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Design Procedures for Fiber Composite Structural Components: Panels Subjected to Combined In-Plane Loads

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DESIGN PROCEDURES FOR FIBER COMPOSITE STRUCTURAL COMPONENTS:

PANELS SUBJECTED TO COMBINED IN-PLANE LOADS

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SUMMARY

Step-by-step procedures are described which can be used to design panels made from fiber composite angleplied laminates and subjected to combined in-plane loads. The procedures are set up as a multi-step sample sample design. Steps in the sample design procedure range from selection of the laminate configuration to the subsequent analyses required to check design requirements for (1) displacement, (2) ply stresses, and (3) buckling. The sample design steps are supplemented with appropriate tabular and graphical data which can be used to expedite the design process.

INTRODUCTION

The design of fiber composite structural components requires analysis methods and procedures which relate the structural response of the component to the specified loading and environmental conditions. The structural response is eventually compared to given design criteria for strength, displacement, buckling, vibration frequencies, etc, in order to ascertain that the component will perform satisfactorily.

Though there are several recent books on composite mechanics available (refs. 1 to 6), none covers design procedures for fiber composite structural components in any detail. A sample design is presented herein in step-by-step detail to illustrate procedures for designing structural components such as panels and other similar components subjected to combined in-plane loads (fig. 1). This is accomplished by assuming a laminate configuration for the component and then checking to verify that it meets all the specified design requirements. The laminate selected is not particularly unique. In describing the sample design, it is assumed that the reader has some familiarity with mechanics of materials and fiber composites. Allowable stress (strength) as used herein denotes fracture stress. The safety factor is applied to the specified load to obtain the design load.

Limiting design requirements considered include displacements, ply stresses, and panel buckling. Procedures are briefly outlined which can be used to design panels for hygrothermal environments, cyclic loads, and lamination residual stresses. The sample design is based on a graphite-fiber/epoxy-resin composite (AS/E). Unidirectional composite (ply) data for this and other typical composites are summarized in table I. Specific AS/E data used to expedite the numerical calculations are graphically

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presented in figures 2 to 5. The theoretical concepts and most of the equations used are from references 7 to 13. These references provided general background information, correlation with available experimental data as well as graphical information similar to figures 2 to 5 for several other composite materials, including hybrids. The notation employed is defined when used and summarized in the SYMBOLS section. Some repetition is unavoidable for the sake of clarity. Collectively, this multi-step sample design provides an illustrative step-by-step procedure which is described for the first time and which is a sequel to those presented previously for rods, beams, and beam columns (ref. 14).

SYMBOLS

APL	angleplied laminate
AS/E	AS graphite-fiber/epoxy-matrix composite
a	panel x-dimension
B	cyclic load degradation coefficient
b	panel y-dimension
E	modulus-equivalent
E_C	laminate modulus - subscripts x,y denote structural axis directions
E_ℓ	ply modulus - subscripts 1,2 denote ply material axis directions
E_θ	$\pm\theta$ laminate modulus - subscripts 1,2 denote ply material axis directions
FVR	fiber volume ratio
G_{Cxy}	laminate shear modulus (x-y plane)
$G_{\ell 12}$	ply shear modulus (1-2 plane)
$G_{\theta 12}$	$\pm\theta$ laminate shear modulus (1-2 plane)
$\mathcal{I}$	ply stress influence coefficient - subscripts denote ratio ply-stress/laminate-stress
M	moisture, percent by weight; subscripts: ℓ-ply, C-laminate
MOS	margin of safety
N	number of cycles
N_C	in-plane loads - subscripts x,y denote structural axis direction

N_ℓ number of plies - subscripts, 0, 90, $\pm\theta$ denote respective orientations

P_ℓ ply property; subscripts: 0-reference, HT-hygrothermal

Q_c reduced laminate stiffness - subscripts x,y denote structural axis directions

Q_ℓ reduced ply stiffness - subscripts 1,2 denote ply material axis directions

Q_θ reduced stiffness for $\pm\theta$ symmetric laminate - subscripts 1,2 denote material axis directions

S_ℓ ply strength - subscripts 1,2 denote ply material axis directions; subscripts T, C, and S denote, respectively, tension, compression, and shear

$S_{\ell N}$ fatigue strength - subscripts: 0-reference, N-fatigue cycles

$S_{\ell NA}$ fatigue strength allowable for N cycles

T use temperature

T_{GD} glass transition temperature, dry conditions

T_{GW} glass transition temperature, wet conditions

T_0 reference temperature

ΔT temperature change

t thickness - subscripts: c-laminate, ℓ-ply

u in-plane displacement along x-axis

V_p ply thickness ratio - subscripts θ, 0, 90 denote ply to which the ratio applies

v in-plane displacement along y-axis

x,y,z structural axis coordinate directions

1,2,3 material axis coordinate directions - one taken along the fiber direction

$[-/-/-]_S$ laminate configuration designation - numbers in the blanks denote ply stacking sequence and orientation - subscript S denotes symmetry about ply in last blank space

α_c laminate thermal expansion coefficient - subscripts x,y denote laminate structural axis directions

α_ℓ ply thermal expansion coefficient - subscripts 1,2 denote ply material axis directions

α_θ	$\pm\theta$ laminate thermal expansion coefficient - subscripts 1,2 denote material axis directions
β_c	laminate moisture expansion coefficient - subscripts x,y denote laminate structural axis directions
β_ℓ	ply moisture expansion coefficient - subscripts 1,2 denote ply material axis directions
β_θ	$\pm\theta$ laminate moisture expansion coefficient - subscripts 1,2 denote material axis directions
ε_c	laminate strain - subscripts x,y denote structural axis directions
ε_ℓ	ply strain - subscripts 1,2 denote material axis directions
θ	ply orientation angle measured from the x-laminate structural axis to the 1-ply axis and taken positive
$\Delta\theta$	in-plane rotation due to shear
ν_c	laminate Poisson's ratio - subscripts x,y denote structural axis directions
ν_ℓ	ply Poisson's ratio - subscripts 1,2 denote ply material axis directions
σ_c	laminate stress - subscripts x,y denote structural axis directions
σ_ℓ	ply stress - subscripts 1,2 denote material axis directions, ST-static and CYC-cyclic
$\sigma^{(cr)}$	buckling stress - subscripts denote type

Conversion factors:

MPa	6.89 ksi
ksi	0.145 MPa
°C	5/9 (°F - 32)

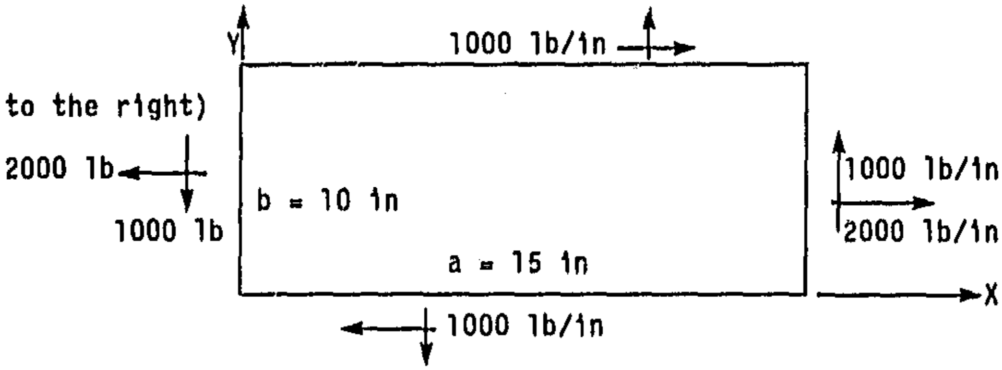
COMPOSITE PANELS - GENERAL

Composite panels (membranes) are structural components which generally have a rectangular shape. They can be used individually (fig. 1) or as members of built-up structural components (fig. 6). They usually are designed to support combined in-plane loading conditions (fig. 1). The loading conditions can include: (1) static loads, (2) static with superimposed cyclic loads, (3) hot-wet (hygrothermal) environmental effects, and (4) lamination residual stresses. We will present a sample design for static loads only, and we will briefly outline the procedures to be used for analyzing for loading conditions (2) to (4).

