NASA Contractor Report 3873

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GRANT NAG1-334
FEBRUARY 1985

NASA

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Prepared for
Langley Research Center
under Grant NAG1-334

NASA
National Aeronautics
and Space Administration

Scientific and Technical
Information Branch

1985

SUMMARY

The problem of sidewall boundary-layer effects in airfoil testing is treated by considering the changes in the flow area due to boundary-layer thinning under the influence of the airfoil flowfield. Using von Karman's momentum integral equation, it is shown that the sidewall boundary-layer thickness in the region of the airfoil can reduce to about half the undisturbed value under the conditions prevailing in testing of supercritical airfoils. A Mach number correction due to this increased width of the flow passage is proposed. Using the small disturbance approximation, the effect of the sidewall boundary-layers is shown to be equivalent to a change in the test Mach number and also in the airfoil thickness. Comparison of the results of this approach with other similarity rules and correlation of the experimental data demonstrate the applicability of the analysis presented from low speeds to transonic speeds.

INTRODUCTION

Numerous studies have been reported in the literature to assess the wall interference effects in wind tunnel testing of models. These studies are generally based on inviscid calculations and the influence of wall boundary-layers on interference effects often ignored. However, in testing of airfoil sections in two-dimensional wind tunnels, recent studies at ONERA (ref. 1) have indicated that the presence of the boundary layers on the sidewalls of the tunnel can influence the measurements even on the airfoil midspan. To keep this influence within acceptable limits, it is desirable to use suitable sidewall boundary layer control. In the absence of such a control, appropriate corrections are to be made to the measured data to account for the effects of the sidewall boundary layer. Particularly, with the current development of flexible walls to nearly eliminate top and bottom interference in airfoil testing, the necessity to assess the extent of side-wall boundary-layer influence is a matter of prime concern.

Methods to account for the sidewall boundary-layer effects have been primarily based on two different considerations. Firstly, as proposed by Preston (ref. 2) in 1944, it is assumed that the trailing vorticity due to loss of lift in the boundary-layer is shed and this in turn causes a change in the effective angle of incidence. In the second approach, proposed independently by Barnwell (References 3 and 4), and Winter and Smith (Reference 5), the effects are attributed to changes in the sidewall boundary-layer thickness due to the model induced pressure field. In the Barnwell's method, based on the small disturbance approximation both the sidewall boundary-layer effects and the compressibility effects are combined into a single factor similar to Prandtl-Glauret rule. This was later extended to transonic speeds by Sewall (Reference 6) by using the von Karman similarity rule.

In the Barnwell-Sewall method, it is assumed that the sidewall boundary-layers induce a spanwise velocity which varies linearly across the test section width. Recent investigations (Reference 7) on a super-critical airfoil in the NASA Langley 0.3-m Transonic Cryogenic Tunnel have indicated that correction to the test Mach number based on the Barnwell-Sewall method accounted for most of the blockage effects and was sufficient to give a good match between the measured pressure distribution and the predictions of an advanced computer code. Particularly, on supercritical airfoils, the pressure distribution is nearly constant over the forward portion and if the growth of the sidewall boundary-layer due to viscous effects is small, it is reasonable to expect the sidewall boundary-layer thickness in this region to be nearly constant at a value lower than the undisturbed thickness. Hence the primary effect of the sidewall boundary-layers appears to be due to change in the area of the flow passage rather than the presence of significant spanwise velocities across the width of the tunnel. In the present report, a new method to account for the change in the effective free-stream Mach number due to change in the sidewall boundary-layer thickness is proposed. Further, it is demonstrated using the small disturbance approximations, that the effect of the sidewall boundary-layers can be considered as causing changes in both the free stream Mach number and the thickness of the airfoil.

NOMENCLATURE

A	Area
a	Speed of sound
b	Width of the two-dimensional tunnel
c	Airfoil chord
C_p	Pressure coefficient
H	Shape factor of the sidewall boundary-layer
k	Constant, $= (2 + \frac{1}{H} - M_\infty^2) \frac{2\delta_u^*}{b}$
M	Local Mach number
M_c	Corrected Mach number
M_e	Effective Mach number in the equivalent 2-D flow
M_∞	Free-stream Mach number
ΔM	Correction for freestream Mach number ($= M_c - M_\infty$)
U	Local velocity
U_e	Effective free-stream velocity in the equivalent 2-D flow

U_∞	Free-stream velocity
u	Perturbation velocity
u_b	Perturbation velocity due to sidewall boundary-layers
u_w	Perturbation velocity in the wind tunnel
x, y	Streamwise and normal coordinates
γ	Ratio of specific heats
δ^*	Boundary-layer displacement thickness
δ_u^*	Undisturbed value of displacement thickness
θ	Boundary-layer momentum thickness
ξ, η	Transformed coordinates
ρ	Density
τ	Airfoil thickness ratio (geometrical)
τ_e	Effective thickness in the equivalent 2-D flow
τ_w	Wall shear stress
ϕ	Velocity potential
ϕ_w	Velocity potential corresponding to wind tunnel flow

ANALYSIS

Consider the flow over an airfoil spanning the width of a two-dimensional wind tunnel (figure 1). It is assumed that the boundary-layer thickness on the sidewalls of the empty tunnel is nearly constant along the length of the tunnel and the corresponding value of the displacement thickness is δ_u^*. With the airfoil present in the tunnel, the boundary-layer thickness changes depending on the local pressure field. An estimate of the change in the boundary-layer thickness can be obtained from the von Karman momentum integral equation

$$\frac{d\theta}{dx} + \frac{\theta}{U}\frac{dU}{dx}\left(2 + H - M^2\right) = \frac{\tau_w}{\rho U^2} \tag{1}$$

The shearing stresses represented by the term on the right hand side of equation (1) can be ignored for attached boundary-layers in comparison with the pressure gradient effects since the model chord c is much smaller than the boundary-layer equivalent length at the model station (References 4, 6 and 8). Further, since the tunnel walls are normally diverged to compensate for the flat plate boundary-layer growth, neglecting the shearing stress term would not have an appreciable effect on the results of the following analysis. With this assumption and noting $\theta = \delta^*/H$, equation (1) can be written in terms of the displacement thickness δ^*

$$\frac{d\delta^*}{\delta^*} = \frac{dH}{H} - \left(2 + H - M^2\right)\frac{dU}{U} \quad . \tag{2}$$

The local Mach number M is related to the local velocity U by

$$M = U/a \quad . \tag{3}$$

Differentiating equation (3) and assuming total temperature to be constant, we get

$$\frac{dU}{U} = \frac{1}{1+.2M^2}\frac{dM}{M} \quad . \tag{4}$$

Combining equations (2) and (4), the variation of δ^* with Mach number and shape factor can be expressed as

$$\frac{d\delta^*}{\delta^*} = \frac{dH}{H} - \frac{2+H-M^2}{1+.2M^2}\,\frac{dM}{M} \quad . \tag{5}$$

The shape factor variation with Mach number for high Reynolds number transonic flows can be approximated by (Reference 4)

$$H = 1 + (\gamma - 1)\, M^2 \ . \tag{6}$$

Using this relation, equation (5) can be integrated in closed form to give (see Appendix)

$$\frac{\delta^*}{\delta^*_u} = \frac{1 + .4M^2}{1 + .4M^2_\infty} \left(\frac{1 + .2M^2}{1 + .2M^2_\infty}\;\frac{M_\infty}{M} \right)^3 \tag{7}$$

δ^*_u and M_∞ refer to the undisturbed value of the displacement thickness and the free-stream Mach number; usually known from the tunnel test conditions. The value of δ^* can then be calculated from equation (7) for the corresponding local Mach number M.

