A NONLINEAR FILTER FOR COMPENSATING FOR TIME DELAYS IN MANUAL CONTROL SYSTEMS

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ABSTRACT

The existence of time delays in manual control systems can have a significant and deleterious effect upon closed-loop system performance and stability. Modern flight control systems often exhibit such delays owing to digital control law implementation and higher-order control system dynamics. Modern flight simulators also share this problem owing to computational delays associated with computer-generated graphics. Thus, the need for an effective method for time delay compensation is becoming increasingly urgent. Linear methods of compensation provide needed phase lead but also introduce a significant gain distortion. To date, little research has been directed toward possible nonlinear compensation methods. This study analyzes and experimentally evaluates a nonlinear filter configured to provide phase lead without accompanying gain distortion. The nonlinear filter is superior to a linear lead/lag compensator in its ability to maintain system stability as open-loop crossover frequency is increased. Test subjects subjectively rated the filter as slightly better than a lead/lag compensator in its ability to compensate for delays in a compensatory tracking task. However, the filter does introduce unwanted harmonics. This is particularly noticeable for low-frequency pilot inputs. A revised compensation method is proposed which allows such low-frequency inputs to bypass the nonlinear filter. A brief analytical and experimental evaluation of the revised filter indicates that further evaluation in more realistic tasks is justified.

INTRODUCTION

Control systems which incorporate a human as a component in the system, such as an aircraft, are called manual control systems. Manual control systems have certain characteristics which make them highly sensitive to time delays in the system. This paper will discuss the sources and effects of time delays, some basics of manual control theory, and the results of an analysis on the effectiveness of a nonlinear filter as compared to a lead-lag filter for time delay compensation in manual control systems.

TIME DELAY SOURCES AND EFFECTS

Aside from the operator's reaction time, delay in manual control systems have three basic sources. One source is computational delays in processing input signals. Examples would be modern high-performance aircraft with sophisticated digital control systems, and flight simulators with computer generated imagery[1,2]. Another source is the sampling delay caused by analog-to-digital conversion in digital control systems. This delay can be shown to be T/2 seconds where T is the sampling interval. The final source of time delays would be apparent delays introduced into the system by higher-order high-frequency system components. These delays are termed apparent because they are not actual time delays; however, the phase lags introduced into the system by these components are perceived by the operator to be time delays.

Time delays effect manual control systems in two ways: first in demanding tasks, delays cause a reduction in closed-loop stability and hence handling qualities, and second in less demanding tasks, delays induce fundamental changes in pilot characteristics[3].

In demanding tasks such as mid-air refueling in an aircraft, pilots tend to increase their open-loop gain. This important characteristic of manual control systems makes time delays a serious concern, as the reduction in gain margin caused by the time delay may cause the pilot to drive his aircraft unstable as he increases his gain[4].

The changes in pilot characteristics caused by time delays in less demanding tasks are also serious particularly when considering flight simulators. When acting as compensatory elements in single loop tasks, Hess[3] has shown that pilot's attempt to generate lead (evidenced by stick pulsing) to compensate for the time delay. Pilots are often asked to rate the handling qualities of a particular simulation configuation. When the pilot is forced to alter his flying technique to compensate for time delays caused by the simulator, he is unable to give an accurate rating of the aircraft being simulated[4].

SOME BASICS OF MANUAL CONTROL THEORY

McRuer and Krendel[5] have shown that, in single-loop man-machine control systems such as Figure 1, pilots adopt compensatory equalization so that the forward loop transfer function, Y_pY_c, resembles ω_c/s in the region of the crossover frequency ω_c. If in this configuration $Y_c = K/S$, the pilot would tailor his own dynamics so that the combined open-loop transfer function, Y_pY_c, would exhibit ω_c like behavior at crossover, i.e. $Y_p = K_p$, a pure gain. If Y_c itself were a pure gain, K, the pilot would adopt a transfer function of the form,

$$Y_p \simeq \frac{K_p}{T_I S + 1} \tag{1}$$

Pilots also prefer to have this crossover to take place around 2-4 rad/sec depending upon the controlled element dynamics and the input bandwidth[6].

When considering the design of a time delay compensator, this range of crossover frequency becomes one of the design parameters. This is because we are interested in providing the maximum amount of phase lead at the point where it is most important, namely at the crossover frequency. Another design parameter would be the amount of time delay for which one is attempting to compensate. For piloted aircraft flight control systems, it has been shown that handling qualities reach the unacceptable region (pilot ratings beyond 6.5) at delays of approximately 0.225 to 0.250 seconds[1].

Much attention has been devoted to finding the best method of compensating for time delays in manual control systems. The technique most commonly used to date is simple lead-lag compensation. The reminder of this paper will compare, both computationally and experimentally, the effectiveness of a nonlinear and lead-lag filter as time delay compensators in manual control systems.

LEAD-LAG FILTER DESIGN

A typical lead-lag filter can be given by the transfer function

$$G_f(s) = K_d \frac{T_n S + 1}{T_d S + 1} \tag{2}$$

Phase lead is generated when $1/T_d > 1/T_n$. A Bode plot is shown in Figure 2. As can be seen from the figure the phase lead generated is always accompanied by a gain distortion. This gain distortion has several undesirable effects[4]. Any gain increase can cause an amplification of high frequency noise and disturbance input making accurate control more difficult. In flight simulators a gain increase will corrupt the replication of aircraft dynamics, so that the simulation is not an accurate reproduction of the aircraft handling qualities.

Crane outlines a simple technique to design a lead-lag compensator for manual control systems. For the transfer function given as Equation (2), the design process goes as follows:

1) Locate the filter zero, $1/T_n$, at the estimated crossover frequency, ω_c.

2) Determine T_d using the following equation which equates the amount of phase lead generated to the phase lag produced by the time delay at ω_c.

$$\tan^{-1}\omega_c T_n - \tan^{-1}\omega_c T_d = \omega_c t_d \tag{3}$$

3) Choose K_d so that the gain of G_f is unity at ω_c.

This design process results from attempting to minimize the effects of the gain distortion while providing the amount of phase lead necessary to compensate for the time delay. The gain distortion is forced to stay within an envelope of least perceived changes in plant dynamics[2]. Such envelopes result from studies done on simulating high order systems with low order models. Changes in the system dynamics are made at various frequencies, and pilot ratings are used to determine in what frequency ranges the changes are most noticeable[7]. Choosing a conservative crossover frequency of 2 rad/sec and a time delay of 0.250 seconds, this design process leads to a lead-lag filter of

$$G_f(d) = \frac{0.737(0.50S+1)}{(0.1467S+1)} \tag{4}$$

THE SPLIT-PATH NONLINEAR FILTER

The problems encountered with the gain distortion of the lead-lag filter suggest that an ideal time delay compensator would provide phase lead with no gain change. Foster, Gieseking, and Waymeyer[8] propose a nonlinear filter which is capable of providing independent magnitude. The filter they propose is called a split-path nonlinear filter (SPAN filter), a block diagram of which is shown in Figure 3.

The filter input is processed through two branches. One branch adjusts phase; the other adjusts magnitude. The phase branch is composed of a linear filter, F_1, and a nonlinear bistable element. The parameters of F_1 are adjusted to provide the desired phase change. This signal is then input to the bistable element which destroys all amplitude effects and retains only the phase changes. The magnitude also consists of two elements: a linear filter, F_2, and an absolute value. Parameters of F_2 are adjusted to provide the desired magnitude changes. The absolute value of this signal is then multiplied by the output of the bistable element to form the SPAN filter would produce output as shown in Figure 4. The describing function for this configuration shows phase lead without gain increase but instead a slight gain attenuation. For the analysis performed in this paper the SPAN filter was configured with a lead-lag filter having zero at -1 and pole at -10 for F_1, and unity gain for F_2. The Bode plots of the SPAN filter just described and the lead-lag compensator described in the previous section are shown in Figure 5.

