AN OPTIMIZATION STUDY TO MINIMIZE SURFACE DISTORTIONS OF A HOOP/COLUMN ANTENNA

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INTRODUCTION

The objective of this study was to develop a computer-based procedure to minimize the surface distortions in a hoop/column antenna. The approach uses mathematical optimization techniques to select a set of control cable tensions which minimize the distortions. The motivation for this study was a need for an automated procedure to lessen the tedium of the manual approach currently used to solve this problem. The overall method uses three fundamental elements. The Engineering Analysis Language (EAL) finite element analysis program (ref. 1) is used to calculate the antenna surface distortions due to externally applied loads and the derivatives of surface distortions with respect to the control cable tensions. The CONMIN general purpose optimization program (ref. 2) is used to determine the set of control cable tensions which minimize the antenna surface distortions. A program based on ref. 3 is used to calculate the best fit parabola passing through a distorted antenna shape and to calculate the RMS distortion error. This paper discusses the interim results of a feasibility study in which the procedure is demonstrated by correcting antenna distortions due to externally applied loads. These loads are useful for check purposes, but do not necessarily represent realistic loads found in orbit. (See fig. 1.)

- Problem - Develop a procedure to optimize control cable tensions in a hoop/column antenna to minimize surface distortions
- Motivation - Need for a systematic approach
- Overall method
    - Use EAL finite element analysis
    - Use CONMIN optimizer
    - Use RMS surface distortion algorithm
- Feasibility study in progress
- Loads cases - simulated to test method

Figure 1

ARTIST'S CONCEPT OF HOOP/COLUMN ANTENNA

Figure 2 shows an earth orbiting hoop/column antenna which requires a parabolic surface to reflect radio frequency energy properly. Once in orbit, externally applied loads, such as nonuniform thermal loads, distort the reflector surface from its desired shape. The surface control cables and hoop support cables can be pulled in a particular arrangement such that the antenna surface distortions will be minimized for that particular external load.

