

NASA-CR-175665

NASA-CR-175665
19850015958

108524269

CONFIDENTIAL

LIBRARY COPY

JUN - 2 1997

LANGLEY RESEARCH CENTER
LIBRARY NASA
HAMPTON, VIRGINIA

A Service of:



NF00723



National Aeronautics and
Space Administration

**Scientific and Technical
Information Program Office**
Center for AeroSpace Information

NOTICE



3 1176 01431 7664

**THIS REPRODUCTION WAS MADE FROM THE BEST AVAILABLE
COPY OF WHICH A NUMBER OF PAGES WERE OF POOR
REPRODUCTION QUALITY.**

JPL PUBLICATION 85-2

(NASA-CR-175665) GEOMETRIC ERROR ANALYSIS
FOR SHUTTLE IMAGING SPECTROMETER EXPERIMENT
(Jet Propulsion Lab.) 172 p EC A08/MP AJ1

CSCI 14B

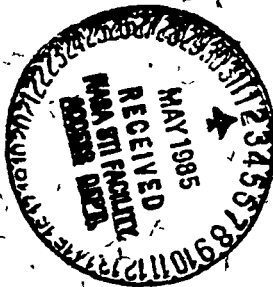
N85-24269

G3/35 Unclas
14802

Geometric Error Analysis for Shuttle Imaging Spectrometer Experiment

Shyh Jong Wang
Che-Hang Charles Ih

December 15, 1984



NASA

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

JPL PUBLICATION 85-2

Geometric Error Analysis for Shuttle Imaging Spectrometer Experiment

Shyh Jong Wang
Che-Hang Charles Ih

December 15, 1984

NASA

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

The research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration

Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not constitute or imply its endorsement by the United States Government or the Jet Propulsion Laboratory, California Institute of Technology

V. 1112

Acknowledgements

The authors wish to express their gratitude to Y. H. Lin and J. M. Cameron for their contributions at the early stage of this work, and to M. Herring, P. N. Kupferman, and J. B. Wellman for their support.

ABSTRACT

The demand of more powerful tools for remote sensing and management of earth resources has been steadily increasing over the last decade. With the recent advancement of area array detectors, high resolution multichannel imaging spectrometers can be realistically constructed.

This report documents the error analysis study for the Shuttle Imaging Spectrometer Experiment system for the purpose of providing information for design, tradeoff, and performance prediction.

Error sources including the Shuttle attitude determination and control system, instrument pointing and misalignment, disturbances, ephemeris, earth rotation, etc., have been investigated. Geometric error mapping functions were developed, characterized, and illustrated extensively with tables and charts. Selected ground patterns and the corresponding image distortions have been generated for direct visual inspection of how the various error sources affect the appearance of the ground object images.

TABLE OF CONTENTS

	<u>Page</u>
List of Illustrations	viii
List of Tables.	xi
I. INTRODUCTION	1
A. BACKGROUND	1
B. APPROACH	3
C. SUMMARY OF MAJOR FINDINGS.	5
II. ORBITAL AND IMAGING SPECTROMETER CONFIGURATIONS.	8
A. SHUTTLE NOMINAL FLIGHT CONFIGURATIONS.	8
B. IMAGING SPECTROMETER SYSTEM DESCRIPTION.	10
III. ERROR SOURCES AND SHUTTLE ATTITUDE DYNAMICS.	14
A. ERROR SOURCES.	14
B. SHUTTLE MASS PROPERTY.	15
C. ASSESSMENT OF DISTURBANCES	18
C.1 Aerodynamic Drag Torque	18
C.2 Gravity Gradient Torques and Gyroscopic Torques	25
D. MISALIGNMENT AND MEASUREMENT NOISE	27
D.1 Misalignment Errors	27
D.2 IMU Model	28
E. SHUTTLE DYNAMICS AND INSTRUMENT POINTING ERRORS.	29
E.1 The Equations of Motion	29
E.2 The Space Shuttle State Estimator	29
E.3 The System Pointing Errors.	30
E.4 The Instrument Pointing Error PSD	31
F. SUMMARY OF MAJOR FINDINGS.	36
F.1 The Major Environmental Disturbances.	36

	<u>Page</u>
F.2 The Measurement Uncertainties	36
F.3 Ground Track Errors and Navigation Uncertainties.	36
IV. PARAMETRIC ANALYSIS OF GEOMETRIC ERRORS	43
A. COORDINATE CONFIGURATIONS	43
B. GEOMETRIC ERRORS INDUCED BY EPHEMERIS UNCERTAINTIES	45
B.1 Geometric Errors Due to Altitude Uncertainties	45
B.2 Geometric Errors Due to Intrack and Crosstrack Prediction Errors.	50
C. GEOMETRIC ERRORS INDUCED BY ATTITUDE UNCERTAINTIES.	53
C.1 Geometric Errors Induced by Roll Error	53
C.2 Geometric Errors Induced by Pitch Error	55
C.3 Geometric Errors Induced by Yaw Error.	58
C.4 Geometric Errors Induced by Roll, Pitch, and Yaw Attitude Errors.	60
D. GEOMETRIC ERRORS INDUCED BY ATTITUDE RATE ERRORS	68
E. GEOMETRIC ERRORS INDUCED BY MISALIGNMENT ERRORS.	68
F. GEOMETRIC ERRORS INDUCED BY EARTH ROTATION	69
G. NUMERICAL RESULTS: GEOMETRIC ERRORS AND ERROR SENSITIVITIES	72
G.1 Geometric Errors Due to Altitude Uncertainties.	72
G.2 Geometric Errors Due to Roll Uncertainties.	73
G.3 Geometric Errors Due to Pitch Uncertainties	74
G.4 Geometric Errors Due to Yaw Uncertainties	74
G.5 Geometric Errors Due to Attitude Uncertainties.	74
G.6 Ground Point Shift Due to Shuttle Motion and Earth Rotation.	75
H. SUMMARY OF MAJOR FINDINGS.	94
V. GROUND PATTERNS AND IMAGE DISTORTIONS.	101



	<u>Page</u>
VI. CONCLUSIONS.	111
REFERENCES.	113
APPENDIX A - GEOMETRIC ERROR MAPPING FUNCTION TABLE	115
APPENDIX B - NAPIER'S RULES FOR RIGHT SPHERICAL TRIANGLES	124
APPENDIX C - COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER POINTING ERROR POWER SPECTRAL DENSITIES FOR CONFIGURATION A -- PAYLOAD-BAY NADIR POINTING	125
APPENDIX D - COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER POINTING ERROR POWER SPECTRAL DENSITIES FOR CONFIGURATION B -- NOSE-DOWN NADIR POINTING.	129
APPENDIX E - GEOMETRIC ERROR ANALYSIS PROGRAM LISTINGS.	133
APPENDIX F - GROUND PATTERN AND IMAGING DISTORTIONS GENERATION PROGRAM LISTINGS	150



List of Illustrations

	<u>Page</u>
1. IS Error Analysis System Block Diagram.	4
2. Payload-Bay Nadir Pointing Configuration (A).	9
3. Nose-Down Nadir Pointing Configuration (B).	9
4. Imaging Spectrometer Basic Operating Principles	11
5. Shuttle Imaging Spectrometer Optical System Configuration	12
6. Error Sources	14
7. Orbiter Coordinate System	16
8. Shuttle Body and Principal Coordinates (Payload-Bay Nadir Pointing Shown).	17
9. The Three-Plate Aerodynamic Model in Body Frame	19
10. Simplified IMU Model.	28
11. Pitch-Axis Block Diagram (Configuration A).	31
12. Roll and Yaw Axes Block Diagram (Configuration A)	32
13. Instrument Pointing Error PSD, (rad) ² -sec	35
14. Instrument Pointing Rate Error, (rad/sec) ² -sec.	35
15. Instrument Pointing Error PSD, (rad) ² -sec	38
16. Instrument Pointing Rate Error, (rad/sec) ² -sec.	38
17. IS Ground Track Error and Rate Error PSD.	40
18. Payload-Bay Nadir Pointing Configuration and Coordinates.	44
19. Ground Point Shift in X and Y Direction Due to Altitude Error	46
20. Change of View Angle of a Ground Point Due to Altitude Error. . . .	51
21. Geometric Errors Induced by Intrack and Crosstrack Prediction Errors.	52
22. Shift of Ground Point Induced by Roll Error	54
23. Shift of Ground Point Induced by Pitch Error.	56



	<u>Page</u>
24. Shift of Ground Point Induced by Yaw Error	59
25. The Shift of Ground Projection of IS Line-of-Sight Due to Attitude Error.	61
26. Side View of Fig. 25	63
27. Shift of Ground Point Induced by Attitude Errors	65
28. Linear Velocity of the Earth at the Nadir Point:	70
29. Earth Spherical Geometry	71
30. Geometric Error Due to Altitude Uncertainties (Δh)	77
31. Geometric Error Due to Roll Attitude Error (ϕ)	79
32. Geometric Error Due to Roll Uncertainty (ϕ) About 20° Roll Offset.	80
33. Geometric Error Due to Pitch Attitude Error (θ).	82
34. Geometric Error Due to Pitch Uncertainty (θ) About 22.5° Pitch Offset	83
35. Geometric Error Due to Pitch Uncertainty (θ) About 45° Pitch Offset	84
36. Geometric Error Due to Yaw Attitude Error (ψ).	86
37. Geometric Error Due to Roll (ϕ), Pitch (θ), and Yaw (ψ) Errors	88
38. Geometric Error Due to Pitch (θ) and Yaw (ψ) Uncertainties About 20° Roll Offset	89
39. Geometric Error Due to Roll (ϕ) and Yaw (ψ) Uncertainties About 22.5° Pitch Offset	90
40. Geometric Error Due to Roll (ϕ) and Yaw (ψ) Uncertainties About 45° Pitch Offset	91
41. Geometric Error Caused by Earth Rotation for the Time Period of 1 Second.	93
42. Effects of Earth Curvature	94
43. Ground Pattern Distortions With No Earth Rotation Effect	102
44. Ground Pattern Distortions With 20° Roll Offset and No Earth Rotation Effect	104

	<u>Page</u>
45. Ground Pattern Distortions with 45° Pitch Offset and No Earth Rotation Effect	105
46. Ground Pattern Distortions With Constant Rotation Rate and No Earth Rotation Effect	106
47. Ground Pattern Distortions With Sinusoidal Rotation Rate and No Earth Rotation Effect.	108
48. Ground Pattern Distortions With Effect of Altitude Change (Increase) of 40 km and No Earth Rotation Effect.	109
49. Ground Pattern Distortions With Effect of Earth Rotation for 40° Latitude and Orbit Inclination of 85°	110

List of Tables

	<u>Page</u>
1. Sensor Performance Requirements	2
2. Modeled Environmental Disturbances.	39
3. Modeled Measurement Uncertainties	39
4. Expected On-Orbit Navigation Accuracies (3σ).	42
5. Geometric Errors and Error Sensitivity Due to Altitude Uncertainties.	76
6. Geometric Errors and Error Sensitivity Due to Roll Attitude Error	78
7. Geometric Errors and Error Sensitivity Due to Pitch Attitude Error	81
8. Geometric Errors and Error Sensitivity Due to Yaw Attitude Error	85
9. Geometric Errors Induced by Yaw, Pitch, and Roll Errors	87
10. Ground Point Shift Induced by Shuttle Motion and Earth Rotation . .	92
11. Geometric Errors Due to Expected On-Orbit Navigation Uncertainty. .	97
12. Expected On-Orbit Navigation Accuracies (Ref. 12)	98
13. Geometric Error Sensitivity	99
14. Geometric Error Induced by Shuttle and Imaging Spectrometer Measurement Uncertainties	100

I. INTRODUCTION

A. BACKGROUND

Earth resource management and utilization have experienced great success over the last decade through the Landsat programs. In more recent years, both NASA and user communities have envisioned the need for development of better and more powerful instruments for surveying and managing earth resources. The Landsat D's new sensor, Thematic Mapper (TM), the proposed utilization of Tracking and Data Relay Satellite System (TDRSS), and a more advanced ground system represent an advancement in the earth resource satellite development [1].

The Thematic Mapper of the Landsat D (launched in 1982) has seven spectral bands, two more than those of the Multispectral Scanner associated with the earlier Landsats. However, study shows that the reflectance spectrum of earth surface materials contains a significant amount of information which can only be identified with spectral resolution much finer than those of the Thematic Mapper [2]. With the advancement of area array detectors, a push broom imaging spectrometer can be realistically constructed for simultaneous imaging and registration of hundreds of spectral bands. For the case of Shuttle Imaging Spectrometer Experiment, 128 spectral channels have been proposed to cover the spectral range of 0.4 to 1.0 μm for VNIR (visible and near infrared) and 1.0 to 2.5 μm for SWIR (short-wavelength infrared) with instantaneous field of view of 30m. Table 1 shows the required sensor performance [2].

The purpose of this report is to document the error analysis study for the imaging spectrometer experiment. Error analysis is an important aspect of the overall remote sensing system, since errors from many sources, including

the spectrometer itself, the spacecraft that carries the instrument, knowledge limitations on the true spacecraft attitude and locations, earth rotation, curvature, and terrain variations, etc., will all contribute to image distortion, shift, rotation, and misregistration. Error correction or compensation are necessary and are an integral part of the image processing. This work covers the analysis of fundamental and geometric errors and error sensitivities, the development of geometric mapping functions, and the computation of ground

Table 1. Sensor Performance Requirements

Parameter	Value	Comments
Spectral Coverage	0.4 to 2.5 μm	Although the entire spectrum is probably not required for any one discipline, in the aggregate of all remote sensing disciplines, the entire region is required
Spectral Sampling Interval		
VNIR (0.4 to 1.0 μm)	0.01 μm or better	
SWIR (1.0 to 2.5 μm)	0.02 μm or better	
Instantaneous Field of View	30 m	Adequate for most research topics
Swath Width	at least 10-12 km	This is adequate for research if pointing capability is provided to assure target access
Pointing Mirror Range		
Along track	at least \pm 45 deg	Essential for atmospheric and BRDF (Bidirectional Reflectance Distribution Function) Studies
Cross track	not more than \pm 25 deg	
Radiometric Performance (NE Δ R)		
VNIR	\leq 0.5%	
SWIR	\leq 1.0%	

pattern distortions. The results are translated into many tables, plots, and patterns for visual apprehension. It is believed that the results reported here are important for design, trade-off, and performance prediction.

B. APPROACH

This study has been performed in four progressive stages as shown in block diagram form in Fig. 1. In Stage I, the error sources were identified, dynamic disturbances and the Space Shuttle error dynamics were modeled, and the error power spectral densities for two in-orbit configurations were developed. Stage II of this study concentrated on the development of geometric error mapping functions and error sensitivity analysis. Geometric errors due to ephemeris uncertainties, attitude deviations, earth rotation, etc., were studied. In Stage III, ground pattern image distortions caused by various error effects and forward and side looking angular offsets, altitude change, and effect of earth rotation were generated. Stage IV consists of the analysis of the imaging spectrometer instrument errors. These errors include optical jitter, nonlinearities, processing errors, and repeatability. The study of Stages I, II, and III has been completed and the results are included in this report. The study called for by Stage IV has not been planned. It is emphasized here that the imaging spectrometer error model development is an important step for the overall performance prediction and design of the imaging spectrometer system.

Major findings of this work are summarized in the following subsection. In Section II, the orbital and imaging spectrometer configurations are described. The attitude dynamics and error power spectral densities are documented in Section III; and parametric analysis of geometric errors are treated in Section IV. Section V deals with the ground pattern image distortions. Conclusions are summarized in Section VI.