COMPUTER ANALYSIS

Harmonic Analysis

Because SPAN is a nonlinear filter it is capable of generating sub- and higher harmonics. To assess the nature of these harmonics a Fourier analysis was performed on the output of the SPAN filter where the input consisted of a single sinusoid of variable frequency. The results are shown in Figure 6. Notice that the harmonics contribute most at the 3-4 rad/sec frequency range which is the region where the SPAN filter produces maximum lead. A Fourier analysis was also performed to check for the existence of subharmonics. None were found.

Steady-State Stability Comparison

To compare the effectiveness of the lead-lag filter vs. the SPAN filter in maintaining system stability, a computer simulation was performed using the configuration in Figure 7. The time delay was chosen to be 0.39 seconds to include the contribution of the pilot's reaction time. The input consisted of a sum of twelve sine waves as shown in Table 1. Compensation was accomplished using the lead-lag and SPAN filters configured as described earlier. For comparison a simulation was also performed using no compensation. Mean square values of r(t), c(t), c(t), and e(t) were calculated. These are denoted R^2, C^2, and E^2 respectively. Values of R^2, C^2, and E^2 were calculated for values of crossover frequency ranging from 0.5 to a value where stability was lost. The results are shown in Figures 8 through 10.

Examining the figures shows the SPAN filter superior in maintaining system stability. The lead-lag filter actually becomes unstable before the case of a time delay with no compensation. This is a result of the gain distortion of the lead-lag filter. The design process described by Crane is "strictly applicable to constant parameter linear systems[2]." A lead-lag filter design based on constant system parameters is unsatisfactory when the design crossover frequency is exceeded.

Closed-Loop Power Ratio

The harmonic analysis described previously gave an indication of the nonlinear nature of the SPAN filter in the open-loop case. To better understand now these nonlinearities would affect closed-loop performance an additional computer analysis was performed on the system of Figure 7. The input to the system was the same as that described in Table 1. The crossover frequency was increased from one to a value where stability was lost. The total power contained in the output was calculated and divided by the power in the output at the input frequencies. Table 2 shows the values obtained. This "power ratio" is an indication of the nonlinear nature of the closed-loop output. It indicates the amount of power in the closed-loop not at input frequencies, and thus attributes to the nonlinearity. The steady increase in the values of Table 2 indicates that, as the crossover frequency is increased, SPAN itself introduces increasing power in the output.

Transient Response

As a final step in the computer analysis the closed-loop step responses were calculated for the system in Figure 7 using the same three configurations as in the steady-state stability analysis. A unit step was the input. The crossover frequency, ω_c, was increased in unit increments from one up to a value which caused the response to diverge. Figures 11 through 13 show the step responses for the case of SPAN compensation for ω_c equal to 1, 2, and 3 rad/sec. Notice the jagged discontinuities present for ω_c = 1 rad/sec. This effect dies out for larger values of ω_c. Figures 14 and 15 shown the step responses for the cases of no compensation and lead-lag compensation respectively when ω_c = 3 rad/sec. Notice that the SPAN filter's response is less oscillatory.

The existence of discontinuities in the output of the SPAN filter, in effect, introduces high frequency noise into the system. The lead-lag filter on the other hand amplifies only existing high frequency noise. Since most physical systems have large reductions in gain at high frequencies, the effects of this noise injection or amplification may be mitigated. to demonstrate this with the SPAN compensation, the ω_c/S plant of Figure 7 was replaced with

$$\frac{\omega_c}{S(0.1S + 1)^2} \tag{5}$$

and the step responses for this new system were calculated. The step response for ω_c = 1 rad/sec is shown in Figures 16. The smoothing effect of additional dynamics is evident compared to Figure 11.

EXPERIMENTAL ANALYSIS

Description

Next, an experiment was performed to obtain subjective and objective measures of the effectiveness of the two compensation methods. The experiment was a single-axis compensatory tracking task involving a human operator, as shown in Figure 17. The error was displayed on a CRT as shown in Figure 18. The test subject was provided with an isometric control stick, his task being to null the error in the presence of a disturbance input.

Procedure

Four different combinations of delay and compensation method were used in the experiment: no delay, no compensation (nominal case); 0.25 second delay, lead-lag second delay, no compensation. A total of five subjects were used. Performance measures included mean square error, mean error, mean square stick output, and mean stick output. Each subject performed five data runs after adequate training.

In addition to the quantitative data obtained, a subjective comparison of each of the different configurations was also performed. Each subject was asked to rate each of the off-nominal configurations on a scale of 0 to 10, based on how closely each approximated the nominal case in terms of response characteristics, etc. The nominal case was given a value of zero.

Results

Figure 19 shows typical root-mean-square error scores for the subjects. Figure 20 shows typical data for root-mean-square stick output. Figure 21 shows data obtained in the comparison test.

Examining the graph of average RMS error scores shows that the three off-nominal cases result in RMS error scores which are approximately equal and larger than the nominal case. No significant reduction in error scores is seen for either the lead-lag or SPAN filter. For the lead-lag filter this is probably due to the amplification of remnant injected by the pilot. For the SPAN filter the larger RMS error scores probably are a result of harmonics produced by SPAN itself.

The graphs of the RMS stick output show that in most cases the SPAN filter has the largest value followed in order by the nominal case, lead-lag compensation, and delay with no compensation. These values of RMS stick output are in a logical order when the phase and gain characteristics of each configuration are considered under the assumption of constant crossover frequency. The case of a time delay with no compensation causes a reduction in system gain margin as compared to the nominal; therefore, the pilot must reduce his gain to maintain adequate stability. The lead-lag filter causes an increase in the "effective plant" gain as compared to the nominal. Thus the operator can reduce his gain with resulting lower RMS stick output scores. The SPAN filter causes a slight reduction in the "effective plant" gain thus allowing a larger pilot gain with accompanying lower RMS stick output.

Examining the graph of subject ratings shows the SPAN filter being ranked most like the nominal case. The average performance for SPAN may be due to subjects disliking the reduced gain margin for the no compensation case and the noise amplification of the lead-lag filter. This seems like the only reasonable explanation since the error scores for the SPAN filter show no significant improvement over either the lead-lag or no compensation cases.

A NEW CONFIGURATION FOR THE SPAN FILTER

The experimental results just obtained indicate that the SPAN filter increases system stability. However, SPAN does not increase tracking accuracy over the lead-lag filter. The results of the computer analysis indicate that this is caused by the harmonics produced by the SPAN filter which degrade closed-loop performance. The step responses also indicate that this detrimental influence of the harmonics on closed-loop performance is most noticeable for low frequency inputs. These results suggest a new configuration for the SPAN filter as shown in Figure 22.

The input to the filter is passed through two branches. The lower branch contains a low-pass filter which allows low frequency signals to by-pass the SPAN filter. Frequencies above a value of $1/T_1$ rad/sec are passed through the SPAN filter with parameters set as before. The output of the SPAN filter is then passed through a filter with break frequency at 20 rad/sec to reduce the amplitude of higher harmonics generated. Finally, the signals in the two paths are added together. A Bode plot of this configuration with $T_1 = 1.5$ is shown in Figure 23.

Calculating the closed-loop power ratios for the new configuration shows no improvement over the original filter. Closed-loop step responses were also calculated. Figures 24 shows the step response for $\omega_c = 1$ rad/sec. The new filter seems to significantly improve the form of the step response. Limited experimentation with the filter implemented in the simulation described earlier showed no improvement in tracking random command signals, but smoother responses in following step-like commands.