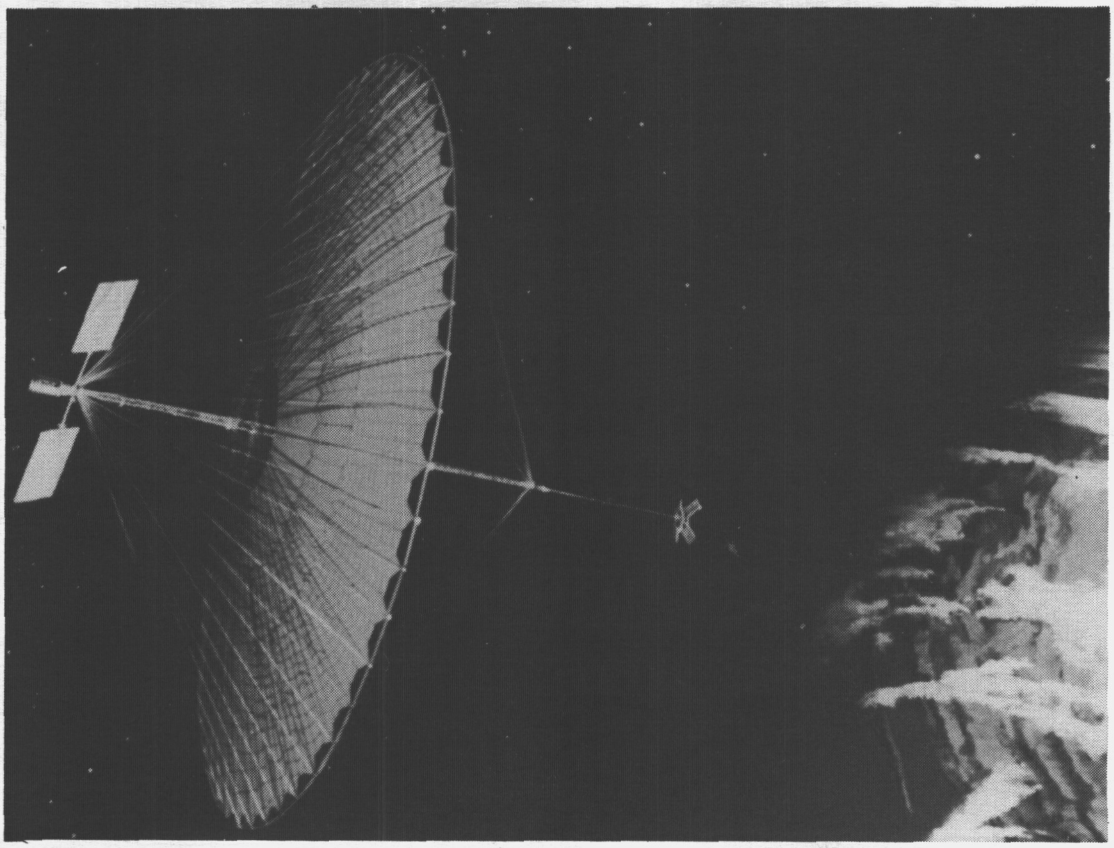


Figure 2

122 METER HOOP/COLUMN ANTENNA CONFIGURATION

The cross-section view of figure 3 shows the major components of a 122 meter hoop/column antenna. It is based on an antenna studied by the Harris Corporation and NASA Langley Research Center (ref. 4). The reflector surface must be kept as nearly parabolic as possible. The surface control cables are connected to the cable truss network located just below the reflector surface. The hoop support cables are connected to the rigid hoop, which is connected to the outer edge of the reflector surface. The shape of the reflector surface is highly dependent on the tensions in these cables.