SAMPLE DESIGN FOR STATIC LOADS

Structural component:	Rectangular panel, 15 by 10 in

Specified loads:
(see schematic to the right)



Displacement limits:	0.5-percent of edge dimensions and 1°-shearing angle
Safety factor:	2.0 on specified load
Composite system:	AS/E, about 0.6 fiber volume ratio (FVR)
Design procedure:	Rectangular panel designed to <u>not</u> exceed displacement limits, or ply strengths, or buckle at design load. Specified-load ply stresses may be used instead of design load ply stresses to compute matrix-controlled ply strength margins when the fiber-controlled stress margins are relatively large.
STEP 1. Design variables:	Number of plies, ply orientations, and ply stacking sequence.
STEP 2. Design loads:	Safety factor times specified loads – N_{cxx} = 2 x 2000 lb/in = 4000 lb/in N_{cyy} = 2 x 1000 lb/in = 2000 lb/in N_{cxy} = 2 x 1000 lb/in = 2000 lb/in
STEP 3. Composite material, properties (ply and angleply)	AS/E, table I and figures 2 and 3

STEP 4. Select laminate configuration

a. Number of 0°-plies = Design load (N_{cxx})/(longitudinal tensile strength ($S_{\ell 11T}$ = 220 000 psi) x ply thickness (t_ℓ = 0.005 in))

$$N_{\ell 0} = \frac{N_{cxx}}{S_{\ell 11T} t_\ell} = \frac{4000 \text{ lb/in}}{220\ 000 \text{ lb/sq in} \times 0.005 \text{ in}} = 3.64 \sim 4$$

Use $N_{\ell 0}$ = 8 (double because of the combined loading)

b. Number of 90°-plies = Design load (N_{cyy}/(longitudinal tensile strength ($S_{\ell 11T}$ x ply thickness (t_ℓ))

$$N_{\ell 90} = \frac{N_{cyy}}{S_{\ell 11T} t_\ell} = \frac{2000 \text{ lb/in}}{220\ 000 \text{ lb/sq in} \times 0.005 \text{ in}} = 1.82 \sim 2$$

Use $N_{\ell 90} = 4$ (double because of the combined loading)

c. Number of ±45° plies = Design load (N_{cxy}) x one-half the ratio of the ply longitudinal ($E_{\ell 11}$ = 18.5 mpsi) to ±45° composite shear modulus ($G_{\theta 12}$ 5.8 mpsi)/(longitudinal compressive strength ($S_{\ell 11C}$ = 180 000 psi) x ply thickness (t_ℓ = 0.005 in))

$$N_{\ell \pm 45} = \frac{N_{cxy} \times (1/2)(E_{\ell 11}/G_{\theta 12})}{S_{\ell 11C} \times t_\ell} = \frac{2000 \text{ lb in } (1/2)(18.5/5.8)}{180\ 000 \text{ lb/sq in} \times 0.005 \text{ in}} = 3.5$$

Use $N_{\ell \pm 45} = 8$ (double because of the combined loading)

Therefore, the laminate is 20 plies (8 at 0°, 8 at ±45, and 4 at 90°). The laminate thickness (t_ℓ) is 20 x 0.005 in = 0.10 in.

d. And the required laminate configuration (using the conventional designation) is:

$[\pm 45/0/90/0]_{2S}$

Notes:
1. The laminate was initially sized using fiber-controlled properties. The number of plies in each orientation was doubled in order to approximately account for the combined loading stresses which are resisted by matrix-controlled properties.
2. The ±45°-plies were placed on the outside for increased shear buckling resistance.
3. The longitudinal compression strength was selected for determining the number of ±45° plies because this is less than the longitudinal tensile strength (180 000 psi < 220 000 psi, table I).

STEP 5. Determine the laminate reduced stiffness coefficients. These coefficients are given by the following formulas (ref. 8):

$$Q_{cxx} = V_{P\theta}\, Q_{\theta 11} + V_{P0}\, Q_{\ell 11} + V_{P90}\, Q_{\ell 22}$$

$$Q_{cyy} = V_{P\theta}\, Q_{\theta 22} + V_{P0}\, Q_{\ell 22} + V_{P90}\, Q_{\ell 11}$$

$$Q_{cyz} = Q_{cxy} = V_{P\theta}\, Q_{\theta 12} + V_{P0}\, Q_{\ell 12} + V_{P90}\, Q_{\ell 21}$$

$$G_{cxy} = V_{P\theta}\, Q_{\theta 33} + V_{P0}\, Q_{\ell 33} + V_{P90}\, Q_{\ell 33}$$

$$V_{P\theta} = \frac{\text{thickness of } \pm\theta \text{ plies}}{\text{thickness of APL}} = 8/20 = 0.4$$

$$V_{P0} = \frac{\text{thickness of 0°-plies}}{\text{thickness of APL}} = 8/20 = 0.4$$

$$V_{P90} = \frac{\text{thickness of 90°-plies}}{\text{thickness of APL}} = 4/20 = 0.2$$

Check: $V_{P\theta} + V_{P0} + V_{P90} = 1.0$
$0.4 + 0.4 + 0.2 = 1.0$ o.k.

From figure 3 at $\theta = 0°$, $\pm45°$, and $90°$ we have:

$\theta = 0°$	$\theta = \pm45°$	$\theta = 90°$
$Q_{\ell11} = 19$ mpsi $Q_{\ell22} = 2$ mpsi $Q_{\ell12} = 0.5$ mpsi $Q_{\ell33} = 0.5$ mpsi	$Q_{\theta11} = 6$ mpsi $Q_{\theta22} = 6$ mpsi $Q_{\theta12} = 5$ mpsi $Q_{\theta33} = 5$ mpsi	$Q_{\ell22} = 2$ mpsi $Q_{\ell11} = 19$ mpsi $Q_{\ell21} = 0.5$ mpsi $Q_{\ell33} = 0.5$ mpsi

Using the respective equations and V_P values, we obtain:

$$Q_{cxx} = V_{P\theta}\, Q_{\theta11} + V_{P0}\, Q_{\ell11} + V_{P90}\, Q_{\ell22}$$

$$Q_{cxx} = (0.4x6 + 0.4x19 + 0.2x2)\text{mpsi}$$

$$Q_{cxx} = 10.4 \text{ mpsi}$$

$$Q_{cyy} = V_{P\theta}\, Q_{\theta22} + V_{P0}\, Q_{\ell22} + V_{P90}\, Q_{\ell11}$$

$$Q_{cyy} \approx (0.4x6 + 0.4x2 + 0.2x19)\text{mpsi}$$

$$Q_{cyy} = 7.0 \text{ mpsi}$$

$$Q_{cxy} = V_{P\theta}\, Q_{\theta12} + V_{P0}\, Q_{\ell12} + V_{P90}\, Q_{\ell21}$$

$$Q_{cxy} = (0.4x5 + 0.4x0.5 + 0.2x0.5)\text{mpsi}$$

$$Q_{cxy} = 2.3 \text{ mpsi}$$

$$G_{cxy} = V_{P\theta}\, Q_{\theta33} + V_{P0}\, Q_{\ell33} + V_{P90}\, Q_{\ell33}$$

$$G_{cxy} = (0.4x5 + 0.4x0.5 + 0.2x0.5)\text{mpsi}$$

$$G_{cxy} = 2.3 \text{ mpsi}$$

STEP 6. Determine the laminate elastic coefficients (moduli): These are determined by using the values of the Q_c's from STEP 5 in the following equations (ref. 8).

$$E_{cxx} = Q_{cxx} - Q_{cxy}^2/Q_{cyy} + (10.4 - 2.3 \times 2.3/7.0)\text{mpsi} = 9.6 \text{ mpsi}$$

$$E_{cyy} = Q_{cyy} - Q_{cxy}^2/Q_{cxx} = (7.0 - 2.3 \times 2.3/10.4)\text{mpsi} = 6.5 \text{ mpsi}$$