Depending on the value of the local Mach number M, the sidewall boundary-layer displacement thickness will be different from the undisturbed value δ^*_u. On the upper surface of the airfoil where the flow accelerates, the boundary-layer thickness reduces thus increasing the effective width of the flow passage and hence changing the local mean velocity. Alternatively, this flow feature can be viewed as the test airfoil being placed in a channel of increased width. Since the disturbances produced by the airfoil persist for long distances in the lateral direction at transonic speeds, the extent of thinning of the sidewall boundary-layer in the region of the airfoil can be expected to be nearly the same along the test section height in the proximity of the airfoil. Hence the passages above and below the airfoil can, to a first approximation be considered as channels with parallel side walls. Also, generally the sidewall boundary-layer thickness will be much larger than the airfoil boundary-layer except near the trailing edge. Therefore, the corner effects of the airfoil/sidewall junction are not likely to be significant. The changes in the airfoil pressure distribution due to sidewall boundary-layer effects will be largely influenced by the change in thickness in the vicinity of the surface. Hence to a first approximation, the effective change in the free stream Mach number at a representative chordwise location on the airfoil can be obtained by considering the conservation of mass flow through a two-dimensional strip across the tunnel width. Equating the mass flows in the strip, for conditions with and without boundary-layer thinning effects, we get

$$\frac{(b - 2\delta_u^*)\ M_\infty}{(1 + .2M_\infty^2)^3} = \frac{(b - 2\ \delta^*)\ M_c}{(1 + .2M_c^2)^3} \tag{8}$$

where M_c is the corrected Mach number. With δ^* estimated from equation (7), M_c can be calculated from equation (8). For small changes in the area, the correction ΔM to the free-stream Mach number can be obtained by simplifying equation (8), to give

$$\frac{\Delta M}{M_\infty} = - \frac{2\delta_u^*}{b} \left(1 - \frac{\delta^*}{\delta_u^*}\right) \frac{1 + .2M_\infty^2}{1 - M_\infty^2} \quad . \tag{9}$$

SMALL DISTURBANCE APPROXIMATIONS

The variations in the width of the flow passage from the free-stream conditions is small for thin sidewall boundary-layers and its effect consists significantly of introducing perturbations in the stream direction. The associated perturbations in the vertical and spanwise directions are assumed to be small. This assumption implies that there is a core of uniform flow between the tunnel sidewalls where the variation in the flow quantities across the tunnel width is quite small and the airfoil aspect ratio effect can be ignored. From equation (2), we have

$$\frac{d\delta^*}{\delta^*} = \frac{dH}{H} - (2 + H - M^2) \frac{dU}{U} \quad .$$

Using the approximate variation for the shape parameter given by equation (6), the above expression can be expressed in a simplified manner as (Reference 4)

$$\frac{d\delta^*}{\delta^*} = - (2 + 1/H - M^2) \frac{dU}{U} \quad . \tag{10}$$

The change $d\delta^*$ in displacement thickness from the undisturbed value δ_u^* can be obtained to a first approximation by substituting for dU, the perturbation velocity du due to the airfoil in a two-dimensional flow.

$$d\delta^* = -\left(2 + 1/H - M^2\right) \frac{du}{U_\infty} \delta_u^* \quad . \tag{11}$$

Let du_b be the perturbation velocity introduced over the two-dimensional field in the longitudinal direction because of change in the flow area. From one-dimensional isentropic flow relations, u_b is related to the change in the flow area dA by

$$\frac{du_b}{U_\infty} = -\frac{1}{1-M^2_\infty} \frac{dA}{A} \tag{12}$$

where

$$A = b - 2\delta_u^*$$

$$dA = -2\,d\delta^*$$

Combining equations (11) and (12) and integrating

$$\frac{u_b}{U_\infty} = -\frac{k}{1-M^2_\infty} \frac{u}{U_\infty} \tag{13}$$

where k is nearly a contant given by

$$k = \left(2 + 1/H - M^2_\infty\right) \left(2\,\delta_u^*/b\right) . \tag{14}$$

In a wind tunnel flow with sidewall boundary-layers, within the small disturbance approximation the longitudinal perturbation velocity u_W will be the sum of the perturbations due to the airfoil in a two-dimensional flow and that due to change in thickness of the sidewall boundary-layer (u_b), i.e.,

$$u_w = u + u_b$$

or using equation (13), u can be written as

$$u \approx u_w \left(1 + \frac{k}{1 - M_\infty^2}\right) \quad . \tag{15}$$

The linearized small disturbance equation for the perturbation potential of an airfoil in the two-dimensional flow or in a wind tunnel with no sidewall boundary-layer effect, is

$$(1-M_\infty^2)\ \phi_{xx} + \phi_{yy} = 0 \tag{16}$$

where M_∞ is the free-stream Mach number. The airfoil shape and the pressure coefficient on the airfoil are given by

$$y/c = \tau f(x/c) \tag{17}$$

$$C_p = -2u/U_\infty \quad . \tag{18}$$

In a wind tunnel with sidewall boundary-layers, for the same reference conditions (U_∞ and M_∞), the pressure coefficients on the airfoil will be different from the two-dimensional value. Consistent with the small disturbance approximation, the measured pressure coefficients ($C_{p,w}$) are related to the perturbation velocity u_w by

$$C_{p,w} = -2u_w/U_\infty \quad . \tag{19}$$

Combining equations (15) and (16), and assuming that the effect of the sidewall boundary-layers is one dimensional (i.e. $\phi_{yy} = \phi_{w,yy}$), the governing differential equation for the perturbation potential in the wind tunnel flow is written as

$$(1 - M_\infty^2)\left(1 + \frac{k}{1-M_\infty^2}\right)\phi_{w,xx} + \phi_{w,yy} = 0 \tag{20}$$

or

$$(1 - M_\infty^2 + k)\,\phi_{w,xx} + \phi_{w,yy} = 0 \quad . \tag{21}$$

If the free-stream velocity is U_∞ and the airfoil thickness τ, the boundary condition for equation (21) is

$$(\phi_{w,y})_{y=0} = U_\infty\,(dy/dx) = U_\infty \tau f'(x/c) \quad . \tag{22}$$

The boundary condition (22) follows from the assumption that the sidewall boundary-layers introduce negligible vertical and spanwise velocities. It may be noted that the equation (21) is exactly the same as that derived by Barnwell (Reference 4) with the assumption that the spanwise velocity varies linearly across the width of the test-section. The above derivation demonstrates that the Barnwell's assumption of linear variation in the spanwise velocity across the width of the tunnel represents to a first order the effect of the change in the width of the flow passage due to changes in the sidewall boundary-layer thickness. It can be easily shown that the consequence of a gradual change in area is equivalent to a linear variation of the spanwise velocity. Further, the present analysis is applicable to either narrow or wide tunnels as long as the model chord is long enough for the effects to be uniform across the span. Using the coordinate transformation

$$\xi = x \quad \text{and} \quad \eta = y\sqrt{1 + k} \tag{23}$$

equation (21) can be written as

$$(1 - M_e^2)\,\phi_{w,\xi\xi} + \phi_{w,\eta\eta} = 0 \tag{24}$$

where $$M_e = M_\infty / \sqrt{1 + k}$$

From equation (22), the corresponding boundary condition is

$$(\phi_{w,\eta})_{\eta=0} = U_\infty \ \tau f'(x/c) / \sqrt{1 + k} \ . \tag{25}$$

From the above boundary condition, it follows that the flow over an airfoil in a wind tunnel with sidewall boundary-layers is equivalent to a two-dimensional flow at Mach number M_e on an airfoil with the free-stream velocity or the thickness ratio reduced by a factor of $\sqrt{1 + k}$.

a) When the free-stream velocity in the equivalent two-dimensional flow is the same as U_∞, the pressure coefficient is given by

$$C_p = -2\ (\phi_\xi)_{\eta=0} / U_\infty \ . \tag{26}$$

This implies that the pressure coefficient in the wind tunnel flow at test Mach number M_∞ and free stream velocity U_∞ will be the same as that in two-dimensional flow at Mach number M_e and free-stream velocity U_∞ but on a thinner profile with thickness ratio $\tau_e = \tau / \sqrt{1 + k}$.

b) Alternatively, if the free-stream velocity in the equivalent two-dimensional flow is $U_e = U_\infty / \sqrt{1 + k}$, the pressure coefficient will be

$$C_{p,\,e} = - 2 \left(\phi_\xi\right)_{\eta=0} / U_e \tag{27}$$

This implies that the flow in a wind tunnel at Mach number M_∞ will be equivalent to that in a two-dimensional flow at Mach number M_e on the same profile with the pressure coefficients increased by a factor $\sqrt{1 + k}$ over the wind tunnel value.

In summary, it follows that for comparison of the wind tunnel data with theoretical prediction methods, the appropriate corrected values of the various parameters are

Mach number	$M_e = M_\infty / \sqrt{1 + k}$	(28a)
Pressure Coeff.	$C_{p,e} = \sqrt{1 + k} \,.\, C_{p,\text{ wind tunnel}}$	(28b)
Normal force	$C_{n,e} = \sqrt{1 + k} \,.\, C_{n,\text{ wind tunnel}}$	(28c)

For transonic flow, if it is assumed that the one-dimensional effects can be superimposed, it is appropriate that the local values of the Mach number are used instead of the free-stream Mach number in equations (15) and (16). Then, the corresponding small disturbance equation for the wind tunnel flow is

$$(1 - M^2 + k)\ \phi_{w,xx} + \phi_{w,yy} = 0 \qquad (29)$$

where
$$M^2(x,y) = M_\infty^2 + \frac{\gamma+1}{U_\infty}\ M_\infty^2\ \frac{\partial\phi}{\partial x}$$