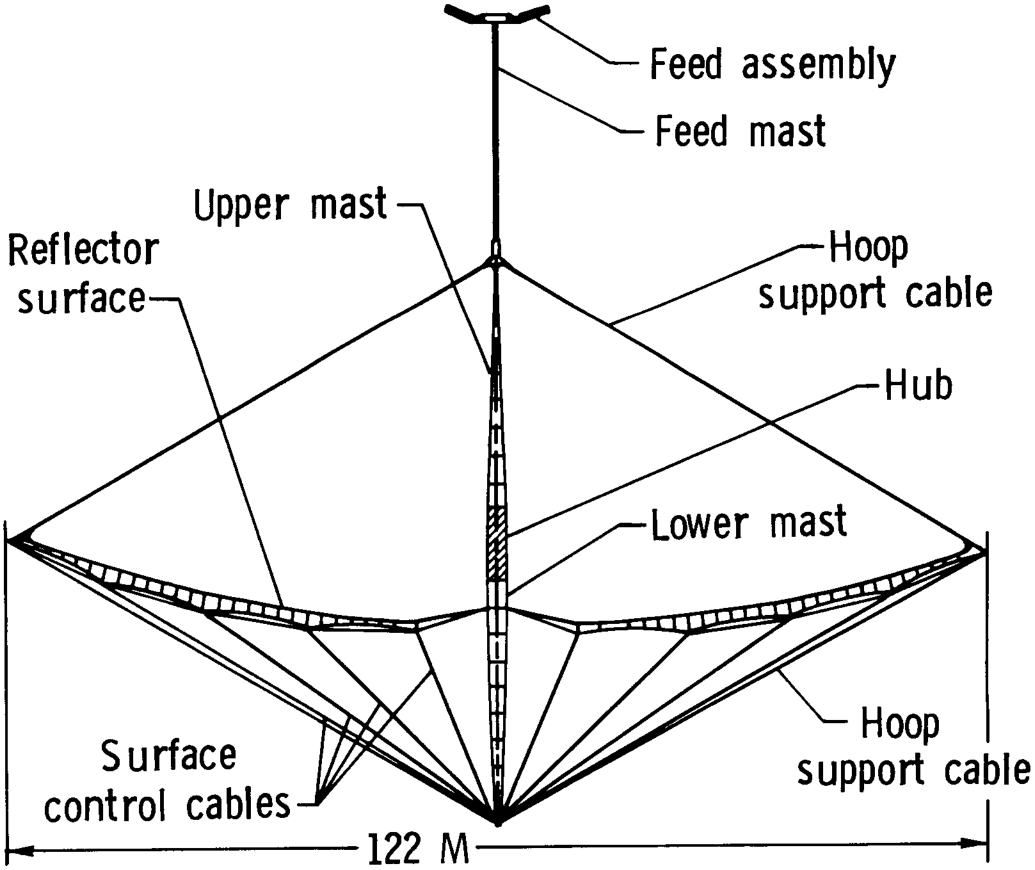


Figure 3

ELEMENTS OF OPTIMIZATION PROCEDURES

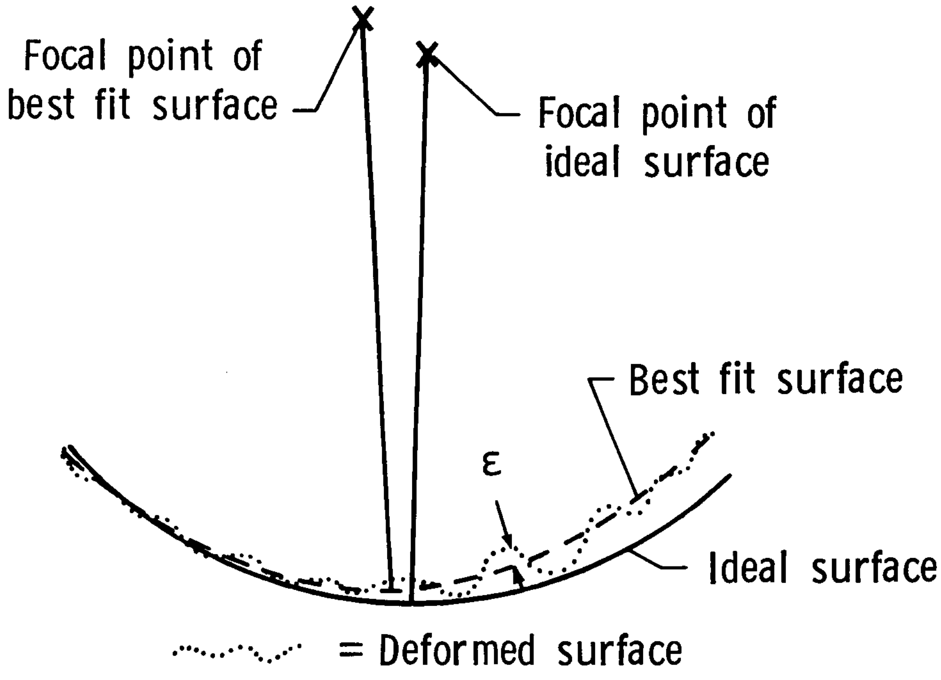
The primary elements of any optimization procedure are shown in figure 4. The objective function is a measure of how good the design is. Its value will either be minimized or maximized for an optimum design. The design variables are the quantities which vary during the optimization process. Their values are the final product of the optimization and represent the quantities which make the objective function optimum. Constraints appear in two forms and are used to place restrictions on the design. Behavior constraints are limits on the response of the system being optimized. A typical example in solid mechanics is a stress constraint limiting the maximum stress allowable in some component of a structure. Side constraints are upper and/or lower limits on the values of individual design variables. An example, also from solid mechanics, is a minimum or maximum gage for a structural component such as a plate or truss element, that is allowed to vary in size.

- Objective function
- Design variables
- Constraints
    - Behavior
    - Side

Figure 4

DEFINITION OF OBJECTIVE FUNCTION

The ideal surface shown in figure 5 represents the parabolic shape of the undeformed antenna. When external loads are applied, the ideal surface deforms to the shape indicated by the dotted line. The dashed line is the best fit parabola that can be passed through the deformed shape. Epsilon is the normal distance between the deformed and best fit surfaces. The epsilons are measured at 66 points over the antenna surface in this study. The objective function is the RMS error measure for the epsilons. Minimizing the objective function forces the deformed surface toward the best fit parabolic shape, which is not necessarily the same parabola as the ideal surface. The best fit parabola's focal point can be different from that of the ideal surface, and restrictions can be put on focal point movement. One restriction could be to force the best fit and ideal focal points to be the same, which means that the best fit surface is the ideal surface. Another restriction could be to require the focal length to be the same for the best fit and ideal surfaces, but to allow the point to move giving boresight errors and translation of the origin of the parabola. A third option might be to impose no restrictions on the focal point. In this study, the focal length was held constant.



ε = Normal distance from deformed surface to the best fit surface

$$\text{Objective function} = \left[\frac{1}{n}\sum \varepsilon^2\right]^{1/2} = \text{RMS error}$$

Figure 5

MODEL OF HOOP/COLUMN ANTENNA SURFACE

Figure 6 shows a plan view of a hoop/column antenna. It is constructed of 48 gores with 4 surfaces of illumination indicated by the dashed circles. The shaded region is the one gore used as the analytical model for this study. Since each of these 48 gores is very much alike, any one of them would be adequately represented by this choice of analytical model. Because of this symmetry, boundary conditions imposed on the analytical model are vertical planes of symmetry along the gore edges. This means that only displacements normal to the antenna surface will be allowed on those two edges.

