the seals including the mass of the housing. To find the dynamic seal coefficients an analytical model, depending on the seal parameters, is fitted to the measured data.

First measurement results are reported in this paper, pointing out the dependence of the seal dynamic coefficients on the rotational frequency and the axial velocity of the fluid in the seals.

DYNAMIC COEFFICIENTS OF ANNULAR TURBULENT SEALS

Black (Ref. 1) was the first, who has developed a dynamic model for short annular pressure seals. In this model he considers

- a turbulent leakage flow in axial direction caused by a pressure drop
- a circumferential fluid flow as a consequence of the shaft rotation
- and small radial motions of the rotor about a centered position (fig. 1)

With the basic equation of continuity, the momentum equation and the friction law for the fluid Black obtains the following linear force-motion relationship, expressed by inertia-, damping- and stiffness coefficients

$$-\begin{bmatrix} F_y \\ F_z \end{bmatrix} = \begin{bmatrix} m_{yy} & 0 \\ 0 & m_{zz} \end{bmatrix} \cdot \begin{bmatrix} \ddot{y} \\ \ddot{z} \end{bmatrix} + \begin{bmatrix} c_{yy} & c_{yz} \\ c_{zy} & c_{zz} \end{bmatrix} \cdot \begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} k_{yy} & k_{yz} \\ k_{zy} & k_{zz} \end{bmatrix} \cdot \begin{bmatrix} y \\ z \end{bmatrix} \qquad (1)$$

The equations are based on the leakage relation from Yamada (Ref. 8) for flow between concentric rotating cylinders

$$\Delta p = (1 + \xi + 2\sigma) \cdot \frac{\rho V^2}{2} \qquad (2)$$

with

Δp pressure difference across the seal
ξ entry loss coefficient; ξ = 0.5 in the whole text
ρ fluid density
V average fluid velocity in axial direction

The friction loss coefficient σ is defined as

$$\sigma = \lambda \frac{L}{\Delta R} \qquad (3)$$

with the seal length L, the radial clearance ΔR and the friction factor

$$\lambda = 0.079 \cdot Ra^{-1/4} \left[1 + \left(\frac{7R_r}{8R_a}\right)^2\right]^{3/8} \tag{4}$$

This factor depends on the radial as well as the axial Reynolds numbers

$$R_a = \frac{2V\Delta R}{\nu} \qquad\qquad R_r = \frac{R\Omega\Delta R}{\nu} \tag{5}$$

R   seal radius
Ω   rotational speed of the rotor
ν   kinematic viscosity of the fluid

It is shown in Black's analysis that the dynamic coefficients of equation (1) can be represented in the following form

Inertia coefficients:

$$m_{yy} = m_{zz} = \pi R \Delta p \cdot \mu_2 T^2/\lambda \tag{6}$$

Damping coefficients:

$$c_{yy} = c_{zz} = \pi R \Delta p \cdot \mu_1 T/\lambda$$
$$c_{yz} = -c_{zy} = \pi R \Delta p \cdot \mu_2 \Omega T^2/\lambda \tag{7}$$

Stiffness coefficients:

$$k_{yy} = k_{zz} = \pi R \Delta p \cdot (\mu_0 - \mu_2\Omega^2 T^2/4)/\lambda$$
$$k_{yz} = -k_{ty} = \pi R \Delta p \cdot \mu_1 \Omega T/2\lambda \tag{8}$$

They are dependent on the pressure difference Δp, the seal radius R, the friction factor λ, the rotational speed Ω, the average flow time T = L/V and finally on the quantities μ_0, μ_1, μ_2, which depend on the friction loss coefficient σ and the entry loss coefficient ξ (Ref. 2). Black's classical formulas correspond to short seals. An extension to finite length seals is possible by using correction terms, which had been determined by measurements (Ref. 2). Changes of λ for the circumferential direction as a fact of the radial shaft displacements are corrected by another factor.

Child (Ref. 3,4) has derived expressions for the dynamic coefficients of high pressure annular seals. In his analysis he assumes completely developed turbulent flow in both the circumferential and axial directions. His model is based on Hirs turbulent lubrication equations (Ref. 9). The short seal dynamic coefficients, derived by Childs (Ref. 3) are similar to the previous mentioned solutions from Black. In addition to Black's formulas the influence of an inlet swirl can be calculated. Childs (Ref. 4) has continued his analysis by deriving expressions for finite length seals. For use in the rotordynamic analysis of pumps the Hirs based models from Childs are the best currently available models.

IDENTIFICATION OF THE DYNAMIC SEAL COEFFICIENTS

The test rig, used for the experimental determination of the dynamic seal coefficients, consists of a very stiff rotating shaft and an elastically mounted rigid housing with two symmetric seals between rotor and housing (fig. 2). The housing is excited by test forces (input signal) and the system response is a relative motion between housing and shaft (output signal). From measured input and output signals mobility frequency response functions can be calculated. Finally an identification method, working in the frequency domain, is applied to the measurement data. It is possible to model a linear mechanical system corresponding to the real test rig. It consists of a rigid mass (housing) and the stiffness and damping coefficients of the seals. We assume that the equations of the model are known, but that the unknown parameters (seal coefficients) have to be determined. This can be done by a procedure requiring a good correlation between measured frequency response functions and analytical frequency response functions (fig. 3).

