How Many Earth-Approaching

Asteroids Are There?

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ABSTRACT

This paper formulates the discovery process of Earth-approaching asteroids as a periodic sampling problem from an urn with replacement. It presumes an unchanging number of such objects over recent historical times, a fixed limiting magnitude/angular speed combination for detection probability, and allows for the possibility of the lack of detection or the lack of sufficient observations to refine an orbit at the first noticed apparition. While simple, the model is sufficiently powerful to show that an estimate of the number of Earth-approaching minor planets is impossible. This conclusion is insensitive to uncertainties in perturbing influences, celestial mechanics, or a model for the distribution of the orbital element sets of these minor planets.

I. INTRODUCTION

In this paper I present a method, based upon the standard urn models of probability theory, which attempts to calculate the number of Earth-approaching asteroids. This approach is similar to that of Whipple's (1967) and the discussion of Kresak (1978). Earth-approaching minor planets can come very close to the Earth and they are defined to include all asteroids with perihelion distances less than that of Mars's. The critical assumption herein is that the number of such objects is not changing on a short (i.e., decades) timescale.

II. FORMULATION

Suppose that there existed a total of N Earth-approaching asteroids at some recent epoch (e.g., 1950). On occasion one or more of these minor planets becomes visible to us. We have some probability, q, of detecting it and recognizing its nature. Once our awareness is raised we have a conditional probability p of observing it sufficiently well to purposefully recover it at the next apparition. The probability 1-q reflects the fact that we will not always perceive the fact that we are seeing an Earth-approaching asteroid, or may not see it at all. Thus, when we apparently successfully detect a new minor planet of this type we can not distinguish between those capable of being seen for the first time (number N_1) and those which we might have previously detected but did not (number N_2 after n Earth-approaching minor planets have crossed the celestial sphere). The reasons for our failure do not concern me here but some obvious ones are an apparition of short duration, an apparition during the Summer or a Full Moon, coming from the Sun (i.e, in the daytime sky) or the southern hemisphere, on a cloudy night, missed on the photographic plate, and so on.

The probability 1-p reflects the fact that we will not always be able to obtain enough precise observations over a long enough arc to refine its orbit. The list above could serve as reasons for this failure as well as others that will occur to the reader. I will assume that a previously observed asteroid is always cataloged on its second sighting and never lost thereafter. Of course we have a simple way of calculating p from the current sample of known Earth-approaching minor planets; it is $p \simeq N_3/(N_3 + N_4)$ wherein N_3 is the number of cataloged Earth-approaching minor planets and N_4 is the number of once seen but with a poor orbital element set. Note that this sample is a biased one.

We can compute q too as follows: For those currently cataloged Earth-approaching asteroids one could compute the circumstances of their previous apparitions. This should not be pushed too far back in time, especially as these objects may have a long least common multiple of their sidereal (i.e., return time to perihelion opposition) and synodic periods, because the equipment and personnel of the astronomical community has changed considerably on a decades-long timescale. From a careful analysis of their lack of discovery eariier we should be able to estimate q. This author does not have, immediately available, the resources necessary to perform these calculations at the required level of accuracy. Hence, I have not done them. The Minor Planet Center, for instance, could easily execute the computations.

After n Earth-approachers have crossed the celestial sphere there are N_3 cataloged ones and N_4 detected but not cataloged. Clearly $N_1 + N_2 + N_3 + N_4 = N$ and $N - n = N_1$. Also we obviously know what N_3 and N_4 are but neither what N_1, N_2, nor n are. Finally, because we have not yet removed one of these objects from interplanetary space and planetary (or lunar) impacts have not occurred over the decades-long timescale under consideration, the sampling is that of with replacement. See Feller (1957) for a fuller discussion on urn models in general. Note too that this is formulated as if it were a Markov process.