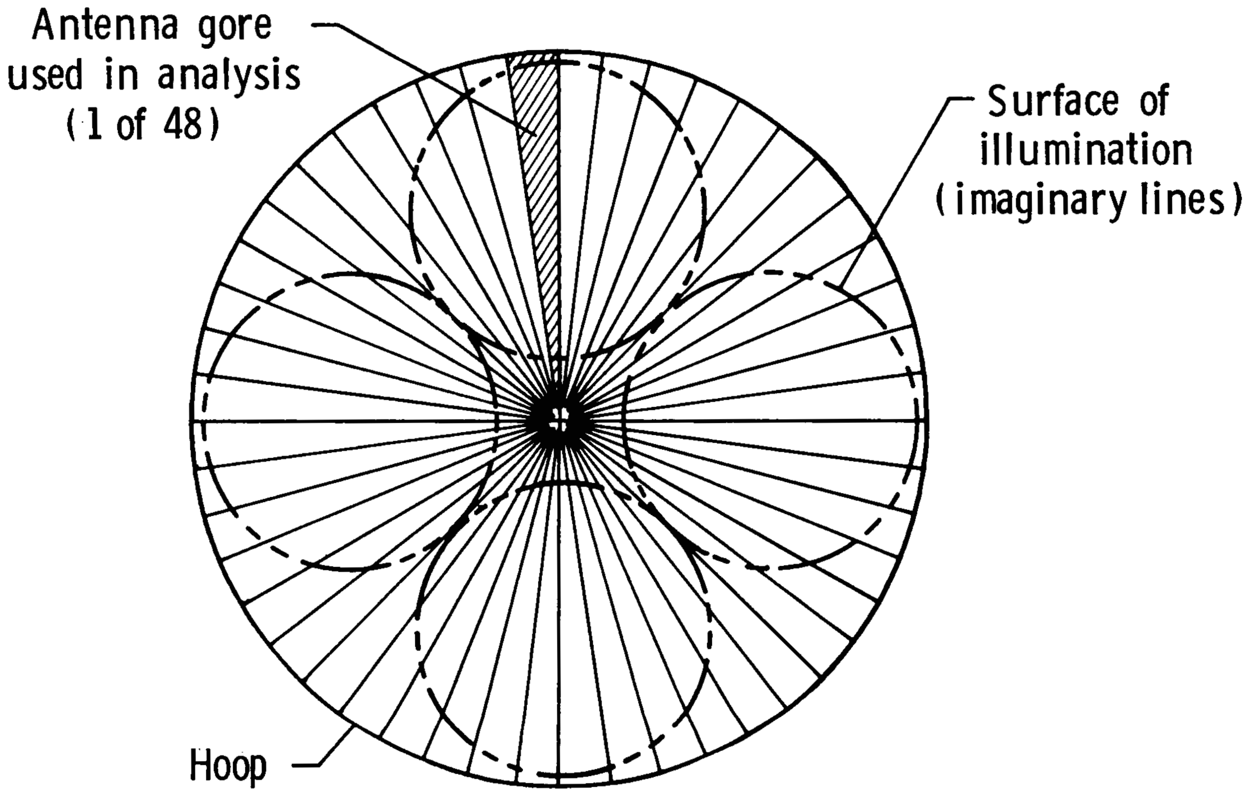


Figure 6

DEFINITION OF DESIGN VARIABLES

An exploded view of the one gore analytical model is shown in figure 7. Simplified representations of the cable trusses connect the surface control cables to the reflector surface. The hoop bar is connected to the reflector surface at the corner points. The design variables used in this study are the tensions in the 12 numbered cables. Cables 1-8 are surface control cables and cables 9-12 are hoop support cables. The only constraints used in this study are side constraints which prevent the control cables from being in compression. The EAL finite element model of this gore is comprised of 166 joints, 234 rod elements, 92 surface membrane elements, and 498 degrees of freedom. The model is initially pre-tensioned, which is represented by a geometric stiffness matrix. The geometric stiffnesses would ordinarily change when tensions are changed in the model, leading to a nonlinear analysis problem. These changes have been neglected for this study, which means the tensions in the control cables are applied as external forces in the equilibrium equation. In addition, the values of the design variables are tension increases above that of the pre-tensioned state, not the total tensions in the control cables.

