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Linearization of Digital Derived Rate Algorithm for Use in Linear Stability Analysis

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NASA

LINEARIZATION OF DIGITAL DERIVED RATE ALGORITHM

FCR USE IN LINEAR STABILITY ANALYSIS

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SUMMARY

The digital derived rate (DDR) algorithm is used to calculate the rate of rotation of the Centaur upper-stage rocket. The DDR is a highly nonlinear algorithm, and classical linear stability analysis of the spacecraft cannot be performed without linearization. The performance of this rate algorithm is characterized by a gain and phase curve that drop off at the same frequency. This characteristic is desirable for many applications.

This paper deals with the investigation of a linearization technique for the DDR algorithm. The linearization method is described. Examples of the results of the linearization technique are illustrated, and the effects of linearization are described. A linear digital filter may be used as a substitute for performing classical linear stability analyses, while the DDR itself may be used in time response analysis.

INTRODUCTION

The powered-phase autopilot (ref. 2) is a digital control system for the Centaur vehicle (fig. 1). It is used to control the vehicle during full thrust operation. Control is maintained by using hydraulic actuators to rotate, or gimbal, the main engines. This control system is illustrated by block diagram for one plane of motion (the pitch plane) in figure 2.

The attitude error of the Centaur vehicle (the difference between actual and desired flight direction) is sampled and digitized according to the A/D conversion in figure 3. The attitude error is then available in the discrete domain.

The control law is a proportional-integral-derivative (PID) controller with a first-order digital integrator. The autopilot operates at a sampling frequency of 50 Hz. Discrete rate is a function of attitude error (n), sampling period (T), and A/D quantization resolution (RES).

$$\dot{n} = \frac{RES(n_i - n_{i-1})}{T} \tag{1}$$

RES is equal to 0.02919 deg/b, so the minimum change in attitude error over one sampling period results in a rotation rate of 1.4595 deg/sec. Rate changes of <0.1 deg/sec are desired during one period. For this reason, a 50 Hz autopilot must use a higher-order algorithm to maintain good rate information during transient conditions (engine startup, etc.). There are also limits on available memory storage and execution time in the digital flight computer.

Centaur flights use the DDR algorithm to calculate rotation rate by tracking changes in attitude error. The proper choice of DDR constants (ADDR, FDDR, TAU) will give good information at normal vehicle frequencies. The block diagram for the DDR algorithm is shown in figure 4, and its constants are defined in the nomenclature.

This algorithm has been used successfully in previous Centaur missions. Analytical performance evaluation on these past missions has been dependent on time response simulation. Classical frequency domain analysis requires linearization. Linear analysis computer codes such as EASY5 (ref. 5) use simple single-input, single-output blocks to model elements such as the DDR. EASY5 could not be used to model the system in figure 2, because no connection between input and output was recognized within the DDR algorithm.

To perform linear analysis on the powered-phase autopilot, it was necessary to find a linear approximation to the DDR. The problem was resolved by finding the Fourier transform of the input and output of the DDR algorithm for a simple quantized sine-wave input. The frequency spectra of input and output were used to find a frequency response spectrum. The nonlinearity of the DDR algorithm made it easier to use a series of sine waves of different frequencies as input to the algorithm than to use a series of random inputs to find the frequency response.

The frequency response of the DDR algorithm is shown in figure 5. It is a function of the input amplitude, the DDR constants ADDR, FDDR, and TAU, and of noise.

This work will illustrate: (1) the usefulness of the Fourier transform in solving this sort of nonlinear problem, and (2) the criteria used in selecting a linear approximation to this type of element for use in analytical simulation.

NOMENCLATURE

A   amplitude of input sine wave, deg

ADDR   DDR increment limit, deg/sec

DDR   digital derived rate, deg/sec

F   resolution of FFT output, Hz

FDDR   DDR increment for direction change, deg/sec

K   DDR incremental index

N   number of samples in record

RDDR   reference derived rate, deg/sec

RES   A/D quantization resolution, deg/b

T   sampling period, sec

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t	time duration of record, sec
TAU	DDR multiplicative scale factor
η	current attitude error, b
$\dot{\eta}$	actual rotation rate, deg/sec
η_D	attitude error, deg
η_k	reference attitude error, b
ω	frequency of input sine wave, rad/sec
ω_A	cutoff frequency of linear DDR approximation, rad/sec
ξ_A	damping ratio of linear DDR approximation

THE DIGITAL DERIVED RATE ALGORITHM

The operation of the DDR algorithm is based on constants FDDR, ADDR, and TAU. Attitude error in bits (η) is input to the algorithm and digital derived rate (DDR or $\dot{\eta}$) in deg/sec is output.