Following the same procedure as in the subsonic case, equation (29) can be expressed as

$$(1 - M_{\ell,e}^2)\ \phi_{w,\xi\xi} + \phi_{w,\eta\eta} = 0 \qquad (30)$$

where
$$M_{\ell,e} = M / \sqrt{1 + k} \qquad (31)$$

and
$$M_{\ell,e}^2 = M_e^2 + \frac{\gamma+1}{U_\infty}\ M_e^2\ \frac{\partial\phi}{\partial\xi}$$

where $M_{\ell,e}$ is the effective local Mach number. Equation (30) is similar to equation (24) and, therefore, it follows that in addition to the result given by equation 28a, the local Mach numbers are scaled by the factor $\sqrt{1 + k}$. However, at transonic speeds because of the nonlinearity of equation (30), the pressure coefficient values cannot be scaled directly in proportion to change in thickness. This can be done using von Karman's similarity rule as shown in the next section. Since at transonic speeds, thickness distribution is more important than the thickness, equation (30) and (31) can be used in an approximate manner to provide a scaling for the local Mach number. Equation (30) can also be expressed approximately as

$$(1 - M_e^2)\ \phi_{w,\xi\xi} + \phi_{w,\eta\eta} = (\gamma + 1)\ (M_e^2/U_\infty)\ \phi_{w,\xi}\ \phi_{w,\xi\xi} \qquad (32)$$

RESULTS AND COMPARISON WITH EXPERIMENTAL DATA

From equation (7), it follows that to a first order the relative thinning of the sidewall boundary-layer is only a function of the local Mach number M and the free-stream Mach number M_∞. Figure 2 shows the expected reduction in the displacement thickness under different conditions. It may be noted that a significant reduction occurs even for local Mach numbers slightly higher than the free-stream value and with further increase in the local Mach number, the rate of decrease of the boundary-layer thickness is much slower. Of particular interest is the shaded region shown in Figure 2 which corresponds to conditions prevailing in testing of supercritical airfoils. The relative reduction in the displacement thickness is about fifty percent on the upper surface of the airfoil where the local Mach numbers can be about 1.2 for free-stream Mach numbers between 0.7 and 0.8. Due to this increased width of the flow passage, the flow development over the upper surface of the airfoil which is supercritical and is important in determining the drag divergence Mach number, will be equivalent to that in a two-dimensional flow at a lower free-stream Mach number. The extent of the correction for the measured free-stream Mach number upstream in a wind tunnel is shown in Figure 3. This correction is obtained by solving equation (8). For a given free-stream Mach number M_∞, the correction increases with an increase in local Mach number. This increase is significant for thick sidewall boundary-layers and at higher free-stream Mach numbers. With relatively thin boundary-layers, about 2% of the test section width, the correction ΔM is constant over a wide range of local Mach numbers on the airfoil surface. This feature can be explained by the fact that the flow in the wind tunnel is confined and the area change is not significant with increase in local Mach number on the airfoil. Since the shock position on the airfoil is sensitive to the free-stream Mach number, the perturbations introduced because of the change in the thickness of the sidewall boundary-layer can be transferred to free-stream conditions. This procedure is in a manner similar to the match point technique used by Kemp (Reference 9) to correct for the top and bottom wall interference effect using the measured boundary data. Correction for the measured pressure coefficients on the airfoil surface can be obtained from small disturbance theory by multiplying the measured values by the ratio of the tunnel free-stream velocity to the corrected freestream velocity.

It may be noted that the analysis is valid for airfoil model chords of the order or greater than the width of the tunnel ($c/b \geq 1$) so that the assumption of spanwise uniformity of the flow over the airfoil is satisfied. This condition is easily met in practice with the current trend of testing long chord airfoil models in adaptive wind tunnels having negligible top and bottom wall interference. With short chord or high aspect ratio models ($c/b \ll 1$), it is likely that the midspan measurement, will be relatively free of the end wall effects. For intermediate aspect ratios, a suitable correction may be required.

The results of the small disturbance theory provides much simpler relations for the global effects of the sidewall boundary-layer influence. In summary, it was shown that the wind tunnel flow at subsonic and transonic speeds over an airfoil spanning the width can be considered equivalent to another two-dimensional flow at a different effective Mach number $\left(M_e = M_\infty/\sqrt{1 + k}\right)$ on a thinner airfoil with thickness $\tau_e = \left(\tau/\sqrt{1 + k}\right)$. The equivalent two-dimensional flow can be related to a number of other two-dimensional flows using familiar similarity rules for high speed flow and results of Barnwell and Sewall can be obtained as particular cases. This is schematically represented in Figure 4.

a. Subsonic flow: Using the Prandtl-Glauret similarity rule the equivalent two-dimensional flow at Mach number M_e on airfoil with thickness τ_e can be converted to another two-dimensional flow corresponding to Mach number M_∞ and airfoil thickness τ. Then the pressure coefficient in the new flow is related to the measured values by

$$C_{p,\text{ corrected}} = \frac{\sqrt{1 - M_e^2}}{\sqrt{1 - M_\infty^2}} \cdot \frac{\tau}{\tau_e} C_p$$

$$= \sqrt{1 + k} \cdot \frac{\sqrt{1 - M_e^2}}{\sqrt{1 - M_\infty^2}} \cdot C_p \quad . \tag{33}$$

The result obtained in equation (33) corresponds to the corrections suggested by Barnwell in Reference 4.

Alternatively, if we consider the pressure coefficient in the new flow to be the same as in the equivalent flow but at a different corrected Mach number M_c on airfoil of thickness τ, then M_c is related to M_e by

$$\sqrt{1 - M_c^2} \cdot \tau_e = \sqrt{1 - M_e^2} \cdot \quad \tau$$

or $$\sqrt{1 - M_c^2} = \sqrt{1 + k} \cdot \sqrt{1 - M_e^2} \tag{34}$$

or $$M_c^2 = M_\infty^2 - k \quad . \tag{35}$$

For $M_\infty^2 < k$, the corrected Mach number M_c is not defined and this case corresponds to the result of Reference 3 and repeated later in Reference 11. This anamoly is due to the fact that the effects of compressibility and that of sidewall boundary layers are of opposite nature. That is, for the pressure coefficient to remain the same, the airfoil thickness τ in the new flow being larger than τ_e in the equivalent flow, the Mach number has to be reduced. Or from equation (34) it follows that for $k << 1$ and for low subsonic flows, the appropriate approximation for M_c is M_e rather than that given by equation (35) or Reference 3.

b. Transonic flow: The von Karman transonic similarity parameter K in the equivalent flow can be written as

$$K = \frac{1 - M_e^2}{[\tau_e \ (\gamma+1) \ M_e^2]^{2/3}} \tag{36}$$

At transonic speeds, it is only possible to compare flows on bodies with different thickness ratios τ_1 and τ_2 at different Mach numbers M_1 and M_2. In particular, choosing the thickness in the new flow to be τ, the corresponding Mach number M_c can be calculated from

$$\frac{1 - M_e^2}{[\tau_e \ (\gamma+1) \ M_e^2]^{2/3}} = \frac{1 - M_c^2}{[\tau \ (\gamma+1) \ M_c^2]^{2/3}} \tag{37}$$

or substituting for M_e and τ_e

$$\frac{1 - M_\infty^2 + k}{M_\infty^{4/3}} = \frac{1 - M_c^2}{M_c^{4/3}} \tag{38}$$

The solution of equation (38) for given M_∞ and k, gives the Mach number M_c in the new flow on airfoil of thickness τ. This result is the same as that obtained by Sewall in Reference 6. The pressure coefficients in the two flow fields are related by

$$\frac{C_{p,c}[(\gamma+1)\ M_c^2]^{1/3}}{\tau^{2/3}} = \frac{C_p\ [(\gamma+1)\ M_e^2]^{1/3}}{\tau_e^{2/3}} \tag{39}$$

$$C_{p,c} = \left(\frac{M_\infty^2}{M_c^2}\right)^{1/3} C_p \quad . \tag{40}$$

Equation (38) for the corrected Mach number M_c can be written in a convenient form, as

$$\frac{M_c}{(1-M_c^2)^{3/4}} = \frac{M_e}{(1-M_e^2)^{3/4}} \ \frac{1}{(1+k)^{1/4}} \quad . \tag{41}$$

This shows that for small k, the corrected Mach number M_c is equal to M_e $(= M_\infty / \sqrt{1 + k})$ and the corresponding expression for the pressure coefficient is

$$C_{p,c} \approx C_p\ (1+k)^{1/3} \quad . \tag{42}$$

In Figure 5, a comparison has been made of the results obtained using different forms of the small disturbance theory for calculating the correction to the free stream Mach number due to sidewall boundary-layer effects. The results of the present approach show a continuing increase in the correction from incompressible to transonic speeds. At higher Mach numbers, the difference between the present results and that of Sewall (Equation 38) are not significant. Equation (41) suggests that at lower Mach numbers $M_c \approx (1+k)^{-3/4} M_\infty$ and the trend will be the same for both the present and Sewall's results. For small values of $2\, \delta_u^*/b$, all the different approaches give nearly the same results at higher Mach numbers. The various approaches represent equivalent flows, either of them can be considered valid. In the form proposed in the present report (Equations 24 and 30), the equivalent flow has been considered without directly invoking the similarity rules. This method of representing the effect of the sidewall boundary layer as causing changes in the airfoil thickness as well as in the test Mach number is valid from low speeds to transonic flows. At low speeds, the thickness effect will be dominant, whereas at transonic speeds the Mach number effect prevails and affects the drag divergence Mach number. It may be noted from Figure 5, that the numerical values of the Mach number correction obtained from small disturbance approximations agree closely with the more exact calculations (Fig. 3) for the changes in the width of the flow passage. Perhaps, this explains for the good agreement reported in Reference 7 between the measured pressure distribution and the calculations accounting for the sidewall boundary layer effects by the method of Reference 6.