SUMMARY AND CONCLUSION

The results of the analysis described in this paper show that the SPAN filter's main strength lies in its ability to maintain system stability as the open-loop crossover frequency is increased. The lead-lag filter actually reduces system stability as the crossover frequency increases due to the gain distortion it introduces into the system. The SPAN filter's relative insensitivity to increase in crossover frequency is an important attribute since in manual control systems the crossover frequency is task dependent. The nonlinear nature of the SPAN filter, which enables it to perform so favorably in maintaining system stability, unfortunately also degrades its closed-loop performance. These effects are partially mitigated by replacing the K/S plant of the simulation with one more typical of those found in physical systems. Finally, a new arrangement for the SPAN filter was proposed which allows low frequency inputs.

The research described in this paper indicates that the split-path nonlinear filter shows definite promise as a compensation method for time delays in manual control systems. The next step would be actual implementation of the filter in a sophisticated system simulation and evaluation of its performance and pilot acceptability.

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Table 1. Sum of sinusoids input

$$r(t) = \sum_{i=1}^{12} A_i \sin(\omega_i t + \theta_i)$$

ω_i rad/sec	A_i/A_1	Number of cycles in run
0.16419	1.0	3
0.27366	1.0	5
0.76624	1.0	14
1.25883	1.0	23
1.86087	1.0	34
2.68185	0.1	49
3.66702	0.1	67
5.03531	0.1	92
7.16984	0.1	131
9.79695	0.1	179
13.73763	0.1	251
20.96219	0.1	383

Table 2. Closed-loop power ratios

Crossover Frequency ω_c	Power Ratio
1.0	1.02
2.0	1.03
3.0	1.04
4.0	1.06
5.0	1.11
6.0	2.07

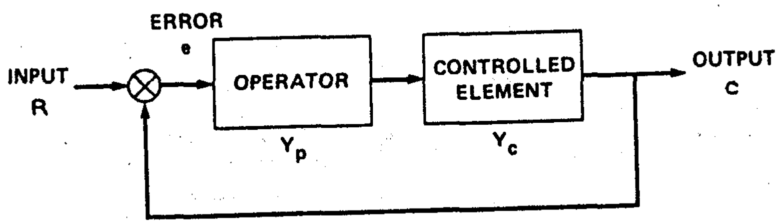

Figure 1. Single-loop compensatory task (from reference 3).

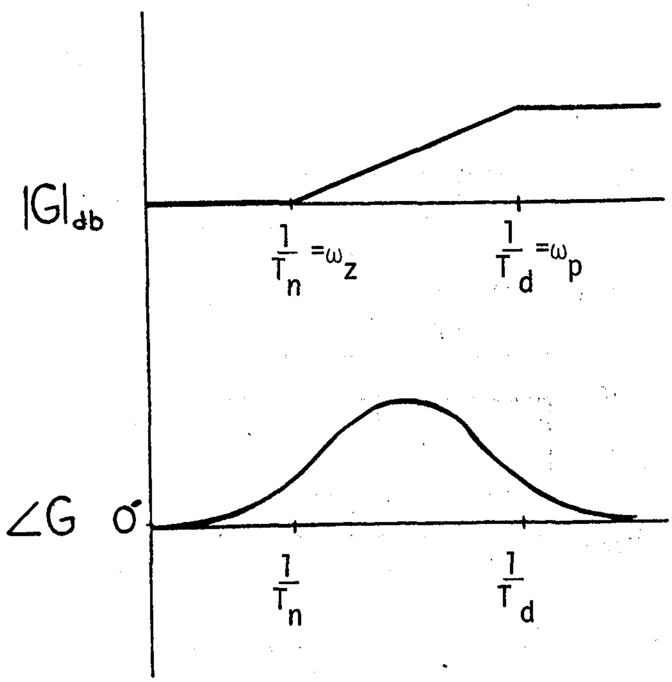

Figure 2. Bode plot of lead-lag filter (from reference 4).

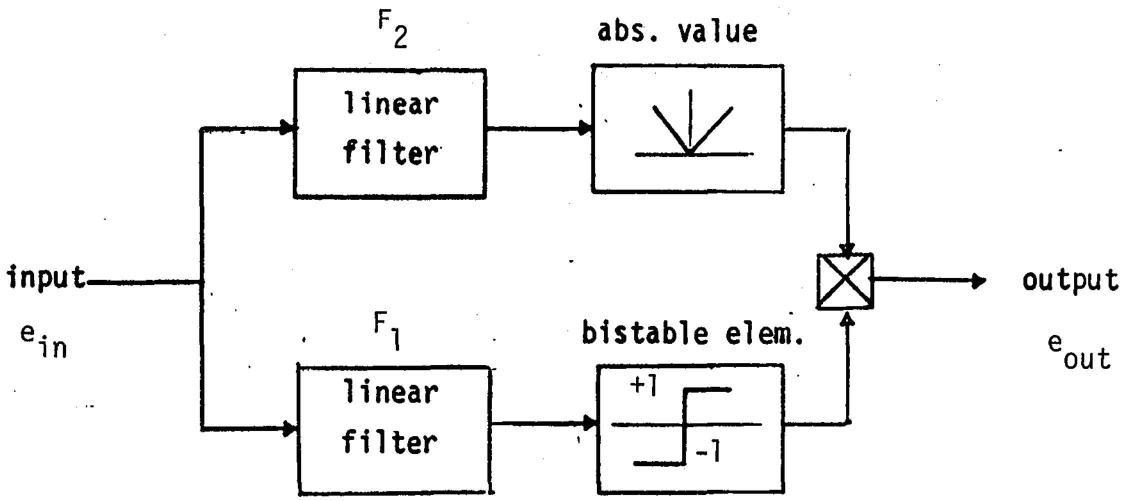

Figure 3. SPAN filter (from reference 8).

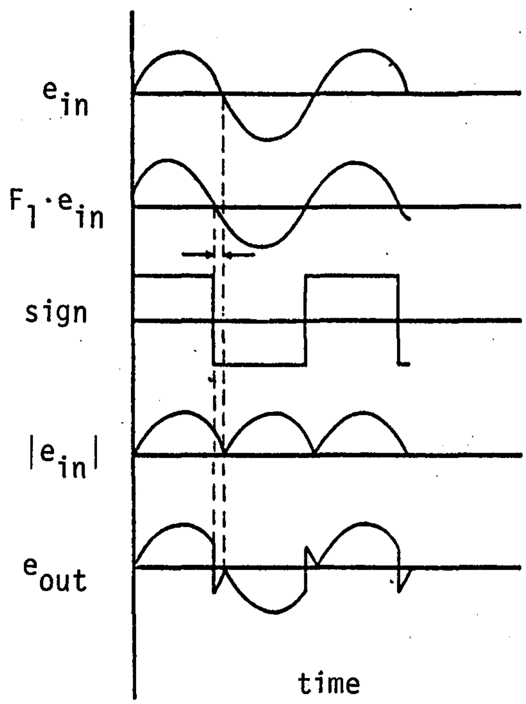

Figure 4. Filter waveform characteristics with lead-lag filter in F_1 (from reference 8).