Mechanical and Mathematical Model

Fig. 4 shows the mechanical model with a rigidly supported very stiff shaft, the rigid mass m of the housing and the stiffness and damping elements corresponding to the seals, respectively to the flexible spring supporting the casing. Other forces acting on the housing, e.g. forces of the pipes, forces of the additional seals etc. are considered to be small. Applying test forces in the middle of the housing, the system responds only with translatory motion in the two directions y and z. The displacements can be described by the movements of the centre of gravity. The equations of motion for the model are

$$m \cdot \ddot{y} + 2(m_{yy} \cdot \ddot{y} + c_{yy} \cdot \dot{y} + c_{yz} \cdot \dot{z} + k_{yy} \cdot y + k_{yz} \cdot z) = F_y$$

$$m \cdot \ddot{z} + 2(m_{zz} \cdot \ddot{z} + c_{zy} \cdot \dot{y} + c_{zz} \cdot \dot{z} + k_{zy} \cdot y + k_{zz} \cdot z) = F_z$$

(9)

With harmonic test forces, acting in both directions, four stiffness frequency functions as well as four mobility frequency functions can be calculated.
If the exciter forces are

$$F_y = \hat{F}_y \cdot \sin\omega t = \hat{F}_y \cdot Im(e^{i\omega t})$$

$$F_z = \hat{F}_z \cdot \sin\omega t = \hat{F}_z \cdot Im(e^{i\omega t})$$

(10)

the system response has the form

$$y = Im(\hat{y} \cdot e^{i\omega t}), \quad z = Im(\hat{z} \cdot e^{i\omega t})$$

(11)

$\hat{y}, \hat{z}$ complex amplitudes

and we obtain the force-displacement relation

$$2 \cdot \begin{bmatrix} k_{yy} - (\frac{m}{2} + m_{yy})\omega^2 + i\omega c_{yy} & \vdots & k_{yz} + i\omega c_{yz} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ k_{zy} + i\omega c_{zy} & \vdots & k_{zz} - (\frac{m}{2} + m_{zz})\omega^2 + i\omega c_{zz} \end{bmatrix} \cdot \begin{bmatrix} \hat{y} \\ \cdots \\ \hat{z} \end{bmatrix} = \begin{bmatrix} \hat{F}_y \\ \cdots \\ \hat{F}_z \end{bmatrix} \qquad (12)$$

The inverse functions of (12) are the complex mobilities

$$\frac{1}{4\Delta} \cdot \begin{bmatrix} k_{zz} - (\frac{m}{2} + m_{zz})\omega^2 + i\omega c_{zz} & \vdots & -(k_{yz} + i\omega c_{yz}) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots & \vdots & \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -(k_{zy} + i\omega c_{zy}) & \vdots & k_{yy} - (\frac{m}{2} + m_{yy})\omega^2 + i\omega c_{yy} \end{bmatrix} \cdot \begin{bmatrix} \hat{F}_y \\ \cdots \\ \hat{F}_z \end{bmatrix} = \begin{bmatrix} \hat{y} \\ \cdots \\ \hat{z} \end{bmatrix} \qquad (13)$$

$$\Delta = \{k_{yy} - (\frac{m}{2} + m_{yy})\omega^2 + i\omega c_{yy}\} \cdot \{k_{zz} - (\frac{m}{2} + m_{zz})\omega^2 + i\omega c_{zz}\}$$

$$- \{k_{yz} + i\omega c_{yz}\} \cdot \{k_{zy} + i\omega c_{zy}\}$$

Measurements of the Mobility Frequency Response Functions

Contrary to the complex stiffness functions (12) the mobility functions (13) are easy to measure. For the determination of these functions from measured input and output time data, we take advantage of the fact, that the ratios of the Fourier transformed signals are equal to the frequency responses. The force and response signals are measured in the time domain, transformed to the frequency domain by means of Fast Fourier Transformation and the quotient is calculated. This procedure is executed by efficient two-channel Fourier Analyzers. Fig. 5 Shows in principle the measuring equipments. The system is excited by a hammer impact, which is equal to an impuls force acting on the housing as a broadband excitation. Pulse duration, frequency content and force amplitude can be influenced by the variation of the hammer mass, the flexibility of the impact cap and the impact velocity. The relative displacements between housing and shaft are measured with inductive pick-ups. The time signals are amplified, digitized by the Fourier Analyzer and the frequency response functions are calculated. A bus system transfers the measured data to a digital computer, where an identification procedure calculates the unknown seal coefficients.

Estimation of the Dynamic Seal Coefficients

Different possibilities exist to determine the dynamic seal coefficients from the measured mobility response functions. The first idea is to fit analytical response functions to the measured ones. This can be done either directly with the flexibility functions or after the inversion of the measured curves, with the stiffness functions.

Another method will be presented in this paper. From a theoretical point of view the product of the complex mobility matrix $\underline{H}_{kin}$ (eq. 13) and the complex stiffness matrix $\underline{K}_{kin}$ (eq. 12) should be the unity matrix $\underline{E}$. By combining the measured matrix $\underline{H}_{kin}$ with the analytical Matrix $\underline{K}_{kin}$ the result will be $\underline{E}$ and an additional error matrix $\underline{S}$ because of measurement errors (noise, etc).

$$\underline{K}_{kin} \cdot \underline{H}_{kin} = \underline{E} + \underline{S} \tag{14}$$

or, with

$$\underline{K}_{kin} = \underline{K} - \omega^2 \underline{M} + i\omega \underline{C} \tag{15}$$

$$[\underline{K} - \omega^2 \underline{M} + i\omega \underline{C}] \cdot \underline{H}_{kin} = \underline{E} + \underline{S} \tag{16}$$