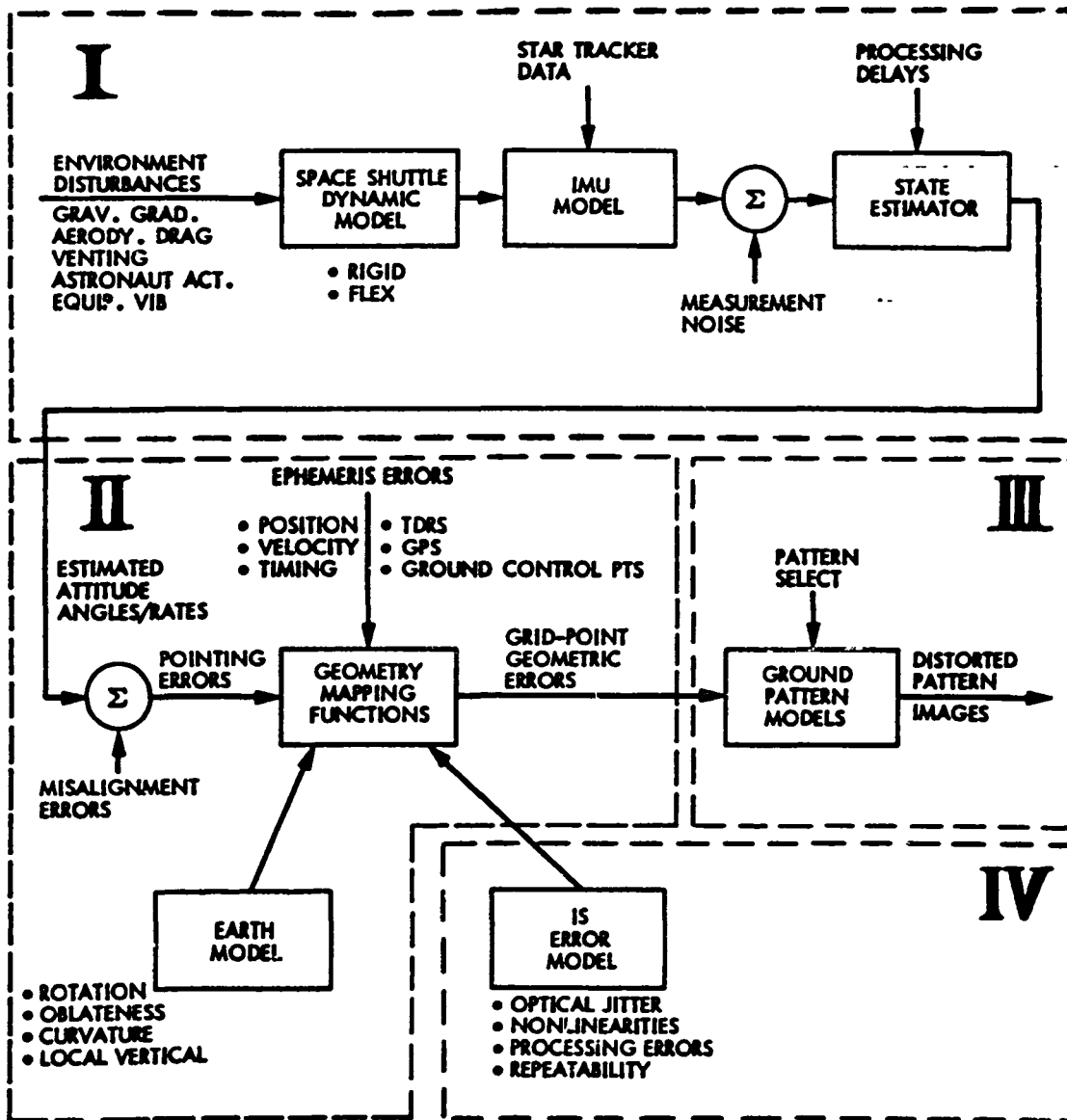


Figure 1. IS Error Analysis System Block Diagram

C. SUMMARY OF MAJOR FINDINGS

The following is a summary of major findings. The details of these are treated in the sections to follow.

1. The results show that the IS Experiment with image pickup period of 20 seconds at a time is feasible with the shuttle properly phased inside the control deadband. The error PSD (power spectral density) characteristic reveals that the system resonates at very low frequencies (in the 10^{-5} to 10^{-2} Hz region). Excitations at these frequencies must be avoided through design precautions. The analysis also showed that errors below 0.01 Hz are dominated by the shuttle dynamics reacting to disturbances, whereas those above 0.01 Hz are dominated by the shuttle inertial measurement system uncertainties and its inherent noise. These high frequency errors limit pointing performance and result in a one-sigma ground track error of 54.8 meters per axis. One-sigma rate errors are shown to be less than 4 meters per second per axis in the frequency range of 10^{-5} to 4×10^{-2} Hz. The image smear is not significant because of the short millisecond-level line time.
2. The effects of earth curvature are very small for the application here (see Fig. 42).
3. Altitude uncertainties cause only moderate geometric errors. The worst 1 σ geometric errors are 11.71 m in position and 0.133 m/sec in rate with STDN and large unmodeled perturbations at 200 km orbit. The performance improves with TDRSS. For the 300 km orbit and with small unmodeled perturbations, the 1 σ geometric errors will reduce to 0.22m and 0.0032 m/sec (see Table 11).

4. The effects of other navigation errors are significantly greater. The 1σ downtrack errors range from 203m (300 km orbit) to 8128m (200 km orbit); and those for the crosstrack are 152m to 508m (see Table 12).
5. The effects of roll and pitch attitude errors are relatively large compared with, for instance, those caused by yaw errors and altitude uncertainties. The error sensitivity is 1.94 m/arc sec or approximately 7000 m/degree. The yaw sensitivity is 0 for 0° view angle and 0.028 m/arc sec for the maximum view angle of $\pm 0.825^\circ$ (see Table 13).
6. The error sensitivities of attitude errors increase significantly for large attitude offsets. For 20° side looking, the sensitivity increases to 2.28 m/arc sec for roll errors and to 0.75 m/arc sec for yaw errors and the pitch error sensitivity is almost unaffected. For the 45° forward looking case, the sensitivities for the pitch, roll, and yaw errors increase to 2.83, 4.36, and 1.97 m/arc sec, respectively (see Table 13).
7. The performance of the Shuttle Imaging Spectrometer is limited by the Shuttle IMU (Inertial Measuring Unit) accuracy, instrument misalignment, etc., Shuttle RCS (Reaction Control Subsystem) deadband, etc. unless some means of error reduction are employed. For instance, ground control points may be used to reduce navigation prediction errors; and precision point mounts, such as AGS (ASPS* Gimbal System) and IPS (Instrument Pointing System), may be used to reduce the

*Annular Suspension Pointing System

attitude errors.

The single axis geometric errors due to the combined Shuttle/IS misalignment, for instance, for normal nadir pointing, 20° side looking, and 45° forward looking are 169m, 198m, and 379m, respectively (refer to Table 10 for detailed breakdowns).

8. Earth rotation causes shifts of images toward the direction of rotation. The magnitude of this shift depends on the latitude of the object. For instance, at the equator the object moves approximately 462m in one second (for 400 km orbit), and at 60° latitude it moves only 231m in one second.

II. ORBITAL AND IMAGING SPECTROMETER CONFIGURATIONS

A. SHUTTLE NOMINAL FLIGHT CONFIGURATIONS

Two shuttle in-orbit configurations have been considered, the Payload-Bay Nadir and the Nose-Down Nadir, as illustrated in Figures 2 and 3, respectively. It will be shown later that the Nose-Down Configuration is gravity gradient stabilized and the Payload-Bay Nadir Configuration is not. However, the Payload-Bay Nadir Configuration offers simpler instrument mounting and less aerodynamic drag. Besides, for certain experiments that require large forward looking angles, the Nose-Down Configuration will be unsuitable.

A circular orbit of 400 km altitude has been selected for this analysis. This selection is consistent with the SIS-B parameters.*

There are a number of possible instrument mounting options that will affect the pointing and the geometric errors of the instrument. These options include:

- a) Direct mount
- b) AGS (ASPS Gimbal System) mount
- c) IPS (Instrument Pointing System) mount

Since both the AGS and IPS systems are capable of providing precision payload pointing and measurements, the system performance will be improved at the expense of significant extra cost. The direct mount approach is the least complex and most cost effective, provided that the errors are within the tolerance. In this report the direct mount approach is considered. Furthermore, the shuttle IMU

*During the period when this part of the work was performed, SIS-B (Shuttle Imaging Spectrometer-Configuration B) was considered. However, the methods used here are mostly generic, and hence, can be applied to systems of similar configurations.

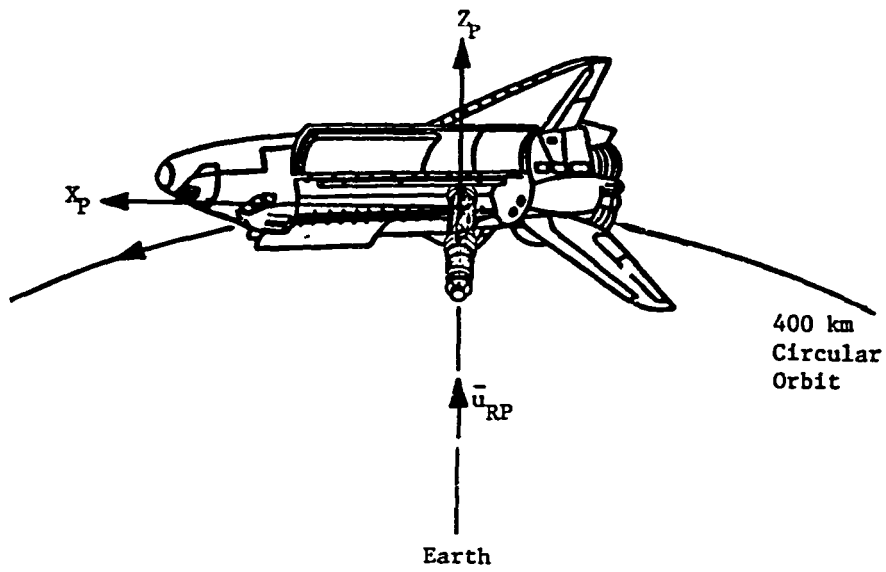


Figure 2. Payload-Bay Nadir Pointing Configuration (A)

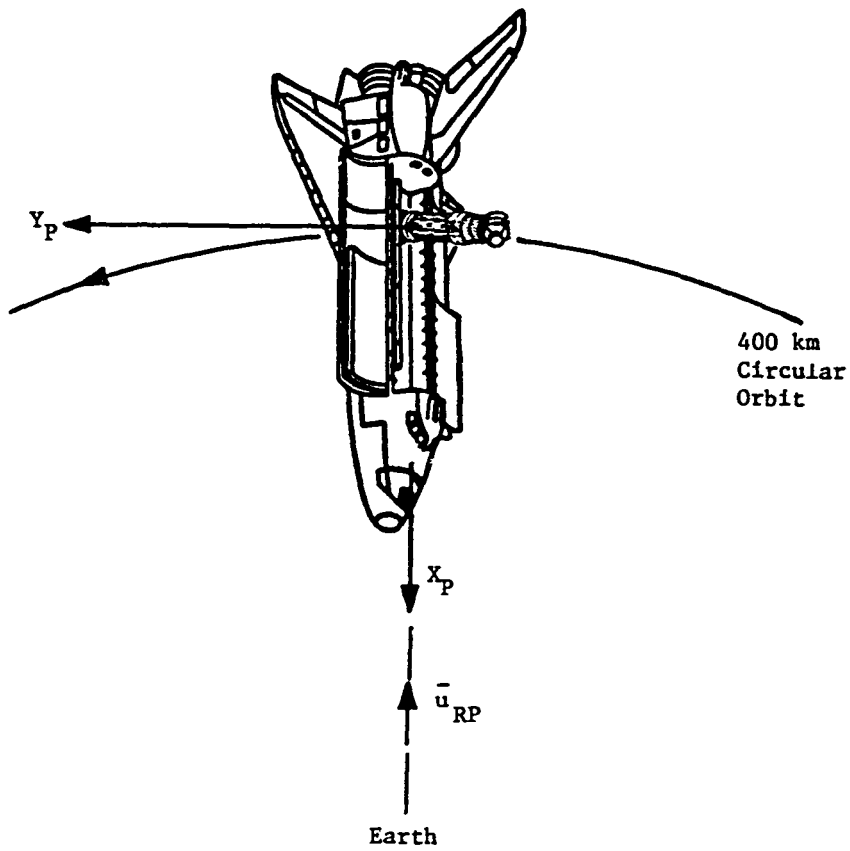


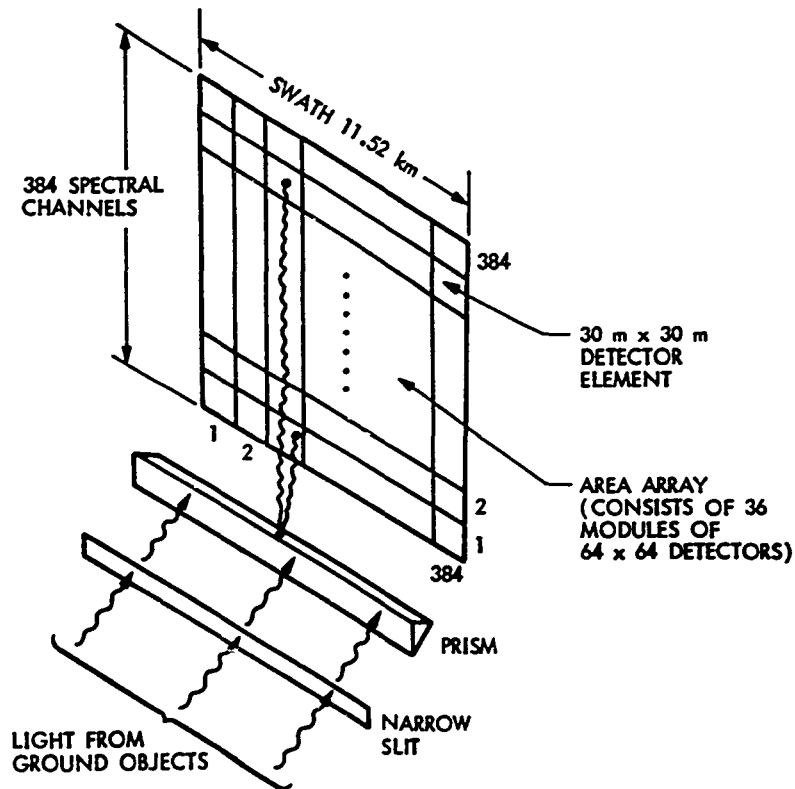
Figure 3. Nose-Down Nadir Pointing Configuration (B)

(Inertial Measurement Unit) and the shuttle state estimator unit are used for obtaining attitude and rate information, without additional instrumentation. Other options may be studied in the future if necessary.

B. IMAGING SPECTROMETER SYSTEM DESCRIPTION

In order to correlate the analysis reported here to the IS applications, it is desirable to understand the basic operating principle of the IS instrument. Figure 4 shows a sketch of the basic elements of the IS, except the memory banks, registers, and the processors. Some of the IS parameters that are relevant to the geometric error analysis, along with definitions of the IS terminologies, are also listed in Fig. 4. It is understood that the sketch and the parameter values used are for illustrative purposes only since the parameters may change as the development of the IS system is finalized.

Referring to Fig. 4, as the reflection from the ground objects passes through a narrow slit, it strikes the incident surface of the prism. The prism separates the incident light into spectral images projected onto an area array of light detectors. The area array consists of 384 linear arrays, corresponding to 384 spectral channels. Each linear array consists of 384 detector elements. Each detector element corresponds to an image area of 30m x 30m on the ground. Hence, each linear array corresponds to an image of a specific spectral channel of the same ground area of 30m (along-track) x 11520m (cross-track). This 11.52 km cross-track measure, referred to as the swath width, defines the IS coverage for each orbit pass. The 30m x 30m area is referred to as the pixel (picture element) which defines the resolution of the image, i.e., within this element no features can be resolved. The IS is operated based on a push broom principle. That is, a specific detector element on the instrument (moving with the spacecraft) collects photons from the 30m x 30m moving window for a specific period



KEY PARAMETERS

- LINE TIME = 4 ms
- SPECTRUM:
 - VISIBLE REGION 0.4 - 1.0 μm
 - SHORT WAVELENGTH IR 1.0 - 2.5 μm
- FIELD OF VIEW: $\pm 0.825^\circ$
- SWATH WIDTH: 11.52 km
- POINTING ANGLES (RANGE):
 - $\pm 45^\circ$ (PITCH)
 - $\pm 20^\circ$ (ROLL)
- ORBIT: 400 km CIRCULAR

DEFINITIONS

- GIFOV (GROUND INSTANTANEOUS FIELD OF VIEW): PROJECTION ON THE GROUND OF EACH SQUARE DETECTOR
- LINE TIME: TIME TO MOVE ALONG THE GROUND A DISTANCE OF 1 GIFOV
- DN: DATA NUMBER, THE "BRIGHTNESS" OF THE ASSOCIATED PIXEL
- PIXEL: PICTURE ELEMENT (30 m x 30 m HERE, IT DEFINES THE RESOLUTION OF THE IMAGE)
- SWIR: SHORT WAVE LENGTH IR
- VIS: VISIBLE WAVE LENGTH

Figure 4. Imaging Spectrometer Basic Operating Principles

of time called the line time. The line time in this case is the time required to move 30m along-track, which is determined by the orbit. For a 400 km circular orbit, it is about 4 ms. At the end of each line time, the total number of photons collected by each detector is recorded and processed and the detector/register is reset and a new 4 ms cycle is repeated. For digital processing, the amount of light collected by a collector is assigned a number called DN (data number), which is proportional to the number of photons accumulated. Therefore, the processor has to record and process 384×384 , or approximately 1.475×10^5 DN's every 4 ms.

As the shuttle flies over an area, banks after banks of DN's are collected. By spacing the banks of DN's 30m apart, the features of the ground image emerge. The ratios of the DN's of various channels for the same area are of special interest, as these ratios are closely correlated to geological and ecological states of the earth including mineral deposits, forestry, crops,

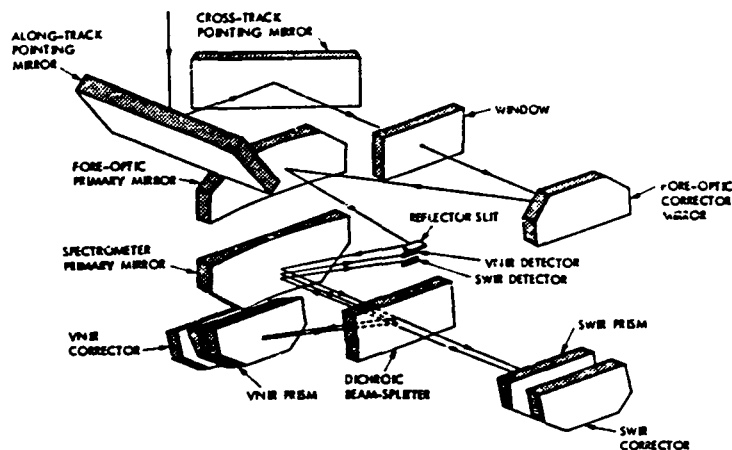


Figure 5. Shuttle Imaging Spectrometer Optical System Configuration

disease and insect infestations, land and soil erosion, precipitation in the form of snow and ice, air and water quality, etc.