$\nu_{cxy} = Q_{cxy}/Q_{cyy} = (2.3/7.0) = 0.33$

$G_{cxy} = G_{cxy} = 2.3$ mpsi

$\nu_{cyx} = \nu_{cxy}\, E_{cyy}/E_{cxx} = 0.33 \times 6.5/9.6 = 0.22$

STEP 7. Determine the composite stresses at design loads: The composite stresses are:

$\sigma_{cxx} = N_{cxx}/t_c = 4000$ lb/in $\div$ 0.10 in $= 40\ 000$ psi

$\sigma_{cyy} = N_{cyy}/t_c = 2000$ lb/in $\div$ 0.10 in $= 20\ 000$ psi

$\sigma_{cxy} = N_{cxy}/t_c = 2000$ lb/in $\div$ 0.10 in $= 20\ 000$ psi

STEP 8. Check displacement limits: These are determined from the composite strain/stress relationship as follows:

a. Along X (u/a):

$(u/a) = \epsilon_{cxx} = (\sigma_{cxx}/E_{cxx} - \nu_{cyx}\, \sigma_{cyy}/E_{cyy}) \times 100$

$(u/a) = (40\ 000/9\ 600\ 000 - 0.22 \times 20\ 000/6\ 500\ 000) \times 100 = 0.35$ percent

Check: $(u/a) \leq 0.50$ percent
0.35 percent < 0.50 percent o.k.

The margin of safety (MOS) is:

$$MOS = \frac{0.50}{0.35} - 1 = 0.43$$

b. Along y (v/b):

$(v/b) = \epsilon_{yy} = (-\nu_{cxy}\, \sigma_{cxx}/E_{cxx} + \sigma_{cyy}/E_{cyy}) \times 100$

$(v/b) = (-0.33 \times 40\ 000/9\ 600\ 000 + 20\ 000/6\ 500\ 000) \times 100 = 0.17$ percent

Check: $(v/b) \leq 0.50$ percent
0.17 percent < 0.50 percent o.k.

$$MOS = \frac{0.5}{0.17} - 1 = 1.94$$

c. Change in angle ($\Delta\theta$)

$\Delta\theta \approx \tan^{-1}(a\epsilon_{cxy}/b) = \tan^{-1}(a\sigma_{cxy}/bG_{cxy})$

$\Delta\theta \approx \tan^{-1}(3 \times 20\ 000/2 \times 2\ 300\ 000) = 0.75°$

Check: $\Delta\theta \leq 1.0°$
0.75° < 1.0° o.k.

$$MOS = \frac{1.0}{0.75} - 1 = 0.33$$

Note: The ratio (a/b) was used to provide an overestimate on $\Delta\theta$. Therefore, the selected laminate satisfies the displacement design requirements at design load.

STEP 9. Check ply stress limits: These are checked by performing ply stress analysis using the ply stress influence coefficients (PSIC). The PSIC are denoted by $\mathscr{I}_{\alpha/\beta}$ where α/β denotes the ply-stress to laminate stress ratio. Specifically, α denotes the ply stress to be calculated (α =: L for longitudinal ($\sigma_{\ell 11}$), T for transverse ($\sigma_{\ell 22}$), and S for intralaminar shear ($\sigma_{\ell 12}$)) due to laminate stress β (β =: X for σ_{cxx} Y for σ_{cyy} and S for σ_{cxy}). The equations for these coefficients (ref. 8) are given below as each ply stress is calculated.

a. Stresses in the 0°-ply -- longitudinal. - The general equation for the ply longitudinal stress ($\sigma_{\ell 11}$) in contracted and expanded form is:

$$\sigma_{\ell 11} = \mathscr{I}_{L/X}\, \sigma_{cxx} + \mathscr{I}_{L/Y}\, \sigma_{cyy} + \mathscr{I}_{L/S}\, \sigma_{cxy}$$

$$\sigma_{\ell 11} = (E_{\ell 11}/E_{cxx})(\cos^2\theta - \nu_{cxy} \sin^2\theta)\sigma_{cxx}$$

$$+ (E_{\ell 11}/E_{cyy})(\sin^2\theta - \nu_{cxy} \cos^2\theta)\sigma_{cyy}$$

$$+ (E_{\ell 11}2/G_{cxy})[(1 - \nu_{\ell 21})\sin 2\theta]\sigma_{cxy}$$

Using previous values for E_c and ν_c (STEP 6) and σ_c (STEP 7), $\theta = 0°$ and figure 2 at $\theta = 0°$ for $E_{\ell 11}$ (18.5 mpsi) and $\theta = 90°$ for $\nu_{\ell 21}$ (0.03) in the above equation, we calculate:

$$\sigma_{\ell 11} = \{(18.5/9.6)(1.0 - 0.33x0)40\ 000$$

$$+ (18.5/6.5)(0 - 0.33x1)20\ 000$$

$$+ (18.5/2x2.3)[(1 - 0.03) \times 0]\ 20\ 000\}\ psi$$

$$\sigma_{\ell 11} = [77\ 083 - 18\ 785 + 0]\ psi = 58\ 298\ psi$$

Check: $\sigma_{\ell 11} \leq S_{\ell 11T}$
58 298 psi < 220 000 psi o.k.

The margin of safety (MOS) is

$$MOS = \frac{S_{\ell 11T}}{\sigma_{\ell 11}} - 1 = \frac{220\ 000}{58\ 298} - 1.0 = 2.77$$

Note: The MOS is an additional factor of safety on the ply stresses at design load. In this case it is greater than two because of the stresses resisted by the ±45° plies and the Poisson's stresses due to σ_{cyy}.

Stresses in the 0°-ply -- transverse. - The general equation for the ply transverse stress ($\sigma_{\ell 22}$) in contracted and expanded for is:

$$\sigma_{\ell 22} = \mathscr{S}T/X\ \sigma_{cxx} + \mathscr{S}T/Y\ \sigma_{cyy} + \mathscr{S}T/S\ \sigma_{cxy}$$

$$\sigma_{\ell 22} = (E_{\ell 22}/E_{cxx})[(\nu_{\ell 12} - \nu_{cxy})\cos^2\theta + (1 - \nu_{cxy}\ \nu_{\ell 12})\sin^2\theta]\sigma_{cxx}$$

$$+ (E_{\ell 22}/E_{cyy})[(1 - \nu_{cxy}\ \nu_{\ell 12})\cos^2\theta + (\nu_{\ell 12} - \nu_{cxy})\sin^2\theta]\sigma_{cyy}$$

$$- (E_{\ell 22}/2G_{cxy})[(1 - \nu_{\ell 12})\sin 2\theta]\sigma_{cxy}$$

Proceeding as for $\sigma_{\ell 11}$ above and obtaining $E_{\ell 22}$ (2 mpsi) and $\nu_{\ell 12}$ (0.25) from figure 2 at $\theta = 0°$, we calculate:

$$\sigma_{\ell 22} = \{(2/9.6)[(0.25 - 0.33) \times 1.0 + (1 - 0.33 \times 0.25) \times 0] \times 40\ 000$$

$$+ (2/6.5)[(1 - 0.33 \times 0.25) \times 1.0 + (0.25 - 0.33) \times 0] \times 20\ 000$$

$$- (2/2 \times 2.3)[(1 - 0.25) \times 0] \times 20\ 000\}\ \text{psi}$$

$$\sigma_{\ell 22} = [-667 + 5646 - 0]\ \text{psi} = 4979\ \text{psi}$$

Check: $\sigma_{\ell 22} \leq S_{\ell 22T}$
4979 psi < 8000 psi o.k.