The last formula can be rearranged to

real part: $\quad \underline{K}\,\underline{H}^r_{kin} - \omega^2 \underline{M}\,\underline{H}^r_{kin} - \omega\,\underline{C}\,\underline{H}^i_{kin} = \underline{E} + \underline{S}^r$

$$\tag{17}$$

imaginary part: $\underline{K}\,\underline{H}^i_{kin} - \omega^2 \underline{M}\,\underline{H}^i_{kin} + \omega\,\underline{C}\,\underline{H}^r_{kin} = \underline{0} + \underline{S}^i$

respectively

$$\underline{A}\,\underline{X} = \underline{E}' + \underline{S}' \tag{18}$$

where $\underline{A}$ consists of the measured frequency response functions and of the related frequencies ω_i, $\underline{X}$ represents the unknown coefficients $\underline{M}$, $\underline{C}$ and $\underline{K}$, and $\underline{E}'$ is a modified unity matrix with some zero elements because of the seperation of the complex values into two real values. Applying the Euclidean norm to this equation $\underline{S}'$ shall become minimal. In practice there will be carried out a matrix multiplication from the left with the transpose matrix of $\underline{A}$.
By this we find the so-called normal equations, a determined system of equations for the unknown seal coefficients.

$$\underline{A}^T \cdot \underline{A} \cdot \underline{X} = \underline{A}^T \cdot \underline{E}' \tag{19}$$

In our case, measuring four frequency functions, the product $\underline{A}^T\underline{A}$ is a 6 x 6 matrix and $\underline{X}$ and $\underline{A}^T\underline{E}'$ are 6 x 2 matrixes. No assumptions are made in the procedure concerning the special structure of the matrices (skewsymmetry etc.). The advantages of the described algorithm are the fast calculation of the seal coefficients and the minimal requirement of storage space. Preliminary simulations have shown the practicability of this method.

DESCRIPTION OF THE TEST RIG

Mechanical and Hydraulic Design

Two annular sealing surfaces, integrated symmetrically in a stiff housing are the core parts of our testing plant (fig. 2). A shaft, rotating without contact inside the housing, is the second component of the seals. The fluid, in our case water, streams trough these radial clearances in an axial direction primarly. There is one central water supply for both seals and two drains, one for each. With this arrangement the housing cannot be displaced axially and we will measure only a translatorial, no rotational, movement of the casing in the radial direction if the geometries of the seals are identical. The shaft itself is resistant to bending and supported by rigid bearings. Shaft, motor and bottom-plate form the reference system for measuring the motion of the housing (mass m = 14.5 kg) which is fastened with soft springs to the fixed system. The necessary connections between both parts of the test rig are dimensioned in such a way that their influences are small compared with the seal behaviour.

Beside the mechanical component of the test stand we have the hydraulic part. A centrifugal pump (maximal throughput: q_{max} = 4.5 m^3/h) feeds water out of a reservoir into the system. Filter and slide values regulate the flow and the temperature of the water to make sure a steady state system. Through flexible hose pipes the water runs into and out of the housing.
With the geometrical values:

length of the seal:	L =	35 mm,
radius of the shaft:	R_s =	21.o mm,
radius of the housing:	R_h =	21.35 mm,
clearance of the seal:	ΔR =	$R_h - R_s$ = 350 μm,

the range for the number of revolutions U = 0...6000 1/min and the range for the fluid velocity in the seal V = 0...13.5 m/sec, the characteristical Reynolds numbers, R_a and R_r can be varied

$$R_a = 0....11\ 800 \qquad R_r = 0....5\ 800$$

if the fluid temperature is 30°C (this temperature is used in every measurement). These ranges are restricted to

$$R_a = 7000....11\ 800 \qquad (V = 9.o...13.5\ m/sec)$$
$$R_r = 2400.... 5\ 800 \qquad (U = 2500...6000\ 1/min)$$

in order to guarantee the turbulent region of flow in the seals. By changing the water temperature there will be available a different working range because of the temperature sensitivity of the kinematical viscosity ν.

Measured Quantities and Instrumentation

Three different groups of measuring values can be distinguished

- data of the fluid
- data of the exciter
- response data of the housing.

Pressure, temperature, density and viscosity characterize the fluid state. If working with an incompressible medium it is sufficient to measure pressure and temperature as the only variables of state and calculate the others.
Therefore several pressure - and temperature pick-ups are distributed over the test plant. Furthermore the fluid velocity is measured in the supply-line and the shaft speed is displayed by the motor control. With this, all necessary data in order to calculate the seal coefficients, are available. The excitation force of the hammer blow is measured with a piezoelectric accelerometer mounted in the hammer hed and a charge amplifier. The greatest forces obtainable are 120 N to 150 N. The third group of data belongs to the motion of the housing relative to the shaft. Inductive pick-ups with a carrier wave amplifier registrate the distance between shaft and housing contactlessly. Two working planes with two orthogonal measurement directions permit the control of the housing's displacements (translatorial, rotatorial).

The present data of the force and displacement as a function of the time are transformed to the frequency domain by a digital spectrum analyzer which calculates the transfer functions (ratio of displacement to force in the frequency domain) and stores them on tape. The identification of the system parameters $\underline{M}, \underline{C}, \underline{K}$ itself is carried out by a desktop computer with the stored data.

Previous Tests

In order to get to know the behaviour of the test rig itself without any "seal effects" several static tests had been performed: a constant load had been applied to the housing and the related movement had been measured, some kind of stiffness determination. When doing this without shaft rotation and fluid velocity (no seal effect), the stiffness of the system, that are the stiffness of the bolt springs, of the inlet and outlet pipes, of the instrument leads and so on, is measurable (fig. 6). We found a linear force-displacement relation, which differs for the four directions ($\pm$ y, $\pm$ z) but in general we found a stiffness of the set-up of about $5 \cdot 10^4$ N/m for the y and the z directions. This is an additional term similar to the mass of the housing which has to be taken into consideration while calculating the diagonal elements of the stiffness matrix.

Increasing the axial flow while the shaft speed is still zero, caused an increase of the system's stiffness which coincided quite well with the pre-calculated values from the models of Black (Ref. 1,2) and Childs (Ref. 3) (table 1). The constant error of those measurements could be a systematical one and consequently it could cause some problems in identifying the stiffness coefficients out of a dynamic measurement.