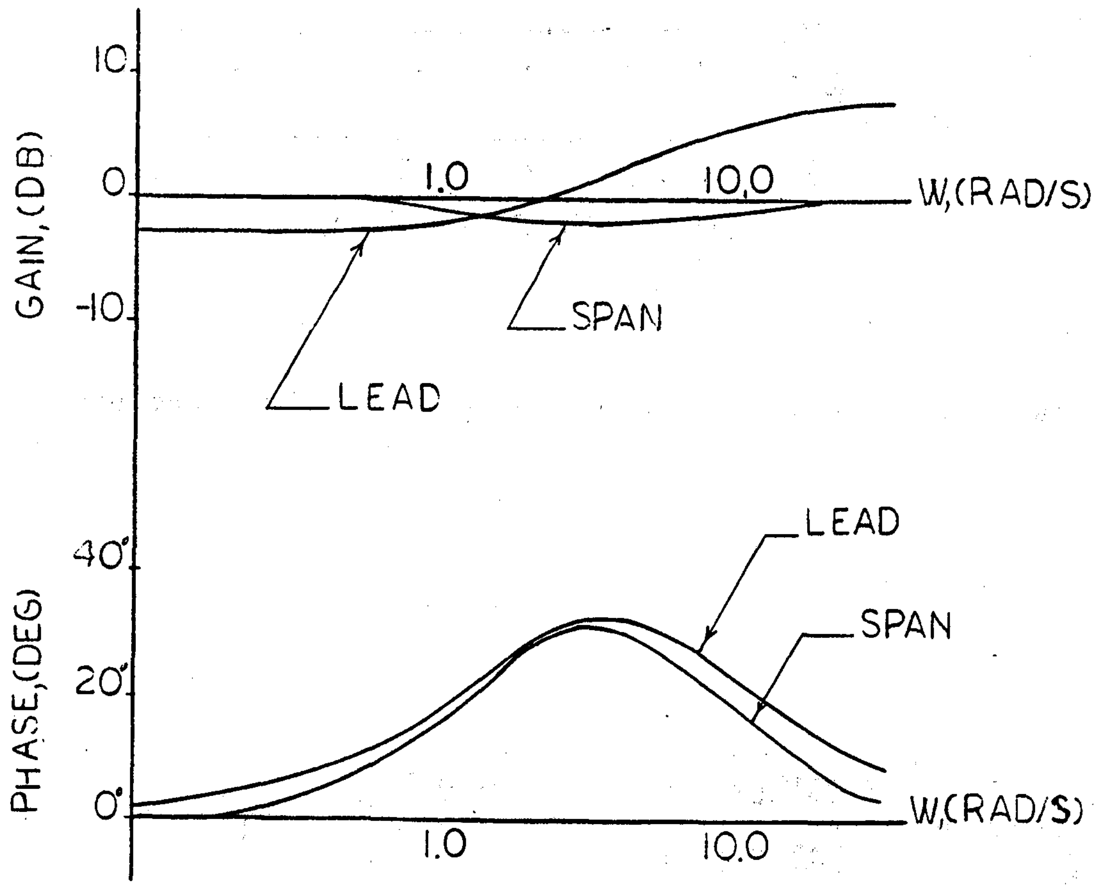

Figure 5. Bode plots for SPAN and lead-lag filters.

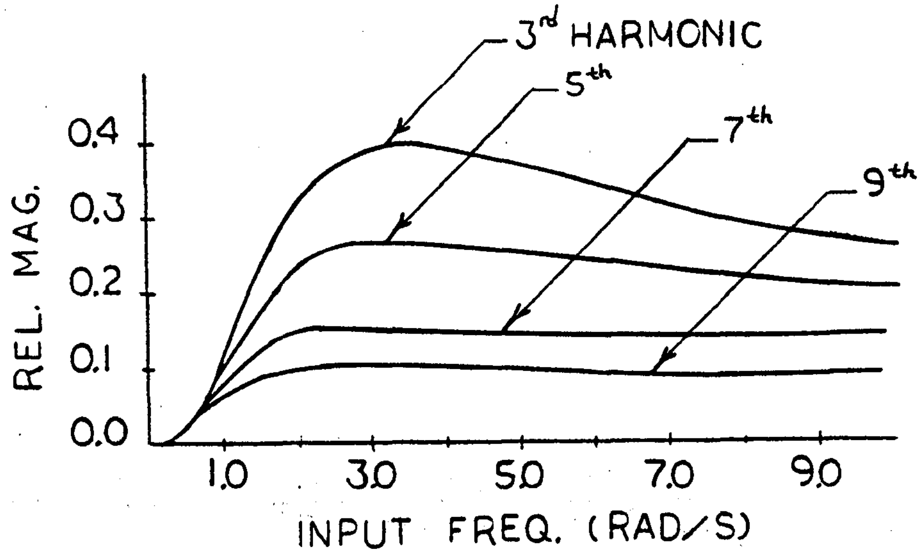

Figure 6. Relative magnitude of nth higher harmonic.

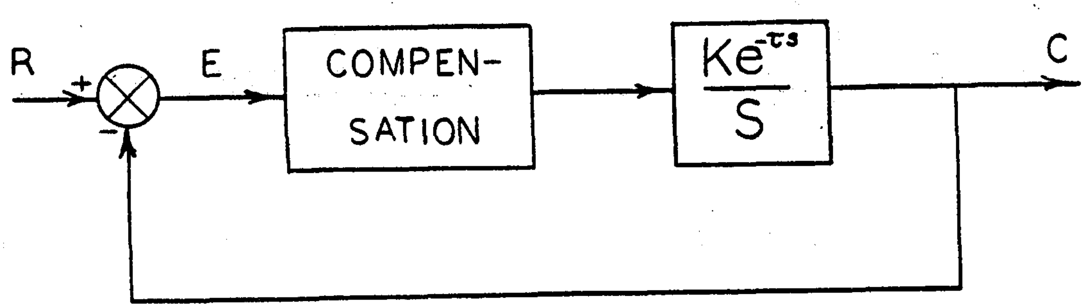

Figure 7. Single-loop system used for computer analysis.

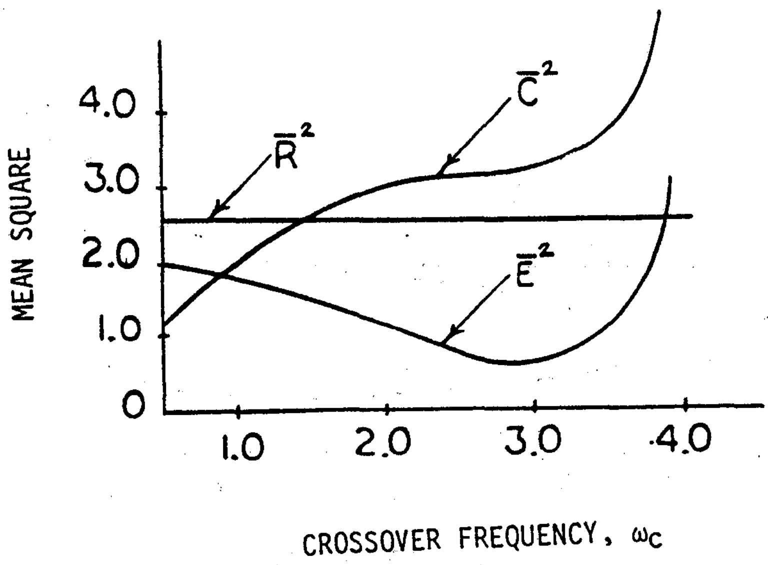

Figure 8. Mean square values for Fig. 7 with a time delay and no compensation.