Design variables are tensions in numbered cables

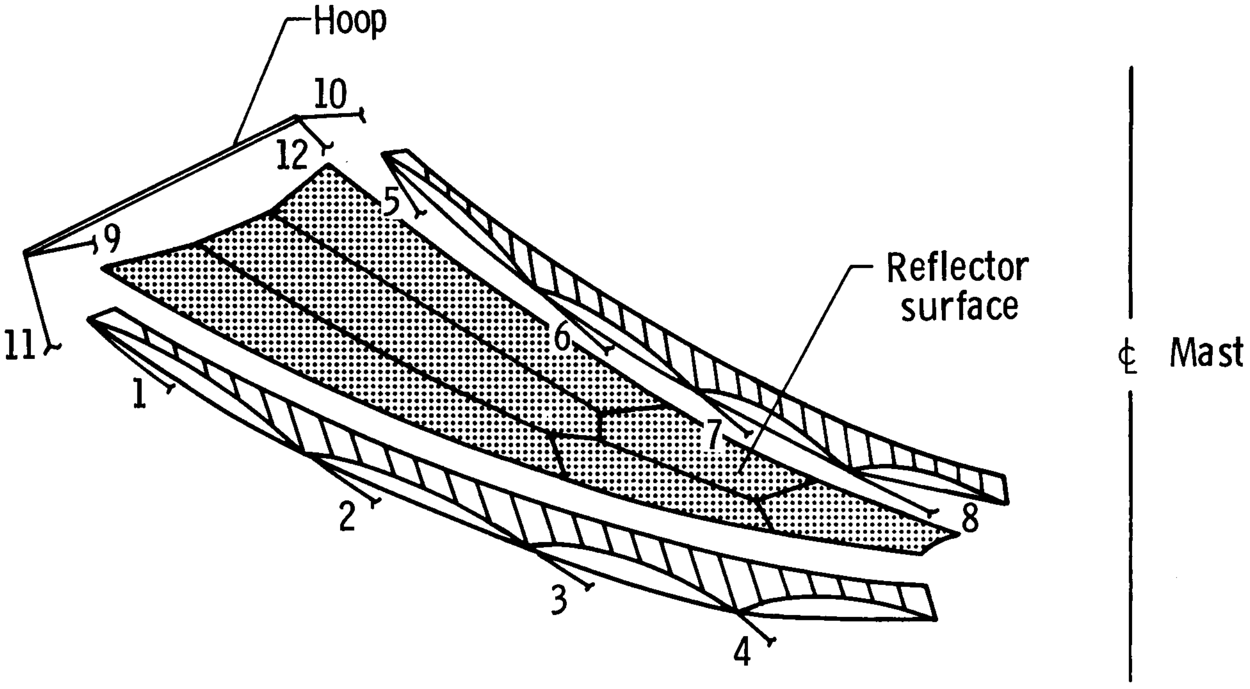


Figure 7

SUMMARY OF OPTIMIZATION PROCEDURE

Figure 8 summarizes the optimization procedure. The procedure finds the set of control cable tensions which results in the best parabolic antenna surface subjected to externally applied loads. The objective function is the RMS error of the surface distortions. The design variables are the values of tensions applied to the control cables. The constraints are side constraints which prevent the control cables from being in compression. The minimization is performed using CONMIN, a widely used general purpose optimization program which employs usable/feasible directions methods and nonlinear programming techniques. The EAL finite element analysis program is used to calculate the surface distortions caused by externally applied loads and derivatives of surface distortions with respect to control cable tensions.

- Find design variables which minimize objective function subject to constraints
  - Objective function is RMS surface distortion
  - Design variables are tensions in control cables
  - Constraints are requirement for positive control cable tensions
- Minimization carried out using CONMIN
  - Usable/feasible directions
  - Non-linear programming techniques
- Method requires derivatives of displacements with respect to control cable tensions
  - Performed in EAL

Figure 8

DISTRIBUTIONS OF TEMPERATURE AND PRESSURE USED TO VALIDATE PROCEDURE

The three shapes in figure 9 show the load distributions used in test cases to validate the procedure. Shape 1 is a uniform distribution of load over the antenna surface. Shape 2 is a linearly varying distribution across the width of the gore. Shape 3 is a linearly varying distribution along the length of the gore. These are the three most logical initial choices for testing purposes and do not necessarily represent load distributions found in orbit. Pressure distributions are used to cause more severe displacements normal to the antenna surface to better test the procedure.