The Imaging Spectrometer optical system configuration and the arrangement of lenses, mirrors, slit, prisms, focal plane detector, etc. are shown in Fig.5 [2]. Additional information on the instrument design and requirements can be found in Refs. 2 and 3.

III. ERROR SOURCES AND SHUTTLE ATTITUDE DYNAMICS

A. ERROR SOURCES

There are two types of errors that are important to the imaging spectrometer experiment — the direct errors and the derived errors. The direct errors are those pertaining to the spacecraft and instrument pointing, ephemeris, and instrument errors as shown in Fig. 6. The derived errors are the geometric errors and pattern distortions of ground objects which result from the direct errors and the effects of earth rotation, curvature, oblateness, and local vertical uncertainties. Fig. 1 shows relationships of the error sources and the system dynamics.

- ATTITUDE DEVIATIONS, RATE ERRORS AND STRUCTURAL VIBRATION
- MEASUREMENT NOISE AND DRIFT
- MISALIGNMENT OF SHUTTLE IMU AND ATTITUDE REFERENCE FRAME
- MISALIGNMENT BETWEEN SHUTTLE AND IS INSTRUMENT REFERENCES
- EPHEMERIS PREDICTION ERRORS
- EARTH ROTATION, CURVATURE, AND OBLATENESS
- IS INSTRUMENT ERRORS INCLUDING
 - OPTICAL JITTER
 - NONLINEARITIES
 - PROCESSING
 - REPEATABILITY

Figure 6. Error Sources

In this section, the steady state analysis is performed in the frequency domain. The PSD (power spectral density) for the pointing errors and the rate errors were obtained by considering the dynamics of the Space Shuttle Orbiter, the IMU (Inertial Measurement Unit), the attitude control state estimator, the measurement noise, the misalignment of reference frames, and the disturbances including gravity gradient, gyroscopic torques, and aerodynamic drag torques.

B. SHUTTLE MASS PROPERTY

The shuttle mass properties employed here were obtained from the Shuttle Operational Data Book [4] for OV-102/STS-3. The mass and the c.m. are, respectively:

Shuttle mass: 102,153.73 kg (224,738.21 lbs)

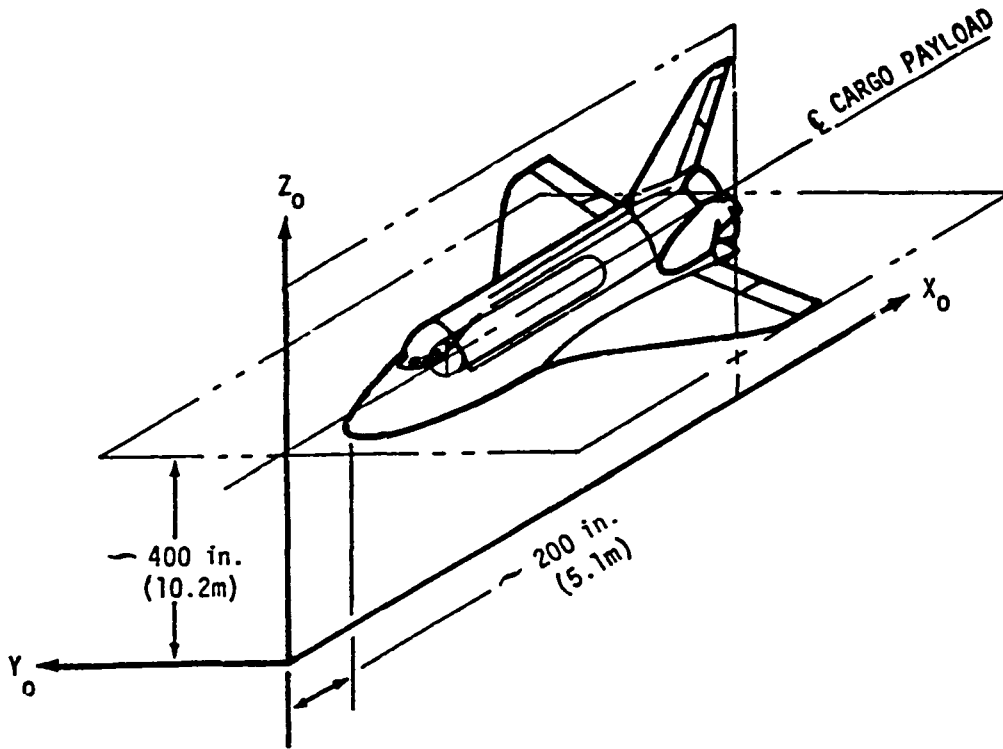
Shuttle c.m.: $X_o = 1105.5"$, $Y_o = 0$, $Z_o = 374.3"$

where the coordinates X_o , Y_o , and Z_o are the Orbiter Coordinates [5] defined in Figure 7. The moment of inertia matrix (kg-m^2) is, in the shuttle body frame (refer to Fig. 8),

$$I_B = \begin{bmatrix} 1.36 \times 10^6 & -4.69 \times 10^3 & -3.49 \times 10^5 \\ -4.69 \times 10^3 & 1.00 \times 10^7 & -3.32 \times 10^3 \\ -3.49 \times 10^5 & -3.32 \times 10^3 & 1.05 \times 10^7 \end{bmatrix}$$

$$= \begin{bmatrix} 1.36 \times 10^6 & 0 & -3.49 \times 10^5 \\ 0 & 1.00 \times 10^7 & 0 \\ -3.49 \times 10^5 & 0 & 1.05 \times 10^7 \end{bmatrix} \quad (i)$$

The magnitude of the off-diagonal terms of the inertia matrix I_B suggests strong couplings exist especially between the X_B and the Z_B -axis. To simplify the dynamic equations, it is convenient to consider principal-axis



ORBITER COORDINATES

TYPE: Rotating, Orbiter referenced

ORIGIN: Approximately 200 inches (5.1m) ahead of the nose and approximately 400 inches (10.2m) below the centerline of the cargo bay

ORIENTATION AND LABELING:

The X-axis is parallel to the centerline of the cargo bay, negative in the direction of launch

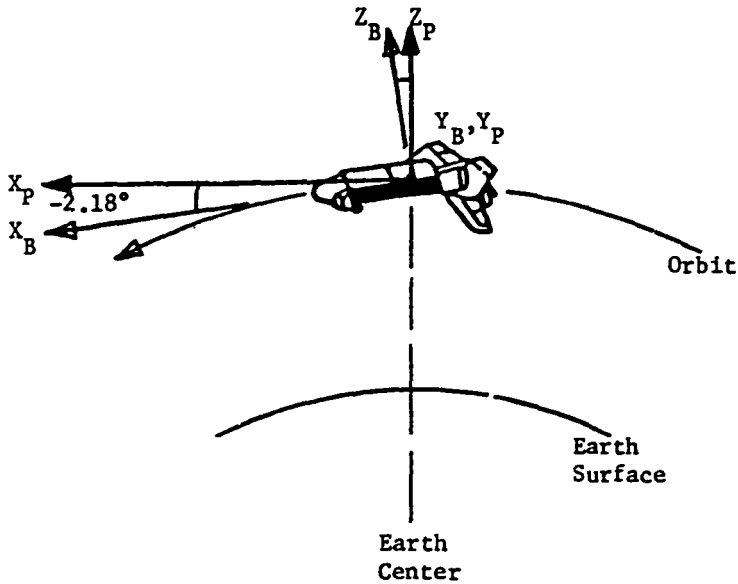
The Z-axis is positive upward in landing attitude

The Y-axis completes the right-handed system

The standard subscript is 0

Figure 7. Orbiter Coordinate System

pointing instead of body-axis pointing, e.g., pointing $-Z_P$ rather than $-Z_B$ as illustrated in Figure 8.



(X_B, Y_B, Z_B) -- Shuttle Body Coordinates

(X_P, Y_P, Z_P) -- Shuttle Principal Axes

Figure 8. Shuttle Body and Principal Coordinates
(Payload-Bay Nadir Pointing Shown)

The orientations of the principal axes can be determined by rotating the body frame an angle α about the Y_B -axis, i.e., let $B: X_B \rightarrow X_P$, then

$$B = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad (2)$$

and

$$I_P = B I_B B^T \quad (3)$$

It can be shown that

$$\alpha = \frac{1}{2} \tan^{-1} \left(\frac{-2 I_{BXZ}}{I_{BZZ} - I_{BXX}} \right) \quad (4)$$

provided that $I_{BXY} = I_{BYZ} = 0$.

For $I_{BXX} = 1.36 \times 10^6 \text{ kg-m}^2$, $I_{BZZ} = 1.05 \times 10^7 \text{ kg-m}^2$, and $I_{BXZ} = 3.49 \times 10^5 \text{ kg-m}^2$, the angle $\alpha = -2.18^\circ$. Therefore, in order to do principal-axis nadir pointing, the shuttle has to rotate 2.18° about the orbit normal (see Figure 8). The moments of inertia about the principal axes are

$$I_p = \text{diag} (1.38 \times 10^6, 1.00 \times 10^7, 1.05 \times 10^7), \text{ kg-m}^2.$$

C. ASSESSMENT OF DISTURBANCES

The shuttle motion is characterized in part by the environmental disturbances, the major sources of which are the aerodynamic drag, gravity gradient, gyroscopic effect, solar radiation, and on-board causes such as astronaut activities, equipment vibrations, and venting. On-board activities may be partially eliminated or reduced through mission planning and their effects will be assessed in the future. In this subsection, the gyroscopic torques, the gravity gradient torques, and the aerodynamic drag torques are estimated. The solar pressure is at least one order of magnitude less than the aerodynamic drag forces and, hence, it is not included in this analysis.

C.1 The Aerodynamic Drag Torques

To estimate the aerodynamic drag torques, an approach referred to as the three-plate model [6] has been used. Referring to Figure 9, let \bar{n}_1 , \bar{n}_2 , and \bar{n}_3 be the unit normal vectors for the equivalent plates 1, 2, and 3.

Where \bar{n}_1 , \bar{n}_2 , and \bar{n}_3 are in the direction of X_B , Y_B , and Z_B -axis, respectively.

To express \bar{n}_1 in body frame,

$$\bar{n}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \bar{n}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \bar{n}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (5)$$

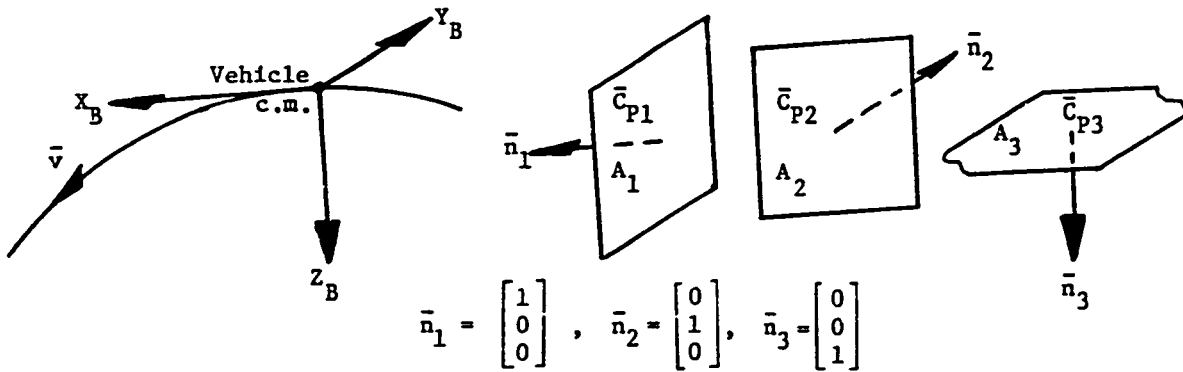


Figure 9. The Three-Plate Aerodynamic Model in Body Frame

Let A_1 , A_2 , and A_3 be the corresponding areas on which the aerodynamic pressure applies, and \bar{C}_{P1} , \bar{C}_{P2} , and \bar{C}_{P3} the three corresponding centers of pressure, respectively. The model assumes that the aerodynamic force applies to each area in the direction opposite to the vehicle velocity vector and assumes no shading among the plates. It is further assumed that the drag coefficient C_D is constant and the lift coefficient C_L is zero.

Let $\bar{v}_B = (v_{B1}, v_{B2}, v_{B3})^T$ be the inertial velocity vector in the body frame, with magnitude v . The force and torque applied on A_1 are

$$\bar{F}_{B1} = - \left(\frac{1}{2} C_D \rho v^2 A_1 \right) |(\bar{n}_1, \bar{u}_B)| \bar{u}_B \quad (6)$$

$$\bar{T}_{B1} = \bar{r}_{B1} \times \bar{F}_{B1} = - \left(\frac{1}{2} C_D \rho v^2 A_1 \right) |(\bar{n}_1, \bar{u}_B)| \bar{r}_{B1} \times \bar{u}_B \quad (7)$$

where ρ is the atmospheric density at the orbital altitude, $\bar{u}_B = \bar{v}_B/v$ the unit velocity vector, and \bar{r}_{B1} the vectors of the center of pressure of A_1 relative to the vehicle center of mass.

The total aerodynamic forces and torques on the vehicle are

$$\bar{F}_B = \sum_{i=1}^3 - \left(\frac{1}{2} C_D \rho v^2 A_i \right) | (\bar{n}_i, \bar{u}_B) | \bar{u}_B \quad (8)$$

$$\bar{T}_B = \sum_{i=1}^3 - \left(\frac{1}{2} C_D \rho v^2 A_i \right) | (\bar{n}_i, \bar{u}_B) | \bar{r}_{Bi} \times \bar{u}_B \quad (9)$$

The value of C_D depends on the shape of the vehicle. In Ref. 7, the values of 2.5 to 3.0 were suggested. The value of $C_D = 2.0$ is used here as it was used in Ref. 6 for Space Shuttle Simulation.

The atmospheric mass density ρ can be found in a JPL internal memorandum. For 1985 mission time, the peak density is expected to occur in April for the 400 km orbit; the densities are

$$2.64 \times 10^{-12} \text{ kg/m}^3 \quad (\text{predicted})$$

and

$$3.77 \times 10^{-12} \text{ kg/m}^3 \quad (97.7 \text{ percentile})$$

The orbital velocity v for a 400 km circular orbit is computed as $v = 7669.60 \text{ m/sec}$ or $v^2 = 5.882 \times 10^7 \text{ (m/sec)}^2$.

The plate areas, according to the attachment (SSFS On-Orbit Aero Data, 7/24/80) to Ref. 6, with Cargo Bay doors open, are

$$\begin{aligned} A_1 &= 119.45 \text{ m}^2 \\ A_2 &= 229.92 \text{ m}^2 \\ A_3 &= 454.46 \text{ m}^2 \end{aligned} \quad (10)$$

However, a different set of values are given in Ref. 8.

$$\begin{aligned} A_1 &= 64.1 \text{ m}^2 \\ A_2 &= 212.7 \text{ m}^2 \\ A_3 &= 367.0 \text{ m}^2 \end{aligned} \quad (11)$$

The values in (10) were used in this work.

The position vectors \bar{r}_{Bi} may be obtained using data given in Ref. 4 and Ref. 6. The values are,

$$\begin{aligned} \bar{r}_{B1} &= \begin{bmatrix} 3.797 \\ 0 \\ -.231 \end{bmatrix} \text{ m} \\ \bar{r}_{B2} &= \begin{bmatrix} .927 \\ 0 \\ -.742 \end{bmatrix} \text{ m} \\ \bar{r}_{B3} &= \begin{bmatrix} 1.166 \\ 0 \\ -1.074 \end{bmatrix} \text{ m} \end{aligned} \quad (12)$$

The torques \bar{T}_{Bi} for the predicted density, are

$$\begin{aligned} \bar{T}_{B1} &= -1.855 \times 10^{-2} |u_{B1}| \begin{bmatrix} -r_{B13}u_{B2} + r_{B12}u_{B3} \\ r_{B13}u_{B1} + r_{B11}u_{B3} \\ -r_{B12}u_{B1} + r_{B11}u_{B2} \end{bmatrix} \\ \bar{T}_{B2} &= -3.570 \times 10^{-2} |u_{B2}| \begin{bmatrix} -r_{B23}u_{B2} + r_{B22}u_{B3} \\ r_{B23}u_{B1} - r_{B21}u_{B3} \\ -r_{B22}u_{B1} + r_{B21}u_{B2} \end{bmatrix} \\ \bar{T}_{B3} &= -7.057 \times 10^{-2} |u_{B3}| \begin{bmatrix} -r_{B33}u_{B2} + r_{B32}u_{B3} \\ r_{B33}u_{B1} - r_{B31}u_{B3} \\ -r_{B32}u_{B1} + r_{B31}u_{B2} \end{bmatrix} \end{aligned} \quad (13)$$

In Eq. (13), the torques are known once the unit velocity vector \bar{u}_B is specified; \bar{u}_B varies with the pointing configuration and spacecraft attitude.