In Figure 6 and 7, correlation of the normal force coefficient and the local Mach number data using the transformation derived in Equations 28a, b and c is presented. The experimental data measured with different sidewall boundary layers thicknesses in the Langley 6" x 19" transonic tunnel for these figures were obtained from Sewall (Ref. 10). It is seen from Figure 6b that the corrected normal force coefficient $C_n\sqrt{1+k}$ and Mach number $M_\infty/\sqrt{1+k}$ correlate well for $2\,\delta^*_u/b = .028$ and .07. For the thickest boundary layer $2\,\delta^*_u/b = .10$, the correlation is somewhat poor possibly because of the higher order effects not accounted for by the small disturbance approximation. The validity of the present approach at transonic speeds is further demonstrated in Figure 7 by scaling of the local Mach numbers given by equation (31). Remarkably good agreement is obtained for the local Mach number distribution for $2\,\delta^*_u/b = .028$ and .07, on the upper surface of the airfoil where the assumptions made in the present analysis are nearly satisfied. Good correlation of the experimental data suggests that equation (28) can be used to correct the wind tunnel data for the attached sidewall boundary layer effects.

CONCLUDING REMARKS

A simple method is presented to estimate the relative thinning of the sidewall boundary layer due to airfoil flowfield in two-dimensional wind tunnel testing of airfoils. It is shown that the increase in width of the flow passage due to thinning of the boundary-layer can be considered as a correction for the test Mach number. Further, using the small disturbance approximation, it is demonstrated that the effect of sidewall boundary layers as causing changes in both the test Mach number and the airfoil thickness that the latter effect is dominant at low speeds. Based on this analysis, the wind tunnel flow over an airfoil with sidewall boundary layers is represented by an equivalent two-dimensional flow with the test Mach number M_∞ reduced by a factor $1/\sqrt{1 + k}$. The measured pressure coefficients are increased by $\sqrt{1 + k}$ at subsonic speeds, and to a first approximation by $(1+k)^{1/3}$ at transonic speeds, where k is a constant calculated using the undisturbed sidewall boundary layer thickness and shape parameter. Comparison of the results of the present analysis with the available experimental data on airfoils showed good agreement.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the following NASA/LaRC personnel:

- Dr. R. A. Kilgore, Head, Experimental Techniques Branch, and C. B. Johnson, Technical Monitor for their encouragement to pursue this research effort.
- Dr. R. W. Barnwell, Head, NTF Aerodynamics Branch, for helpful discussions.
- W. G. Sewall, NTF Aerodynamics Branch, for providing wind tunnel test data on airfoils for Figures 6 and 7.

APPENDIX

$$\frac{d\delta^*}{\delta^*} = \frac{dH}{H} - \frac{2 + H - M^2}{1 + 0.2M^2} \cdot \frac{dM}{M} \tag{A.1}$$

and $H = 1 + 0.4M^2$

Integrating A.1.

$$\log \delta^* = \log H - 3 \int \frac{1 - .2M^2}{1 + .2M^2} \cdot \frac{dM}{M} + \log C \tag{A.2}$$

where C is the constant of integration

or
$$\log \delta^* = \log H + \log C - 3 \int \left(\frac{1}{M} - \frac{.4M}{1 + .2M^2}\right) dM \tag{A.3}$$

$$\log \delta^* = \log H + \log C - 3 \log \frac{M}{1 + .2M^2} \tag{A.4}$$

or
$$\delta^* = C.\ H.\ \left(\frac{1 + .2M^2}{M}\right)^3 \tag{A.5}$$

The constant of integration is evaluated by using the condition that for $M = M_\infty$ the value of the δ^* corresponds to the undisturbed value δ_u^*. The value of C is obtained as

$$C = \frac{\delta_u^*}{1 + .4M_\infty^2} \cdot \left(\frac{M_\infty}{1 + .2M_\infty^2}\right)^3 \qquad (A.6)$$

or the ratio of local value of displacement thickness to the undisturbed value can be expressed as

$$\frac{\delta^*}{\delta_u^*} = \frac{1 + 0.4M^2}{1 + 0.4M_\infty^2} \cdot \left(\frac{M_\infty}{M}\right)^3 \left(\frac{1 + .2M^2}{1 + .2M_\infty^2}\right)^3 \qquad (A.7)$$

REFERENCES

1. Bernard-Guelle, René: Influence of Wind Tunnel Wall Boundary Layers on Two-Dimensional Transonic Tests. NASA TT F-17,255, October 1976.

2. Preston, J. H.: The Interference on a Wing Spanning a Closed Tunnel Arising from the Boundary Layers on the Side Walls, with Special Reference to the Design of Two-Dimensional Tunnels. R & M No. 1924, A.R.C., 1944.

3. Barnwell, R. W.: A Similarity Rule for Compressibility and Sidewall Boundary-Layer Effects in Two-Dimensional Wind Tunnels. AIAA Paper No. 79-0108, January 1979.

4. Barnwell, R. W.: Similarity for Sidewall Boundary-Layer Effect in Two-Dimensional Wind Tunnels. AIAA Journal, vol. 18, Sept. 1980, pp. 1149-1151.

5. Winter, K. G.; and Smith, J. H. B.: A Comment on the Origin of Endwall Interference in Wind Tunnel Tests of Airfoils. RAE Tech. Memo. Aero 1816, August 1979.

6. Sewall, W. G.: The Effects of Sidewall Boundary Layers in Two-Dimensional Subsonic and Transonic Wind Tunnels. AIAA Journal, vol. 20, Sept. 1982, pp. 1253-1256.

7. Murthy, A. V.; Johnson, C. B.; Ray, E. J.; Lawing, P. L.; and Thibodeaux, J. J.: Investigations of Upstream Sidewall Boundary Layer Removal Effects on a Supercritical Airfoil. AIAA Paper No. 83-0386, January 1983.

8. Oswatitsch, K.; and Wieghardt, K.: Theoretical Analysis of Stationary Potential Flows and Boundary Layers at High Speed. NACA TM No. 1189, April 1948.

9. Kemp, W. B.: Transonic Assessment of Two-Dimensional Wind Tunnel Wall Interference Using Measured Wall Pressures. Advanced Technology Airfoil Research - Volume I, NASA CP-2045, Part 2, 1979, pp. 473-486.

10. Sewall, W. G.: Application of a Transonic Similarity Rule to Correct the Effects of Sidewall Boundary Layers in Two-Dimensional Transonic Wind Tunnels. M.S. Thesis, George Washington University, August 1982.

11. Mokry, M.; Chan, Y. Y.; and Jones, D. J.: Two-Dimensional Wind Tunnel Wall Interference. L. Ohman, Editor, AGARD-AG-281, November 1983.