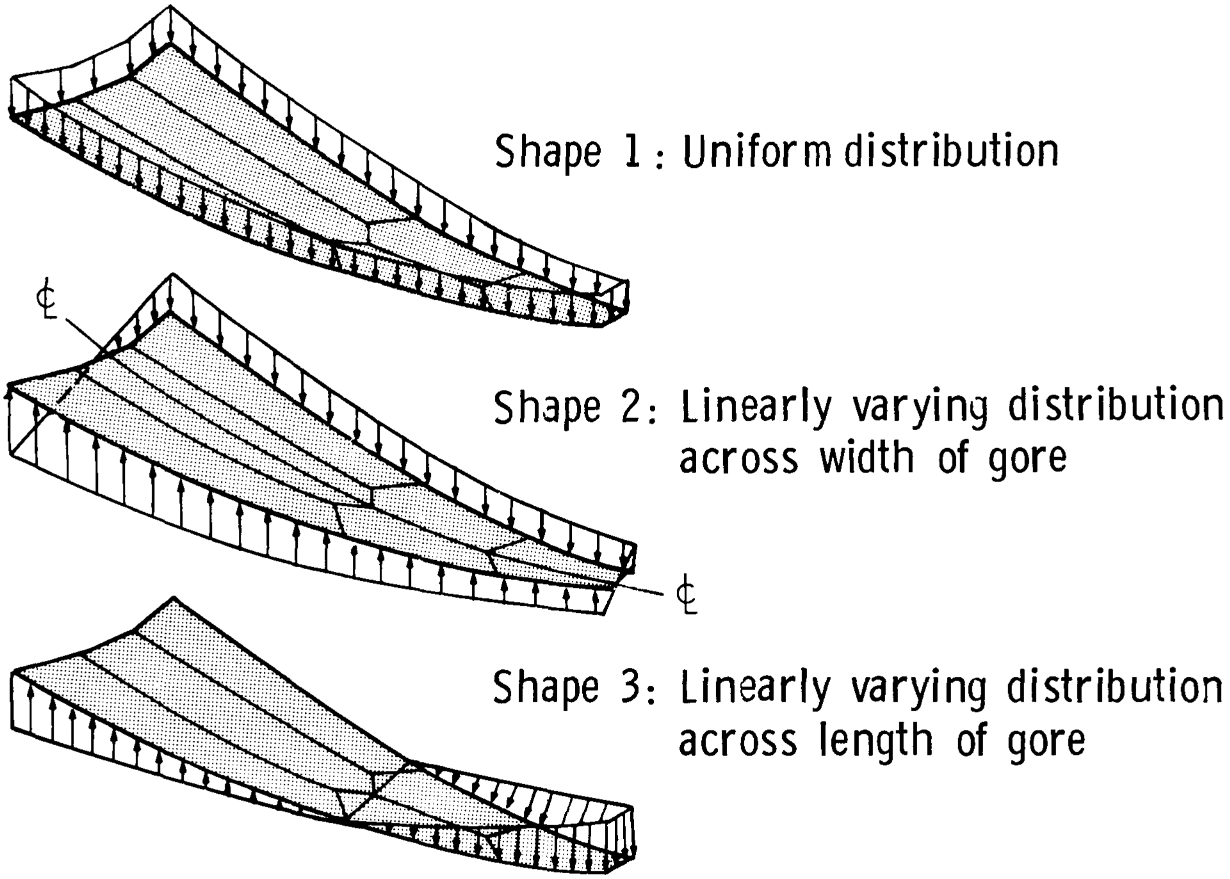


Figure 9

FLOW DIAGRAM FOR OPTIMIZATION PROCEDURE

The flow diagram shown in figure 10 outlines the implementation of the optimization procedure. First, EAL calculates the displacements of the antenna surface due to externally applied loads. EAL then calculates the analytical derivatives of surface displacements with respect to control cable tensions. Since changes in geometric stiffnesses are neglected, these derivatives are constant and need be calculated only once. Next, EAL evaluates the surface displacements due to the current set of control cable tensions with a first order Taylor series approximation using the previously calculated derivatives and the current values of the design variables. The net displacements are the differences between the initial displacements and the displacements due to control cable tensions. A best fit parabola is passed through the net displacements, and the normal distances from the net displacements to the best fit surface are found. The RMS distortion error is computed as the objective function. CONMIN checks whether or not the RMS error is a minimum. If it is not, then CONMIN updates the tensions and the procedure reevaluates the surface displacements. The process continues until CONMIN verifies that the RMS error is a minimum. At this point, the procedure terminates and the result is a set of control cable tensions that produce the best antenna surface. The procedure exhibits slow convergence and typically requires about 40 iterations and 80 cp seconds on a CYBER 175 computer.