C.1.1. Drag Torques for Configuration A

Referring to Fig. 2, since this is "-Z_p" pointing, for nominal attitude, the vehicle moves in the X_p direction. Let \bar{u}_P be the unit velocity vector in the P-frame and let A_α be the rotation matrix due to α, then

$$\begin{aligned} \bar{u}_B &= A_\alpha^T \bar{u}_P = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \cos\alpha \\ 0 \\ -\sin\alpha \end{bmatrix} \end{aligned} \quad (14)$$

Since α (= -2.18° = -.038 rad) is small,

$$\bar{u}_B \approx \begin{bmatrix} 1 \\ 0 \\ -\alpha \end{bmatrix} \quad (15)$$

Using Eqs. (12), (13), and (15), and

$$\bar{T}_P = \sum_{i=1}^3 A_\alpha \bar{T}_{Bi} \quad (16)$$

the torques in the P-Frame, for nominal attitude, are

$$\bar{T}_P = \begin{bmatrix} 0 \\ 9.84 \times 10^{-3} \\ 0 \end{bmatrix} \quad \text{N-m} \quad (17)$$

For small attitude errors, φ, θ, and ψ, from the rotating orbital frame,

$$\bar{u}_P = A \begin{bmatrix} 1 \\ 0 \\ c \end{bmatrix} \quad (18)$$

where,

$$A = \begin{bmatrix} 1 & \psi & -\theta \\ -\psi & 1 & \phi \\ \theta & -\phi & 1 \end{bmatrix}$$

and

$$\bar{u}_B = A_{\alpha}^T A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

Using Eqs. (12), (13), and (20), the predicted aerodynamic torques become, by retaining only the first order terms, in N-m,

$$\begin{aligned} \bar{T}_P &= \sum_{i=1}^3 A_{\alpha} \bar{T}_{Bi} \quad (21) \\ &= \begin{bmatrix} 4.29 \times 10^{-3} \psi \\ 4.29 \times 10^{-3} + 7.04 \times 10^{-2} (\theta - \alpha) + 7.58 \times 10^{-2} |\theta - \alpha| + 2.65 \times 10^{-2} |\psi| \\ 7.04 \times 10^{-2} \psi \end{bmatrix} \end{aligned}$$

C.1.2. Drag Torques for Configuration B

Referring to Figure 3, under this configuration, X_p is in the Nadir direction and Y_p is in the direction of motion for nominal attitude. In this case,

$$\bar{u}_P = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (22)$$

$$\bar{u}_B = A_{\alpha}^T \bar{u}_P = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (23)$$

and from Eqs. (12), (13), and (23),

$$\bar{T}_P = \sum_{i=1}^3 A_{\alpha} \bar{T}_{Bi}$$

$$= \begin{bmatrix} -3.96 \times 10^{-2} \\ 0 \\ -4.53 \times 10^{-2} \end{bmatrix} \text{ N-m} \quad (24)$$

In the presence of small attitude errors, ϕ , θ , and ψ , the aerodynamic drag torques are

$$\bar{T}_P = \begin{bmatrix} -2.45 \times 10^{-2} - 7.58 \times 10^{-2} |\phi| - 4.29 \times 10^{-3} |\phi| + 3.31 \times 10^{-2} \alpha \\ -3.31 \times 10^{-2} \phi + 2.65 \times 10^{-2} \psi \\ -(3.31 \times 10^{-2} + 8.23 \times 10^{-2} |\phi| + 7.04 \times 10^{-2} |\psi| + 2.65 \times 10^{-2} \alpha) \end{bmatrix} \quad (25)$$

C.1.3. Estimation of Disturbance PSD

Before the PSD's are estimated, the uncertainty part of the disturbance torques has to be determined. The torques of Eqs. (21) and (25) consist of static parts and the dynamic parts. The dynamic parts are functions of the attitude errors ϕ , θ , and ψ . The attitude errors are assumed to be random and time-varying with standard deviation of 1° (.01745 rad.) per axis. Therefore, the estimated values of the random disturbance torques are, for Configuration A

$$\bar{\sigma}_{T_{PA}} = \begin{bmatrix} 7.48 \times 10^{-5} \\ 1.86 \times 10^{-3} \\ 1.23 \times 10^{-3} \end{bmatrix} \text{ N-m} \quad (26)$$

and that for Configuration B are,

$$\bar{\sigma}_{T_{PB}} = \begin{bmatrix} 1.33 \times 10^{-3} \\ 7.40 \times 10^{-4} \\ 1.89 \times 10^{-3} \end{bmatrix} \text{ N-m} \quad (27)$$

and the corresponding PSD's are obtained by the following approximation with the correlation time of $T = 180$ seconds,

$$Q_{T_{PA}} = 2T (\bar{\sigma}_{T_{PA}}^2)$$

$$= \begin{bmatrix} 2.01 \times 10^{-6} \\ 1.25 \times 10^{-3} \\ 5.44 \times 10^{-4} \end{bmatrix} \quad (\text{N-m})^2 - \text{sec} \quad (28)$$

$$\begin{aligned} \bar{Q}_{TPB} &= 2T (\bar{\sigma}_{TPB}^2) \\ &= \begin{bmatrix} 6.32 \times 10^{-4} \\ 1.97 \times 10^{-4} \\ 1.29 \times 10^{-3} \end{bmatrix} \quad (\text{N-m})^2 - \text{sec} \quad (29) \end{aligned}$$

C.2 The Gravity Gradient Torques and Gyroscopic Torques

The gravity gradient torques and the gyroscopic torques can be estimated using the following equations, respectively:

$$\bar{T}_{GP} = 3\omega_o^2 \bar{u}_{RP} I_P \bar{u}_{RP} \quad (30)$$

and

$$\bar{T}_{GRP} = \bar{\omega}_{oP} \times \bar{H}_P = \bar{\omega}_{oP} I_P \bar{\omega}_{oP} \quad (31)$$

Where \bar{u}_{RP} and $\bar{\omega}_{oP}$ are the unit earth vector and the spin velocity vector, respectively, in principal frame, and \bar{u}_{RP} is the skew symmetric matrix of the vector \bar{u}_{RP} . For Configuration A,

$$\bar{u}_{RP} = A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (32a)$$

$$\bar{\omega}_{oP} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \omega_o \quad (32b)$$

and for Configuration B,

$$\underline{u}_{RP} = A \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad (33a)$$

$$\underline{\omega}_{oP} = A \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \omega_o \quad (33b)$$

the corresponding torques, to include only the first order effect, are, for Configuration A

$$\bar{T}_{gP} = 3 \omega_o^2 \begin{bmatrix} (I_{PZZ} - I_{PYY})\phi \\ (I_{PZZ} - I_{PXX})\theta \\ 0 \end{bmatrix} = \begin{bmatrix} 1.92 \phi \\ 33.11 \theta \\ 0 \end{bmatrix} \text{ N-m} \quad (34a)$$

$$\bar{T}_{GRP} = \omega_o^2 \begin{bmatrix} -(I_{PZZ} - I_{PYY})\phi \\ 0 \\ (I_{PYY} - I_{PXX})\psi \end{bmatrix} = \begin{bmatrix} -.64 \theta \\ 0 \\ 11.04 \psi \end{bmatrix} \quad (34b)$$

and for Configuration B

$$\bar{T}_{gP} = -3 \omega_o^2 \begin{bmatrix} 0 \\ (I_{PZZ} - I_{PXX})\theta \\ (I_{PXX} - I_{PYY})\psi \end{bmatrix} = \begin{bmatrix} 0 \\ -35.03 \theta \\ 43.71 \psi \end{bmatrix} \quad (35a)$$

$$\bar{T}_{GRP} = \omega_o^2 \begin{bmatrix} -(I_{PZZ} - I_{PYY})\phi \\ (I_{PZZ} - I_{PXX})\theta \\ 0 \end{bmatrix} = \begin{bmatrix} -.64 \phi \\ 11.68 \theta \\ 0 \end{bmatrix} \quad (35b)$$

Since the gravity gradient torques and gyroscopic torques are proportional to the attitude errors, they may be included in the equations of motion

as part of the vehicle dynamics rather than disturbances to the plant.

D. THE MISALIGNMENT AND MEASUREMENT NOISE

In this subsection, the errors that will contribute to pointing uncertainties are estimated. These error sources include the misalignment errors of the IMU and the IS instrument, and the sensor noise.

D.1 The Misalignment Errors

Let b_{MS} be the misalignment between the IMU and the shuttle reference frame; let b_{SI} be the misalignment between the shuttle and the imaging spectrometer reference frame.

Based on the space shuttle performance requirement [5], the IMU 3 σ misalignment uncertainty is $\pm .133^\circ$ /axis, hence,

$$\sigma_{bMS} = .044^\circ/\text{axis} = 159.6 \text{ arc-sec}/\text{axis} \quad (36a)$$

However, based on a JPL internal memorandum, the shuttle flight test performance is much better,

$$\sigma_{bMS} = 82 \text{ arc-sec}/\text{axis} \quad (36b)$$

The performance data Eq. (36b) was used in this report.

The misalignment data for the IS instrument is not directly available. Based on Ref. 9, the estimated LANDSAT D initial alignment bias between the sensor optical axis and the vehicle pointing vector was ± 200 arc-sec. The alignment bias can be measured before launch and can be removed from the image data. The variation part that cannot be removed without ground control points was estimated as ± 30 arc-sec. This latter number was used in this work,

$$\sigma_{bSI} = 30 \text{ arc-sec} \quad (37)$$

Therefore, the total misalignment error becomes

$$\begin{aligned} \sigma_{b_o} &= (\sigma_{bMS}^2 + \sigma_{bSI}^2)^{1/2} \\ &= 87.32 \text{ arc-sec.} \end{aligned} \quad (38)$$

D.2 The IMU Model

The sensor dynamics for the shuttle IMU was modeled approximately by the first order low pass filter as shown in Figure 10.

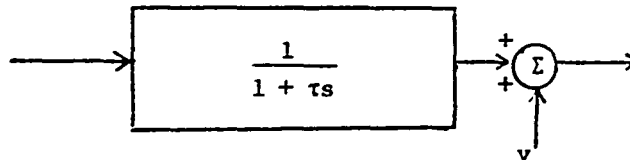


Figure 10. Simplified IMU Model

Where τ is the time constant and v the white gaussian noise. The time constant was estimated from a DRIRU II Model,

$$\tau = \frac{1}{\omega_s} = \frac{1}{14\pi} = .023 \text{ sec} \quad (39)$$

That is, this measurement is assumed to have a 7 Hz bandwidth.

To determine v , from Ref. 5, the 3σ IMU readout error is $\pm .073^c$ /axis. Assume that the measurement noise is the readout error, then

$$\sigma_v = .0243^o/\text{axis} = 87.6 \text{ arc-sec/axis} \quad (40)$$

However, the actual performance of the shuttle IMU was much better; it has a 1σ gyro resolution error (σ_{RESO}) of 20 arc-sec/axis. If we assume the random noise has the same amplitude as that of the resolution noise, then

$$\sigma_v = \sqrt{2} \sigma_{\text{RESO}} \quad (41)$$

The corresponding measurement noise PSD, R , is estimated as,

$$\begin{aligned} R &= 2 \left(\frac{1}{14\pi} \right) \sigma_v^2 \\ &= 8.55 \times 10^{-10} \text{ (rad)}^2 \text{-sec} \end{aligned} \quad (42)$$

E. SHUTTLE DYNAMICS AND INSTRUMENT POINTING ERRORS

E.1 The Equations of Motion

Consider Configuration A. For simplicity, the subscript P for principal axes is dropped from all equations. Again, let ϕ , θ , and ψ be the small roll, pitch, and yaw angles, respectively. Let $\bar{\omega} = (\omega_X, \omega_Y, \omega_Z)^T$ be the angular velocity vector of the shuttle, then for small attitude errors,

$$\begin{aligned}\dot{\omega}_X &= \dot{\phi} + \psi \omega_0 \\ \dot{\omega}_Y &= \dot{\theta} + \omega_0 \\ \dot{\omega}_Z &= \dot{\psi} - \phi \omega_0\end{aligned}\tag{43}$$

and

$$\begin{aligned}\ddot{\omega}_X &= \ddot{\phi} + \dot{\psi} \omega_0 \\ \ddot{\omega}_Y &= \ddot{\theta} \\ \ddot{\omega}_Z &= \ddot{\psi} - \dot{\phi} \omega_0\end{aligned}\tag{44}$$

With this simplified relation between the inertial rates and the attitude error rates, one can show that the equations of motion may be summarized as follows, accounting only for first order effects:

$$\begin{bmatrix} I_X \ddot{\phi} + (I_X - I_Y + I_Z) \omega_0 \dot{\psi} + 4 (I_Y - I_Z) \omega_0^2 \phi \\ I_Y \ddot{\theta} + 3 (I_X - I_Z) \omega_0^2 \theta \\ I_Z \ddot{\psi} - (I_X - I_Y + I_Z) \omega_0 \dot{\phi} - (I_X - I_Y) \omega_0^2 \psi \end{bmatrix} = \bar{T}_d + \bar{T}_c\tag{45}$$

The left-hand side of Eq. (45) accounts for the body dynamics and the gravity gradient and the gyroscopic torques; and on the right-hand side of Eq. (45) \bar{T}_d and \bar{T}_c are the disturbance torques and the control torques, respectively.

E.2 The Space Shuttle State Estimator

The Shuttle State Estimator consists of two parallel Kalman-type filters, one for acceleration estimation and one for rate estimation [10]. The attitude estimate is the extrapolation of the measured attitude and the rate estimate may be approximated by using a second order filter with parameters

determined by the filter gains. With the current baseline filter data, the equivalent damping coefficient and the corner frequency for the rate filter are .8 and .04 Hz (for vernier rate filter), respectively [11].

E.3 The System Pointing Errors

E.3.1. The Pitch Loop (Configuration A)

From Eq. (45) and ignoring the initial conditions, one has the following output function,

$$\theta(S) = \frac{1}{I_Y S^2 + 3\omega_o^2 (I_X - I_Z)} (T_{dY} + T_{cY}) \quad (46)$$

Figure 11 shows the open-loop block diagram for the dynamics of the instrument pointing error excited by the random disturbances. Included in the diagram are the dynamics of the vehicle, the IMU, and the rate filter. Measurement error and the misalignment error are also included.

E.3.2. The Roll and Yaw Loops (Configuration A)

The output equations for the coupled roll and yaw dynamics are, from Eq. (45)

$$\begin{bmatrix} \phi(S) \\ \psi(S) \end{bmatrix} = \frac{1}{D(S)} \begin{bmatrix} I_Z S^2 + \omega_o^2 (I_Y - I_X) & -\omega_o (I_X - I_Y + I_Z) S \\ \omega_o (I_X - I_Y + I_Z) S & I_X S^2 + 4\omega_o^2 (I_Y - I_Z) \end{bmatrix} \begin{bmatrix} T_X \\ T_Z \end{bmatrix} \quad (47)$$

where

$$D(S) = I_X I_Z S^4 + \omega_o^2 (I_X I_Z + I_Y^2 + 3I_Y I_Z - 3I_Z^2) S^2 + 4\omega_o^4 (I_Y - I_X) (I_Y - I_Z) \quad (48)$$

and

$$T_X = T_{cX} + T_{dX} \quad (49a)$$

$$T_Z = T_{cZ} + T_{dZ} \quad (49b)$$

Figure 12 shows the block diagram for the roll and yaw instrument error dynamics. The block diagrams for the pitch and the roll-yaw loops are similar except for

the coupling terms for the roll and yaw axes.