MEASUREMENTS OF THE DYNAMIC COEFFICIENTS

Measurement Procedure

Every measurement set consists of four frequency response functions H_{ik} (input direction k = y, z; response direction i = y, z). As described above our input data result from multiple successive impact signals (in order to reach a high energy level) of an optimized hammer which renders possible an excitation only for a linear seal behaviour. Force and response (distance between shaft and housing as a function of time) signals are measured simultaneously and later on processed in a digital fourier analyzer. The data set, obtained in this way, describes a single working condition of the test system, i.e. fluid velocity, fluid temperature and shaft speed are constant. To get the following results and dependences, only one parameter will be varied during one series of measurements. The temperature and with this the viscosity will be constant for all measurements and equal to $30^{\circ}C$.

By the way, at the moment we are measuring four frequency response functions in order to confirm the skew-symmetrical character of the frequency response matrix, but later on the expenditure of measurements could be probably reduced to two functions.

A desktop computer with a medium storage capacity takes over the subsequent treatment of the stored data. A program which contains the identification procedure itself ($\underline{H} \cdot \underline{K} = \underline{E}$) and an in-core solver for this determinante equation with respect to the unknown coefficients of the seal, calculates the parameters and, with the help of these, the fitted frequency response functions. Both functions, measurement and curve fit, can be displayed and plotted so we can estimate the quality of our curve fit algorithm visually. Furthermore the parameters $\underline{M}$, $\underline{C}$, $\underline{K}$ as functions of fluid velocity and shaft speed are reproduced by approximate equations which are polynominal equations whose order had been found in comparison with theoretical curves calculated with the above mentioned models and a least square method's optimization.

Presentation and Discussion of some Results

The described process is demonstrated for one working condition: V = 12.0 m/sec, U = 3000 min^{-1}, T = $30^{\circ}C$, in order to understand the results discussed later. Figure 7 shows the measured H_{ik}, plotted as magnitude and phase of the complex values, and the analytical frequency response functions too. The correlation of measured and analytical data is more or less good. Especially the additional peak in H_{yy} surprise. Furthermore the signal to noise ratio was expected to be better.

The resultant matrices of the seal coefficients can be read in fig. 8. As mentioned before the values for the complete system with two seals and all additional terms (mass of the housing, soft springs etc.) are presented. It can be seen, that the main diagonal elements are not equal, furthermore the expected skewsymmetry could not be found in the measured results. This is one result of a series of measurements, which point out the influence of the shaft rotation while the axial velocity is constant. The values of the whole series for constand axial fluid velocity V = 12 m/sec are drawn in fig. 9. The coefficients which should be equal in magnitude, when following the theory, are shown in one diagram and treated as two values for the same operating condition. Besides the measured coefficients fig. 9 points out the

corresponding values of the above mentioned theoretical models (Ref. 1,3). There is partly a good correlation, especially for the inertia and damping values. Even the traces of curves of the stiffnesses are similar to the theory, but here the values for k_{yy}, k_{zz} are found out 30 % to 40 % too small. The worst case we find for the cross-coupled stiffnesses k_{yz}, k_{zy}. Differences are possible up to 90 % of the theoretical values.

How can we explain and then correct this problem?
Suppose there is an inlet swirl of the water streaming through the inlet chamber, i.e. the fluid has an axial and, that's important, a circumferential velocity at the beginning of the seal. Applying the theory of Childs (Ref. 3) to our test system with a negative inlet swirl (the senses of rotation of the shaft and of the fluid are different), so the stiffness terms could reduce remarkably. On the other hand the cross-coupled dampings are influenced also very clearly. The problem at the moment is how to determine this inlet swirl and to decide whether or not it is constant with the shaft speed. Some experiments done with the contrary sense of rotation for the shaft, in that case an inlet swirl should change its sign and should effect the coefficients in some different manner, we got stiffness-values slightly different to the first results, but not as much influenced as expected! We suppose: our test rig doesn't agree with the two-degree-of-freedom model which was described earlier in the paper. When looking at fig. 7 the two transfer functions H_{yy} and H_{zy} show a particularity, a break-down in magnitude at about 12 to 14 Hz only by exciting in the y direction. Also the coherences arent't good at all at this frequencies. That characteristic can be found in almost every measurement. In order to eliminate its reason we looked at the whole system and measured a vibration belonging to a motion of the complete test rig at that critical frequency. It's a vibration of the system on the flexible connections between itself and the environment in the y-direction. This causes a motion of the "fixed" reference system during our measurements. An expanded mathematical model for the test set-up (three degrees of freedom) points out a reduction of the stiffness elements. This may be an explanation for the small stiffness values in the measurements. To avoid this problem, a modification of the test rig supports will be carried out.

Fig. 10 points out the influence of the axial velocity V. At first some remarks to the theoretical values. If we increase the axial velocity the seal coefficients will increase too, except the inertia term. Two cases appear: the increasement of velocity results in a parallel shift of the k_{yy}, k_{zz}, c_{yy}, c_{zz} curves or the gradient rises for k_{yz}, k_{zy}.
Now let's look at the measurements. The statements form above are transferable very well. Even the absolute shift values of k_{yy}, k_{zz} and c_{yy}, c_{zz} are agreeing. The difference of the main stiffness element between V = 16.5 m/sec and V = 12.5 m/sec calculated for the measurements is $7 \cdot 10^4$N/m and for the theory $6 \cdot 10^4$N/m.
The main damping values aren't as constant as predicted but the difference between them of about 250 N/msec is comparable with theoretical results.