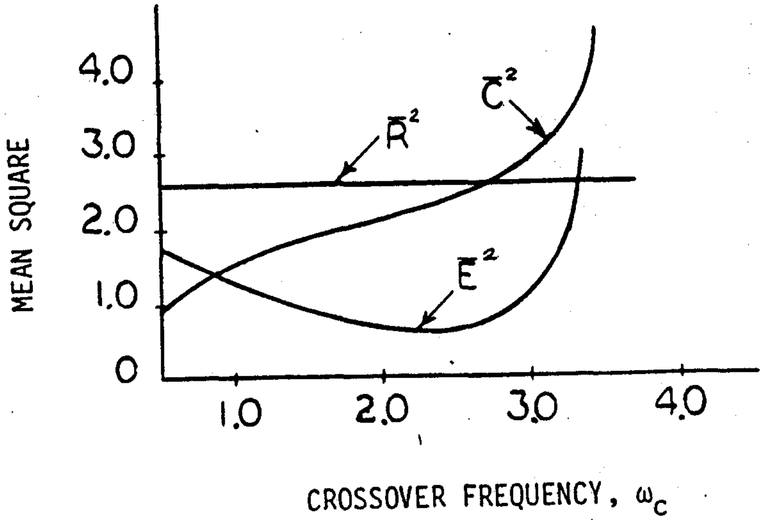

Figure 9. Mean square values for Figure 7 with a time delay and lead-lag compensation.

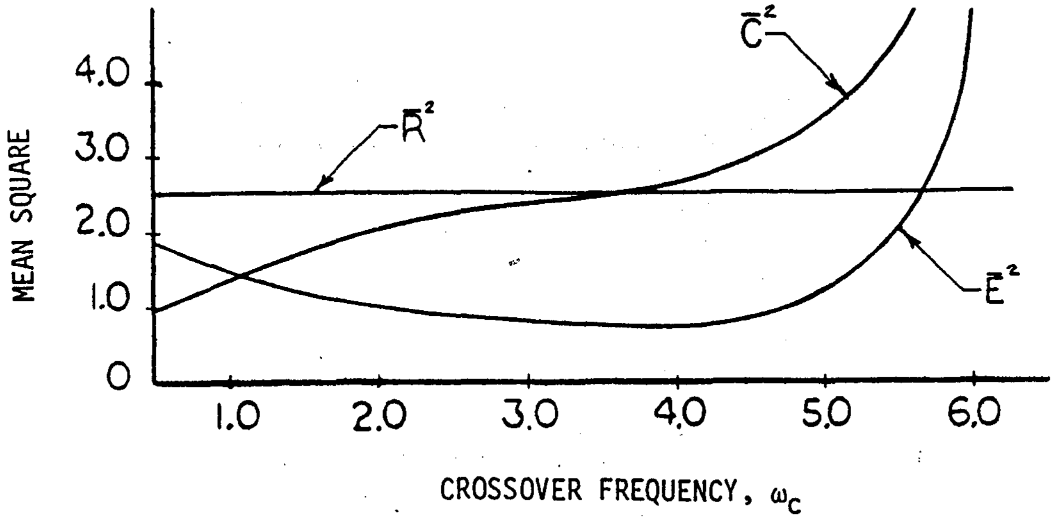

Figure 10. Mean square values for Fig. 7 with a time delay and SPAN compensation.

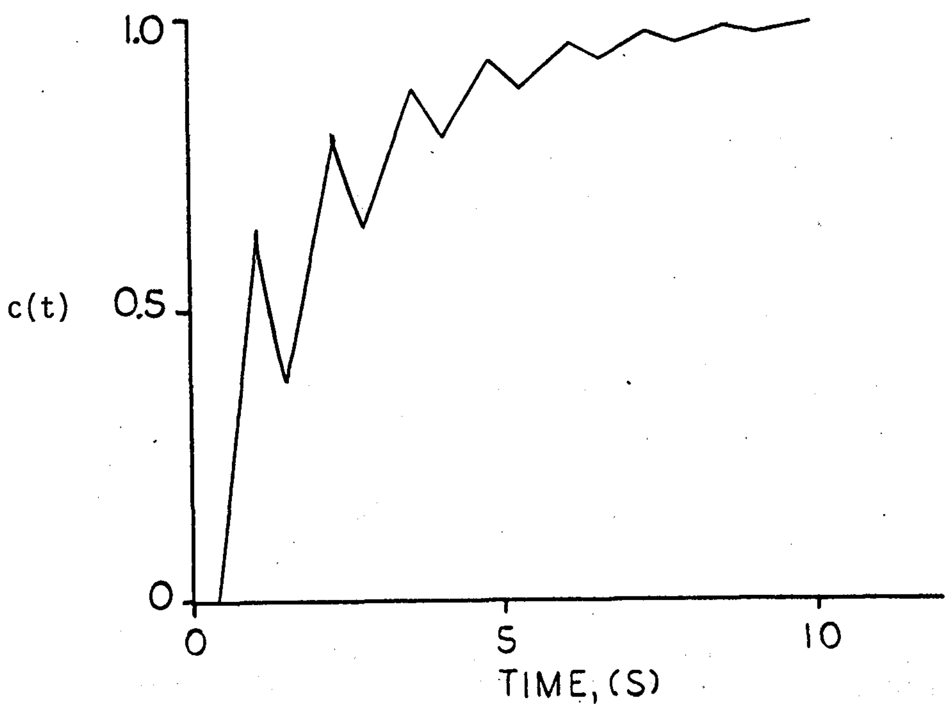

Figure 11. SPAN compensation, ω_c=1.0.

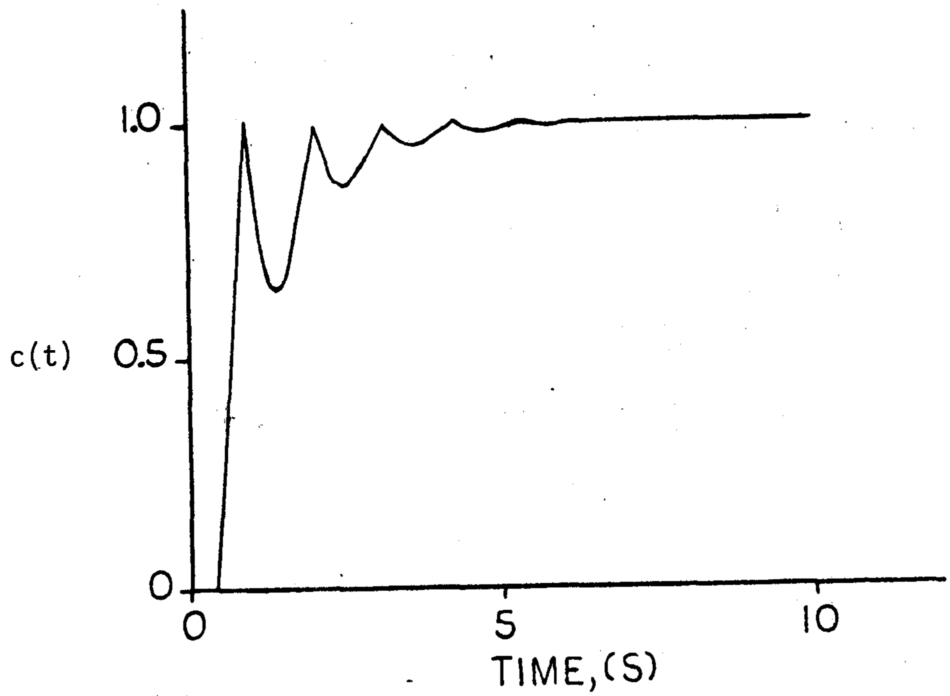

Figure 12. SPAN compensation, ω_c=2.0.