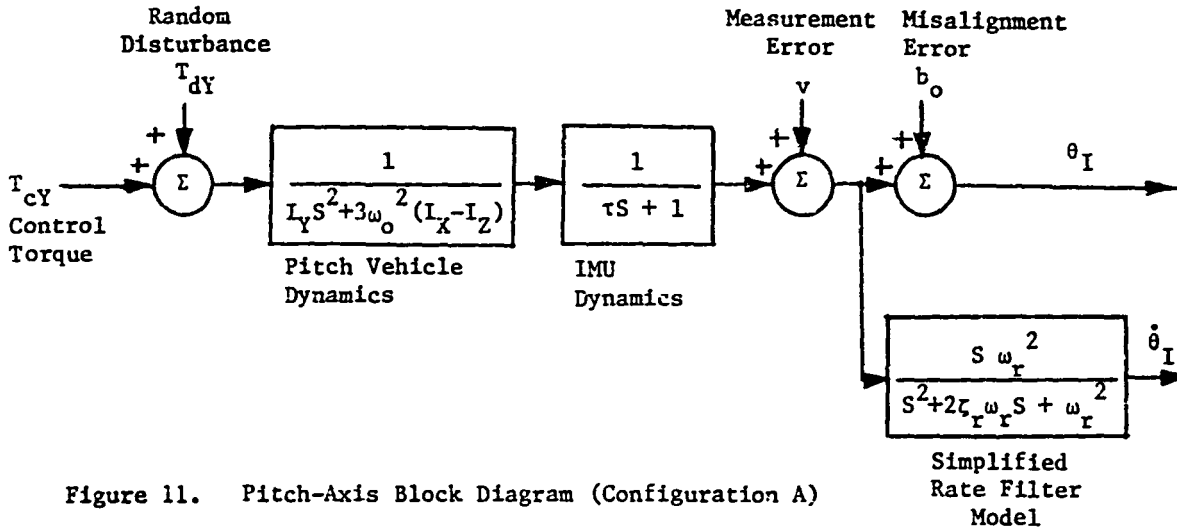


Figure 11. Pitch-Axis Block Diagram (Configuration A)

E.4 The Instrument Pointing Error PSD

E.4.1 The Pitch Error PSD (Configuration A)

The PSD's for the pitch-axis instrument pointing error and the rate error are,

$$P_{\theta}(\omega) = F_{\theta}^*(\omega) F_{\theta}(\omega) Q_Y + R + \sigma_{b_o}^2 \delta(\omega) \quad (50)$$

$$P_{\dot{\theta}}(\omega) = (F_{\theta}^*(\omega) F_{\theta}(\omega) Q_Y + R) H_r^*(\omega) H_r(\omega) \quad (51)$$

where $Q_Y = Q_{T_{PA}^Y}$ and $Q_{T_{PA}^Y}$ is the second component of Eq. (28), and where R , and σ_{b_o} are defined in preceding sections, and $\delta(\omega)$ is the Dirac delta function; and where,

$$F_{\theta}(\omega) = G_{\theta}(\omega) H(\omega) \quad (52a)$$

$$G_{\theta}(\omega) = \left[\frac{1}{I_Y S^2 + 3 \omega_o^2 (I_X - I_Z)} \right]_{S=j\omega} \quad (52b)$$

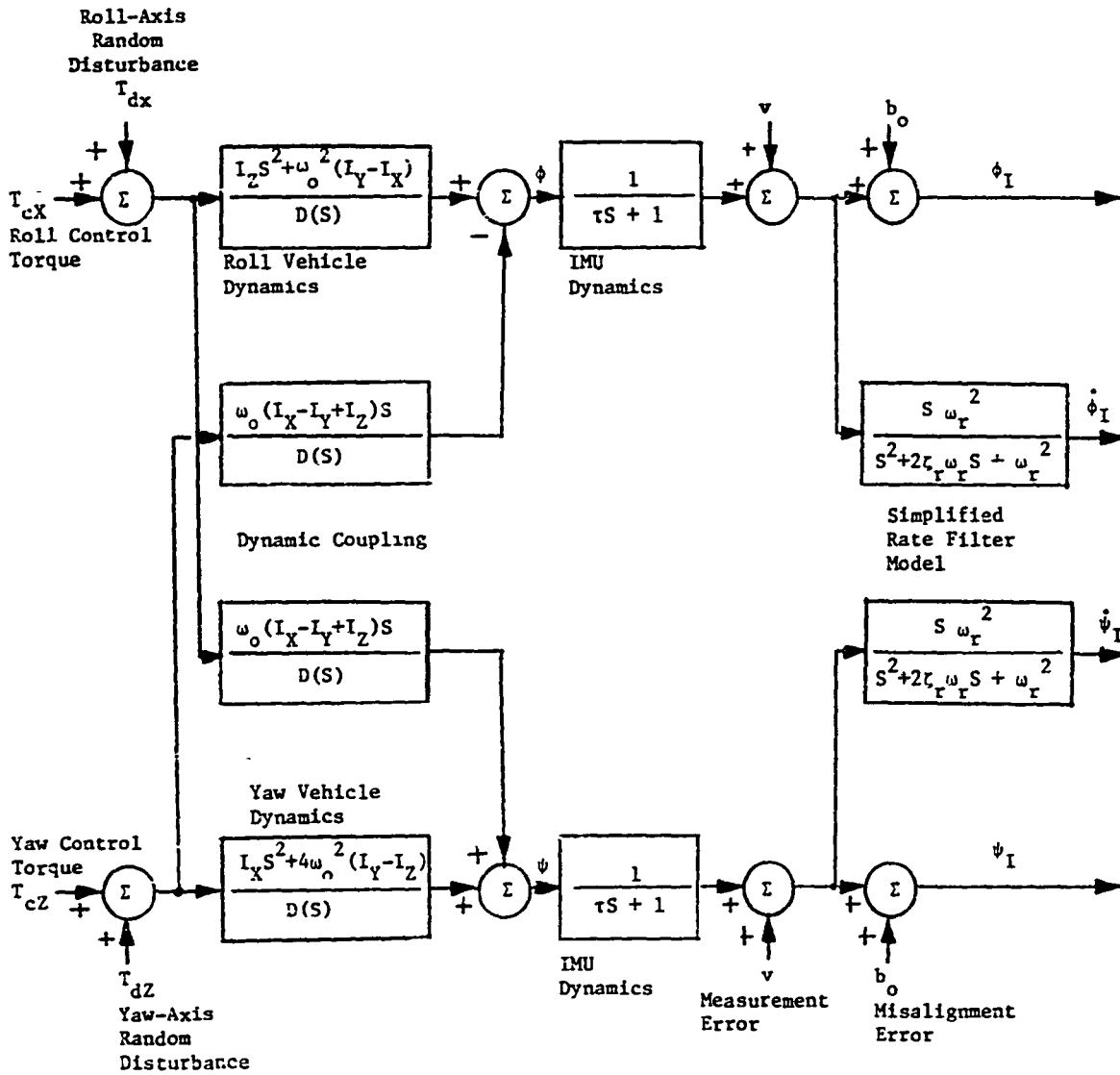


Figure 12. Roll and Yaw Axes Block Diagram. (Configuration A)

$$H(\omega) = \left[\frac{1}{\tau S + 1} \right]_{S=j\omega} \quad (52c)$$

$$F_{\theta}^*(\omega) = F_{\theta}(-\omega) \quad (52d)$$

$$H_r(\omega) = \left[\frac{S \omega_r^2}{S^2 + 2\zeta_r \omega_r S + \omega_r^2} \right]_{S=j\omega} \quad (52e)$$

The output power spectral density for the system is closely related to the frequency response of the system, which characterizes the steady state dynamics of the system, and, hence, it is meaningful only if the system is stable. Unfortunately, in Eq. (52b), $I_x < I_z$ which implies that the system is unstable. The destabilizing term comes from the gravity gradient because the nadir pointing axis is not the axis of minimum inertia. To proceed, one has to consider the gravity gradient as external disturbance rather than a part of the dynamics. Figure 13 shows the instrument pointing error PSD in $(\text{rad})^2\text{-sec}$ as a function of frequency in Hz. Figure 14 shows the rate error PSD in $(\text{rad/sec})^2\text{-sec}$ as a function of frequency.

E.4.2. The Roll and Yaw Error PSD (Configuration A)

The instrument pointing error PSD's for the roll and yaw axes are,

$$P_{\phi}(\omega) = F_{\phi}^*(\omega) F_{\phi}(\omega) Q_x + F_{\phi\psi}^*(\omega) F_{\phi\psi}(\omega) Q_z + R + \sigma_{bo}^2 \delta(\omega) \quad (53a)$$

$$P_{\psi}(\omega) = F_{\psi}^*(\omega) F_{\psi}(\omega) Q_z + F_{\phi\psi}^*(\omega) F_{\phi\psi}(\omega) Q_x + R + \sigma_{bo} \delta(\omega) \quad (53b)$$

and the instrument pointing rate error PSD's for the roll and yaw axes are,

$$P_{\dot{\phi}}(\omega) = [F_{\phi}^*(\omega) F_{\phi}(\omega) Q_x + F_{\phi\psi}^*(\omega) F_{\phi\psi}(\omega) Q_z + R] H_r^*(\omega) H_r(\omega) \quad (54a)$$

$$P_{\psi}(\omega) = [F_{\psi}^*(\omega) F_{\psi}(\omega) Q_z + F_{\phi\psi}^*(\omega) F_{\phi\psi}(\omega) Q_x + R] H_r^*(\omega) H_r(\omega) \quad (54b)$$

where

$$F_{\phi}(\omega) = G_{\phi}(\omega) H(\omega) \quad (55a)$$

$$G_{\phi}(\omega) = \left[\frac{I_z S^2 + \omega_o^2 (I_y - I_x)}{D(S)} \right]_{S=j\omega} \quad (55b)$$

$$F_{\phi\psi}(\omega) = G_{\phi\psi}(\omega) H(\omega) \quad (56a)$$

$$G_{\phi\psi}(\omega) = \left[\frac{\omega_o (I_x - I_y + I_z)}{D(S)} \right]_{S=j\omega} \quad (56b)$$

$$F_{\psi}(\omega) = G_{\psi}(\omega) H(\omega) \quad (57a)$$

$$G_{\psi}(\omega) = \left[\frac{I_x S^2 + 4\omega_o^2 (I_y - I_z)}{D(S)} \right]_{S=j\omega} \quad (57b)$$

and Q_x and Q_z are the first and the third components of $\bar{Q}_{T_{PA}}$ in Eq. (28).

For the same reason discussed earlier, the roll and yaw dynamics are also unstable (referring to Eq. (48), $D(s)$ has roots in the right-half complex plane). The PSD's for this case are obtained again by treating gravity gradient and gyroscopic torques as disturbances. Figures 13 and 14 show the roll and yaw power spectral densities.

E.4.3. The Error PSD's for Configuration B

The stability problem may be resolved by reorienting the shuttle from payload-bay nadir (Configuration A) to nose-down nadir (Configuration B). The advantage of this new configuration is that it is a gravity gradient stabilized system. However, there are drawbacks for this configuration. First, it will require a larger support-tower for the IS instrument, and the second drawback is that the aerodynamic forces and torques have increased significantly over the payload-bay nadir case.



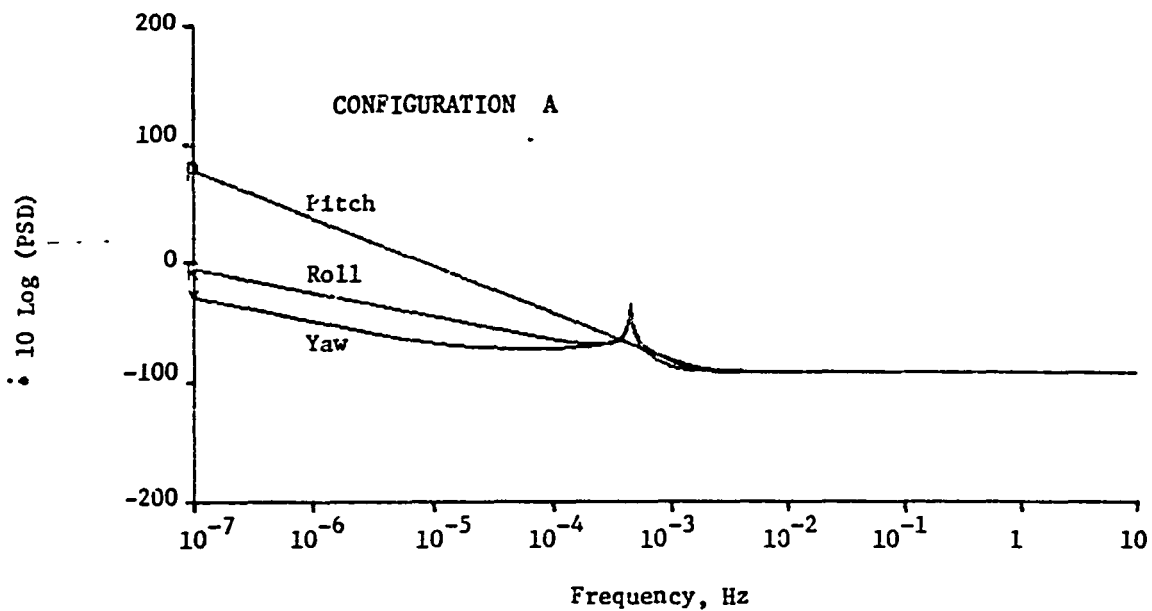


Figure 13. Instrument Pointing Error PSD, $(\text{rad})^2\text{-sec}$

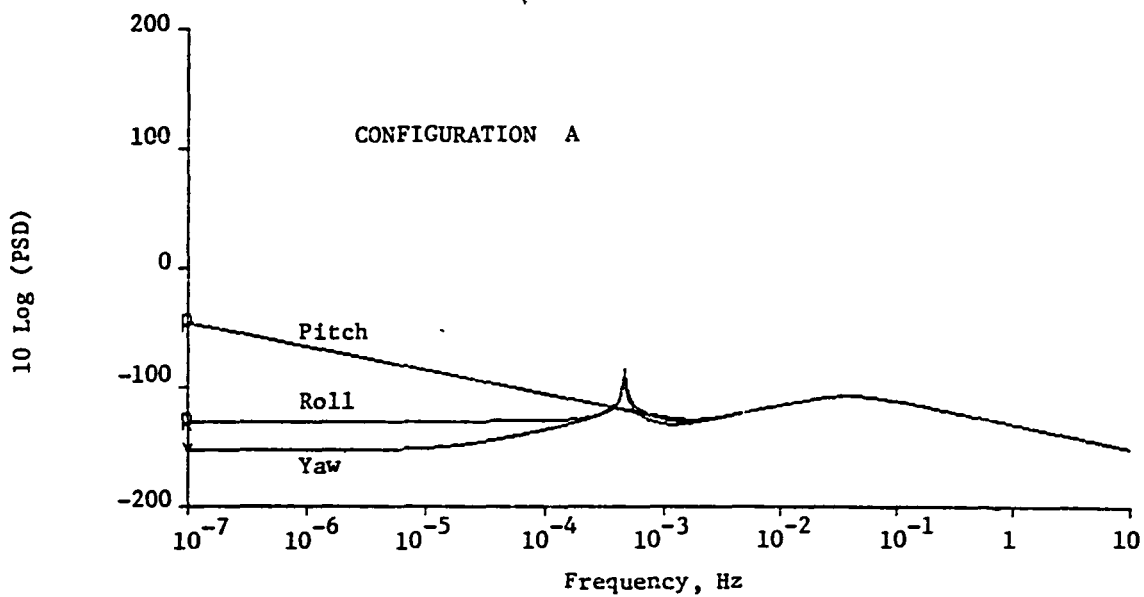


Figure 14. Instrument Pointing Rate Error, $(\text{rad/sec})^2\text{-sec}$

To obtain the error power spectral density for Configuration B, it is only necessary to replace $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_A$ and the corresponding notations of the equations in this section by $\begin{bmatrix} X \\ Y \\ Z \\ -X \end{bmatrix}_B$. That is, for instance, Eq. (52b) becomes,

$$G_{\theta}(\omega) = \left[\frac{1}{I_Z S^2 + 3 \omega_o^2 (I_Y - I_X)} \right]_{S=j\omega} \quad (58)$$

where $G_{\theta}(\cdot)$ here still represents the pitch dynamics. Since $I_Y > I_X$, Eq. (58) is stable.

The instrument pointing error PSD's are shown in Figure 15 and the rate error PSD's are illustrated in Figure 16.

The computer programs that are used for generating these results are included in Appendices C and D.

F. SUMMARY OF MAJOR FINDINGS

F.1 The Major Environmental Disturbances

The major disturbance sources that are modeled are the aerodynamic drag torques, the gravity gradient torques, and the gyroscopic torques. The unmodeled disturbances are the solar pressure torques, the on-board equipment vibrations, crew motions, and venting. Table 2 shows the static disturbance torques in N-m and the stochastic torque PSD's in $(N-m)^2$ -sec for both Configurations A and B with a circular orbit of 400 km altitude.

F.2 The Measurement Uncertainties

The modeled measurement uncertainties are summarized in Table 3.

F.3 Ground Track Errors and Navigation Uncertainties

The power spectral densities for the ground track errors and the rate errors due to instrument pointing uncertainties for Configurations A and B are shown in Figure 17. It is important to note that strong resonances occur within

the frequency band of 10^{-5} Hz to 10^{-3} Hz. The ground track errors at higher frequencies, .01 Hz and above, are dominated by the measurement noise. Recall that the 1σ measurement noise is 28.28 arc-sec/axis. The corresponding ground track error is 54.84 m/axis or 77.56 m/lateral motion.

The 3σ ground track errors due to navigational uncertainties are about one order of magnitude greater than the attitude errors with the aid of TDRSS; and the error is greater with STDN as indicated in Table 4 [12]. Note that the values tabulated in Table 3 are the 3σ rms values.