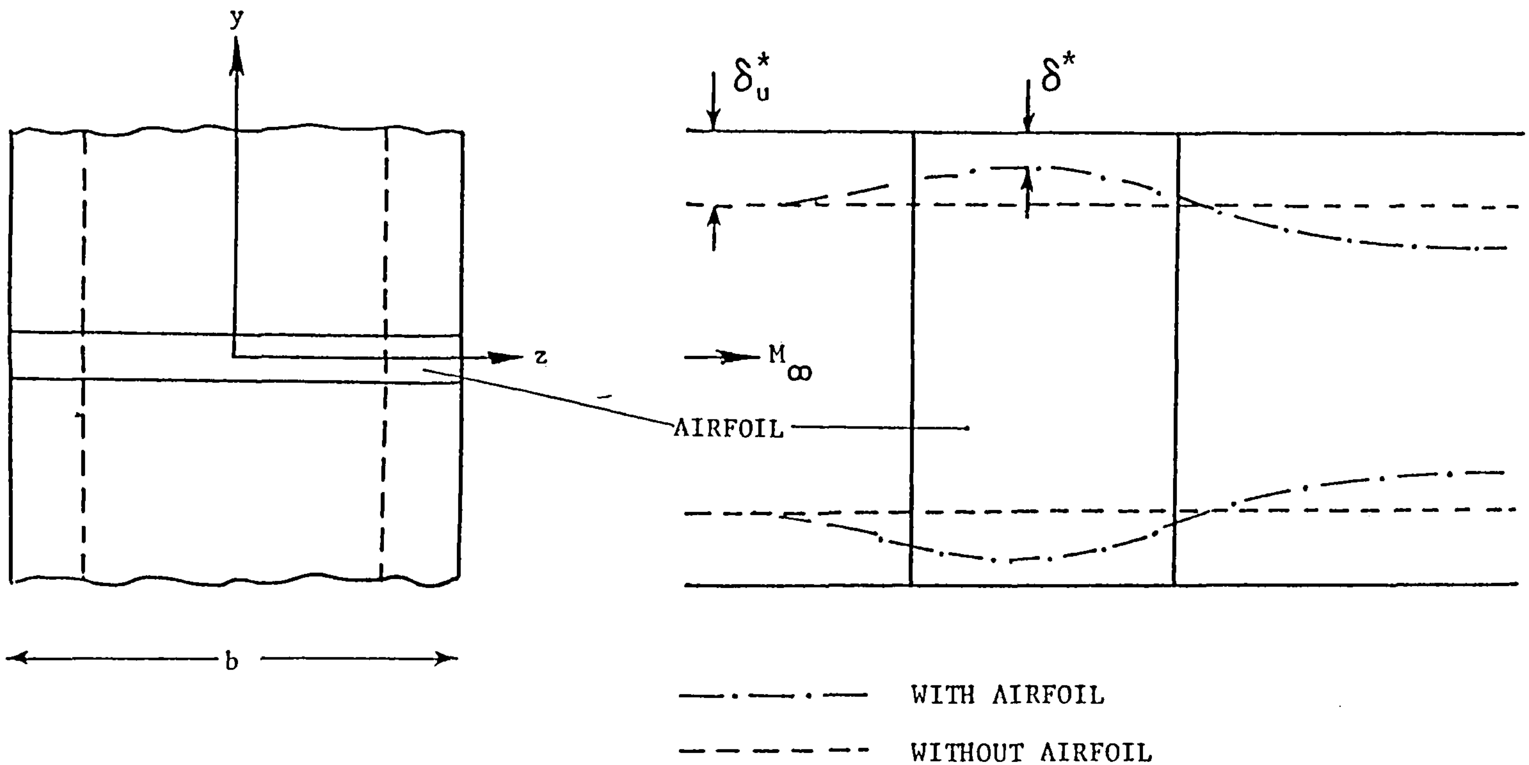

Figure 1. Typical sidewall boundary-layer growth above a lifting airfoil in a two-dimensional wind tunnel.

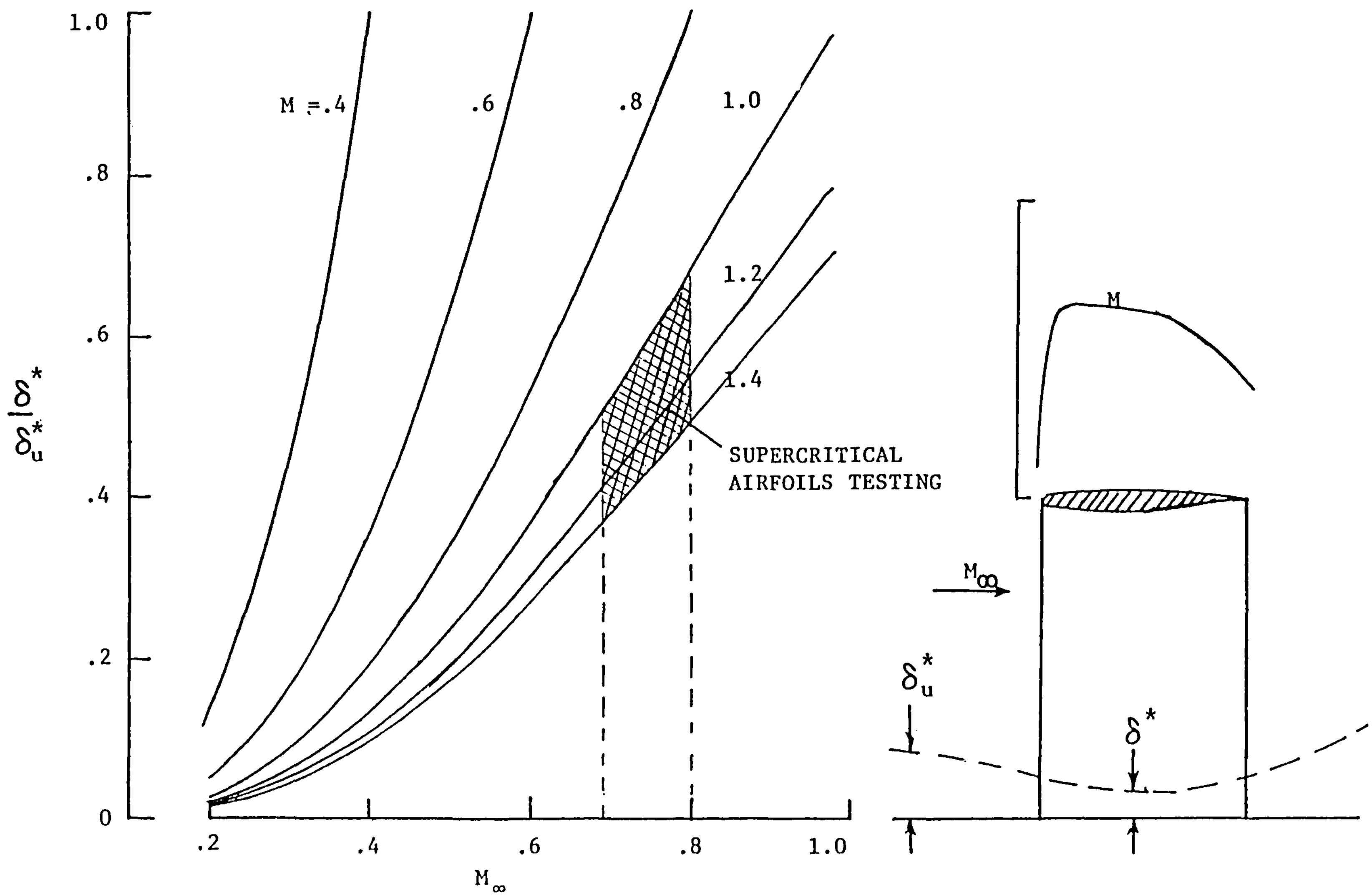

Figure 2. Relative reduction in sidewall boundary-layer thickness due to airfoil pressure field (Eq. 7).

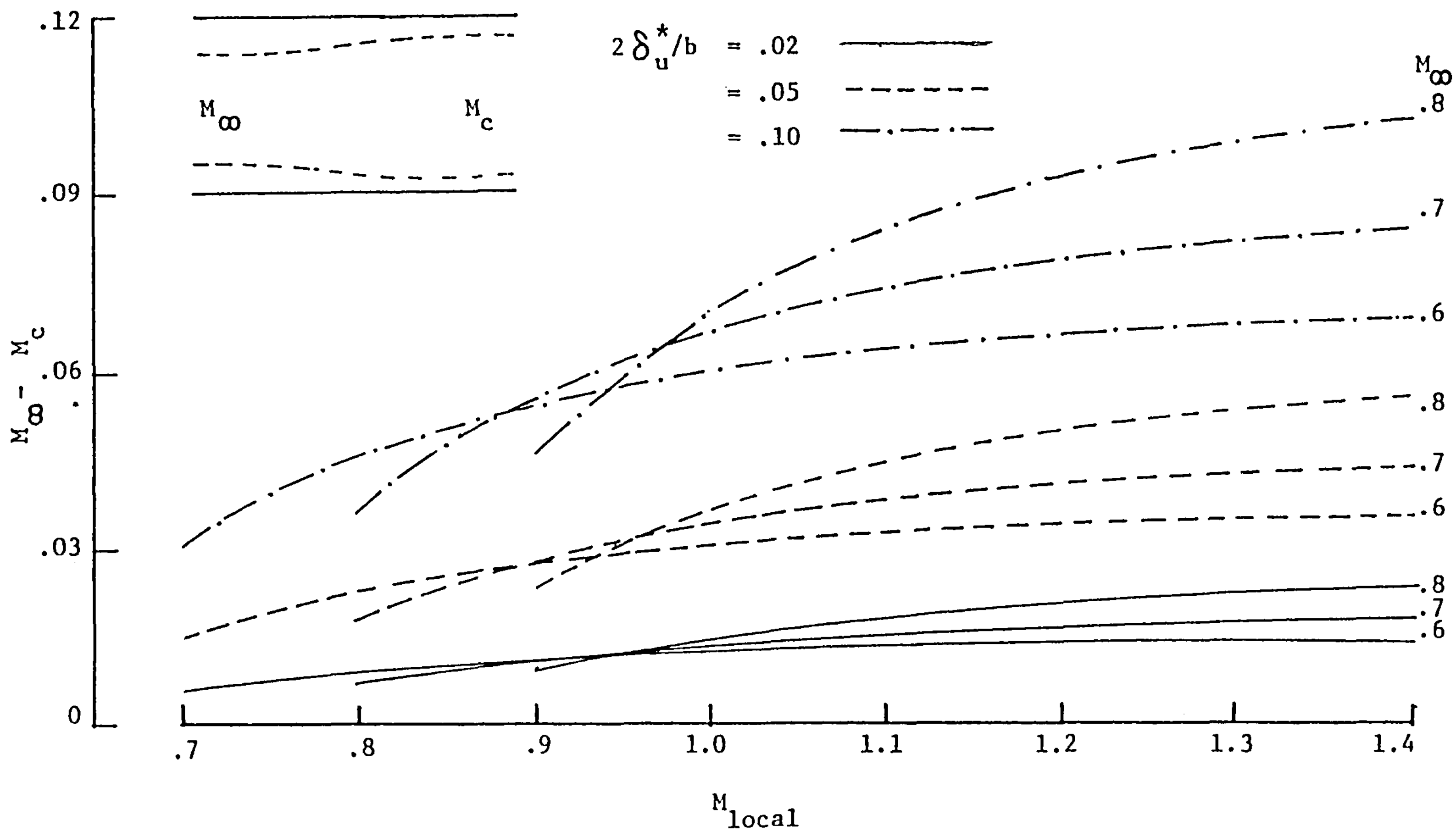

Figure 3. Correction for free-stream Mach number due to sidewall boundary-layer thinning in the airfoil region.