FDDR is a rate increment used in the case of a change in vehicle direction. If the difference in attitude error between current and reference values ($\eta - \eta_k$) is negative and the current rate (DDR) is zero or positive, then DDR is decreased by FDDR. If the error difference is positive and DDR is zero or negative, then DDR is increased by FDDR.

ADDR is a rate increment limit used when the difference in attitude error and DDR are both nonzero and of the same sign. A reference rate is found by equation (2).

$$RDDR = \frac{RES(\eta - \eta_k)}{KT} \qquad (2)$$

The current value of DDR is not permitted to be incremented by more than ADDR, nor is it permitted to be a value less than RDDR. This provides a good deal of attenuation of noise and higher frequency components, since DDR is not permitted to become too large.

TAU is a scale factor used to keep DDR from becoming too large in the case of a change in vehicle direction. If the vehicle did not change direction, then the counter (K) is increased by one and DDR is not changed. Otherwise, DDR is multiplied by TAU (which must be <1).

The counter K defines the number of sampling periods that a value of reference attitude error (η_k) is delayed. When K is incremented, the change in attitude error over K sampling periods has not been significant. When the change in attitude error becomes greater than 1 dB, then DDR is changed according to the rate increments, the current attitude error becomes the new reference and K is reset to one.

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FREQUENCY RESPONSE OF THE DDR ALGORITHM

The steps in solving for the frequency response of the DDR algorithm are shown by flowchart in figure 6.

(1) Input a sine wave of given amplitude, frequency, and time duration to the DDR algorithm with quantization. For a given input sine wave,

$$n_D = A \sin \omega t \tag{3}$$

the ideal derivative is given by

$$\dot{n} = \omega A \cos \omega t$$
$$= \omega A \sin (\omega t + 90°) \tag{4}$$

The transfer function for the ideal differentiator is

$$g(\omega) = \frac{\dot{n}}{n_D} = \frac{\omega \sin (\omega t + 90°)}{\sin \omega t} \tag{5}$$

The magnitude and phase of this transfer function are

$$|g(\omega)| = 20 \log \omega (dB)$$
$$\angle g(\omega) = 90° \tag{6}$$

Since the attitude error is harmonic, the frequency response of the DDR should approximate equations (6) for low frequencies.

(2) Find the DDR output for the given input. A sample of DDR time response is illustrated in figure 7. For this case, the DDR constants were chosen as follows:

$$ADDR = 0.5 \text{ deg/sec}$$
$$FDDR = 0.2 \text{ deg/sec} \tag{7}$$
$$TAU = 0.5$$

along with resolution 0.02919 deg/b and sampling time 0.02 sec. The noise level used was ± 1 dB (RES). The input was defined by the following (used in eq. (3))

$$A = 0.5°$$
$$\omega = 1.26 \text{ rad/sec} \tag{8}$$

From this time response plot, the magnitude and phase of the DDR transfer function can be read directly with some accuracy.

$$|g(\omega)| = 1.8 \text{ dB}$$
$$\angle g(\omega) = 80° \tag{9}$$

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which agrees fairly well with equations (6). However, choosing frequency information from time responses is less reliable as noise becomes more dominant in the time response.

(3) Take the FFT of both input and output signals. The FFT of the input and output signals (1 and 4) will give the frequency spectra of those signals. If the sampling frequency is given, the time duration of the signal (t) must be chosen to give a desired resolution (F) of the frequency spectra. For this example, F was chosen to be 0.01 Hz, so t was 100 sec. The number of points in the record (N) was then taken to be equal to the time duration of the signal divided by the period, or, 5000 points. The FFT returns 2500 valid frequency domain points, and the range of the spectrum was normalized to DC to 25 Hz.

(4) Choose the gain and phase at the input frequency. From the points found in step C, the gain and phase of the DDR are chosen at the point where the frequency is the same as that of the input sine wave.

(5) Repeat steps 1 through 4, increasing the input frequency of the sine wave. This step gives a sweep of frequencies of interest. Frequencies higher than about 7.5 Hz were filtered prior to sampling, and the response for frequencies >5 Hz was not calculated. The frequency response shown in figure 8 is for an input amplitude of 0.5°, for cases both with and without noise.

(6) Repeat steps 1 through 5 for a selection of input amplitudes of interest. For most Centaur flights, attitude error limit cycles are ≤2°. Frequency responses were taken at input amplitudes of 0.1, 0.3, 0.5, 1, and 2°, to get a good representation of this range.