The margin of safety is

$$\text{MOS} = \frac{8000}{4979} - 1.0 = 0.61$$

Stresses in the 0°-ply -- intralaminar shear. - The general equation for the ply intralaminar shear ($\sigma_{\ell 12}$) in contracted and expanded form is:

$$\sigma_{\ell 12} = \mathscr{S}S/X\ \sigma_{cxx} + \mathscr{S}S/Y\ \sigma_{cyy} + \mathscr{S}S/S\ \sigma_{cxy}$$

$$\sigma_{\ell 12} = -\ (G_{\ell 12}/E_{cxx})[(1 + \nu_{cxy})\sin 2\theta]\sigma_{cxx}$$

$$+ (G_{\ell 12}/E_{cyy})[(1 + \nu_{cxy})\sin 2\theta]\sigma_{cyy}$$

$$+ (G_{\ell 12}/G_{cxy})(\cos 2\theta)\sigma_{cxy}$$

Proceeding as for $\sigma_{\ell 11}$ or $\sigma_{\ell 22}$ and obtaining $G_{\ell 12}$ (0.5 mpsi) from figure 2 at $\theta = 0°$, we calculate:

$$\sigma_{\ell 12} = \{ -\ (0.5/9.6)[(1 + 0.33) \times 0] \times 40\ 000$$

$$+ (0.5/6.5)[(1 + 0.33 \times 0] \times 20\ 000$$

$$+ (0.5/2.3)[1.0] \times 20\ 000\}\ \text{psi}$$

$$\sigma_{\ell 12} = (-\ 0 + 0 + 4348)\ \text{psi} = 4348\ \text{psi}$$

Check: $\sigma_{\ell 12} \leq S_{\ell 12S}$
4348 psi < 10 000 psi o.k.

The margin of safety is

$$MOS = \frac{10\ 000}{4348} - 1.0 = 1.30$$

Therefore, the stresses in the 0°-plies meet the ply strength design requirements at design load with substantial margins in both fiber controlled ($\sigma_{\ell 11}$) and matrix controlled ($\sigma_{\ell 22}$ and $\sigma_{\ell 12}$) strengths.

b. <u>Stresses in the ±45° ply -- longitudinal</u>. - The general equation for the ply longitudinal stress ($\sigma_{\ell 11}$) is

$$\sigma_{\ell 11} = (E_{\ell 11}/E_{cxx})(\cos^2\theta - \nu_{cxy} \sin^2\theta)\sigma_{cxx}$$
$$+ (E_{\ell 11}/E_{cyy})(\sin^2\theta - \nu_{cxy} \cos^2\theta)\sigma_{cyy}$$
$$+ (E_{\ell 11}/2G_{cxy})[(1 - \nu_{\ell 21})\sin 2\theta)\sigma_{cxy}$$

Using $\theta = 45°$ in the above equation, laminate properties (E_c, ν_c, and G_c) from STEP 6, laminate stress (σ_c) from STEP 7 and ply properties ($E_{\ell 11} = 18.5$ mpsi and $\nu_{\ell 21} = 0.03$) from figure 2 at $\theta = 0$ and $\theta = 90$, respectively, we calculate:

$$\sigma_{\ell 11} = \{(18.5/9.6)(0.5 - 0.33\text{x}0.5)\ 40\ 000$$
$$+ (18.5/6.5)(0.5 - 0.33\text{x}0.5)\ 20\ 000$$
$$+ (18.5/2\text{x}2.3)[(1 - 0.03) \times 1.0)\ 20\ 000\}\ \text{psi}$$

$$\sigma_{\ell 11} = [25\ 823 + 19\ 069 + 78\ 022]\ \text{psi} = 122\ 914\ \text{psi}$$

Check: $\sigma_{\ell 11} \leq S_{\ell 11T}$
122 914 psi < 220 000 psi o.k.

The margin of safety is

$$MOS = \frac{120\ 000}{122\ 914} - 1.0 = 0.79$$

<u>Stresses in the +45°-ply -- transverse</u>. - The general equation for the ply transverse stress ($\sigma_{\ell 22}$) is

$$\sigma_{\ell 22} = (E_{\ell 22}/E_{cxx})[(\nu_{\ell 12} - \nu_{cxy})\cos^2\theta + (1 - \nu_{cxy}\ \nu_{\ell 12})\sin^2\theta]\sigma_{cxx}$$
$$+ (E_{\ell 22}/E_{cyy})[(1 - \nu_{cxy}\ \nu_{\ell 12})\cos^2\theta + (\nu_{\ell 12} - \nu_{cxy})\sin^2\theta]\sigma_{cyy}$$
$$- (E_{\ell 22}/2G_{cxy})[(1 - \nu_{\ell 12})\sin 2\theta]\sigma_{cxy}$$

Proceeding as for $\sigma_{\ell 11}$ above with $E_{\ell 22} = 2$ mpsi (fig. 2, $\theta = 0°$) and $\nu_{\ell 12} = 0.25$ (fig. 3, $\theta = 90°$), we calculate:

$$\sigma_{\ell 22} = \{(2.0/9.6)[(0.25 - 0.33) \times 0.5 + (1 - 0.33\text{x}0.25) \times 0.5]\ 40\ 000$$

$$+ (2.0/6.5)[(1.0 - 0.33\times 0.25) \times 0.5 + (0.25 - 0.33) \times 0.5]\ 20\ 000$$

$$- (2.0/2\times 2.3)[(1 - 0.25) \times 1.0]\ 20\ 000\}\ \text{psi}$$

$$\sigma_{\ell 22} = [3490 + 2577 - 6522]\ \text{psi} = -455\ \text{psi}$$

$\sigma_{\ell 22} \approx 0$ for all practical purposes.

Stresses in the ±45-ply -- intralaminar shear. - The general equation for the ply intralaminar shear stress is

$$\sigma_{\ell 12} = - (G_{\ell 12}/E_{cxx})[(1 + \nu_{cxy})\sin 2\theta]\sigma_{cxx}$$

$$+ (G_{\ell 12}/E_{cyy})[(1 + \nu_{cxy})\sin 2\theta]\sigma_{cyy}$$

$$+ G_{\ell 12}/G_{cxy})(\cos 2\theta)\sigma_{cxy}$$

Proceeding as for $\sigma_{\ell 11}$ or $\sigma_{\ell 22}$ above with $G_{\ell 12} = 0.5$ mpsi (fig. 2, $\theta = 0°$ or $90°$) we calculate:

$$\sigma_{\ell 12} = \{- (0.5/9.6)[(1 + 0.33) \times 1.0]\ 40\ 000$$

$$+ (0.5/6.5)[(1 + 0.33) \times 1.0]\ 20\ 000$$

$$+ (0.5/2.3)(0)\ 20\ 000\}\ \text{psi}$$

$$\sigma_{\ell 12} = [- 2771 + 2046 + 0]\ \text{psi} = -725\ \text{psi}$$

$\sigma_{\ell 12} \approx 0$ for all practical purposes.

Therefore the stresses in the +45° plies meet the ply strength design requirements. Note the only significant stress in this ply is $\sigma_{\ell 11}$. The other two ($\sigma_{\ell 22}$ and $\sigma_{\ell 12}$) are negligible.

c. Stresses in the -45°-ply. General comment: The numerical calculations for the stresses in this ply (or -0° ply in general) are the same as those for the +45° ply (or +θ° ply in general) except for sign changes in the $\sin 2\theta$ term in the equation. Repeating here the sum for each ply stress and using the proper sign we have:

Stresses in the -45° ply -- longitudinal. (see $\sigma_{\ell 11}$, part b. above)

$$\sigma_{\ell 11} = [25\ 823 + 19\ 069 - 78\ 022]\ \text{psi} = -33\ 130\ \text{psi}$$

Check: $\sigma_{\ell 11} \leq S_{\ell 11C}$
$33\ 130\ \text{psi} < 180\ 000\ \text{psi}$ o.k.

The margin of safety is

$$MOS = \frac{180\ 000}{33\ 130} - 1.0 = 4.43$$

Stresses in the -45° ply -- transverse. - This stress is determined from the $\sigma_{\ell 22}$ stress for the +45° ply but with the correct sign for the $\sin 2\theta$ term. Referring to the calculations for $\sigma_{\ell 22}$ above, we have:

$$\sigma_{\ell 22} = [3490 + 2577 - (-6522)] \text{ psi} = 12\ 589 \text{ psi}$$

Check: $\sigma_{\ell 22} \leq S_{\ell 22T}$
$12\ 589 \text{ psi} \leq 8000 \text{ psi}$ n.g.