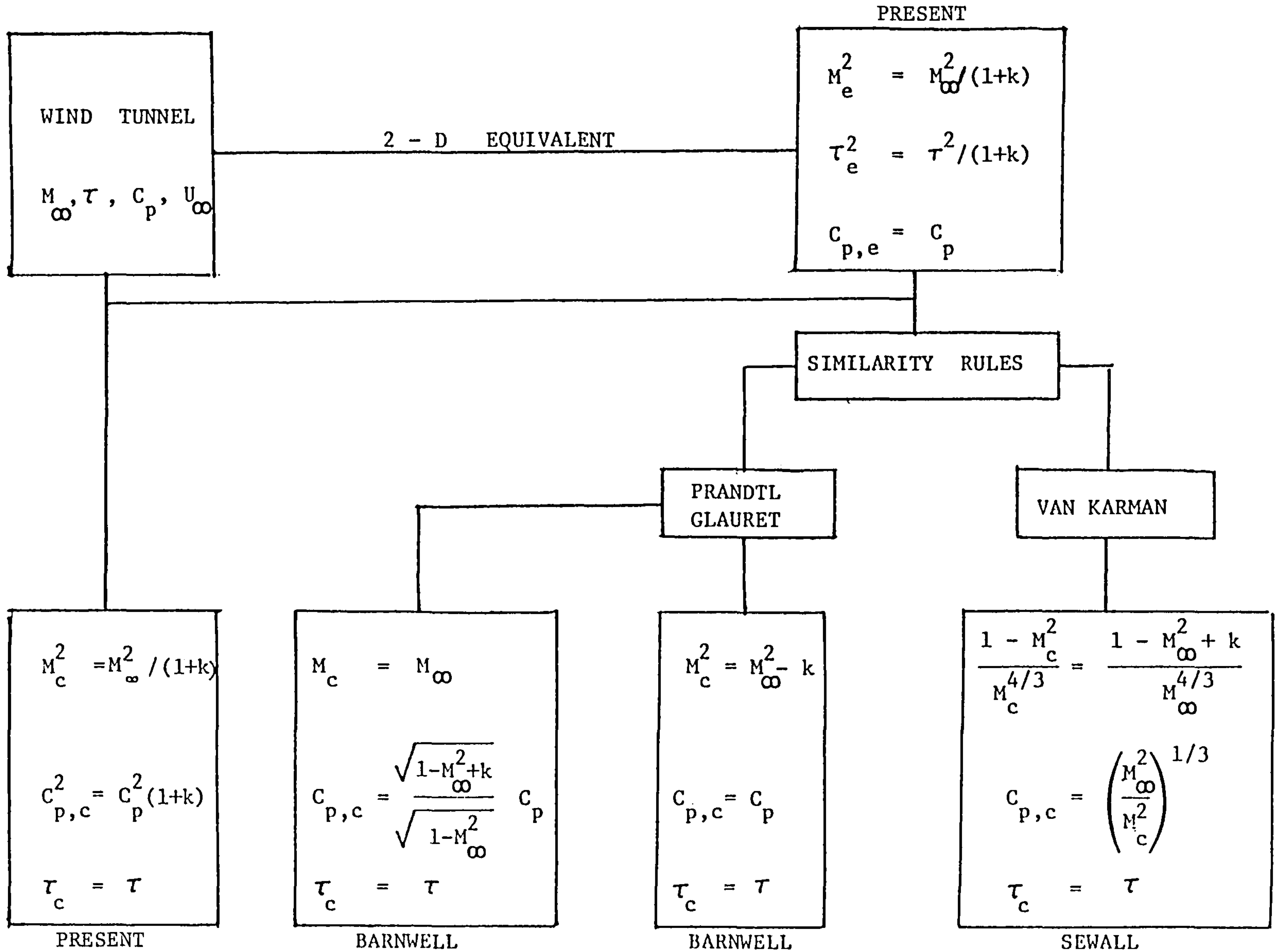

Figure 4. Schematic representation of small disturbance theory results for sidewall boundary-layer effects at subsonic and transonic speeds.

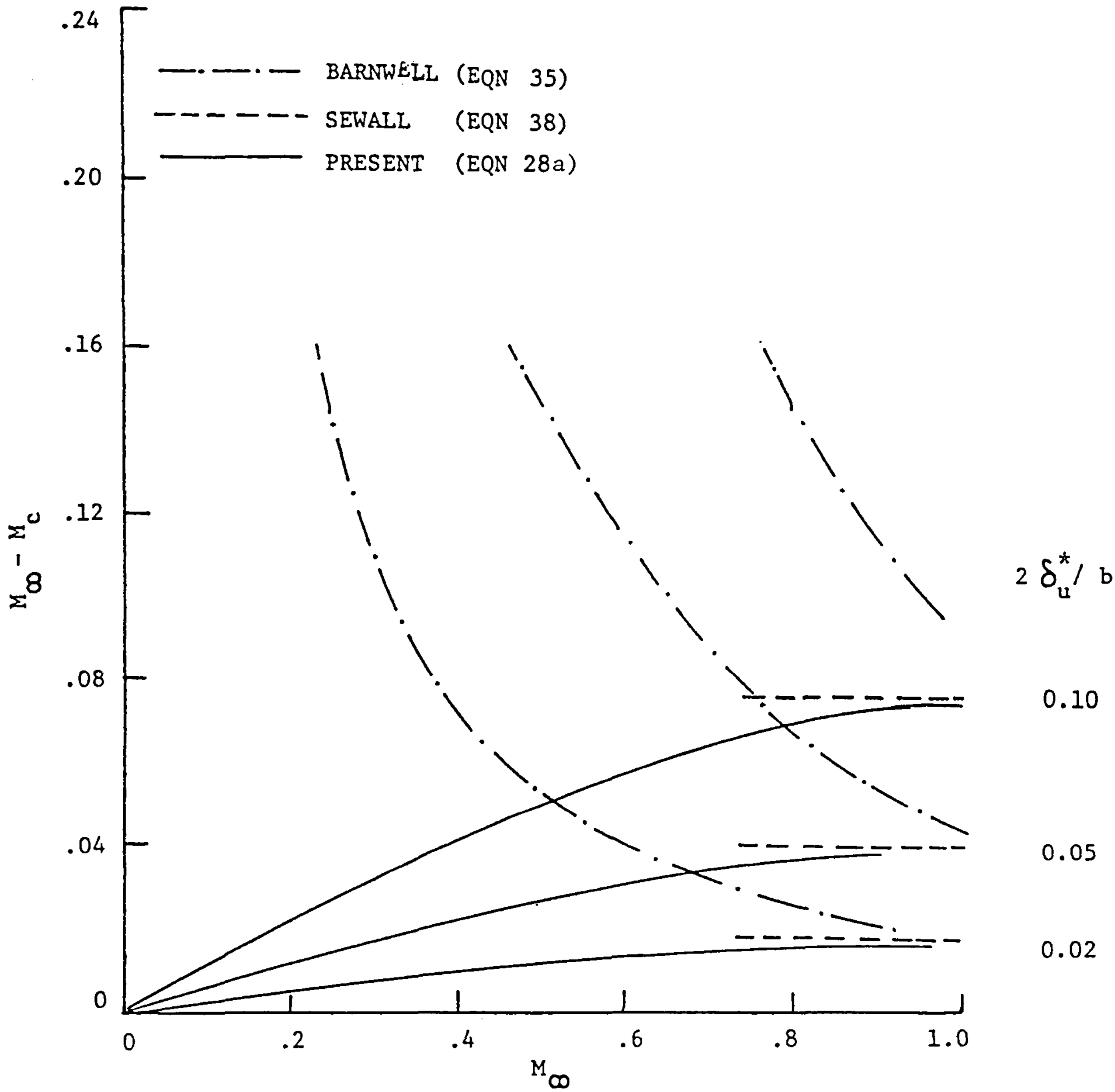

Figure 5. Comparison of correction to free-stream Mach number due to sidewall boundary-layer effects (Small disturbance approximation).