A FORTRAN program was written to perform the steps listed. The input sine wave was sampled and quantized according to the plot in figure 3. In some cases, random noise was added to the quantized signal prior to calculation of DDR. Some cases were also run without noise. Time responses were generated with another program to check the results.

RESULTS

The frequency responses shown in figure 8 are typical for many combinations of DDR parameters and input amplitudes. For all cases observed, the gain and phase began to drop off at the same frequency. The character of the response was also seen to be affected by noise.

The gain and phase plots both give the impression that the DDR may be modeled as an ideal differentiator with a second-order lag,

$$G(s) = \frac{\omega_A^2 S}{S^2 + 2\xi_A \omega_A S + \omega_A^2} \qquad (10)$$

If the approximation is chosen to fit the phase, however, the cutoff frequency will be about a decade greater than if the approximation is chosen to fit the gain. The approximation is therefore accurate over a small frequency range of interest.

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The nonlinearity caused inconclusive results for fitting an equation (10) approximation to the data by Levy's method (ref. 2).

The DDR algorithm as modeled according to the approximation of equation (10), with ω_A set equal to the cutoff frequency observed in figure 8, and ξ_A chosen to be appropriate for the phase characteristic for the case with noise.

$$\omega_A = 9.42 \text{ rad/sec}$$

$$\xi_A = 0.7$$

(11)

This approximation is accurate to within 1 dB in gain and within 250 to 30° in phase in the vicinity of the cutoff frequency. Below the cutoff frequency, the accuracy is even better. For the case of most Centaur flights, where attitude error limit cycles oscillate at <1.5 Hz, this is a good approximation for linear analysis.

CONCLUSIONS

The method shown here generated a suitable frequency response for a linear approximation to the DDR algorithm. The approximation was used in simulations written in EASY5 for root locus and frequency domain analysis.

The method could be used to find frequency response information for other nonlinear components, provided a certain amount of information is known about the fundamental behavior of the component, so the right parameters may be varied, and so the results obtained by this method may have meaning. For instance, it was necessary in this case to know that the effects of the three DDR constants were coupled, and that the response of the DDR might vary with input amplitude.

This technique is only necessary because of the nonlinear nature of the DDR algorithm. If a discrete linear algorithm could be developed that would maintain accuracy at low frequencies and high noise levels, without exceeding computer storage limitations, it would be a great aid to linear stability analysis.

The approximation constants ω_A and ξ_A were found by this technique as functions of the DDR constants, input amplitude, and noise. This would be a more powerful analysis tool if DDR constants could be calculated as functions of ω_A and ξ_A, either analytically or graphically.