The margin of safety is

$$MOS = \frac{8000}{12\ 589} - 1.0 = -0.36$$

Thus, the transverse stress in the -45° ply exceeds the ply strength design requirements at design load. At this point we check the margin at the specified load since (1) this is a matrix-controlled property, and (2) the margin for $\sigma_{\ell 11}$ is 4.43 at design load. The transverse ply stress in the -45° ply at specified load is one-half of that calculated for the design load since the design load is twice the specified load (see STEP 2). Thus at specified load:

$$\sigma_{\ell 22} = 1/2(12\ 589) \text{ psi} = 6294 \text{ psi}$$

Check: $\sigma_{\ell 22} \leq S_{\ell 22T}$
$6294 \text{ psi} < 8000 \text{ psi}$ o.k. (at specified load)

The margin of safety at specified load is

$$MOS = \frac{8000}{6294} - 1.0 = 0.27$$

Stresses in the -45° ply -- intralaminar shear. - This stress is the same as that for the +45° ply but with opposite sign. Referring to the calculation for $\sigma_{\ell 12}$ above, we have

$$\sigma_{\ell 12} = [2771 - 2046 + 0] \text{ psi} = 725 \text{ psi}$$

$\sigma_{\ell 12} \approx 0$ for all practical purposes.

Therefore, the longitudinal and intralaminar shear stresses in the -45° ply meet the design requirements at design load while the transverse stress $\sigma_{\ell 22}$ meets the design requirements at specified load.

d. Stresses in the 90°-ply -- longitudinal. - The general equation for the ply longitudinal stress ($\sigma_{\ell 11}$) is:

$$\begin{aligned}\sigma_{\ell 11} = {} & (E_{\ell 11}/E_{cxx})(\cos^2\theta - \nu_{cxy} \sin^2\theta)\sigma_{cxx} \\ & + (E_{\ell 11}/E_{cyy})(\sin^2\theta - \nu_{cxy} \cos^2\theta)\sigma_{cyy} \\ & + (E_{\ell 11}/2G_{cxy})[(1 - \nu_{\ell 21})\sin 2\theta]\sigma_{cxy}\end{aligned}$$

Using $\theta = 90°$ in the above equation, laminate properties (E_c, ν_c, and G_c) from STEP 6, laminate stresses (σ_c) from STEP 7 and ply properties ($E_{\ell 11}$ = 18.5 mpsi, $\nu_{\ell 21}$ = 0.03) from figure 2 at $\theta = 0°$ and $\theta = 90°$ respectively, we calculate:

$$\sigma_{\ell 11} = \{(18.5/9.6)(0 - 0.33\text{x}1.0)\ 40\ 000$$

$$+ (18.5/6.5)(1.0 - 0.33\text{x}0)\ 20\ 000$$

$$+ (18.5/2\text{x}2.3)[(1 - 0.33)\ 0]\ 20\ 000]\ \text{psi}$$

$$\sigma_{\ell 11} = [-\ 25\ 438 +\ 56\ 923 + 0]\ \text{psi} = 31\ 485\ \text{psi}$$

Check: $\sigma_{\ell 11} \leq S_{\ell 11T}$
31 485 psi < 220 000 psi o.k.

The margin of safety is

$$\text{MOS} = \frac{220\ 000}{31\ 485} - 1.0 = 6.00$$

Stresses in the 90°-ply -- transverse. - The general equation for this ply transverse stress ($\sigma_{\ell 22}$) is:

$$\sigma_{\ell 22} = (E_{\ell 22}/E_{cxx})[(\nu_{\ell 12} - \nu_{cxy})\cos^2\theta + (1 - \nu_{cxy}\ \nu_{\ell 12})\sin^2\theta)]\sigma_{cxx}$$

$$+ (E_{\ell 22}/E_{cyy})[(1 - \nu_{cxy}\ \nu_{\ell 12})\cos^2\theta + (\nu_{\ell 12} - \nu_{cxy})\sin^2\theta]\sigma_{cyy}$$

$$+ (E_{\ell 22}/2G_{cxy})[(1 - \nu_{\ell 12})\sin 2\theta]\sigma_{cxy}$$

Using $\theta = 90°$ in the above equation, laminate properties (E_c, ν_c, and G_c) from STEP 6, laminate stresses (σ_c) from STEP 7, and ply properties ($E_{\ell 22}$ = 2 mpsi, $\nu_{\ell 12}$ = 0.25) from figure 2 at $\theta = 0°$ we calculate:

$$\sigma_{\ell 22} = \{(2.0/9.6)[(0.25 - 0.33)\ 0 + (1.0 - 0.33\text{x}0.25)\ 1.0]\ 40\ 000$$

$$+ (2.0/6.5)[(1.0 - 0.33\text{x}0.25)\ 0 + (0.25 - 0.33)\ 1.0]\ 20\ 000$$

$$+ (2.0/2\text{x}2.3)[(1.0 - 0.25)\ 0]\ 20\ 000]\}\ \text{psi}$$

$$\sigma_{\ell 22} = [7646 - 492 + 0]\ \text{psi} = 7154\ \text{psi}$$

Check: $\sigma_{\ell 22} \leq S_{\ell 22T}$
7154 psi < 8000 psi o.k.

The margin of safety is

$$\text{MOS} = \frac{8000}{7154} - 1.0 = 0.12$$

Stresses in the 90°-ply -- intralaminar shear. - The general equation for the ply intralaminar shear stress ($\sigma_{\ell 12}$) is:

$$\sigma_{\ell 12} = -(G_{\ell 12}/E_{cxx})[(1 + \nu_{cxy})\sin 2\theta]\sigma_{cxx}$$

$$+ (G_{\ell 12}/E_{cyy})[(1 + \nu_{cxy})\sin 2\theta]\sigma_{cyy}$$

$$+ (G_{\ell 12}/G_{cxy})(\cos 2\theta)\sigma_{cxy}$$

Using $\theta = 90°$ in the above equation, laminate properties (E_c, ν_c, and G_c) from STEP 6, laminate stresses from STEP 7 and the ply property ($G_{\ell 12}$ = 0.5 mpsi) from figure 2 at $\theta = 0°$ or 90°, we calculate:

$$\sigma_{\ell 12} = [-(0.5/9.6)[(1 + 0.33)\ 0]\ 40\ 000$$

$$+ (0.5/6.5)[(1 + 0.33)\ 0]\ 20\ 000$$

$$+ (0.5/2.3)(-1.0)\ 20\ 000]\ \text{psi}$$

$$\sigma_{\ell 12} = [-0 + 0 - 4348]\ \text{psi} = -4348\ \text{psi}$$

Check: $\sigma_{\ell 12} \leq S_{\ell 12S}$
$|-4348|$ psi < 10 000 psi o.k.

The margin of safety is

$$\text{MOS} = \frac{10\ 000}{|-4348|} - 1.0 = 1.30$$

Therefore, the stresses in the 90° plies satisfy the ply strength design requirements at design load.

The results of the ply stress analysis for the combined loading conditions, including ply strength and margins of safety, are summarized in table II. From the results presented in this table, several interesting observations/conclusions can be made that can be used as guidelines for selecting $[\pm\theta/0/90]_S$ laminate configurations for combined loadings. Some of these are, (1) fiber stress limits are controlled by the longitudinal strength in the ±45°-plies and generally the +θ plies (θ < 45°); (2) matrix stress limits are controlled by the transverse tensile stress in the -45° plies and generally the -θ plies (θ > 30°); and (3) laminates configured to satisfy matrix-controlled stress limits under combined design load will have substantial margins for fiber-controlled properties.