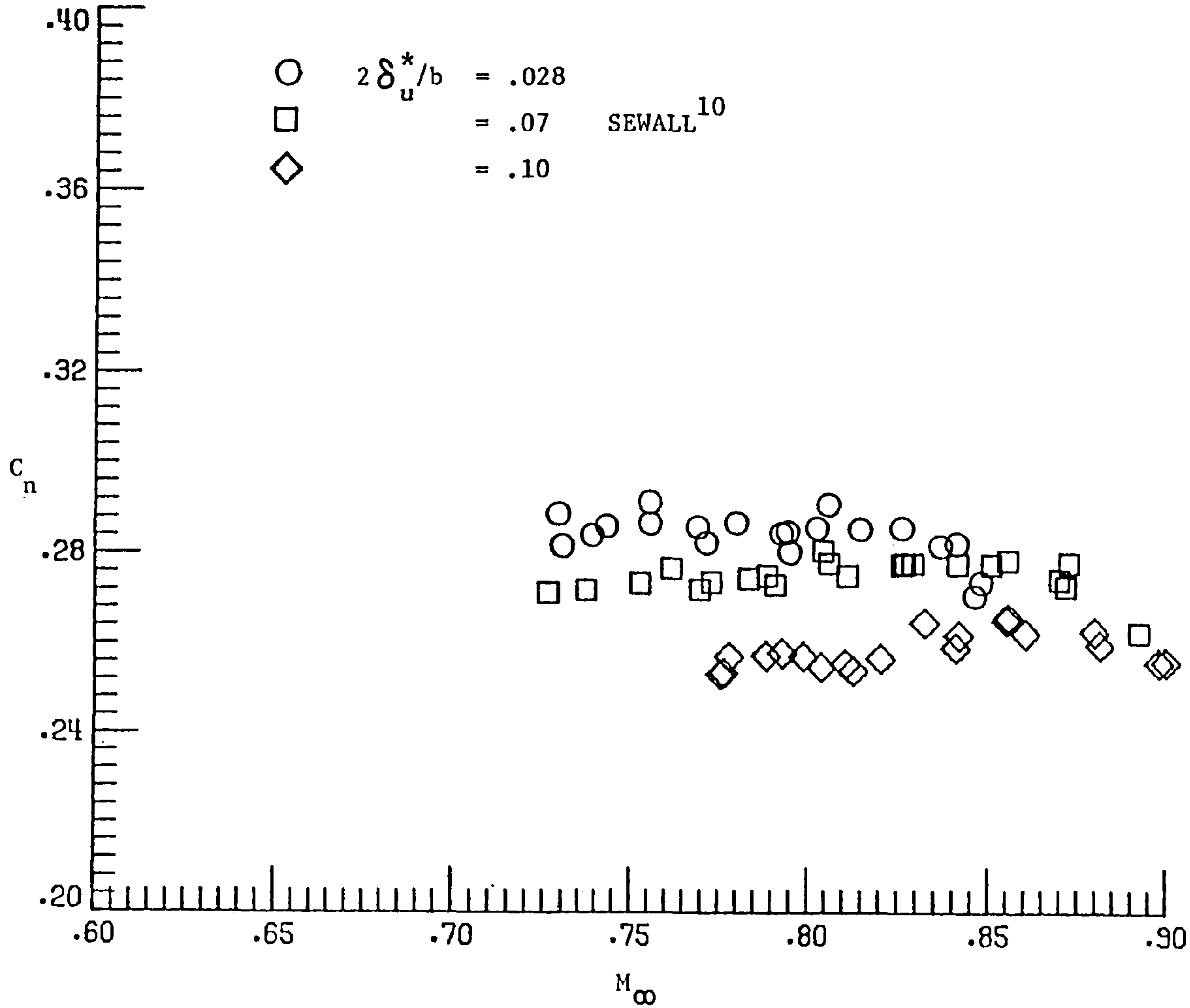

Figure 6a. Measured normal force data on a supercritical airfoil in the LANGLEY 6" x 19" TUNNEL with different sidewall boundary-layer thicknesses (from Sewall, Ref. 10).

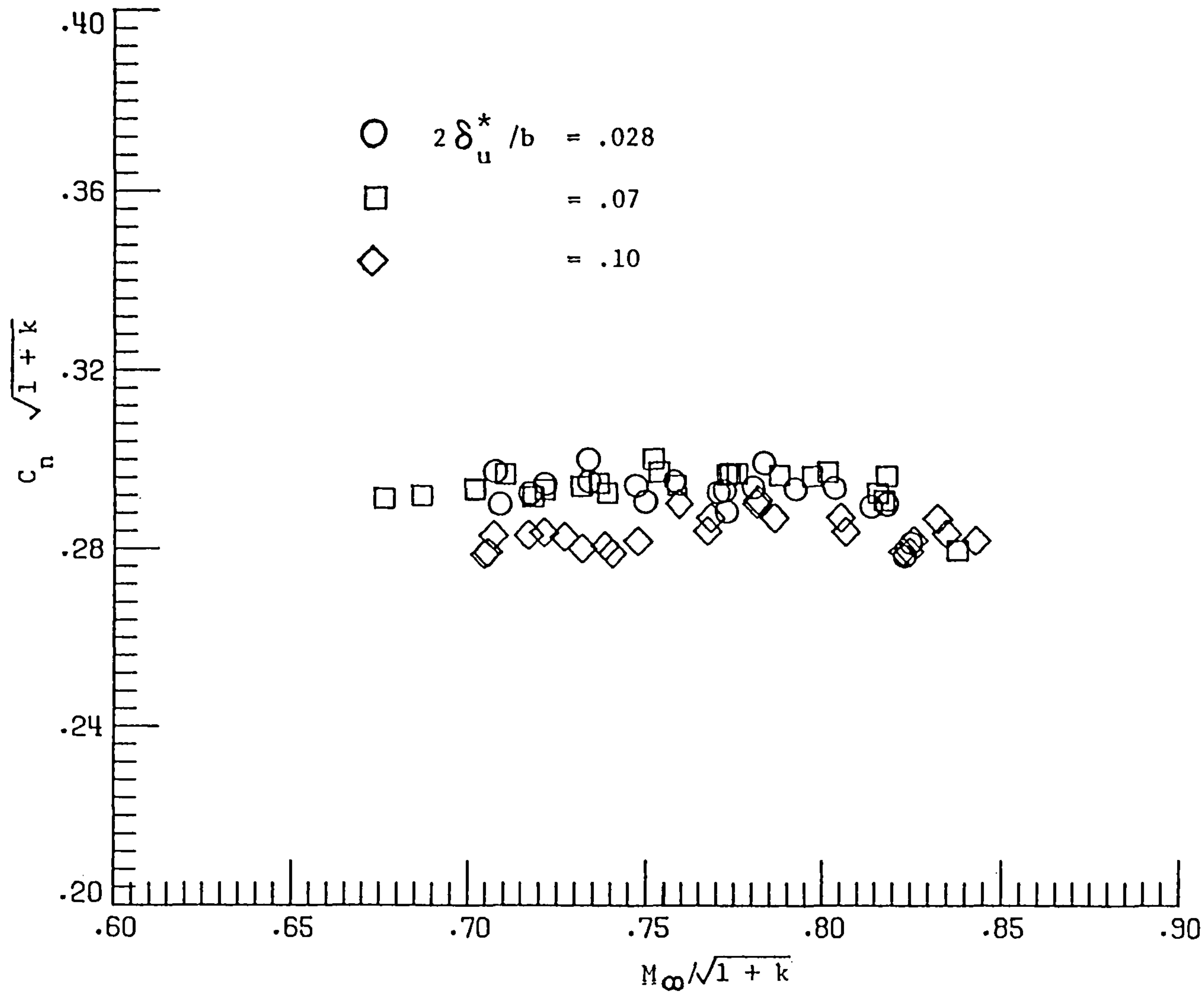

Figure 6b. Correlation of normal force coefficient data of Reference 10 (Sewall) using present small disturbance approximation (Eq. 28c).