The cross-coupled stiffness element is very small but we find, that the main aspect, greater gradients for higher velocities, is right. The cross-coupled damping is approximately a linear function of the shaft speed but there is hardly an influence of the velocity. This fact and the numerical values confirm the annular seal models.
Finally no influence of the axial velocity can be seen for the inertia term, which is also in good correlation with the theory.
After a modification of our test rig further examinations will follow and we hope to obtain new results for the dynamic seal coefficients.

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TABLE I. - STATIC STIFFNESS VALUES FOR U = 0 1/min

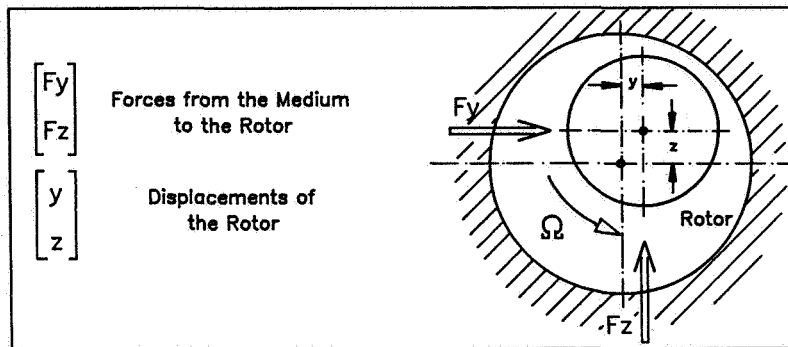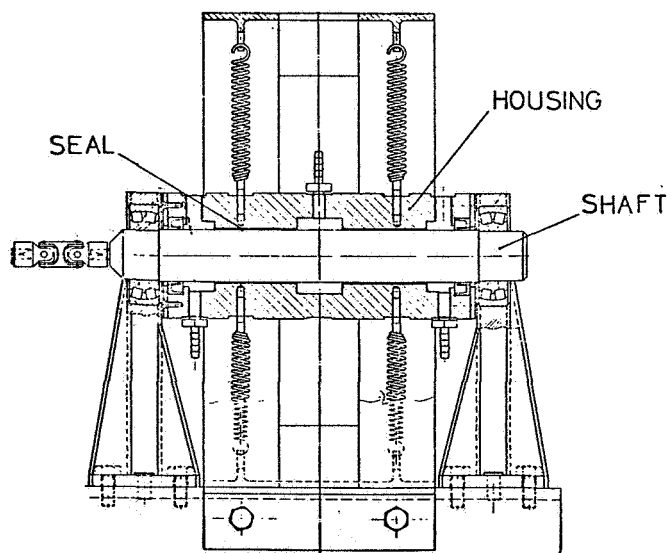| shaft speed [1/min]	axial velocity [m/sec]	measured K [N/m]	theoretical K [N/m]	Error [%]
0.0	7.5	132 500	164 000	−19.8
	9.0	171 800	208 000	−17.4
	10.5	216 000	270 000	−19.8
	12.0	266 600	330 000	−19.4
	13.5	308 100	390 000	−21.0

Figure 1. - Black's annular seal model.
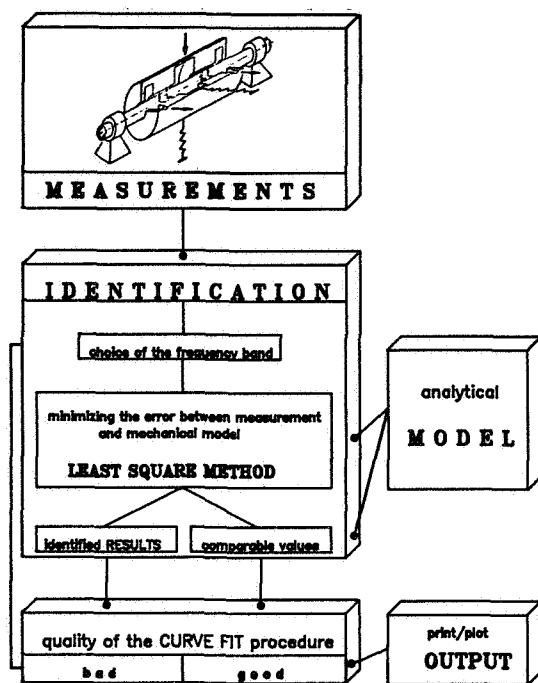

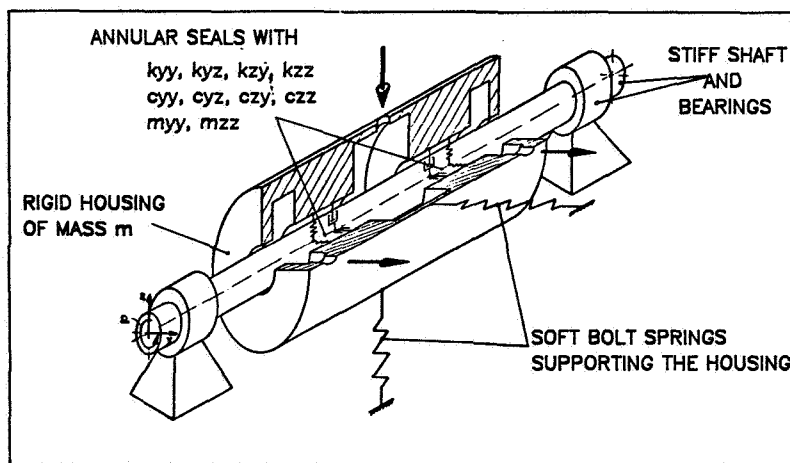Figure 2. - Test rig.

Figure 3. - Identification procedure.