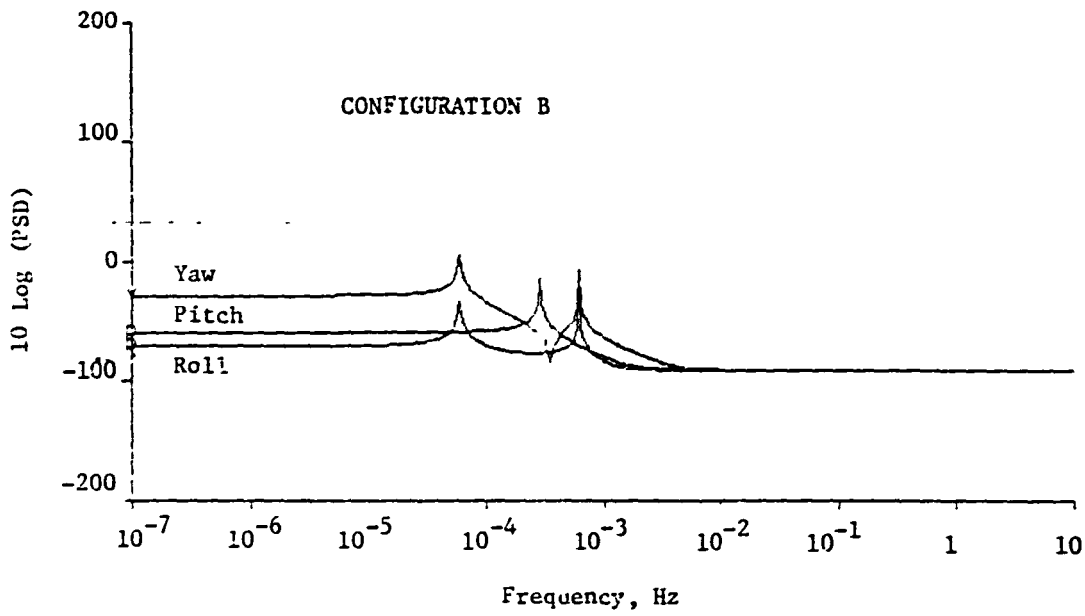


Figure 15. Instrument Pointing Error PSD, $(\text{rad})^2\text{-sec}$

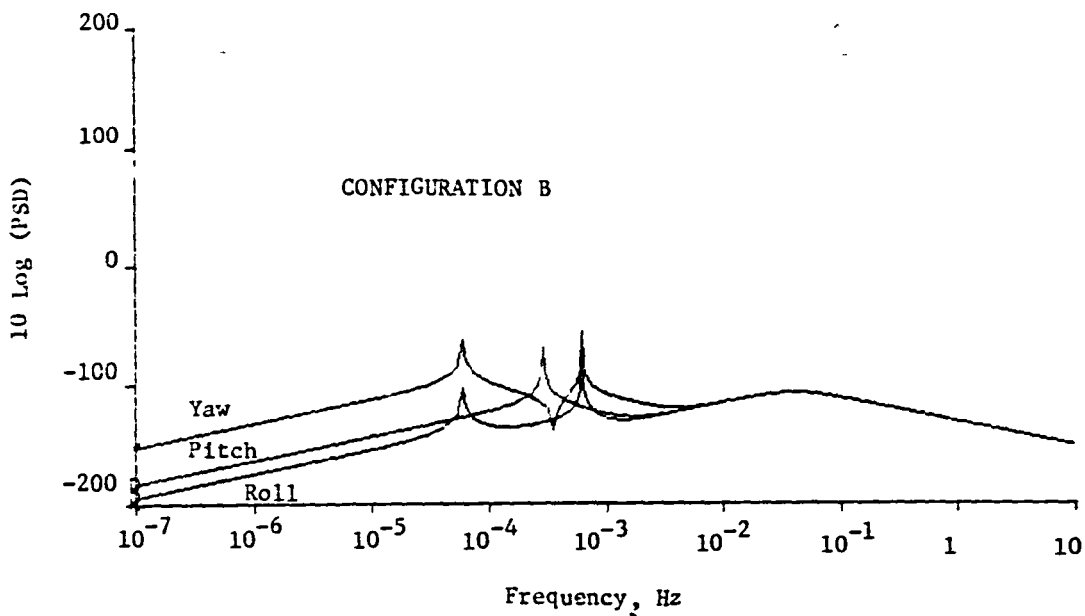


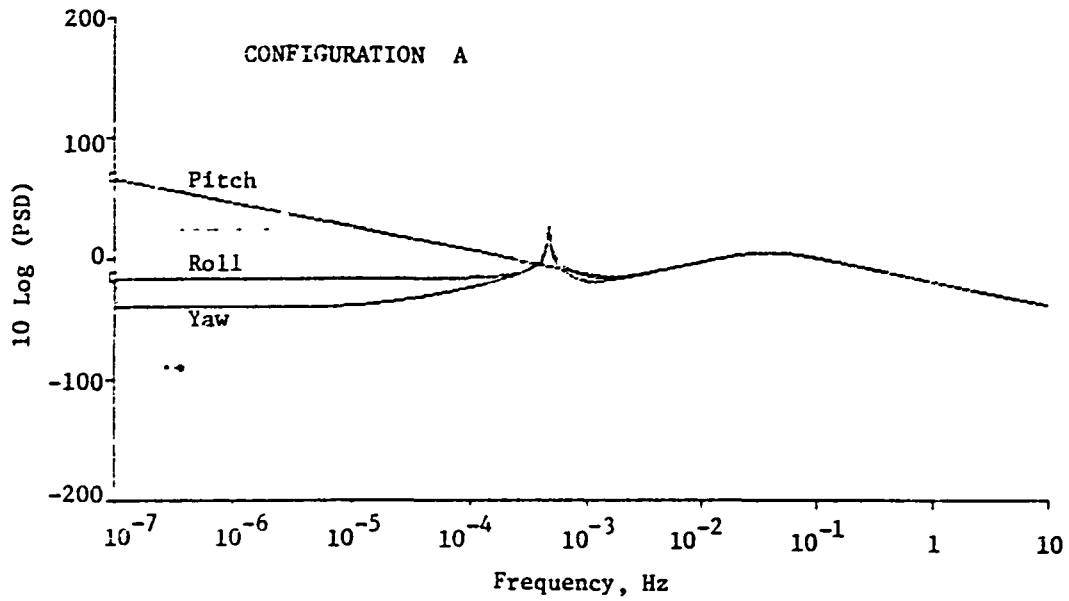
Figure 16. Instrument Pointing Rate Error, $(\text{rad}/\text{sec})^2\text{-sec}$

Table 2. Modeled Environmental Disturbances

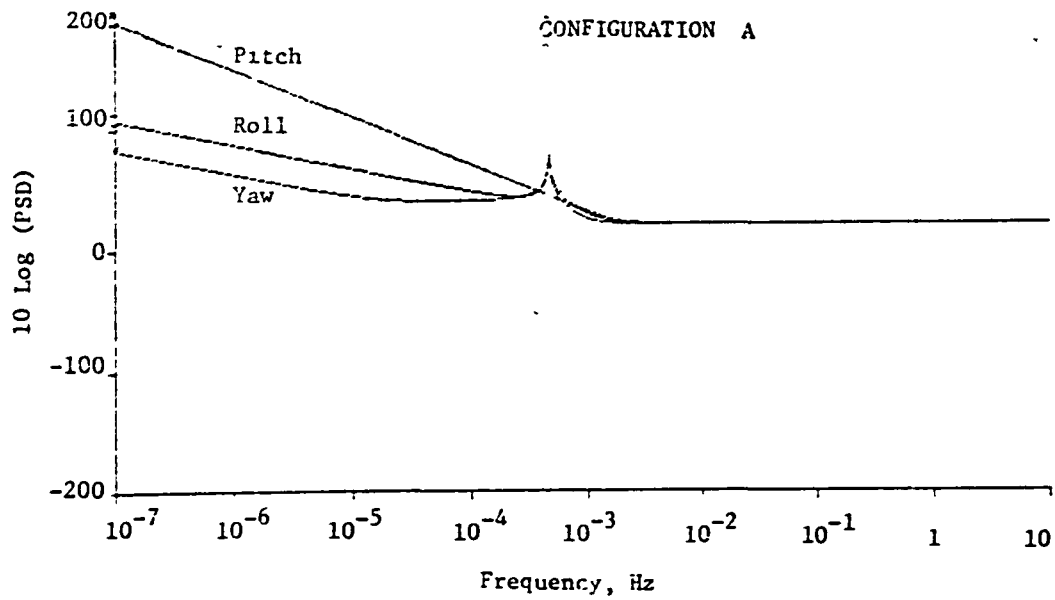
Sources		Static, N-m			Stochastic -- PSD, (N-m) ² -sec		
		X _P	Y _P	Z _P	X _P	Y _P	Z _P
Aerodynamic Drag Torques	Config. A	0	$-.68 \times 10^{-2}$	0	2×10^{-6}	1.2×10^{-3}	5.4×10^{-4}
	Config. B	-2.78×10^{-2}	0	-3.21×10^{-2}	6.3×10^{-4}	1.9×10^{-4}	1.2×10^{-3}
Gravity Gradient and Gyroscopic Torques		0	0	0	Proportional to attitude errors		

Table 3. Modeled Measurement Uncertainties

Sources	Performance (1σ)	Requirement (1σ)
IMU/Shuttle Misalignment	82 arc-sec	160 arc-sec
IMU Resolution	20 arc-sec	20 arc-sec
IMU Noise	(20) arc-sec	-
IMU Derived Rate	See filter dynamic model	
Rate Gyro	-	60 arc-sec/sec
Shuttle/IS Misalignment (1σ)	30 arc-sec	

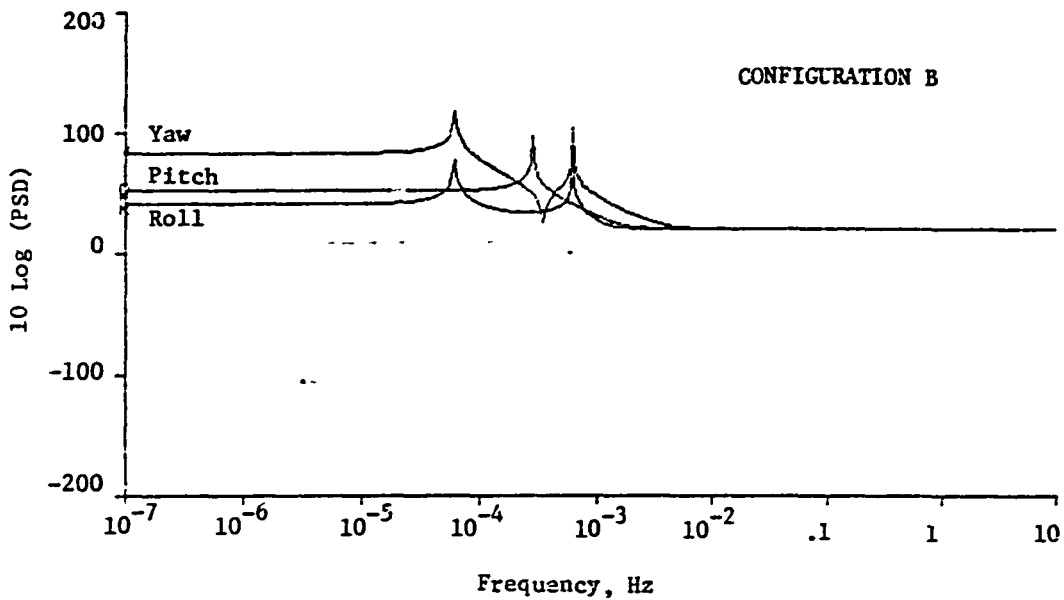


(a) IS Ground Track Error PSD, m^2 -sec

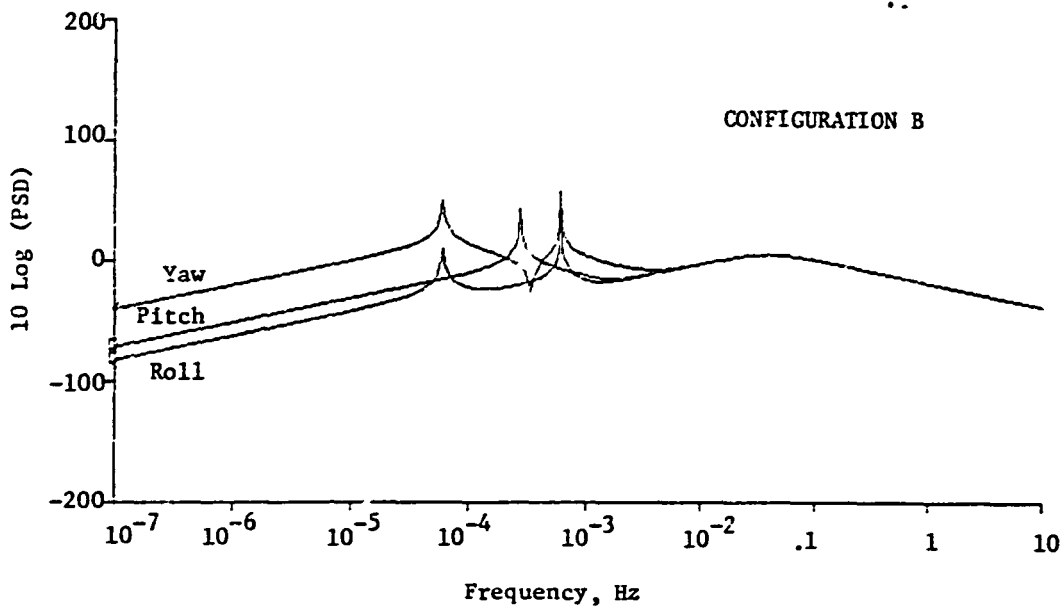


(b) IS Ground Track Rate Error PSD, $(m/sec)^2$ -sec

Figure 17. IS Ground Track Error and Rate Error PSD



(c) IS Ground Track Error PSD, $m^2\text{-sec}$



(d) IS Ground Track Rate Error PSD, $(m/sec)^2\text{-sec}$

Figure 17. (Continued)

Table 4. Expected On-Orbit Navigation Accuracies (3σ)

CASE	NAVIGATION TRACKING SYSTEM	DESCRIPTION		POSITION, FEET			VELOCITY, FPS		
		UNMODELED PERTURBATION	MINIMUM PERIGEE n.mi.	RADIAL	DOWNTRACK	CROSSTRACK	RADIAL	DOWNTRACK	CROSSTRACK
1	STDN*	NOMINAL	105	5,000	34,500	5,000	38.4	5.9	9.8
	TDRSS**		105	3,600	18,500	4,000	22.1	3.3	4.0
2	STDN	SMALL	105	3,000	22,000	1,500	24.6	3.3	2.0
			150	1,200	5,500	1,000	5.8	1.3	1.8
	TDRSS		105	1,800	10,000	2,000	11.0	1.8	2.9
			150	800	2,000	1,500	2.2	0.9	2.1
3	STDN	LARGE	105	8,000	80,000	5,000	91.2	8.0	8.0
	TDRSS		105	6,000	50,000	5,000	56.0	6.2	7.8

NOTE:

The correlation between downtrack position and radial velocity is -0.95

The correlation between radial position and downtrack velocity is -0.80

* Spaceflight Tracking and Data Network

** Tracking Data Relay Satellite System

IV. PARAMETRIC ANALYSIS OF GEOMETRIC ERRORS

In section III, the steady state error dynamics of the Shuttle Imaging Spectrometer system has been analyzed with the major error sources and disturbance effects estimated. In this section, the emphasis is on the geometric error analysis. Geometric errors are consequences of the more direct errors including attitude and rate errors, ephemeris uncertainties, misalignment errors, earth rotation and curvature, etc. Figure 1 shows how the various errors propagate and how the geometric errors and the ground pattern distortions can be generated through dynamic analysis and simulation. The block that is considered in this section is II. Due to mounting and other practical considerations, only the payload-bay principal-axis nadir pointing configuration is considered (see Fig. 2).

The geometric error mapping functions due to the individual error sources as well as the aggregated errors are derived. The earth curvature effect is incorporated in all of the results. For the purpose of quick reference, the key mapping functions are tabulated in Appendix A. A list of the source code of the computer program that has been used for generating the geometric error characteristic curves is given in Appendix E.

A. COORDINATE CONFIGURATIONS

In Fig. 1B, the coordinate frame (X_p, Y_p, Z_p) on the Shuttle c.m. consists of the principal body axes. For the purpose of geometric analysis, another set of coordinates is used, i.e., the (X, Y, Z) frame centered at the nadir point on the ground. This frame is the nadir projection of the orbital rotating frame centered at the Shuttle c.m. Specifically, the X-axis is in the direction of the projected motion on the ground, or the along-track direction, the Z-axis is in the nadir direction, and the Y-axis is in the cross-track direction, so that a right-hand coordinate system is formed.