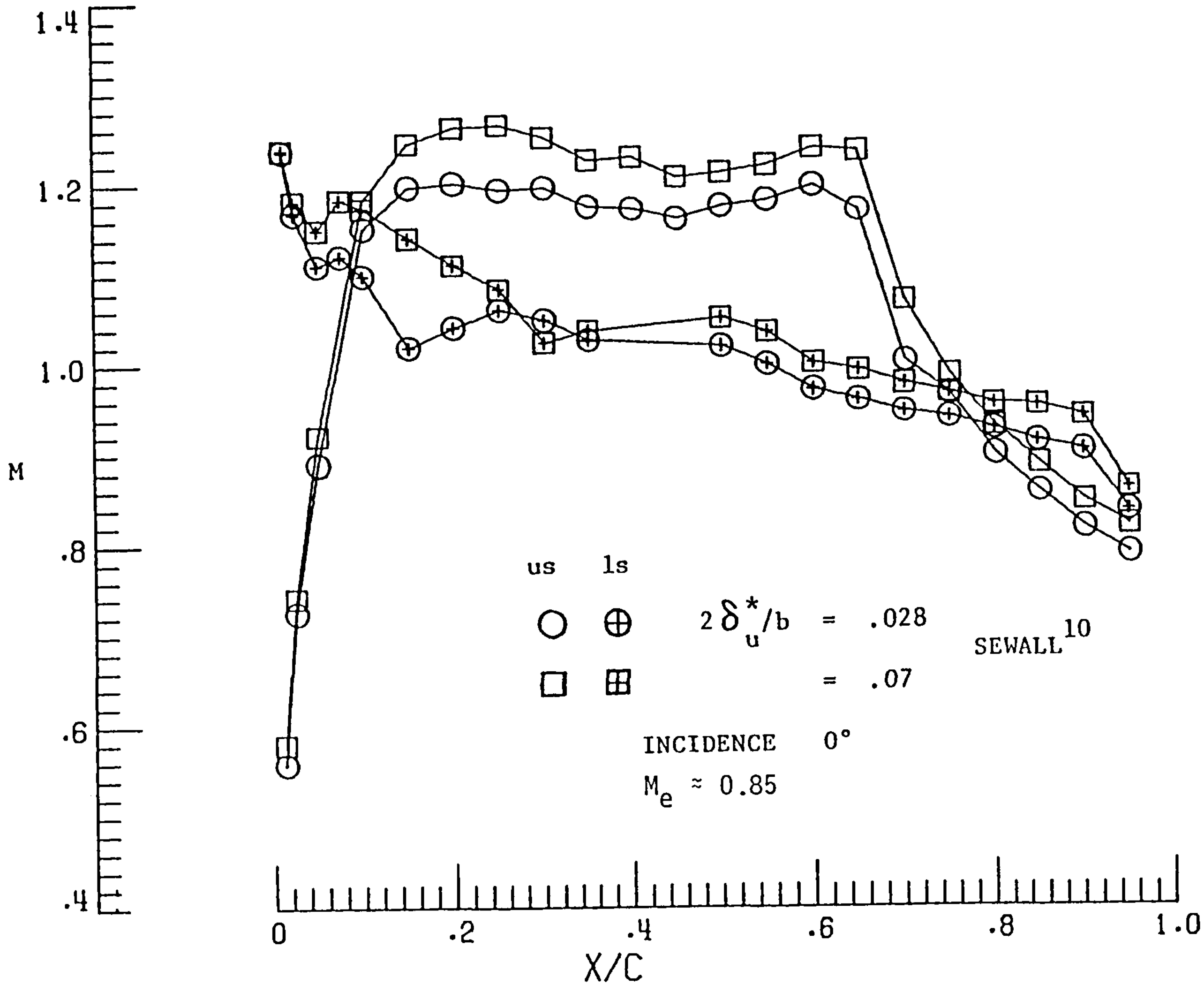

Figure 7a. Measured Local Mach number data on NLR-1 airfoil in the LANGLEY 6" x 19" TUNNEL from Reference 10.

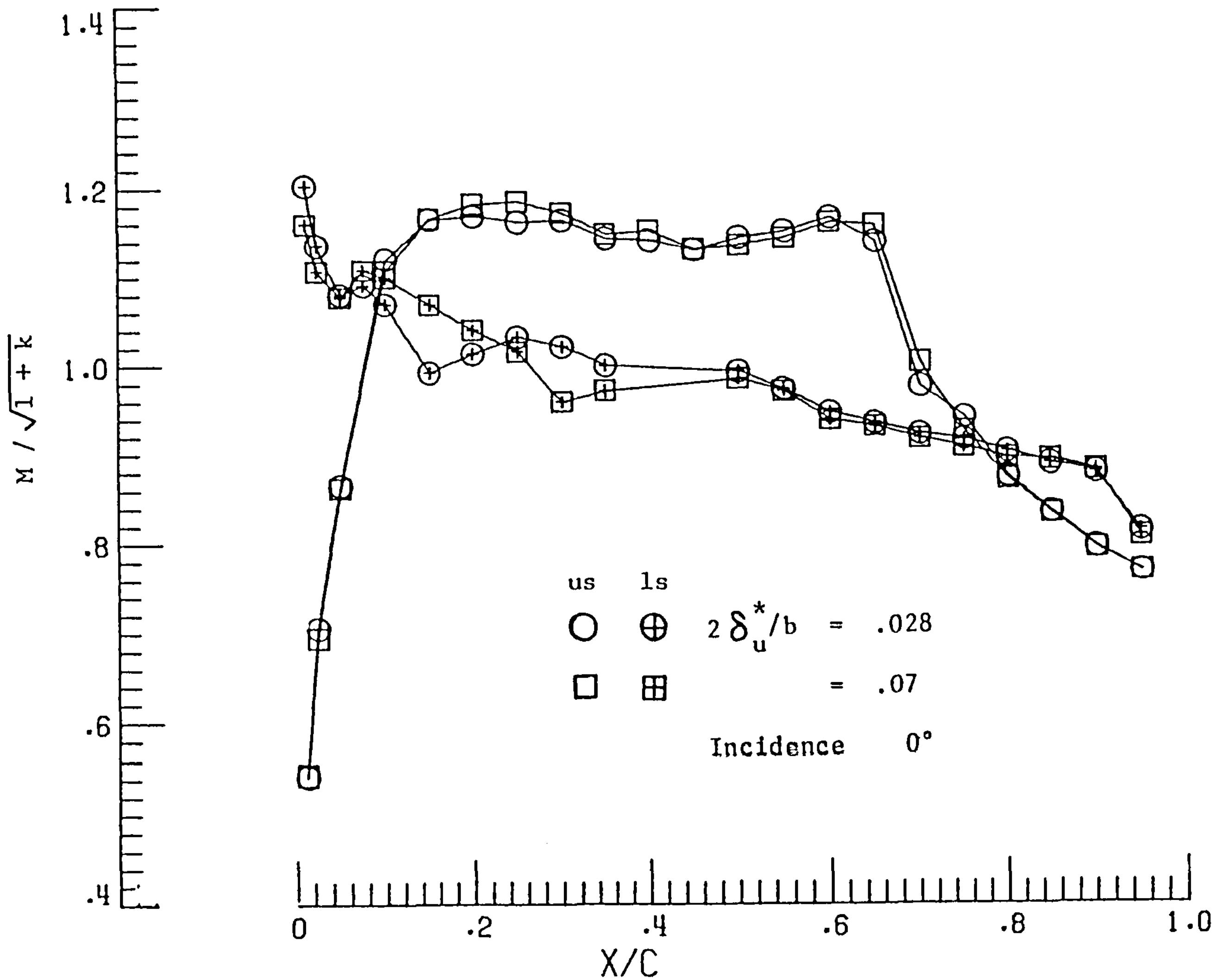

Figure 7b. Correlation of local Mach number distribution on NLR-1 airfoil (Ref. 10) using present small disturbance approximation (Eq. 31).

1. Report No. NASA CR-3873	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle CORRECTIONS FOR ATTACHED SIDEWALL BOUNDARY-LAYER EFFECTS IN TWO-DIMENSIONAL AIRFOIL TESTING		5. Report Date February 1985
		6. Performing Organization Code
7. Author(s) A. V. Murthy		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Old Dominion University Research Foundation P. O. Box 6369 Norfolk, Virginia 23508		11. Contract or Grant No. NAG1-334
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		14. Sponsoring Agency Code 505-31-53-10

15. Supplementary Notes
Technical Monitor: Charles B. Johnson
Principal Investigator: Dr. Gene L. Goglia

16. Abstract

The problem of sidewall boundary-layer effects in airfoil testing is treated by considering the changes in the flow area due to boundary-layer thinning under the influence of the airfoil flowfield. Using von Karman's momentum integral equation, it is shown that the sidewall boundary-layer thickness in the region of the airfoil can reduce to about half the undisturbed value under the conditions prevailing in testing of supercritical airfoils. A Mach number correction due to this increased width of the flow passage is proposed. Using the small disturbance approximation, the effect of the sidewall boundary-layers is shown to be equivalent to a change in the test Mach number and also in the airfoil thickness. Comparison of the results of this approach with other similarity rules and correlation of the experimental data demonstrate the applicability of the analysis presented from low speeds to transonic speeds.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Airfoil Testing Boundary Layers Wind Tunnel Testing	Unclassified - Unlimited Subject Category 02

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	38	A03

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