STEP 10. Check shear buckling. Shear buckling is estimated by using the following approximate equation if the tensile stresses (σ_{cxx} and σ_{cyy}) are neglected

$$\sigma_{cxy}^{(cr)}\ \frac{7J T^2 t_c^2 E}{12b^2(1 - \nu_{cxy}\ \nu_{cyx})}\ (1 \leq a/b \leq 2)$$

$$E = \sqrt[3]{4E_{cxx}E_{cyy}G_{cxy}}$$

From STEP 6 we have, the

$\nu_{cxy} = 0.33$
$\nu_{cyx} = 0.22$
$E_{cxx} = 9.6$ mpsi
$E_{cyy} = 6.5$ mpsi
$G_{cxy} = 2.3$ mpsi

Using these moduli values in the equation for E, we calculate:

$$E = \sqrt[3]{4 \times 9.6 \times 6.5 \times 2.3} \text{ mpsi} = 8.31 \text{ mpsi}$$

Using this value for E, the values for ν_{cxy} and ν_{cyx}, b = 10 in and t_c = 0.1 in, in the equation for $\sigma_{cxy}^{(cr)}$, we calculate:

$$\sigma_{cxy}^{(cr)} = \frac{7J\pi^2(0.1)^2 \text{ in}^2 \times 8.31\times10^6 \text{ lb}}{12 \times 10 \text{ in} \times 10 \text{ in} \times (1 - 0.33\times0.2) \text{ in}^2} = 5159 \text{ psi}$$

Check: $\sigma_{cxy}^{(cr)} \geq \sigma_{cxy}$ (design)
5159 psi < 20 000 psi

$$\text{MOS} = \frac{5159}{20\ 000} - 1.0 = -0.74$$

Therefore, the shear buckling stress needs to be checked in combination with the two normal (σ_{cxx} and σ_{cyy}) tensile stresses.

An estimate of buckling resistance may be obtained from the approximate interaction equation given by

$$\frac{\sigma_{cxx}}{\sigma_{cxx}^{(cr)}} + \frac{\sigma_{cyy}}{\sigma_{cyy}^{(cr)}} - \left(\frac{\sigma_{cxy}}{\sigma_{cxy}^{(cr)}}\right)^2 + 1.0 \geq 0$$

where σ_{cxx}, σ_{cyy}, and σ_{cxy} are the laminate stresses at design load. The buckling stresses $\sigma_{cxx}^{(cr)}$ and $\sigma_{cyy}^{(cr)}$ are roughly approximated from

$$\sigma_{cyy}^{(cr)} = \sigma_{cxx}^{(cr)} \approx \frac{J\pi^2 t_c^2 E}{12b^2(1 - \nu_{cxy}\,\nu_{cyx})}\left(\frac{a}{b} + \frac{b}{a}\right)^2$$

where E and ν_c are the same as before. Using respective values for the moduli, Poisson's ratios b, a, and t_c, we calculate

$$\sigma_{cyy}^{(cr)} = \sigma_{cxx}^{(cr)} \approx \frac{J\pi^2 \times (0.1)^2 \text{ in}^2 \times 8\ 310\ 000 \text{ lb}}{12 \times 10 \text{ in} \times 10 \text{ in} \times (1 - 0.33\times0.22)}\left(\frac{15}{10} + \frac{10}{15}\right)^2 \text{ psi}$$

$$\sigma_{cyy}^{(cr)} = \sigma_{cxx}^{(cr)} \approx 3460 \text{ psi}$$

Substituting the following:

$\sigma_{cxx} = 40\ 000$ psi; $\sigma_{cxx}^{(cr)} = 3460$ psi

$\sigma_{cyy} = 20\ 000$ psi; $\sigma_{cyy}^{(cr)} = 3460$ psi

$\sigma_{cxy} = 20\ 000$ psi; $\sigma_{cxy}^{(cr)} = 5159$ psi

in the interaction equation, we calculate

$$\frac{40\ 000}{3460} + \frac{20\ 000}{3460} - \left(\frac{20\ 000}{5159}\right)^2 + 1.0 > 0$$

$$11.56 + 5.78 - 14.86 + 1.0 = 3.48 > 0 \quad \text{o.k.}$$

For this case the MOS = 0.11.

Therefore, based on the estimate obtained using the interaction equation, the panel should not buckle at the design shear stress, provided that all three loads (N_{cxx}, N_{cyy}, and N_{cxy}) are applied proportionally and simultaneously. This can also be stated as: N_{cyy} and N_{cxy} are proportional to N_{cxx}. It is important to observe the dramatic positive effect of the normal tensile stresses on the shear buckling strength. A more accurate estimate may be obtained by using the equations for the buckling of composite panels given in reference 15 or by performing a finite element analysis. The conclusion from this sample design is that the panel as designed satisfies all the displacement and shear buckling requirements at design loads and also the ply strength requirements, except for the transverse ply stress in the -45° ply which satisfies tne design requirements only at the specified loads, not the design loads.

STEP 11. Sample design results summary (margins given on design load unless otherwise noted).

a. Laminate configuration $[\pm 45/0/90/0]_{2S}$

b. Margins of safety on displacement design requirements

Displacement	Margin
(u/a)	0.43
(v/b)	1.94
$\Delta\theta$	0.33

c. Margins of safety on ply stress limits

Ply	Margins for stress		
	$\sigma_{\ell 11}$	$\sigma_{\ell 22}$	$\sigma_{\ell 12}$
0	2.77	0.61	1.30
+45	0.79	∞	∞
-45	4.43	[a]0.27	∞
90	6.00	0.12	1.30

[a]At specified load; this margin is -0.38 at design load.

d. Margin of safety on shear buckling stress

Case (stress in psi)			Margin for $\sigma_{cxy}^{(cr)}$
σ_{cxx}	σ_{cyy}	σ_{cxy}	
0	0	20 000	-0.74
40 000	20 000	20 000	3.48

DESIGN PROCEDURES FOR: HYGROTHERMAL EFFECTS, CYCLIC LOADS, AND LAMINATION RESIDUAL STRESSES - BRIEF OUTLINE

Due to space limitations, the sample design described was only for combined static loads. However, the procedure and the governing equations used to design for hygrothermal effects, cyclic loads, and lamination residual stresses are the same. The ply strengths used to check stress limits change depending on the environment, the cyclic load history and lamination residual stresses. Some general guidelines are briefly described below.

Hygrothermal Effects

Hygrothermal (hot-wet) degradation of matrix-controlled ply properties ($P_{\ell HT}$) can be estimated using the following equation (refs. 7 and 11). When the use temperature (T) and moisture pickup (M) are known:

$$\frac{P_{\ell HT}}{P_{\ell 0}} = \left[\frac{T_{GW} - T}{T_{GD} - T_o}\right]^{1/2} \tag{1}$$

$$T_{GW} \approx (0.005\ M_\ell - 0.1\ M_\ell + 1.0)\ T_{GD} \tag{2}$$

where T_{GW} is the glass transition temperature of the <u>wet</u> unidirectional composite, T_{GD} is the glass transition temperature of the <u>dry</u> unidirectional composite, T is the use temperature at which $P_{\ell HT}$ is required, T_0 is the reference temperature at which $P_{\ell 0}$ was determined and M_ℓ is the moisture in the ply in percent by weight. Hygrothermal effects on the stiffness of $[\pm\theta/0/90]_S$ angleplied laminates are generally negligible. Since matrix (resin)-controlled ply properties (moduli and strengths) degrade at about the same rate (eq. (1)), the corresponding ply limit stress margins remain practically unchanged. One exception to this is ply longitudinal compression strength which may degrade substantially at elevated temperatures which approach the T_{GW}. It can be concluded that $[\pm\theta/0/90]_S$ angleplied laminates selected to meet design requirements at room temperature conditions will generally satisfy hygrothermal environmental conditions so long as the use temperature does not approach T_{GW}. Calculations for thermal and hygral stresses are expedited using figures 2 to 5.