For the purpose of this report, it is assumed that the Imaging Spectrometer is attached to the payload bay with its optical axis aligned with the body

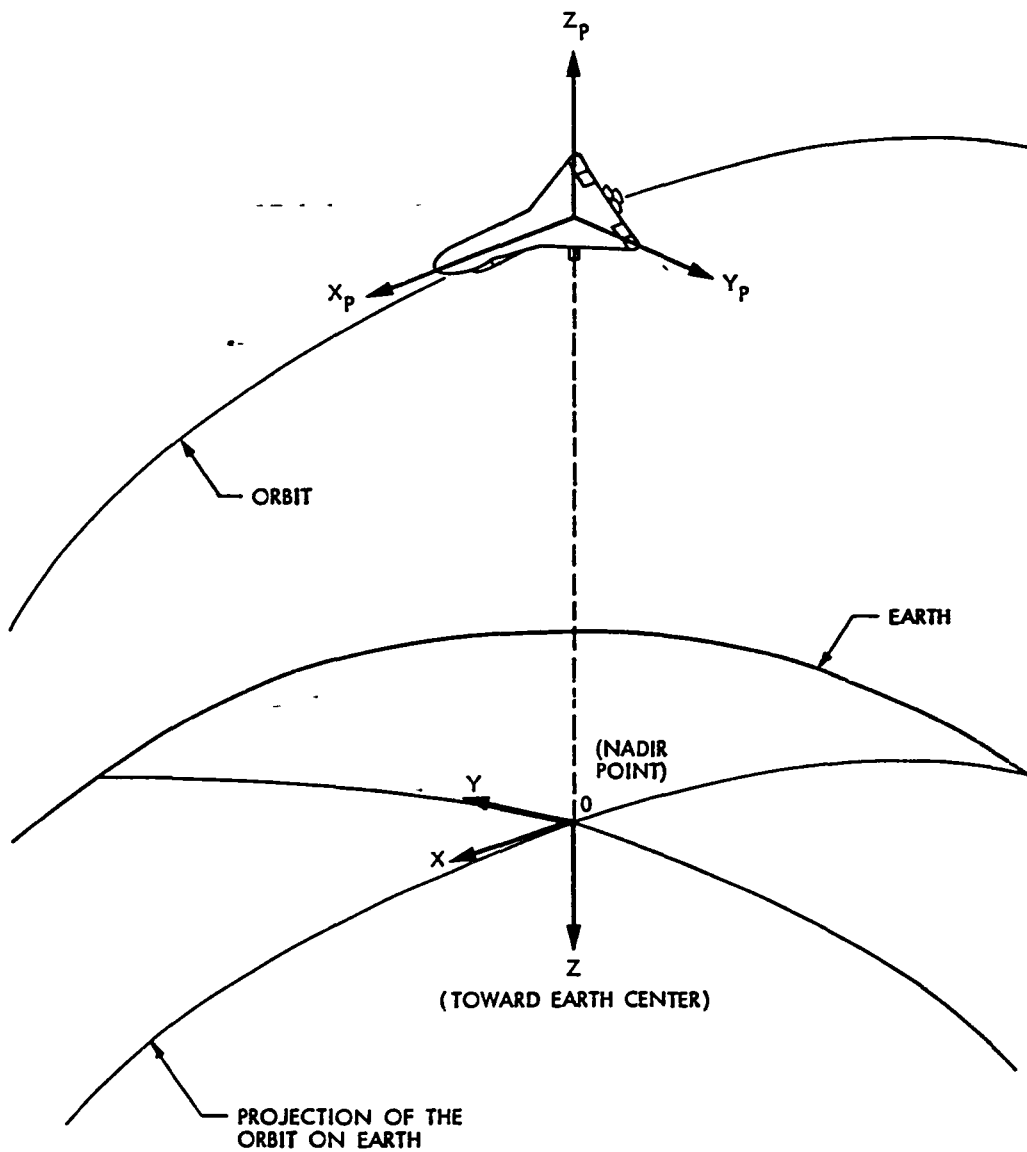


Figure 18. Payload-Bay Nadir Pointing Configuration and Coordinates

Z_p -axis. For normal IS operations, this is the desired configuration. However, for some IS experiments, such as those for assessing the atmospheric effects on the images, the IS instrument is required to point up to $+45^\circ$ about the pitch axis (Y) from nadir (forward and backward looking) or up to $+20^\circ$ about the roll axis (X) (side looking). For those cases, it is assumed that the instrument is gimbal mounted. However, since the analysis performed here is not primarily concerned with dynamics, no specific details are made at this time regarding mounting configurations.

B. GEOMETRIC ERRORS INDUCED BY EPHEMERIS UNCERTAINTIES

Ephemeris uncertainties include radial, along-track, and cross-track prediction errors. The geometric errors induced by ephemeris uncertainties are discussed in the following subsections.

B.1 Geometric Errors Due to Altitude Uncertainties

When the altitude of the shuttle varies, the ground point will shift in both the X and Y directions accordingly. This can be investigated in the following two ways.

B.1.1. With Fixed Viewing Angles

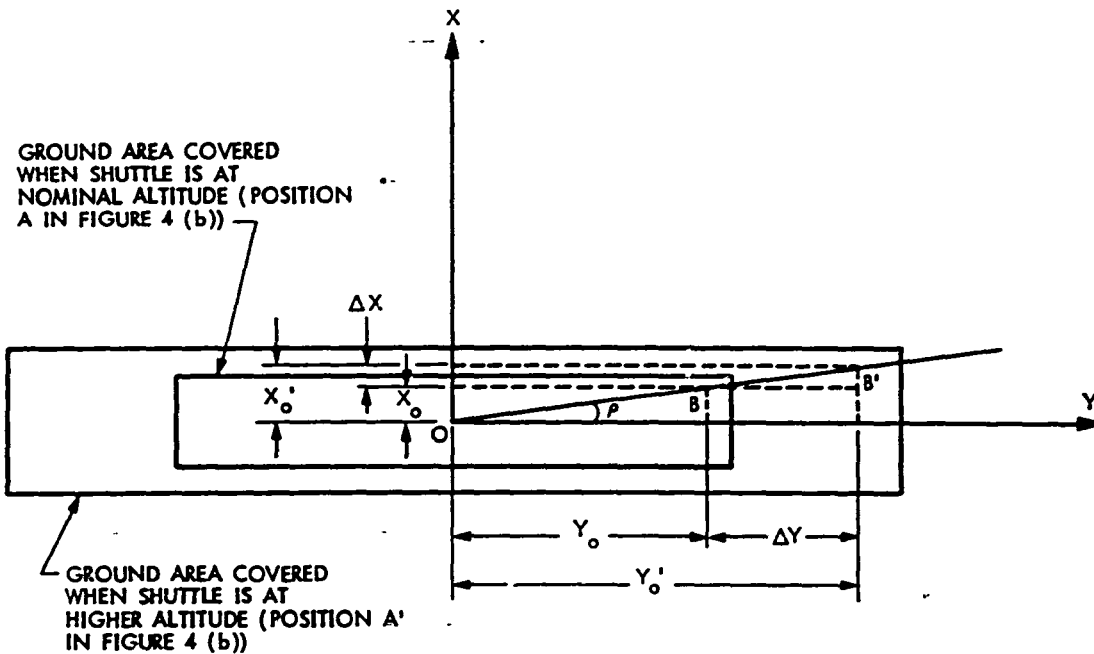
Referring to Figure 19, when the shuttle flies at nominal altitude h (position A), a ground point B with coordinates (X_0, Y_0) corresponding to a view angle λ from the shuttle IS is located. After the shuttle elevates Δh to position A', point B' with coordinates (X_0', Y_0') corresponding to the same view angle λ is located. The problem is to determine the shift of image due to the altitude change. Consider that when the altitude increases, the image of ground objects tends to move toward the nadir point, which causes reduction in image size and increase in field of view. Mathematically, the shift of image may be characterized by computing ΔX and ΔY , where

$$\Delta X = X_0 - X_0'$$

$$\Delta Y = Y_0 - Y_0'$$

Starting by assuming (X_o, Y_o) known, the task is to determine (X_o', Y_o') by first computing the view angle λ . Figure 19b is generated by taking a side view from Fig. 19a in the direction perpendicular to Plane $A'O C_e B'$. Apparently,

$$\sqrt{X_o^2 + Y_o^2} = R \sin \beta$$



LEGEND:

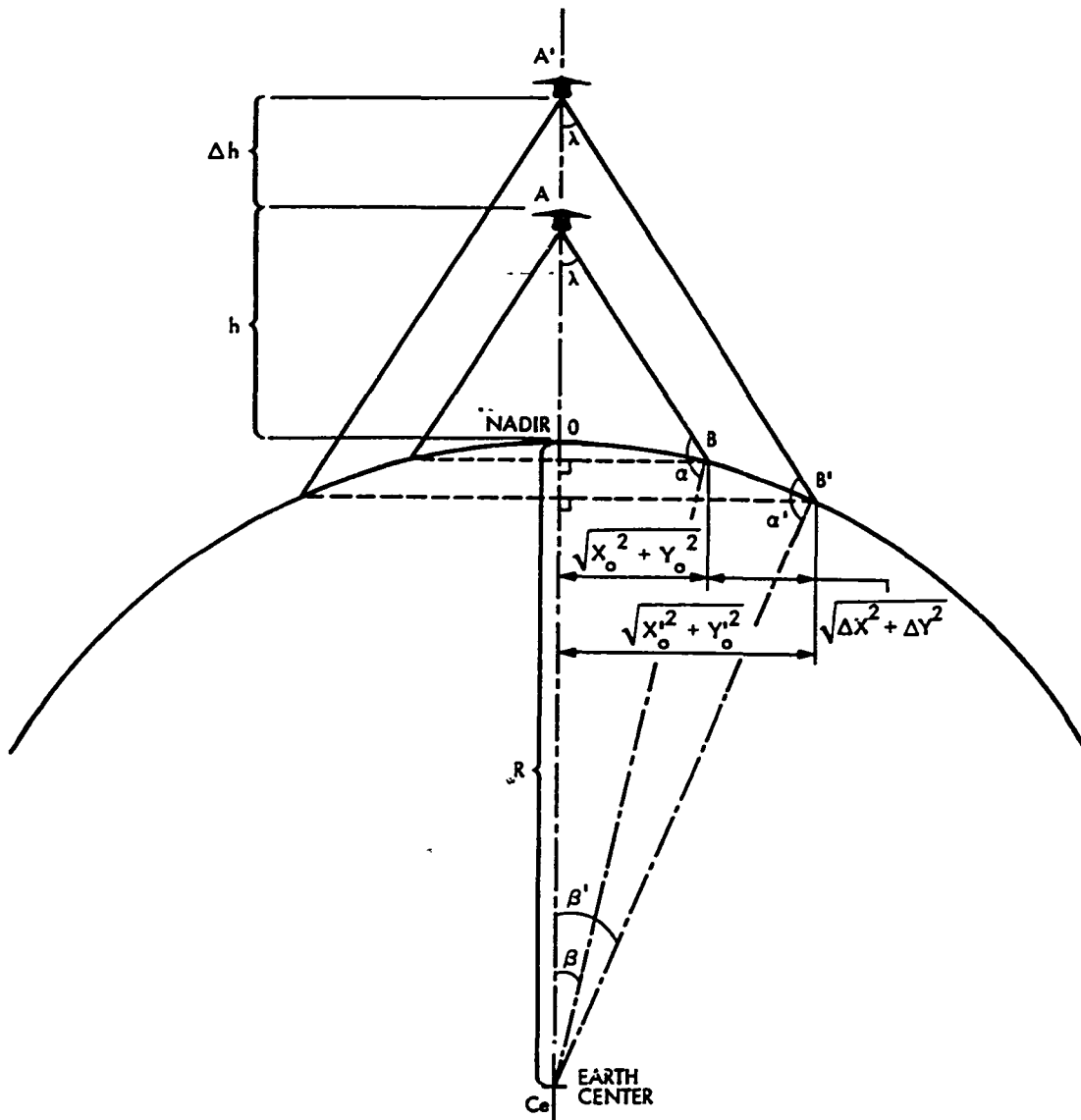
- B = A GROUND POINT CORRESPONDING TO VIEW ANGLE λ WHEN SHUTTLE IS AT NOMINAL ALTITUDE (POSITION A IN FIG. 4(b))
- B' = THE GROUND POINT CORRESPONDING TO THE SAME VIEW ANGLE AFTER THE SHUTTLE ELEVATED Δh (POSITION A' IN FIG. 4(b))
- (X_o, Y_o) = COORDINATES OF POINT B

(X_o', Y_o') = COORDINATES OF POINT B'

$\Delta X, \Delta Y$ = X AND Y COMPONENTS OF THE POSITION CHANGE

(a) TOP VIEW (LOOKING TOWARD NADIR)

Figure 19: Ground Point Shift in X and Y Direction due to Altitude Error



(b) SIDE VIEW

LEGEND:

- h = NOMINAL ALTITUDE
- Δh = ALTITUDE ERROR
- λ = VIEW ANGLE
- A = SHUTTLE'S POSITION AT NOMINAL ALTITUDE
- A' = SHUTTLE'S POSITION AFTER BEING ELEVATED Δh

Figure 19. (Continued)



or

$$\beta = \sin^{-1} \left(\frac{\sqrt{X_o^2 + Y_o^2}}{R} \right) \quad (59)$$

and

$$\alpha = 180^\circ - \lambda - \beta \quad (60)$$

From triangle ABC_e , we have

$$\frac{R}{\sin \lambda} = \frac{R + h}{\sin \alpha} \quad (61)$$

Substituting Eqs. (59) and (60) into (61), it becomes

$$\frac{R}{\sin \lambda} = \frac{R + h}{\sin \left[180^\circ - \lambda - \sin^{-1} \left(\frac{\sqrt{X_o^2 + Y_o^2}}{R} \right) \right]}$$

which leads to

$$\tan \lambda = \frac{\sqrt{X_o^2 + Y_o^2}}{R + h - \sqrt{R^2 - X_o^2 - Y_o^2}}$$

hence,

$$\lambda = \tan^{-1} \left(\frac{\sqrt{X_o^2 + Y_o^2}}{R + h - \sqrt{R^2 - X_o^2 - Y_o^2}} \right) \quad (62)$$

To determine $\sqrt{X_o'^2 + Y_o'^2}$, triangle $A'B'C_e$ is used,

$$\frac{R}{\sin \lambda} = \frac{R + h + \Delta h}{\sin \alpha'}$$

$$\therefore \alpha' = \sin^{-1} \left[\left(1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right] \quad (63)$$

and

$$\begin{aligned} \sqrt{X_o'^2 + Y_o'^2} &= R \sin \beta' \\ &= R \sin (\lambda + \alpha') \end{aligned} \quad (64)$$

Substituting Eq. (63) into (64),

$$\sqrt{X_o'^2 + Y_o'^2} = R \sin \left\{ \lambda + \sin^{-1} \left[\left(1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right] \right\} \quad (65)$$

Expanding the right hand side of Eq.(65), and adopting the "-" sign for $\cos \left\{ \sin^{-1} \left[\left(1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right] \right\}$, since $90^\circ < \sin^{-1} \left[\left(1 + \frac{h + \Delta h}{R} \right) \sin \lambda \right] < 180^\circ$, one has

$$\sqrt{X_o'^2 + Y_o'^2} = R \left[\left(1 + \frac{h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right] \sin \lambda \quad (66)$$

Now, referring to Fig.19(a), since B' is on the line OB, the angle ρ is determined by

$$\begin{aligned} \rho &= \sin^{-1} \frac{X_o}{\sqrt{X_o^2 + Y_o^2}} \\ &= \cos^{-1} \frac{Y_o}{\sqrt{X_o^2 + Y_o^2}} \end{aligned} \quad (67)$$

Hence, from Eqs. (66) and (67),

$$\begin{aligned} X_o' &= \sqrt{X_o'^2 + Y_o'^2} \sin \rho \\ &= \frac{R X_o \sin \lambda}{\sqrt{X_o^2 + Y_o^2}} \left[\left(1 + \frac{h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right] \end{aligned} \quad (68)$$

$$\begin{aligned} Y_o' &= \sqrt{X_o'^2 + Y_o'^2} \cos \rho \\ &= \frac{R Y_o \sin \lambda}{\sqrt{X_o^2 + Y_o^2}} \left[\left(1 + \frac{h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right] \end{aligned} \quad (69)$$

where λ is obtained by Eq.(62).

Hence, from Eqs. (68), (69), and (62) the values for $\Delta X = X_o - X_o'$ and $\Delta Y = Y_o - Y_o'$ can be obtained.

When point A' is below point A, the negative value for Δh should be used in the above equations.

However, if one starts with given view angle λ and ρ , the values of X_o , Y_o , X_o' , and Y_o' can be readily obtained using Eqs. (68) and (69) directly (set Δh to 0 for X_o and Y_o).