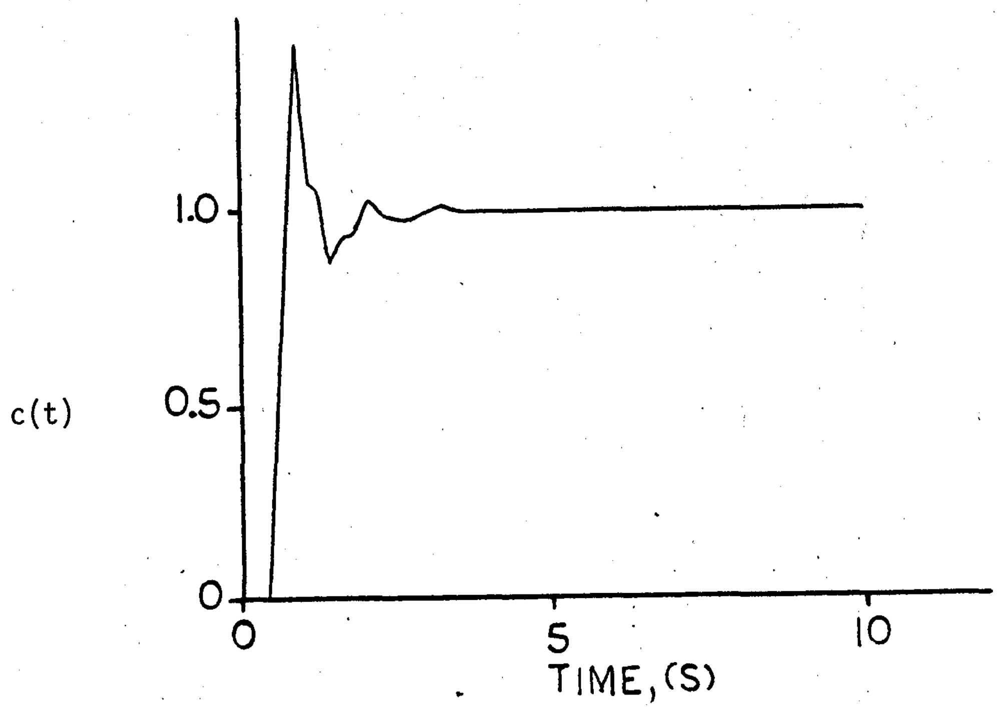

Figure 13. SPAN compensation, ω_c=3.0.

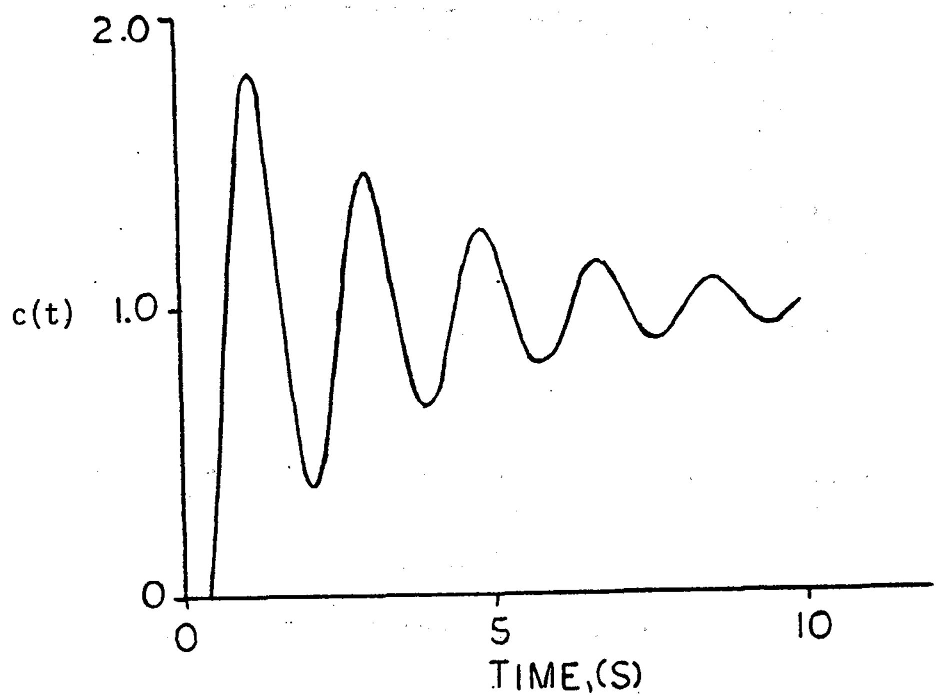

Figure 14. No compensation, $\omega_c=3.0$.

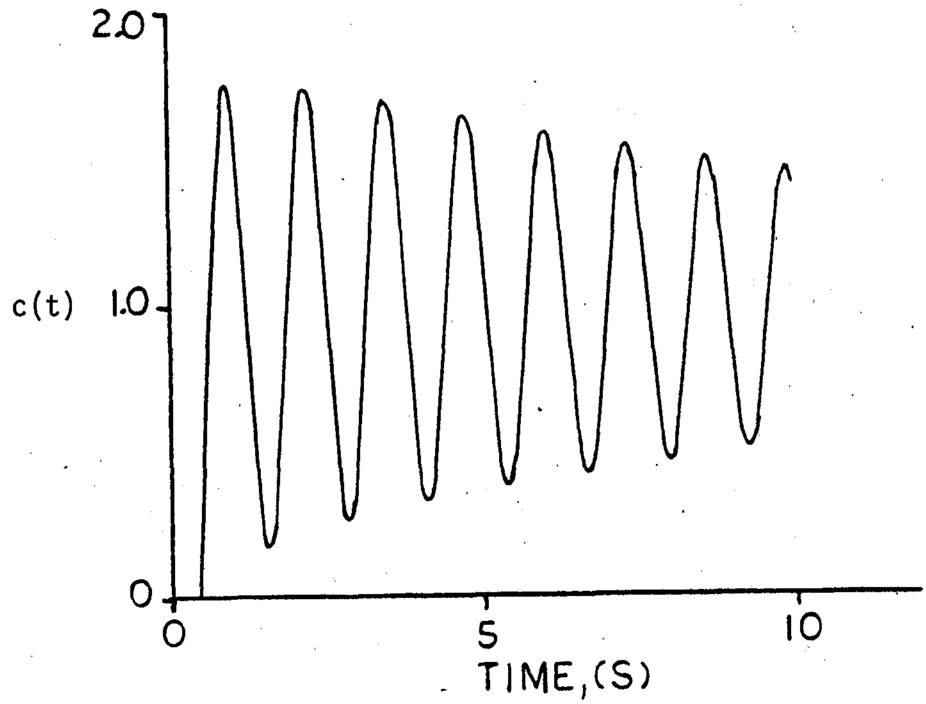

Figure 15. Lead-lag compensation, $\omega_c=3.0$.

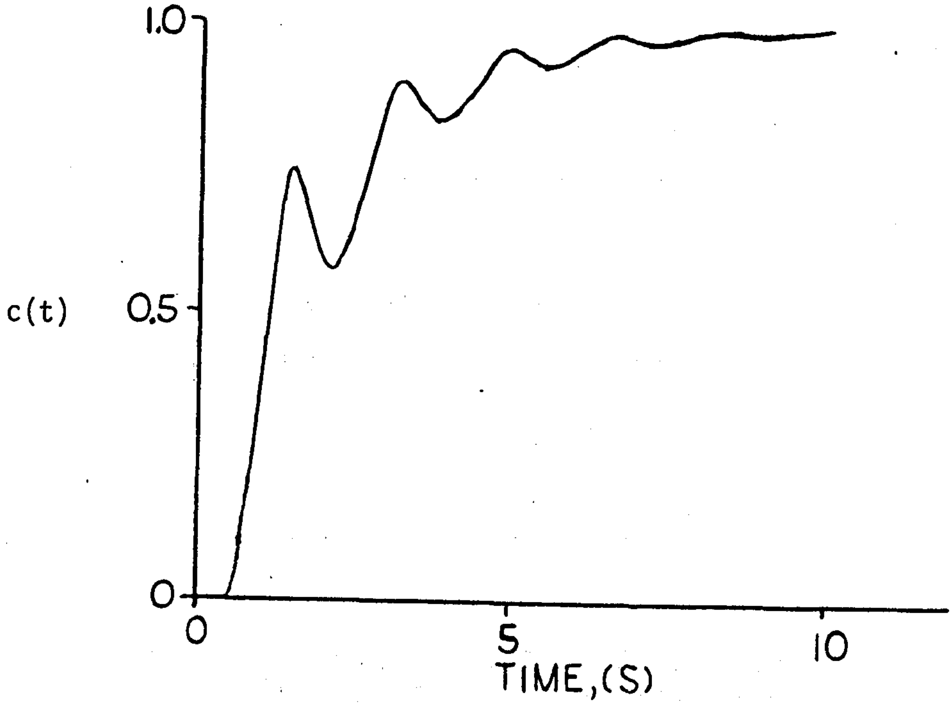

Figure 16. SPAN compensation with $\omega_c/S(0.1S+1)^2$ replacing ω_c/S, $\omega_c=1.0$.