Cyclic Loads

Cyclic loads fatigue the laminate and, therefore, the ply stress limit needs to be checked against the fatigue strength of the ply. The fatigue strength of the ply can be estimated using the following equation (refs. 10 and 14)

$$\frac{S_{\ell N}}{S_{\ell 0}} = 1.0 - B\log N \qquad (3)$$

where $S_{\ell N}$ is the fatigue strength for the specified N cycles; $S_{\ell 0}$ is the reference static strength; B is a constant depending on the composite system (0.1 is a reasonable value, ref. 10); and N is the number of cycles. Usually a safety factor (ranging from 2 to 4) is applied to $S_{\ell N}$ calculated from equation (3). The procedure, then, is to calculate $S_{\ell NA}$ and use this for the ply strength to check for the ply stress limits and to determine the margins of safety. In the presence of combined static and cyclic loads, the ply stress limit is estimated from the following equation (ref. 14)

$$\frac{\sigma_{\ell ST}}{S_{\ell}} + \frac{\sigma_{\ell cyc}}{S_{\ell NA}} \leq 1.0 \qquad (4)$$

where $\sigma_{\ell ST}$ is the ply stress ($\sigma_{\ell 11}$, $\sigma_{\ell 22}$, and $\sigma_{\ell 12}$) due to design static load; $\sigma_{\ell cyc}$ is the corresponding ply stress due to cyclic load; S_{ℓ} is the ply static strength; and $S_{\ell NA}$ is determined from equation (3) with an appropriate safety factor.

Displacement and buckling stress limits are checked at maximum design load (static plus cyclic) magnitude (ref. 14). For these calculations damping and inertial effects are usually neglected.

Lamination Residual Stresses

The lamination residual stresses generally increase the transverse ply stresses. Consideration of these stresses results in thicker laminates in order to meet ply stress design requirements at combined loads. Lamination residual stresses can be determined following the procedures described in reference 9. The lamination ply residual stresses need to be superimposed on the other ply stresses prior to checking for ply limit stresses and margins of safety.

CONCLUDING REMARKS

Step-by-step procedures are described for designing panels made from fiber composite angleplied laminates and subjected to combined in-plane loads. These procedures are set up as a multi-step sample design. The various steps include the governing equations and subsequent calculations required to check that the design requirements are not violated. The sample design steps are supplemented with appropriate tabular and graphical data for expediting the design process. Some guidelines are described which can be used to select configurations for general $[\pm\theta/0/90]_S$ angleplied laminates. Procedures for considering hygrothermal effects, cyclic loads, and lamination residual stresses are briefly outlined.

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TABLE I. – TYPICAL PROPERTIES OF UNIDIRECTIONAL COMPOSITES AT ROOM TEMPERATURE

Properties	Symbol	Units	Boron/ epoxy	Boron/ poly mide	S-glass /epoxy	Modmor I/ epoxy	Modmor I/ polymide	Thornel 300/ epoxy	Kevlar 49/ epoxy	Graphite AS/epoxy
Fiber volume ratio	k_f	———	0.50	0.49	0.72	0.45	0.45	0.70	0.54	0.60
Density	ρ_ℓ	lb/in^3	0.073	0.072	0.077	0.056	0.056	0.058	0.049	0.057
Longitudinal thermal coefficient	$\alpha_{\ell 11}$	10^{-6} in /in/ °F	3.4	2.7	2.1	———	0.0	0.01	-1.60	0.40
Transverse thermal coefficient	$\alpha_{\ell 22}$	10^{-6} in /in/ °F	16.9	15.8	9.3	18.5	14.1	12.5	31.3	16.4
Longitudinal modulus	$E_{\ell 11}$	10^6 psi	29.2	32.1	8.8	27.5	31.3	21.0	12.2	16.0
Transverse modulus	$E_{\ell 22}$	10^6 psi	3.15	2.1	3.6	1.03	0.72	1.5	0.70	2.2
Shear modulus	$G_{\ell 12}$	10^6 psi	0.78	1.11	1.74	0.9	0.65	1.0	0.41	0.72
Major Poissons's ratio	$\nu_{\ell 12}$	———	0.17	0.16	0.23	0.10	0.25	0.28	0.32	0.25
Minor Poissons's ratio	$\nu_{\ell 21}$	———	0.02	0.02	0.09	———	0.02	0.01	0.02	0.34
Longitudinal tensile strength	$S_{\ell 11T}$	psi	199 000	151 000	187 000	122 000	117 000	218 000	172 000	220 000
Longitudinal compressive strength	$S_{\ell 11C}$	psi	232 000	158 000	119 000	128 000	94 500	247 000	42 000	180 000
Transverse tensile strength	$S_{\ell 22T}$	psi	8100	1600	6670	6070	2150	5850	1600	8000
Transverse compressive strength	$S_{\ell 22C}$	psi	17 900	9100	23 500	28 500	10 200	35 700	9400	36 000
Intralaminar shear strength	$S_{\ell 12S}$	psi	9100	3750	6500	8900	3150	9800	4000	10 000
Longitudinal moisture coefficient	$\beta_{\ell 11}$	10^{-2} in	0.003	0.003	0.014	0.003	0.003	0.006	0.008	0.006
Transverse moisture coefficient	$\beta_{\ell 22}$	10^{-2} in	0.168	0.168	0.128	0.129	0.129	0.129	0.151	0.129
Glass transition temperature (estimate)	T_{GD}	°F	420	700	420	420	700	420	420	420

TABLE II. - SUMMARY OF PLY STRESS ANALYSIS FOR COMBINED LOAD, ($[\pm 45/0/90/0]_{2S}$ AS/E ANGLEPLIED LAMINATE)

Load condition /strength /MOS	Ply/ply-stress/strength (ksi), MOS-ratio											
	0°-Ply			+45°-Ply			-45°-Ply			90°-Ply		
	$\sigma_{\ell 11}$	$\sigma_{\ell 22}$	$\sigma_{\ell 12}$	$\sigma_{\ell 11}$	$\sigma_{\ell 22}$	$\sigma_{\ell 12}$	$\sigma_{\ell 11}$	$\sigma_{\ell 22}$	$\sigma_{\ell 12}$	$\sigma_{\ell 11}$	$\sigma_{\ell 22}$	$\sigma_{\ell 12}$
N_{cxx}	77.1	-.7	0	25.8	3.5	-2.8	25.8	3.5	2.8	-25.4	7.6	0
N_{cyy}	-18.1	5.6	0	19.1	2.6	2.0	19.1	2.6	-2.0	56.9	-0.5	0
N_{cxy}	0	0	4.3	78.0	-6.5	0	-78.0	6.5	0	0	0	-4.3
SUM	58.3	5.0	4.3	122.9	-0.5	-0.7	-33.0	12.6	.8	31.5	7.1	-4.3
S_ℓ	220.0	8.0	10.0	220.0	8.0	-10.0	-180.0	8.0	10	220.0	8.0	-10.0
MOS	2.77	0.61	1.30	0.79	∞	∞	4.43	[a]-0.36	∞	6.00	0.12	1.30

[a]At specified load this is +0.27.

Notation:
- N_c panel in-plane loads
- S_ℓ ply strength
- σ_ℓ ply stress
- MOS margin of safety

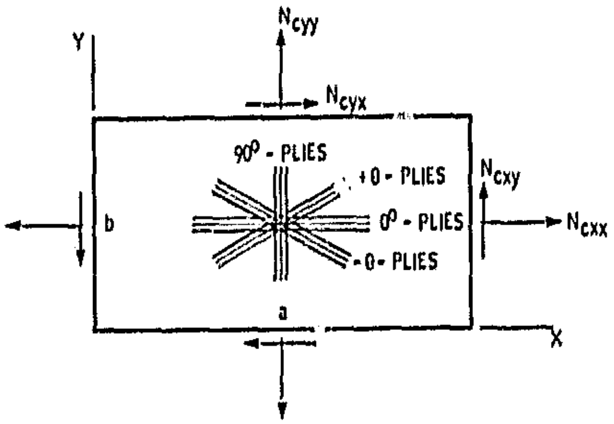

Figure 1. - Schematic of angleplied fiber composite panel subjected to combined in-plane loads.