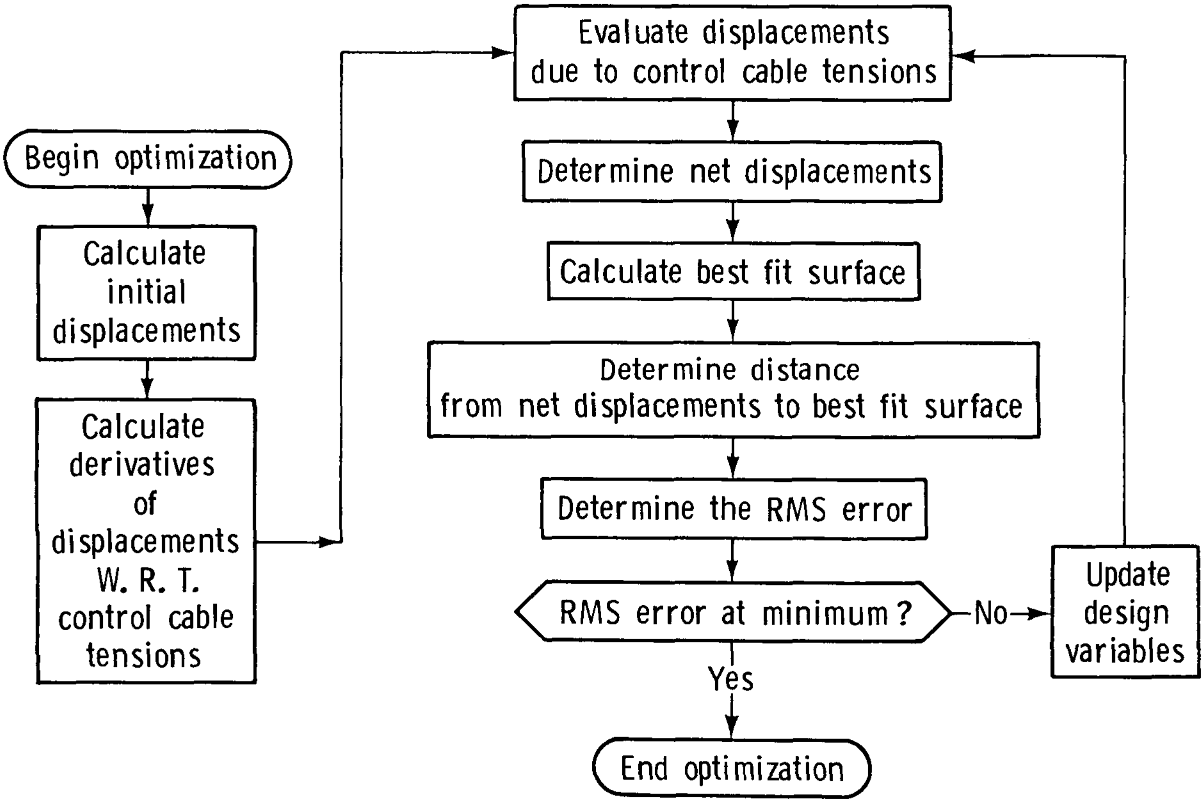


Figure 10

SHAPE CORRECTION FOR ANTENNA-TEMPERATURE LOADING

Representative results are presented in figure 11 to indicate the level of distortion correction attainable by optimally assigning tensions in 12 control cables of a 122 meter hoop/column antenna. The figure shows the results of five test cases using temperature distributions which cause initial surface distortions. Each of the cases had a maximum temperature magnitude of 100° F. The column labeled "Uncorrected RMS distortion" is the initial RMS error of the antenna surface with the indicated temperature load applied. After the optimization procedure is used, the RMS error is changed to that listed in the column labeled "Corrected RMS distortion". The resulting reductions in the RMS error of the surface distortions ranged between 22 and 58 percent.

Load shape	Maximum temperature (^{0}C)	Uncorrected RMS distortion (cm)	Corrected RMS distortion (cm)	Percent reduction
1	38	0.0207	0.0156	25
1	-38	0.0207	0.0161	22
2	38	0.0266	0.0152	43
3	38	0.0511	0.0244	52
3	-38	0.0511	0.0215	58

Figure 11

SHAPE CORRECTIONS FOR ANTENNA-PRESSURE LOADING

Figure 12 shows representative results of five test cases using pressure distributions to cause initial surface distortions. Each of the cases had a maximum pressure magnitude of 10^{-5} pounds per square inch. The sign of the maximum pressure indicates to which side of the antenna surface the load is being applied. The column labeled "Uncorrected RMS distortion" is the initial RMS error of the antenna surface with the indicated pressure load applied. After the optimization procedure is used, the RMS error is changed to that listed in the column labeled "Corrected RMS distortion". The resulting reductions in the RMS error of the surface distortions ranged between 16 and 45 percent.