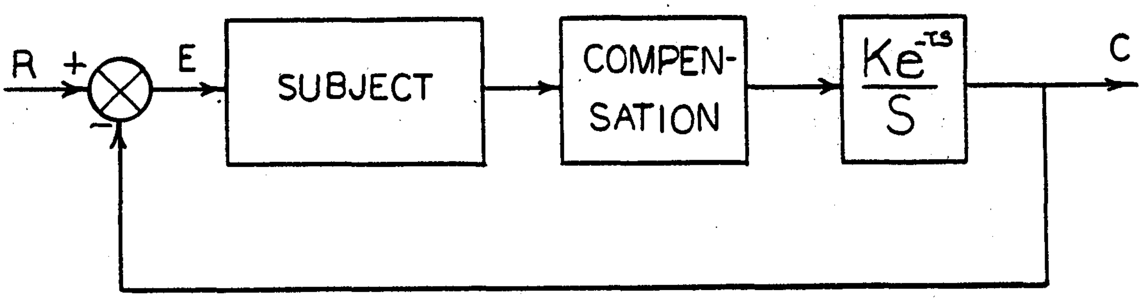

Figure 17. Diagram of single-axis, compensatory tracking task used in experimental analysis.

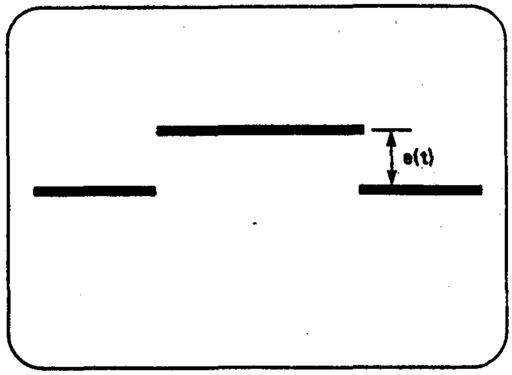

Figure 18. CRT display used in experimental analysis (Reference 3).

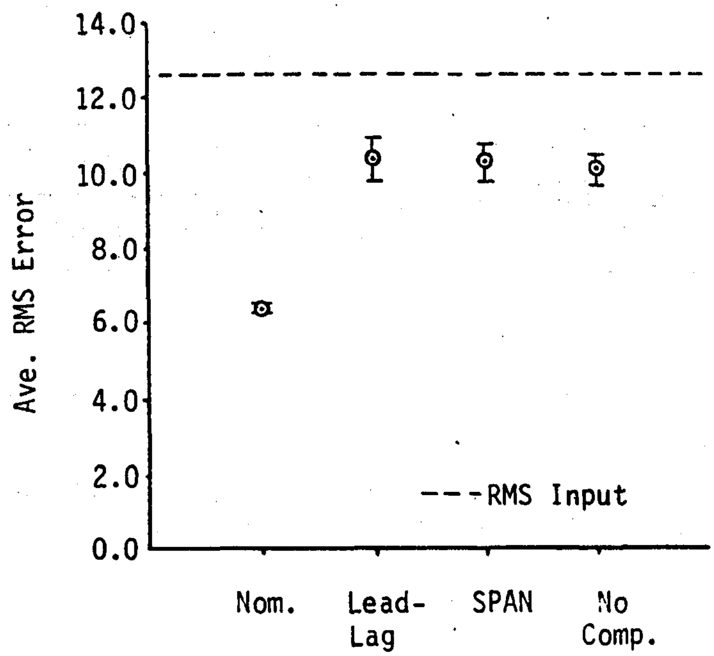

Figure 19. Plot of typical RMS error scores.

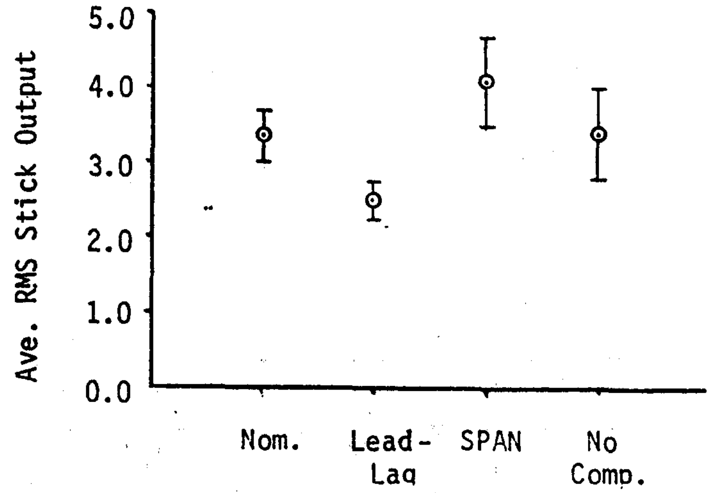

Figure 20. Plot of typical RMS stick output.

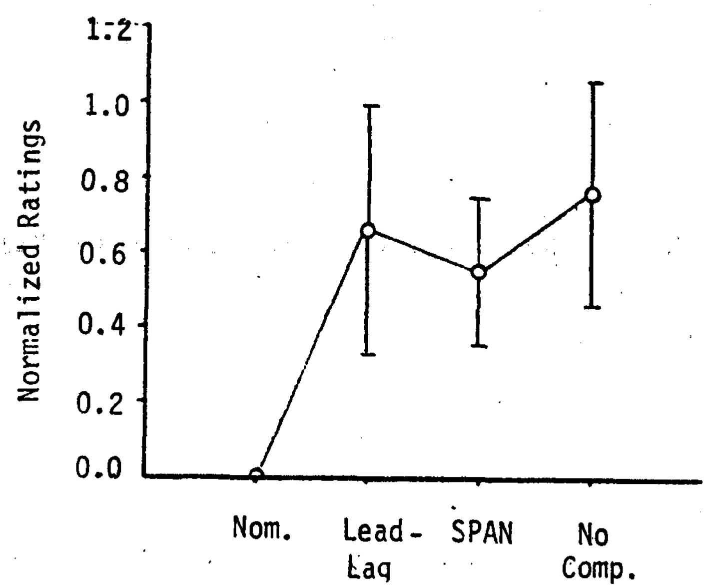

Figure 21 Plot of normalized subject ratings

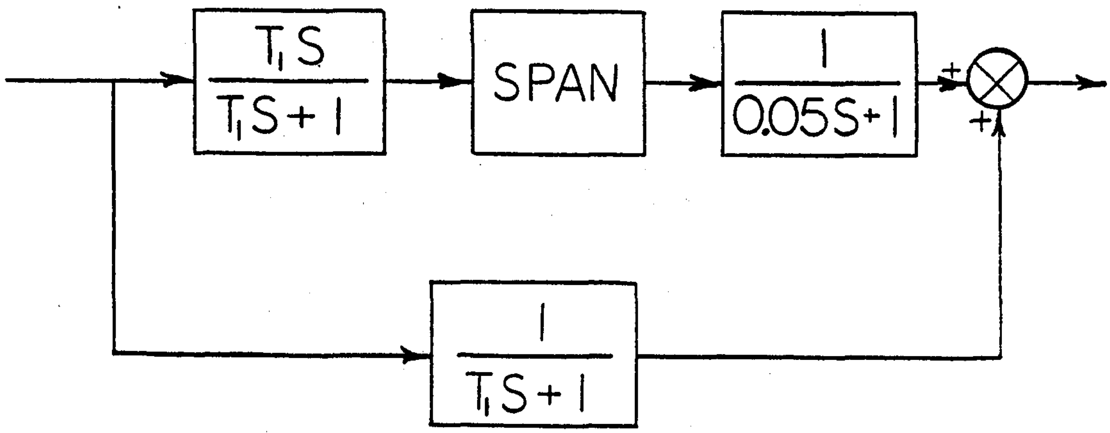

Figure 22. New configuration for SPAN filter.