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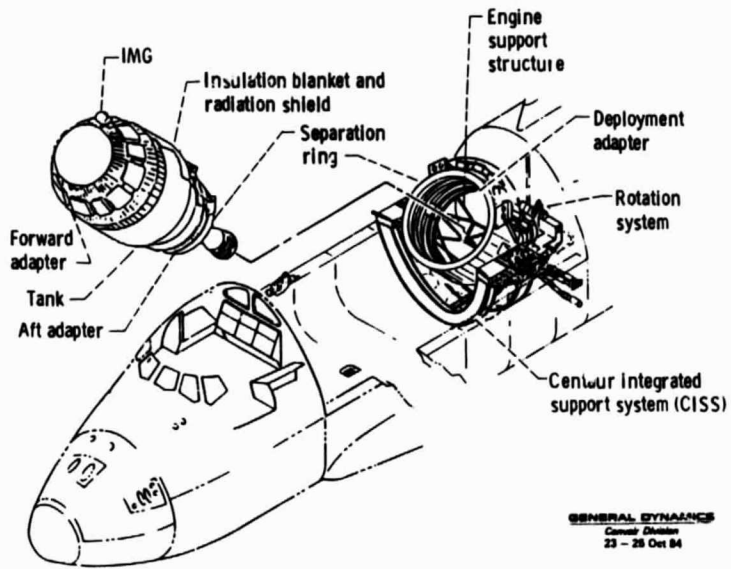
Figure 1. - Centaur upper-stage.
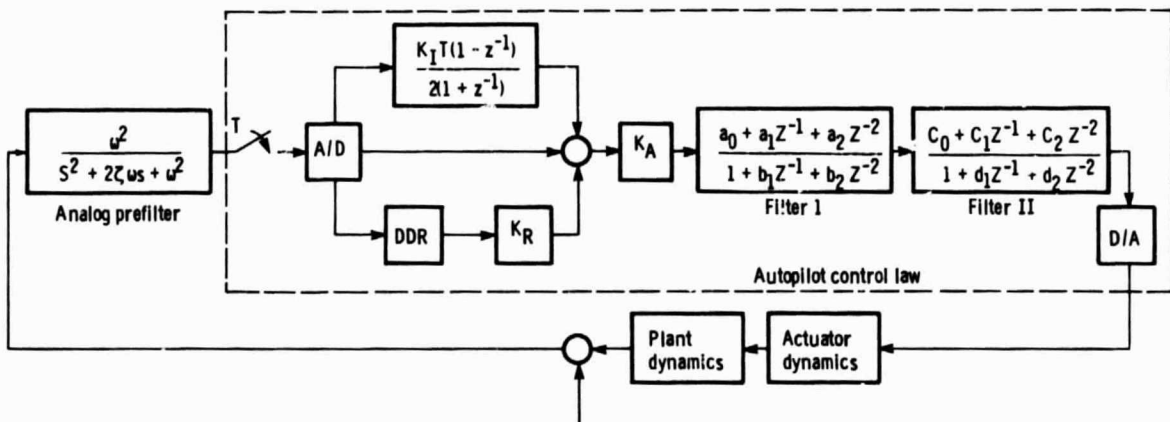
Figure 2. - Centaur powered-phase autopilot.
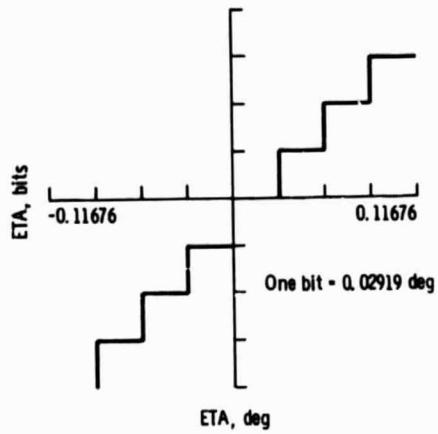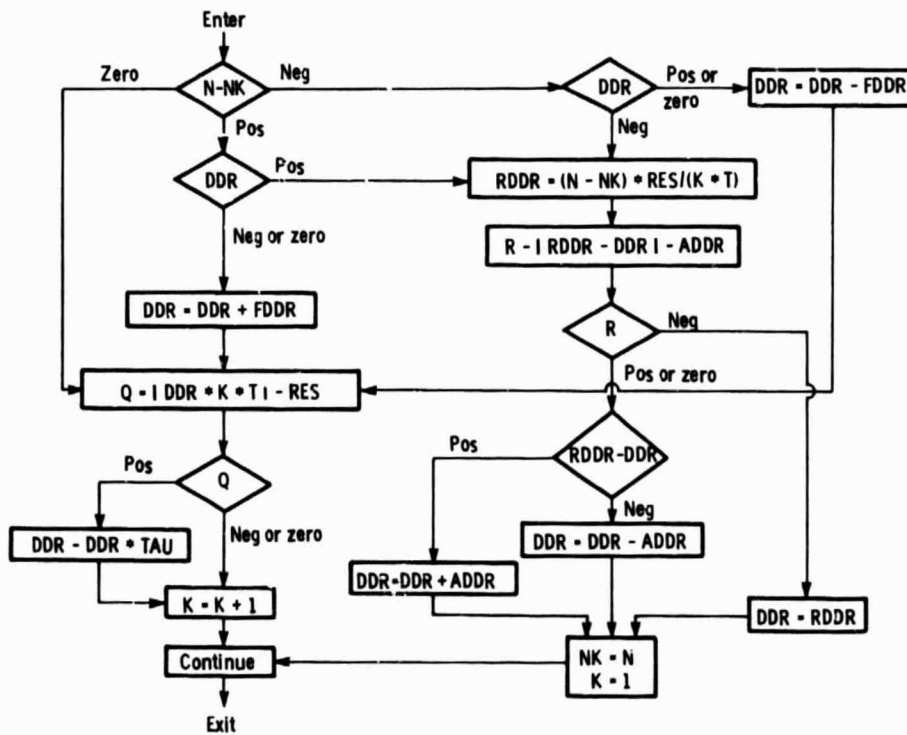
Figure 3. – A/D quantization resolution.