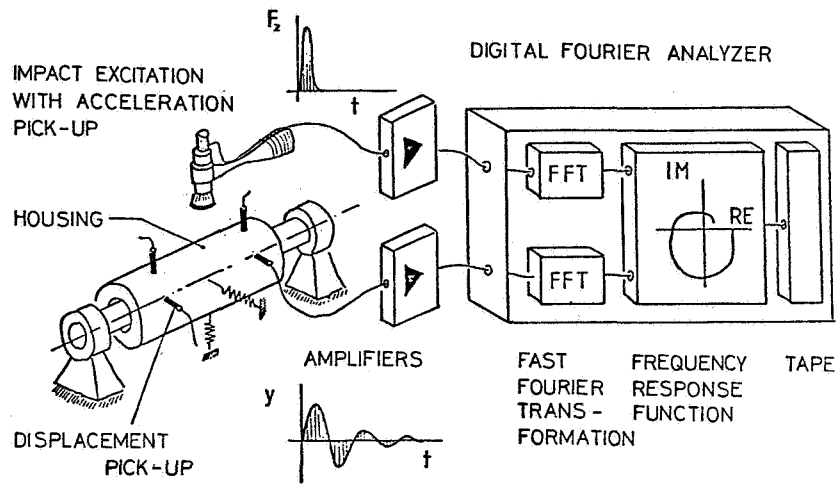Figure 4. - Mechanical model of test rig.

The necessary Measurement Set—up

Figure 5. - Necessary measurement set-up.

Static Load Test

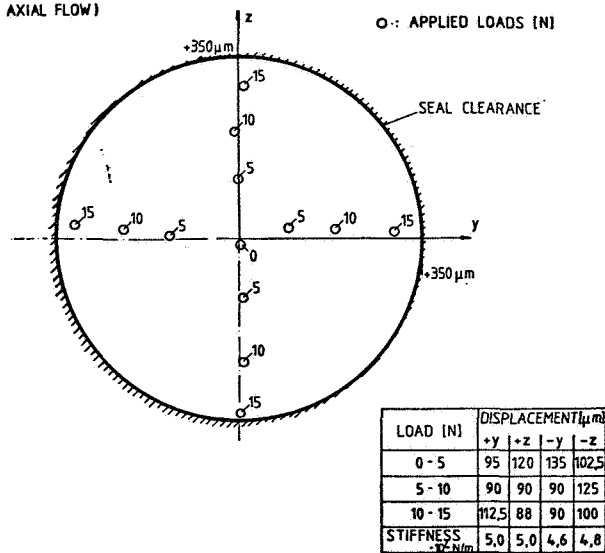| LOAD [N]	DISPLACEMENT[μm]			
	+y	+z	-y	-z
0 - 5	95	120	135	102,5
5 - 10	90	90	90	125
10 - 15	112,5	88	90	100
STIFFNESS ₁₀₂ N/m	5,0	5,0	4,6	4,8

Figure 6. - Static load test (without shaft rotation and axial flow).

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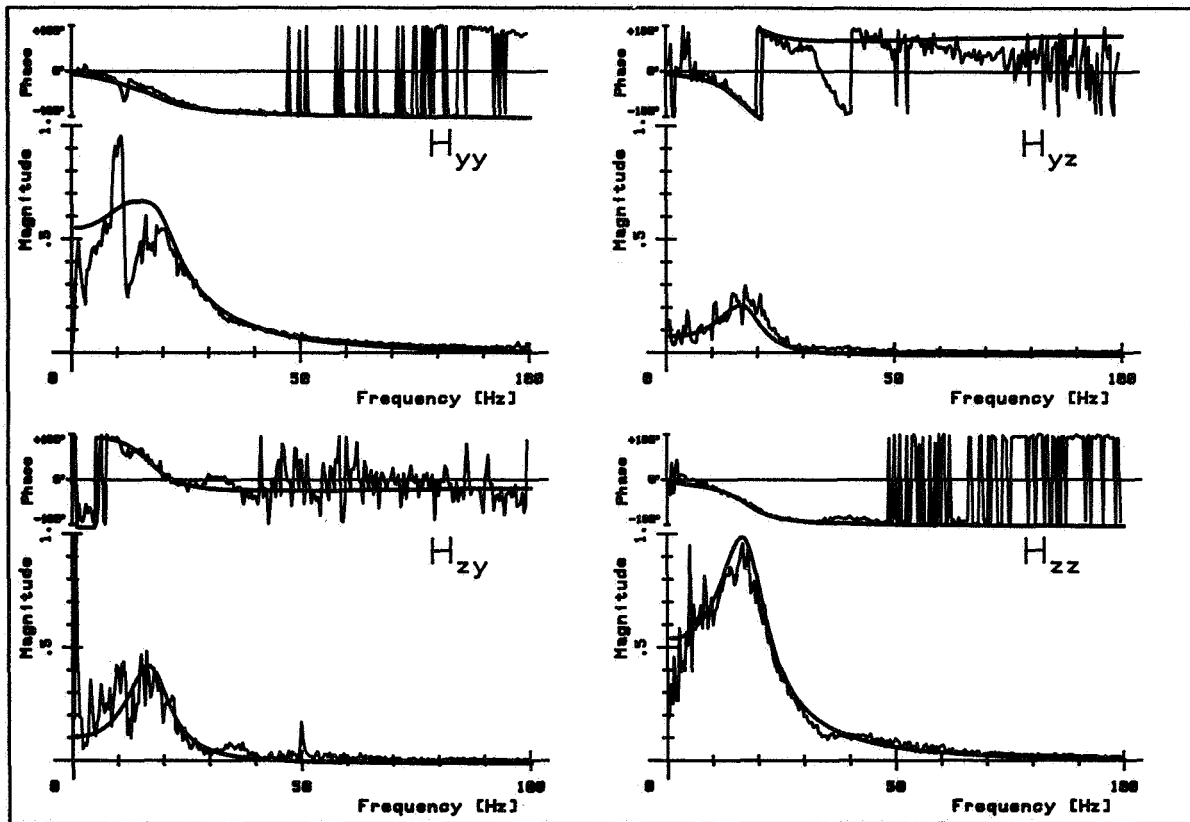
Figure 7. - Complete set of frequency response functions for   U = 3000 1/min,
V = 12.0 m/sec, and T = 30° C.