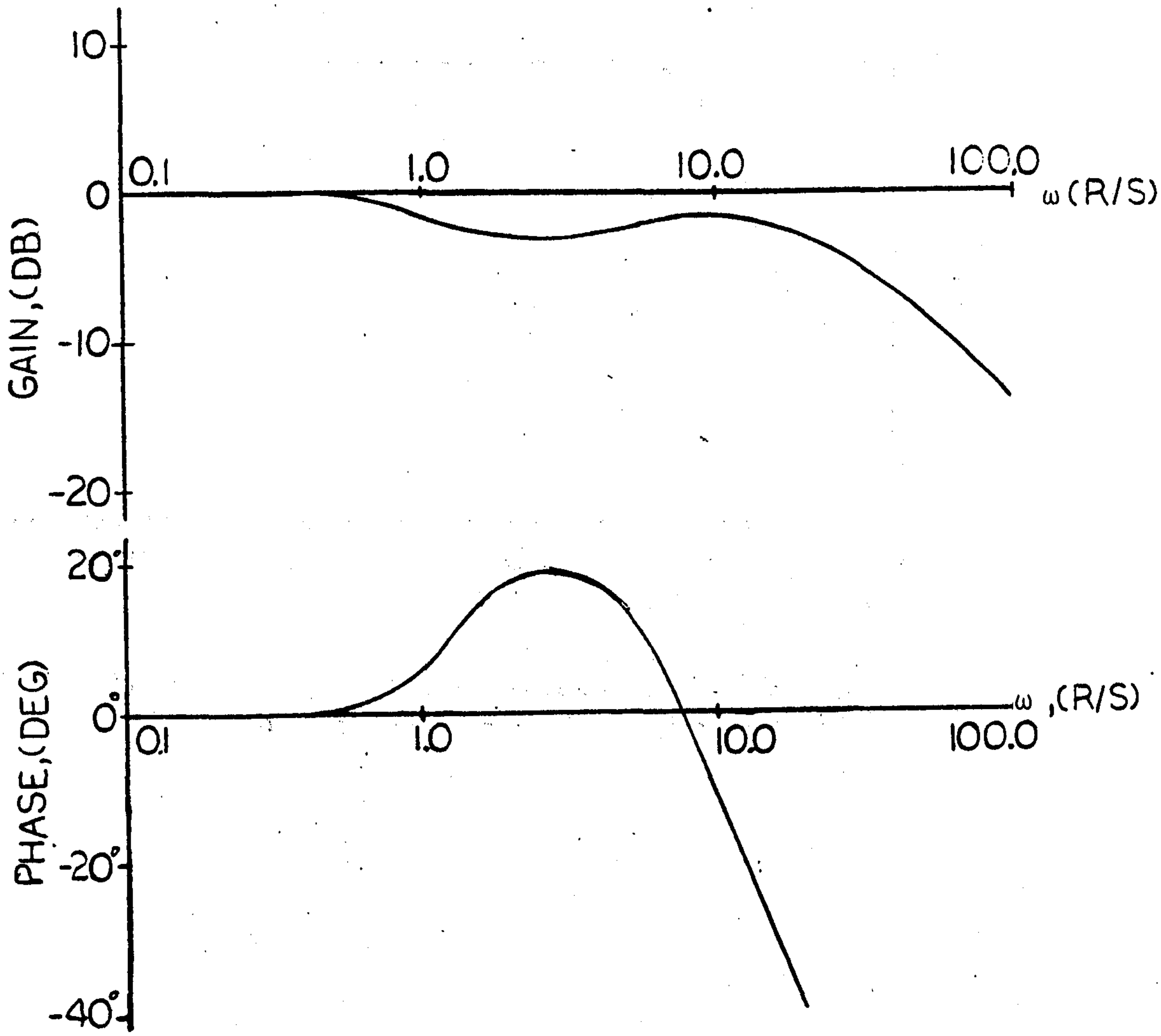

Figure 23. Bode plot of new configuration for the SPAN filter with $T_1 = 1.5$.

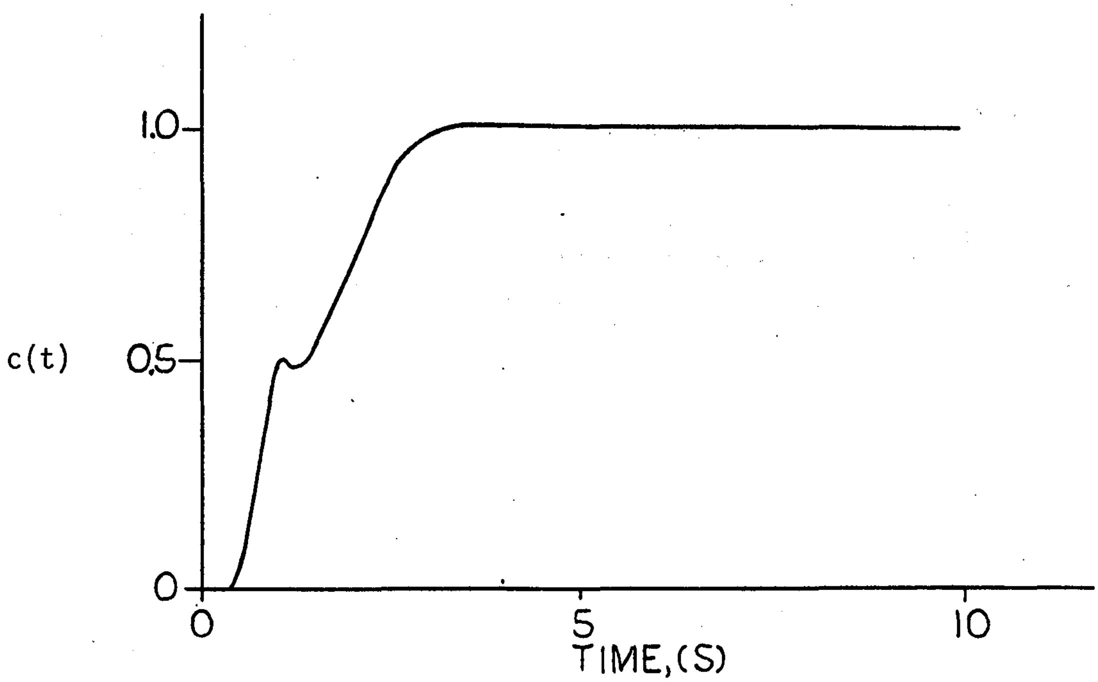

Figure 24. Closed-loop step response of new configuration with $T_1=1.5$, and $\omega_c=1.0$.