Load shape	Maximum pressure (N/sq. m $\times 10^6$)	Uncorrected RMS distortion (cm)	Corrected RMS distortion (cm)	Percent reduction
1	.34875	2.1996	1.8501	16
1	-.34875	2.1996	1.6812	24
2	.34875	.5535	.3023	45
3	.34875	1.0055	.8161	19
3	-.34875	1.0055	.7950	21

Figure 12

UNCORRECTED AND CORRECTED DISTORTIONS ALONG GORE CENTERLINE

As a typical result, figure 13 shows the uncorrected and corrected distortions along the gore centerline for a 100° F temperature distribution which varies linearly along the length of the gore. The vertical axis indicates the normal displacements from the best fit surface (previously defined as epsilon) and the horizontal axis is the nondimensional length of the antenna gore. The horizontal reference line is the best fit parabola. The solid curve is the uncorrected distortion of the antenna surface along the gore centerline. The dashed curve is the corrected distortion of the antenna surface after the optimization procedure determines a set of control cable tensions which minimize the RMS error. It can be seen that the corrected curve closely follows the horizontal reference line and therefore is nearly parabolic, which is the objective of the optimization procedure.

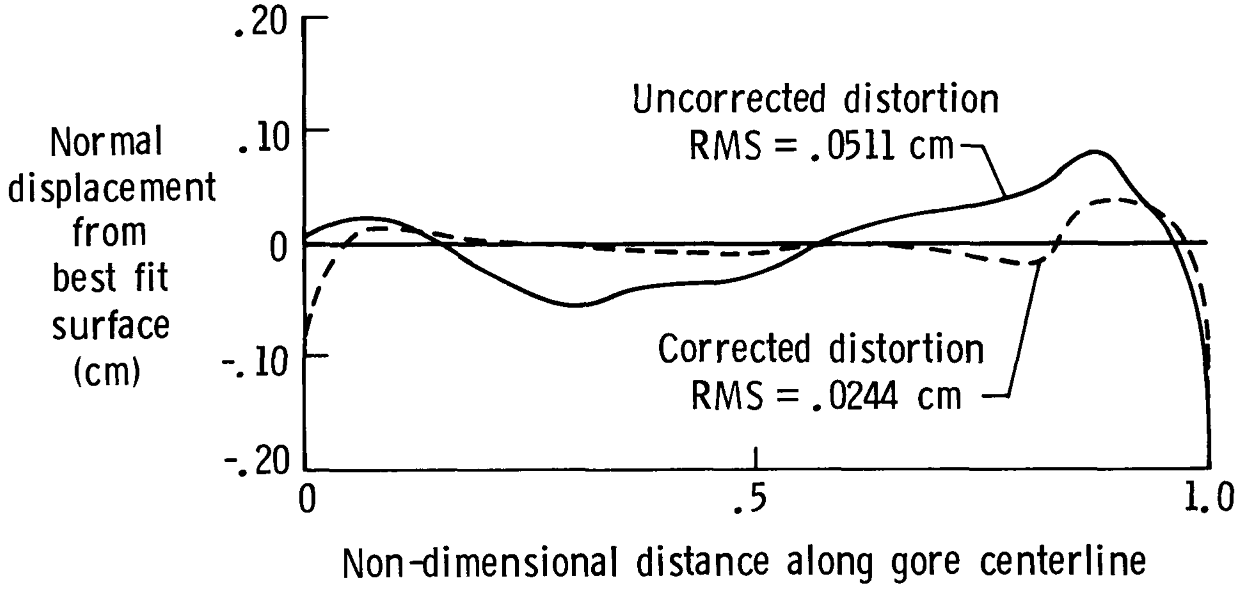


Figure 13

CONCLUDING REMARKS

A systematic, computer-based procedure using mathematical optimization has been developed to minimize the surface distortions of a hoop/column antenna caused by externally applied loads. The procedure is built around the EAL finite element analysis program and the CONMIN general purpose optimization program, both of which are widely used and available. Improvements in RMS error were obtained for every test case attempted during the validation of the procedure. Future plans call for testing the procedure using a finite element model of a complete 48 gore hoop/column antenna, and use of more realistic load conditions that would be encountered in orbit. Also, the inclusion of other parameters in the optimization procedure will be studied. These may initially consist of boresight and focus errors in the objective function or constraints. (See fig. 14.)

- Developed procedure based on mathematical optimization to control antenna distortions
- Used EAL, CONMIN, RMS algorithm
- Successfully tested procedure on hoop/column antenna
- Future work
    - Test procedure on full antenna model
    - Use more realistic loading conditions
    - Include boresight and focus errors in objective function

Figure 14

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