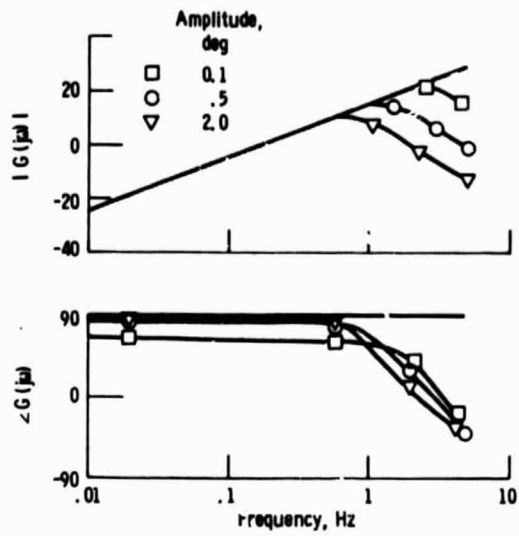Figure 4. – Flowchart of the DDR algorithm.

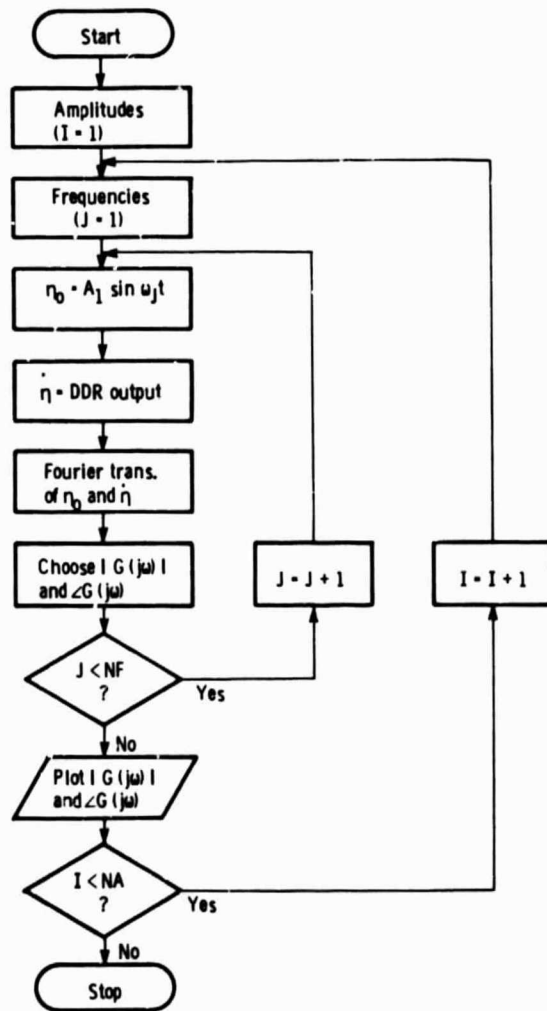Figure 5. - Frequency response of the DDR as a function of input amplitude.

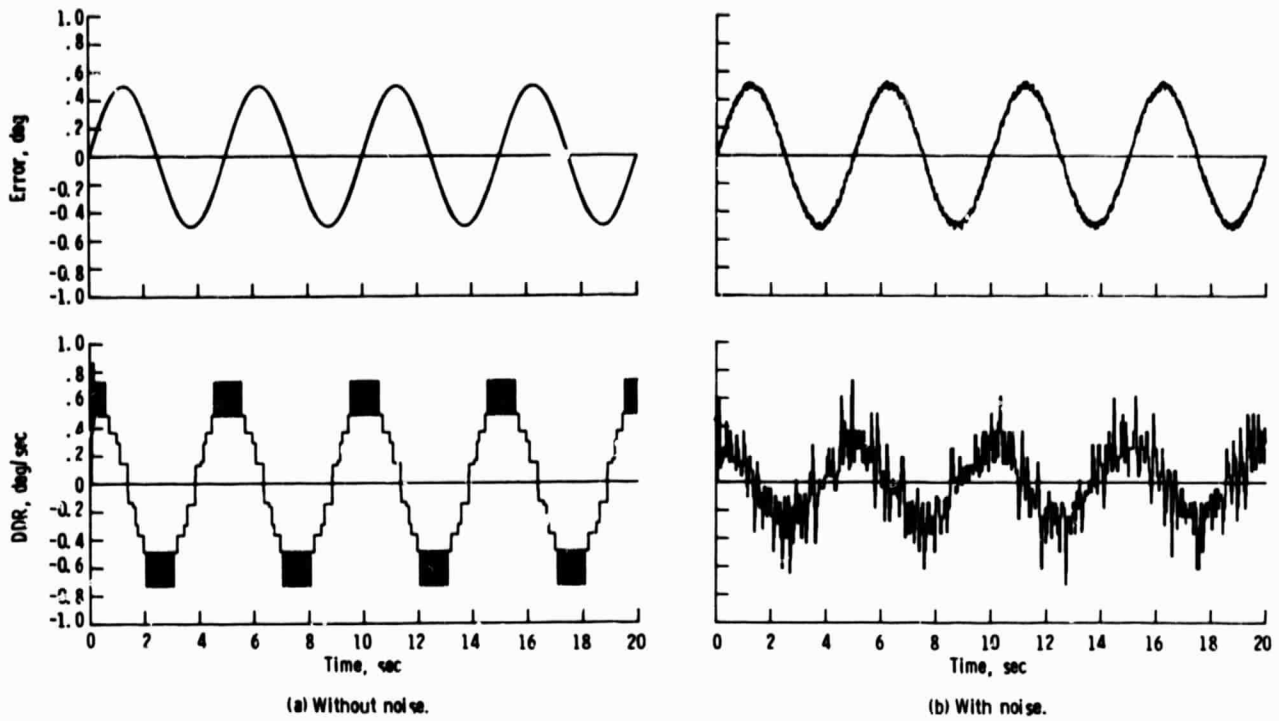Figure 6. - Flowchart of solution method.