Fluidal and geometrical values:

Length of the Seal: 35 mm
Radius of the Seal: 21 mm
Radial Clearance: 0.35 mm

Temperature: 30.0°C
Density: 995 kg/m^3
Viscosity: 8.0E-7 m^2/sec

Pressure drop: 2.23 bar
Fluid Velocity: 12.03 m/sec
Rev. of the shaft: 3000 1/min

Reynolds—radial: 2 887
Reynolds—axial :   10 530

Representation of the identified SEAL—PARAMETERS

Inertia matrix [kg]

$$\begin{bmatrix} +16.40 & -0.70 \\ +1.82 & +19.40 \end{bmatrix}$$

Damping matrix [N/msec]

$$\begin{bmatrix} +1\ 560 & -240 \\ +854 & +1\ 100 \end{bmatrix}$$

Stiffness matrix [N/m]

$$\begin{bmatrix} +197\ 000 & -33\ 300 \\ +47\ 200 & +210\ 000 \end{bmatrix}$$
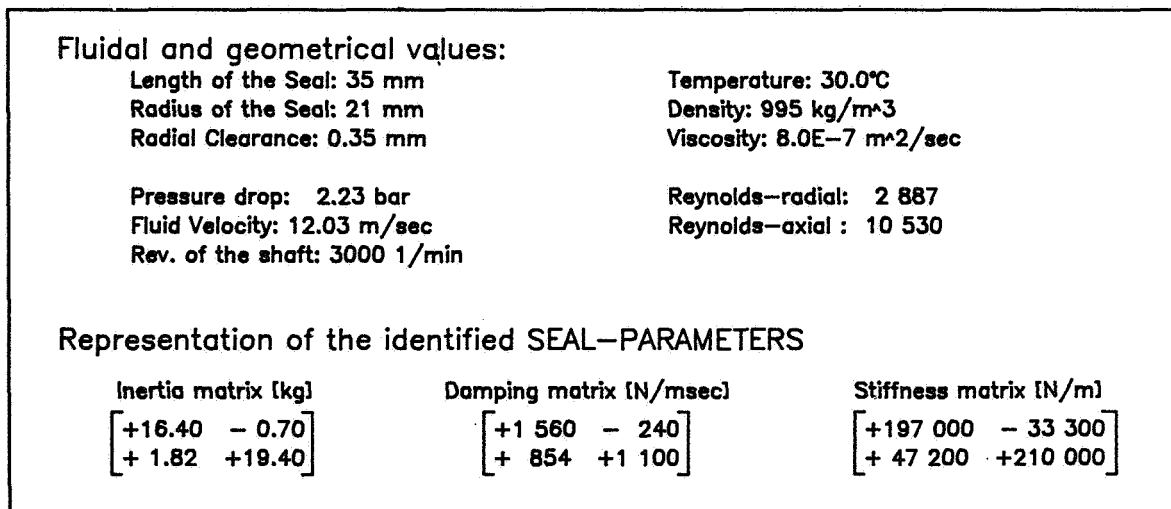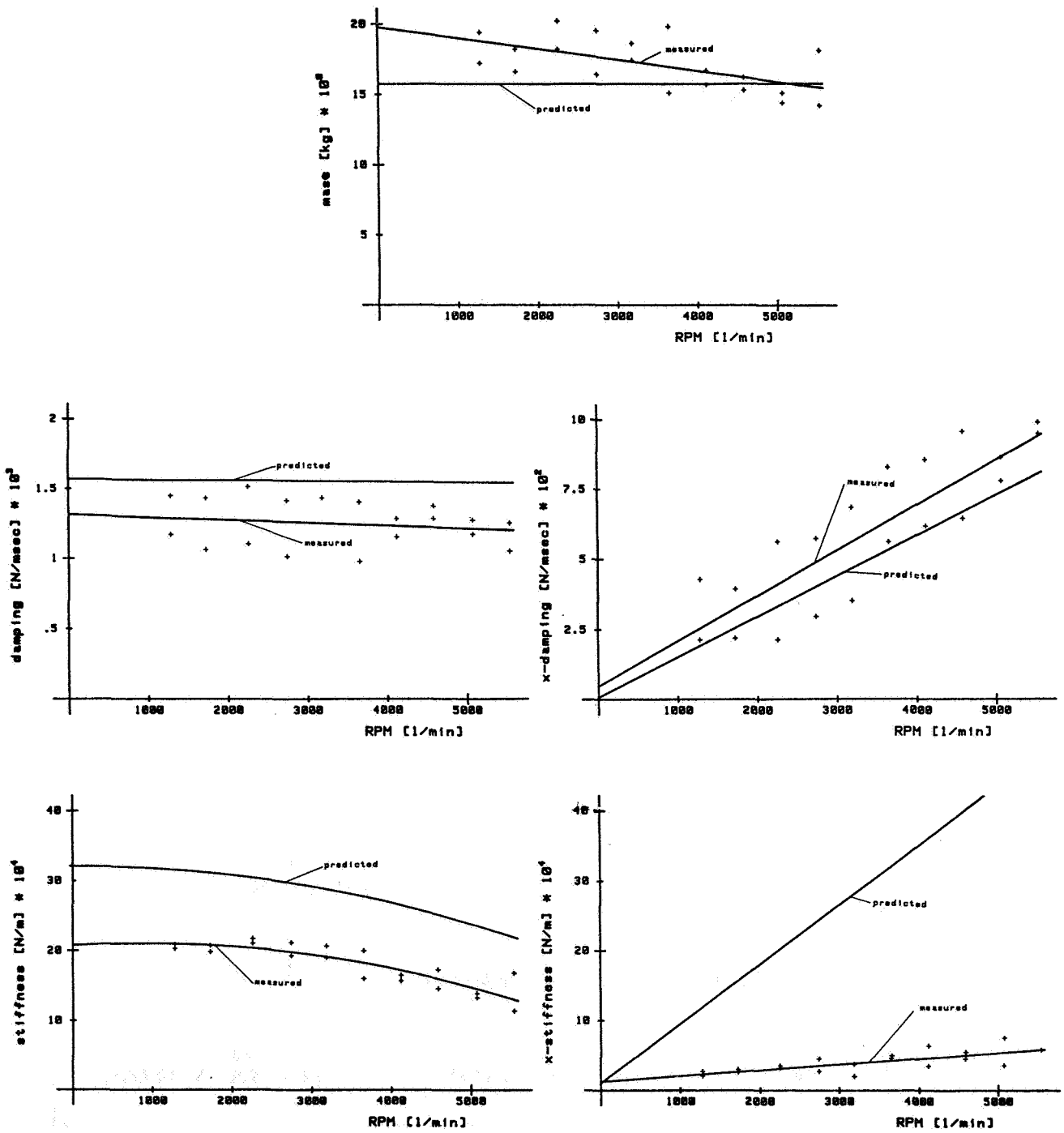
Figure 8. - Numerical results for example in figure 7.