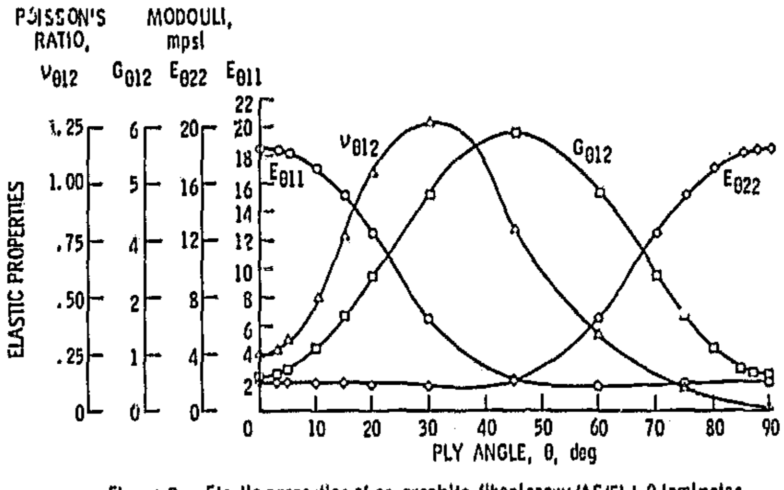

Figure 2. - Elastic properties of as-graphite-fiber/epoxy (AS/E) ± θ laminates.

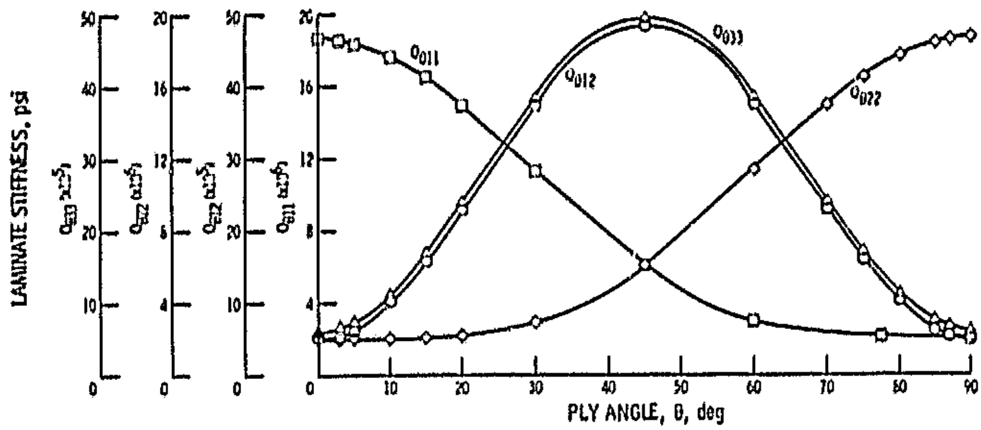

Figure 3. - Reduced stiffnesses of as graphite-fiber/epoxy (AS/E) ± θ laminates.

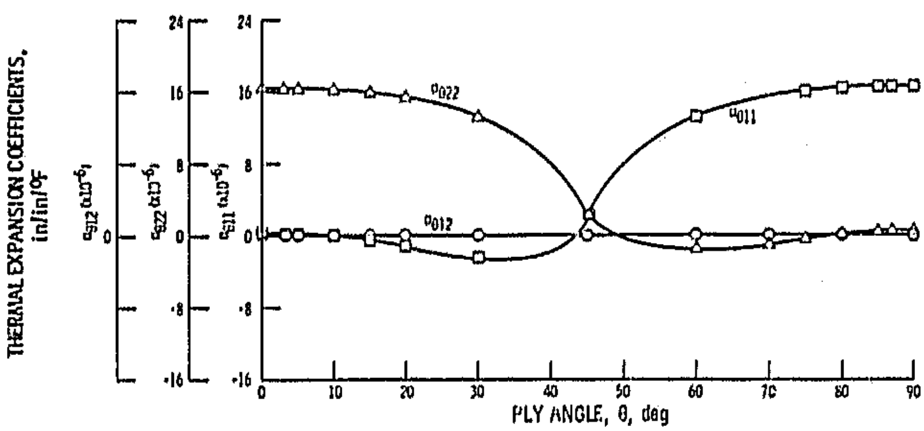

Figure 4. - Thermal expansion coefficients of as graphite-fiber/epoxy (AS/E) ± θ laminates.

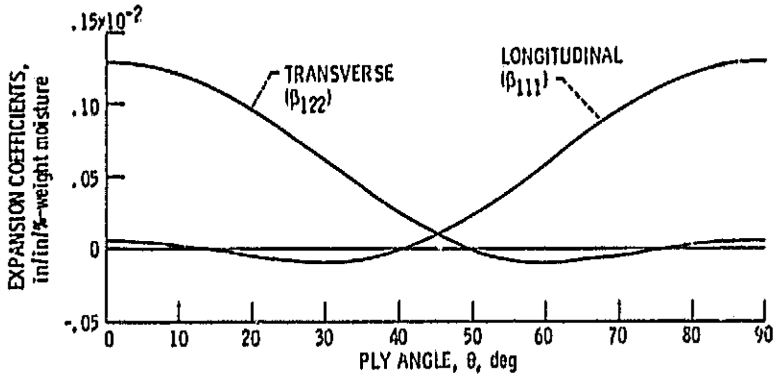

Figure 5. - Moisture expansion coefficients of as graphite-fiber/epoxy (AS/E) ±θ laminates.

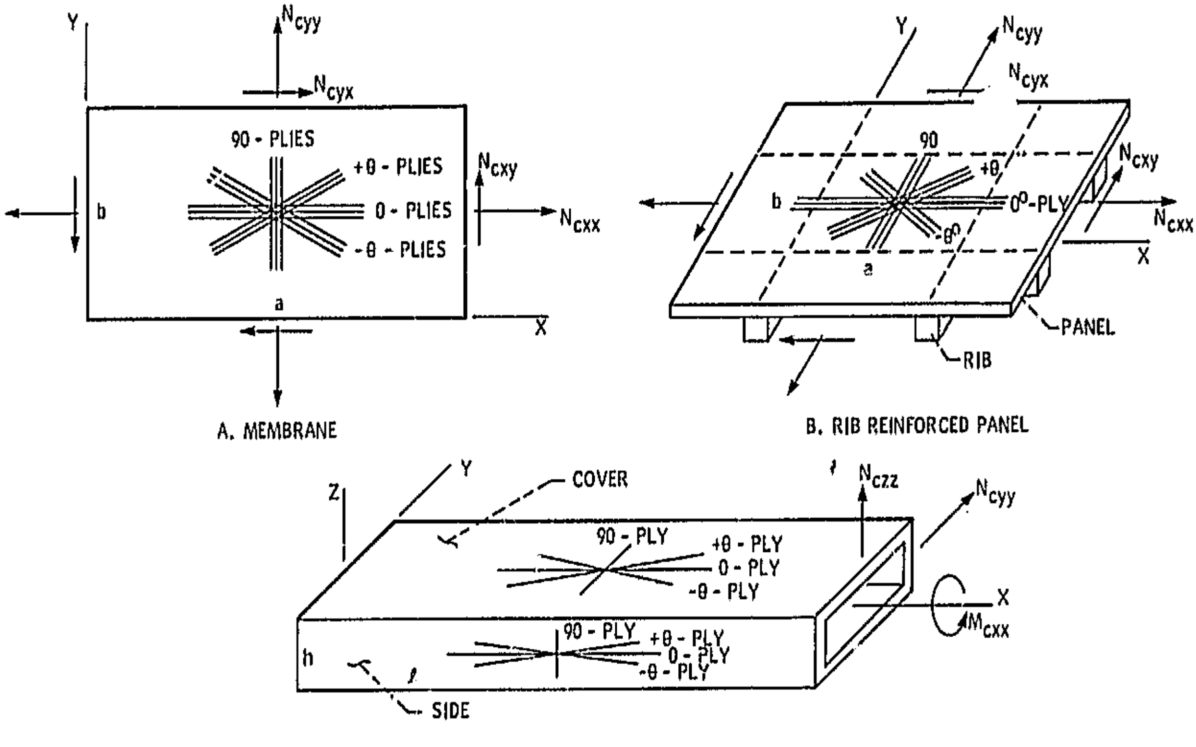

Figure 6. - Schematics of select composite structural components with respective geometry and typical loadings.