III. THE STATE TRANSITIONS

The state of this population of minor planets is completely specified by N_1, N_2, and N_3 (for a fixed N). Using the model postulated above, we can calculate the random transition probabilities from one state to another. There are five transitions of interest shown in Table I along with their respective probabilities of occurrence. Let us consider each of these in turn by considering the probabilities of an asteroid going from one group to another. There are nine interesting transitions: (1) $1 \rightarrow 2$ with probability $N_1(1 - q)/N$, (2) $1 \rightarrow 3$ probability $P = N_1pq/N$, (3) $1 \rightarrow 4$ with $P = N_1q(1 - p)/N$, (4) $2 \rightarrow 2$, $P = N_2(1 - q)/N$, (5) $2 \rightarrow 3$, $P = N_2pq/N$, (6) $2 \rightarrow 4$, $P = N_2q(1 - p)/N$, (7) $3 \rightarrow 3$, $P = N_3/N$, (8) $4 \rightarrow 3$, $P = N_4q/N$, and (9) $4 \rightarrow 4$, $P = N_4(1 - q)/N$.

The values in Table I come from these nine elemental probabilities and the choice of N_1, N_2, and N_3 as the triplet to use to parameterize the state of the Earth-approaching asteroid population. Hidden in the equations is the implicit assumption that the character of our detection capability has not varied over time with limiting magnitude or angular speed. Therefore, if we are to use the known sample of asteroids to estimate p or q, we can not go too far backward in historical time.

Table I. Asteroid State Transitions

Initial State	Final State	Probability
N_1, N_2, N_3	N_1, N_2, N_3	$[(N_2+N_4)(1-q)+N_3]/N$
N_1, N_2, N_3	N_1-1, N_2+1, N_3	$N_1(1-q)/N$
N_1, N_2, N_3	N_1, N_2-1, N_3+1 or N_1-1, N_2, N_3+1	$(N_1+N_2)pq/N$
N_1, N_2, N_3	N_1, N_2-1, N_3 or N_1-1, N_2, N_3	$(N_1+N_2)(1-p)q/N$
N_1, N_2, N_3	N_1, N_2, N_3+1	N_4q/N

IV. THE NUMBER

We certainly know N_3 and N_4, the number of asteroids with good orbital element sets and the number of once seen Earth-approaching asteroids without good orbital element sets. We also know (Taff and Randall, 1985) the circumstances of the discovery of this type of minor planet. They are preferentially discovered in the northern sky, during the Fall and Winter, nearer New Moon then Full Moon, closer to the Earth rather than farther, and brighter or faster rather than fainter or more slowly moving. They also tend to be found at a perihelion opposition (aphelion opposition for those with a < 1 A.U.) and recovered in a similar set of geometrical circumstances.

Finally, although the problem appears to be one of random sampling with replacement, it is not. There exists a fixed set of Earth-approaching minor planets. This set has a certain series of repetitive approaches to the Earth. This series is, to first order, itself repetitive. Thus, the elements of randomness have to do with lunar phase and local weather, not with the apparition of one of these asteroids. When we commence a search, at a random phase of the Earth-approaching minor planets' long-term apparition cycle, we can deduce nothing about n or N from our knowledge of N_3, N_4, p, and q.

The recent history of the discoveries of these objects supports this point of view. Before the commencement of the Helin and Shoemaker search (1979; it started in 1973), this type of asteroid was discovered haphazardly and frequently not recovered (low p and low q). Objects such as 1963UA (=2059), 1960UA (=2061), 1953RA (=1916), 1953EA (=1915), 1950LA (=1980), 1949OA (=1951), 1949EA (=1863), 1959EH = 1980RB1 (=2629), or 1947XC (=2201) are typical. They were recovered after the onset of concentrated search activities. 1951RA (=1620) and 1948OA (=1685) are examples of those few that were held on to earlier. A few objects have yet to be recovered; examples include 1937UB, 1950DA, 1950XA, 1959LM, 1972RB, 1973NA. and some Palomar-Leiden discoveries.

After a competing search (Taff, 1981, 1984) commenced and a concomittant change in the other searches procedures (Helin, 1983), we see the discovery of lots more "new" objects. These are in the N_1 group of this part of the longer-term apparition cycle. As almost all of the older discoveries have been recovered post-search initiation, we can only quantify p and q, not guess the value of N_1 or N_2. Because each of the last few years has produced 5-10 new discoveries, we must conclude that the N_1 reservoir is not empty. Furthermore, as this longer-term apparition cycle must have a period on the order of decades, it would be premature to assert that we have crossed an important threshold. When the current searches run out of new discoveries, then we can claim that N_2 has been driven to zero, equate $N \approx n \approx N_3 + N_4$, estimate N_2 from q, and finally evaluate $N = N_2 + N_3 + N_4$.

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