Figure 9. – Theoretical and experimental results for V = 12.0 m/sec and T = 30° C.
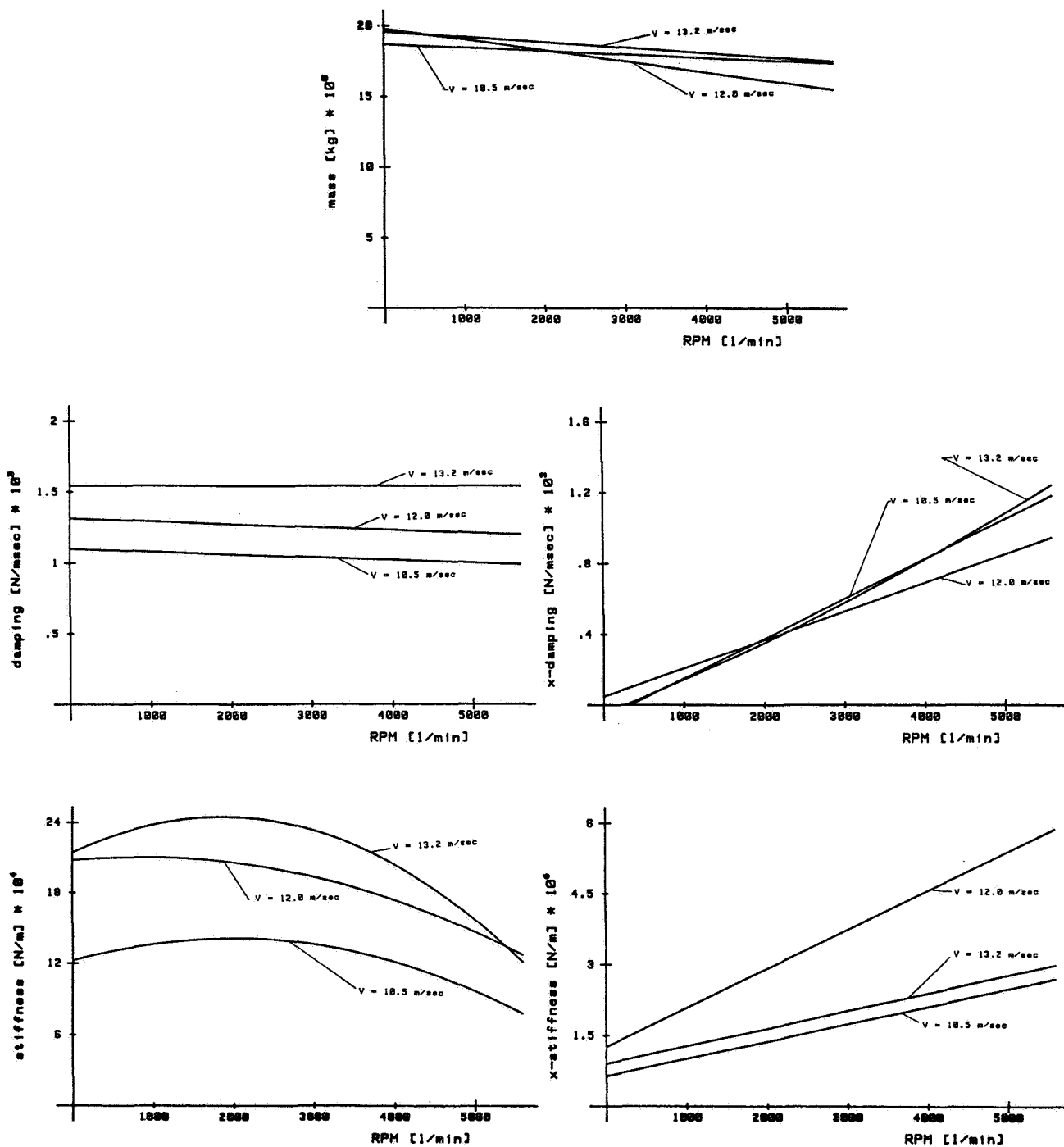
Figure 10. - Fitting curves of identified coefficients for V = 10.5, 12.0 and 13.2 m/sec.