Figure 7. - Time response of DDR algorithm.

(a) Without noise.

(b) With noise.

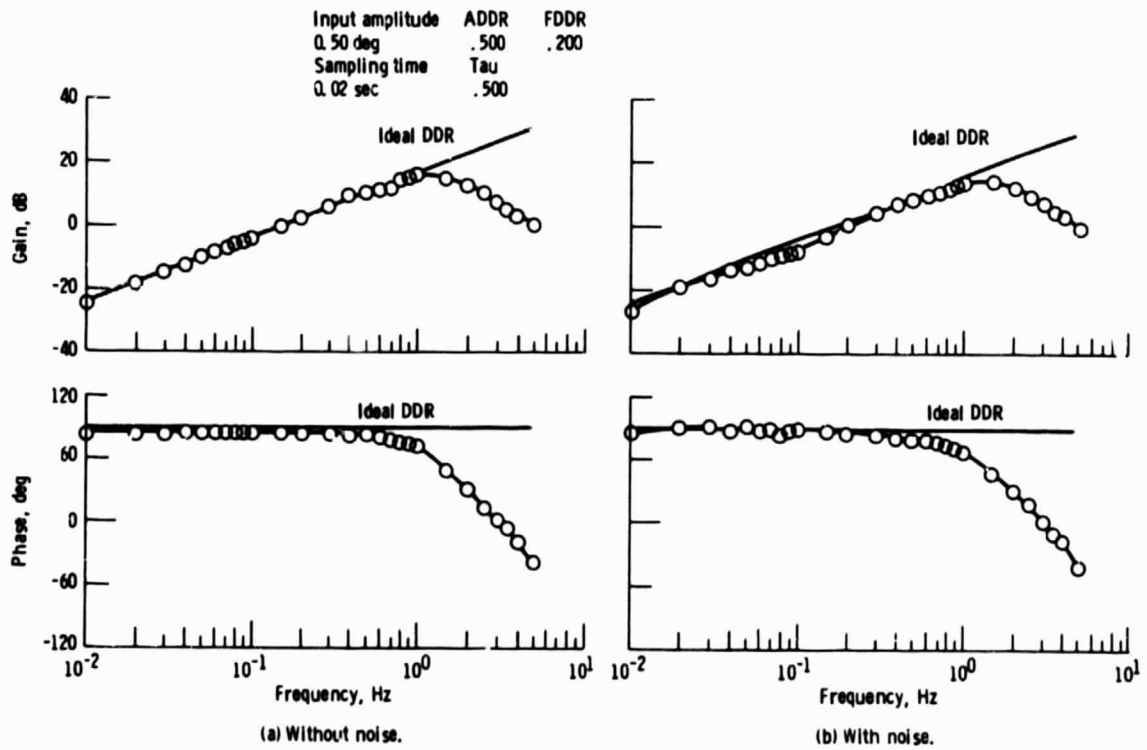| Input amplitude	ADDR	FDDR
0.50 deg	.500	.200
Sampling time	Tau	
0.02 sec	.500	

(a) Without noise.

(b) With noise.

Figure 8. - Frequency response of the DDR algorithm.