B.1.2 With Fixed Ground Points

Referring to Fig.20, the view angle for the fixed ground point B will change as the shuttle altitude varies. Follow the same approach of paragraph B.1.1, the view angles Ω and Ω' can be found,

$$\Omega = \tan^{-1} \left(\frac{\sqrt{X_o^2 + Y_o^2}}{R + h - \sqrt{R^2 - X_o^2 - Y_o^2}} \right) \quad (70)$$

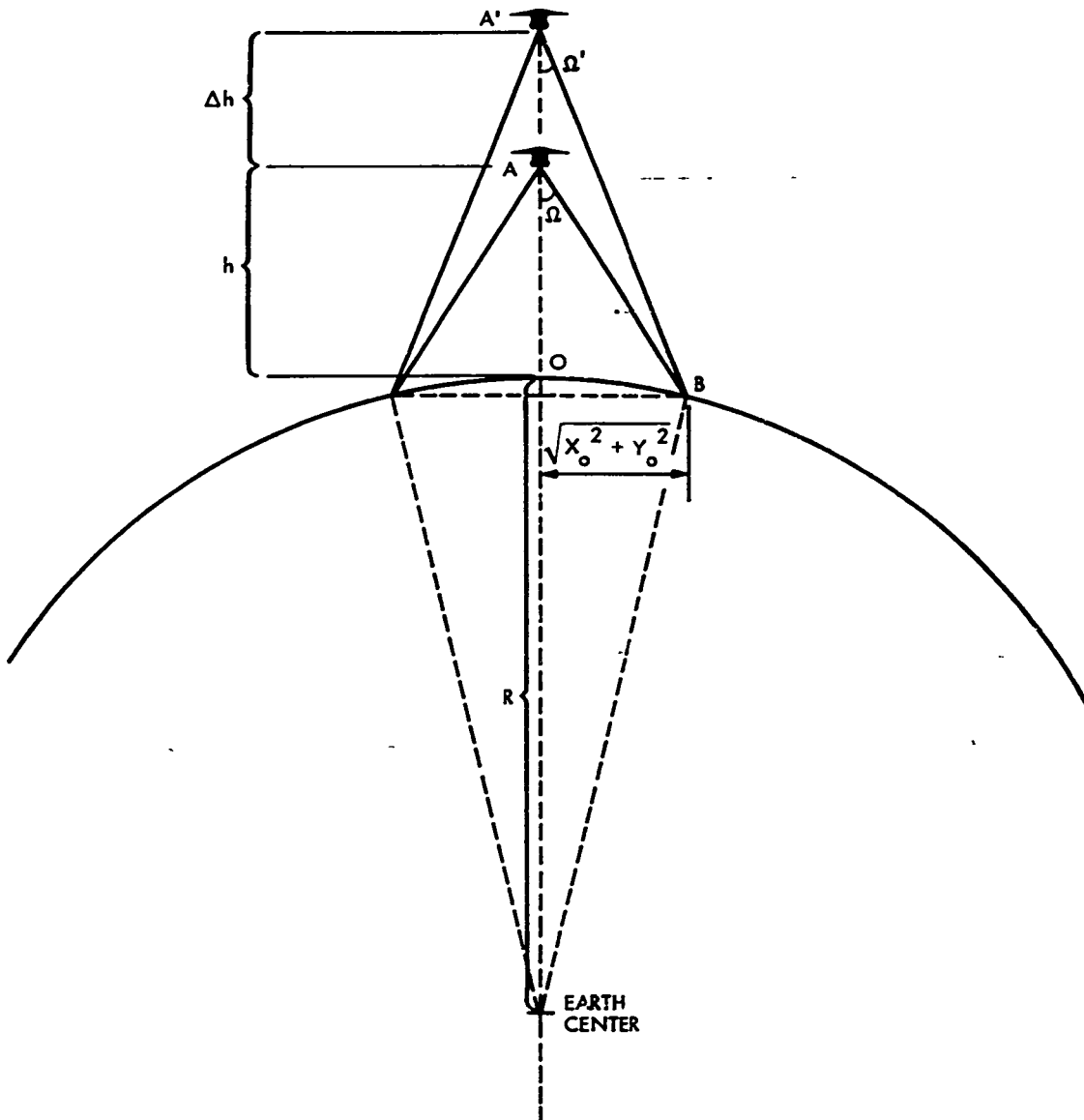
and

$$\Omega' = \tan^{-1} \left(\frac{\sqrt{X_o^2 + Y_o^2}}{R + h + \Delta h - \sqrt{R^2 - X_o^2 - Y_o^2}} \right) \quad (71)$$

Using Eqs. (70) and (71), the value of $\Delta\Omega = \Omega' - \Omega$ can be obtained. Again if point A' is below point A, the negative value for Δh should be used in the above equations.

B.2. Geometric Errors Due to Intrack and Crosstrack Prediction Errors

The intrack (along X-direction or direction of orbit) and the crosstrack (along Y-direction or "-" orbit normal direction) prediction errors will cause the ground objects to shift along the X and Y directions, respectively, as shown in Fig.21. To determine the overall geometric errors associated with ephemeris uncertainties, the intrack error ΔX^* and the crosstrack error ΔY^* can be incorporated in ΔX and ΔY found in Section B.1.1, respectively.



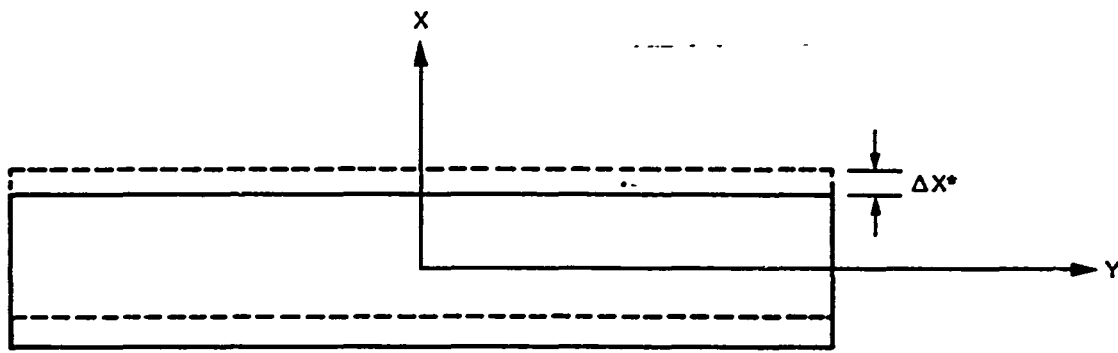
LEGEND:

Ω = VIEW ANGLE FROM SHUTTLE TO POINT B
WHEN SHUTTLE IS AT NOMINAL ALTITUDE h

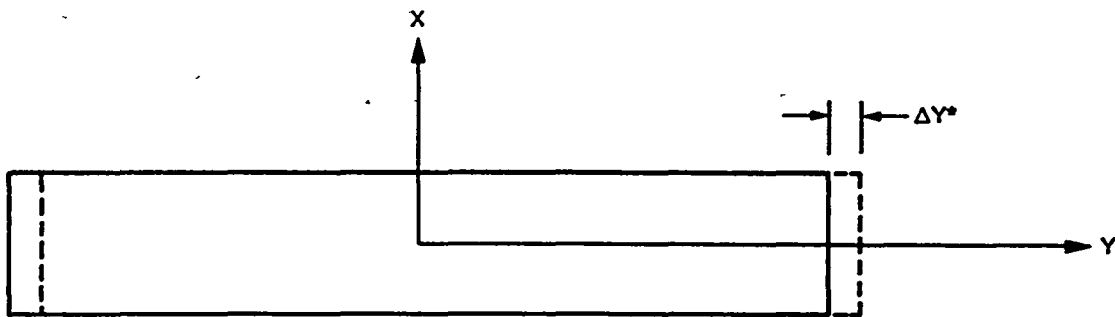
Ω' = VIEW ANGLE FROM SHUTTLE TO THE SAME
POINT B AFTER SHUTTLE ELEVATES Δh

Figure 20. Change of View Angle of a Ground Point due to Altitude Error





(a) INTRACK ERROR (ΔX^*)



(b) CROSTRACK ERROR (ΔY^*)

Figure 21. Geometric Errors Induced by Intrack and Crosstrack Prediction Errors



C. GEOMETRIC ERRORS INDUCED BY ATTITUDE UNCERTAINTIES

Attitude uncertainties refer to the angular errors with respect to the nominal roll (ϕ), pitch (θ), and yaw (ψ) axes. The effects of these errors to the images of ground objects are studied in this subsection first on the individual error basis and then on the aggregated basis. For convenience, in the subsequent discussions, the term nominal flight condition will be used to signify the condition of the shuttle flying in a 400 km circular orbit with the attitude of payload-bay nadir pointing.

C.1 Geometric Errors Induced by Roll Error

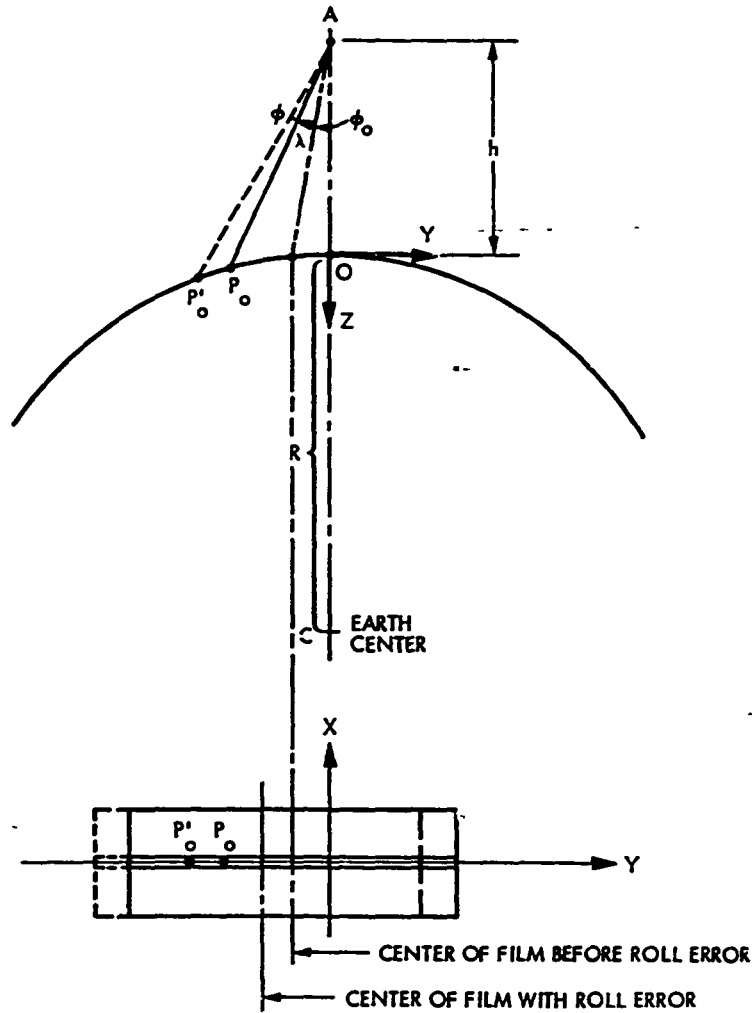
Referring to Fig. 22, consider the nominal flight condition with ϕ_0 offset angle about the roll axis. The IS slit in this case is directed at the Y-axis on the ground. For a given view angle λ , the image point (or pixel) on the ground is P_0 with coordinates $(0, Y^*)$. For the same roll offset and view angle the ground point P'_0 is located after a roll error ϕ is introduced. The coordinates of P'_0 are $(0, Y^{*'})$. Equivalently, by rotating the optical instrument to the "left," the image recorded on the film appears to move to the "right." Therefore, the geometric error may be defined as the displacement of the object image after error is introduced relative to the image before the error is introduced. Mathematically, this is $\bar{P}_0 - \bar{P}'_0$ or $\bar{P}_{\phi_0, \lambda} - \bar{P}_{\phi_0 + \phi, \lambda}$. Since, in the case being considered, the image is on the Y-axis, one has

$$\Delta X = 0 \quad (72)$$

$$\Delta Y = Y^* - Y^{*'} \quad (73)$$

where Y^* and $Y^{*'}$ are,

$$Y^* = -R \sin(\lambda + \phi_0) \left[\left(1 + \frac{h}{R}\right) \cos(\lambda + \phi_0) - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2(\lambda + \phi_0)} \right] \quad (74)$$



LEGEND:

- λ = VIEW ANGLE
- ϕ_o = ROLL OFFSET ANGLE
- ϕ = ROLL ERROR
- P_o = THE GROUND POINT CORRESPONDING TO VIEW ANGLE λ AND ROLL OFFSET ϕ_o
- P'_o = THE GROUND POINT CORRESPONDING TO THE SAME λ AND ϕ_o AFTER ROLL ERROR ϕ IS INTRODUCED

Figure 22. Shift of Ground Point Induced by Roll Error

$$Y^{*'} = -R \sin (\lambda + \phi_o + \phi) \left[\left(1 + \frac{h}{R} \right) \cos (\lambda + \phi_o + \phi) - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 (\lambda + \phi_o + \phi)} \right] \quad (75)$$

C.2 Geometric Errors Induced by Pitch Error

Consider the case of pitch attitude error. Let θ_o be the offset angle about the pitch axis (desired attitude), and θ be the pitch error. Referring to Fig.23, for a given view angle λ , projecting the slit onto the XY-plane forms two line segments: \overline{BC} , corresponding to null pitch error ($\theta = 0$), and $\overline{B'C'}$, corresponding to arbitrary pitch error θ . Projecting along the line of view \overline{AC} onto the ground, $P(X^*, Y^*)$ is obtained; similarly, $P'(X^{*'}, Y^{*'})$ is obtained, corresponding to line of view $\overline{AC'}$. The geometric error here is defined the same way as that for the roll case, i.e., $\overline{P}_{\theta_o, \lambda} - P_{\theta_o + \theta, \lambda}$, or

$$\Delta X = X^* - X^{*'} \quad (76)$$

$$\Delta Y = Y^* - Y^{*'} \quad (77)$$

The formulae for computing X^* , Y^* , $X^{*'}$, and $Y^{*'}$ are derived as follows. In Fig.23, assume that h , θ , θ_o , and λ are known. The lengths of the line segments \overline{OB} and $\overline{OB'}$ are

$$\overline{OB} = h \tan \theta_o \quad (78a)$$

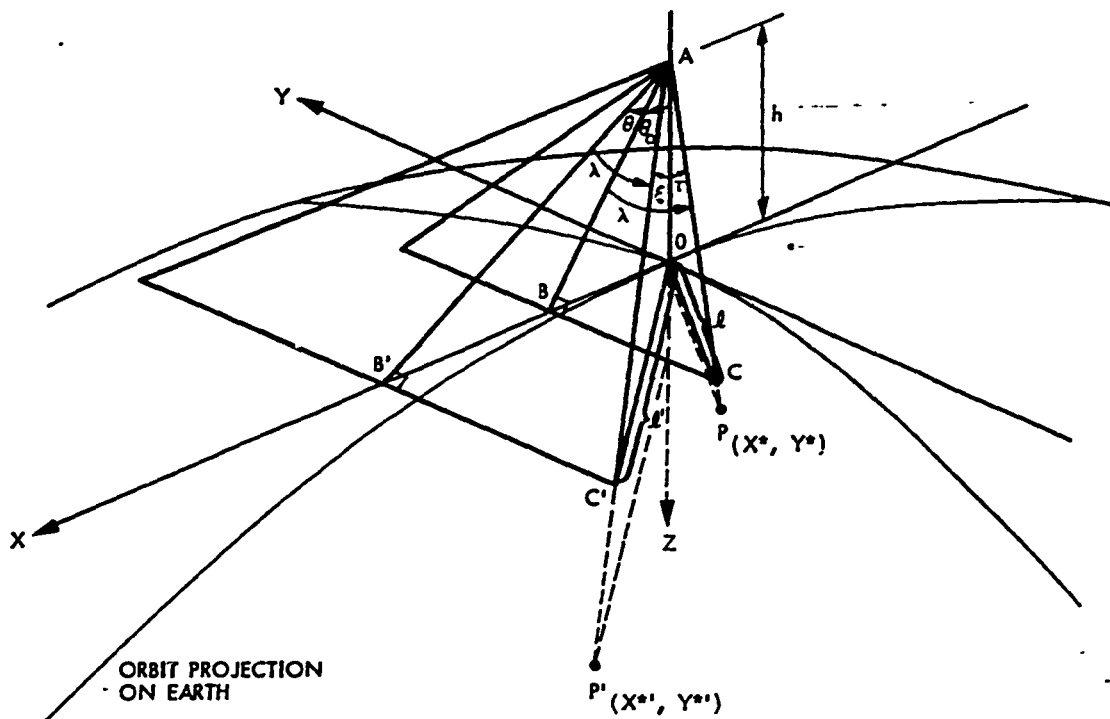
and

$$\overline{OB'} = h \tan (\theta_o + \theta) \quad (78b)$$

and that of \overline{AB} and $\overline{AB'}$ are

$$\overline{AB} = \frac{h}{\cos \theta_o} \quad (79a)$$

$$\overline{AB'} = \frac{h}{\cos (\theta_o + \theta)} \quad (79b)$$



LEGEND:

- λ = VIEW ANGLE
- θ_0 = PITCH OFFSET ANGLE
- θ = PITCH ERROR
- P = THE GROUND POINT CORRESPONDING TO VIEW ANGLE λ AND PITCH OFFSET θ_0
- P' = THE GROUND POINT CORRESPONDING TO THE SAME λ AND θ_0 AFTER PITCH ERROR θ IS INTRODUCED

Figure 23. Shift of Ground Point Induced by Pitch Error

\overline{BC} and $\overline{B'C'}$, with respect to Y-axis, can be obtained as

$$\overline{BC} = -\overline{AB} \tan \lambda = \frac{-h \tan \lambda}{\cos \theta_0} \quad (80a)$$

and

$$\overline{B'C'} = -\overline{AB'} \tan \lambda = \frac{-h \tan \lambda}{\cos(\theta_0 + \theta)} \quad (80b)$$

implies that the signs of \overline{BC} and $\overline{B'C'}$ are opposite to that of λ . l and l' are, referring to Fig. 23,

$$l = \sqrt{\overline{OB}^2 + \overline{BC}^2} = h \sec \theta_0 \sqrt{\sin^2 \theta_0 + \tan^2 \lambda} \quad (81a)$$

and

$$l' = \sqrt{\overline{OB'}^2 + \overline{B'C'}^2} = h \sec(\theta_0 + \theta) \sqrt{\sin^2(\theta_0 + \theta) + \tan^2 \lambda} \quad (81b)$$

the angles τ and ξ (see Fig. 23) can then be determined,

$$\tau = \tan^{-1} \left(\frac{l}{h} \right) = \tan^{-1} \left(\sec \theta_0 \sqrt{\sin^2 \theta_0 + \tan^2 \lambda} \right) \quad (82a)$$

and

$$\xi = \tan^{-1} \left(\frac{l'}{h} \right) = \tan^{-1} \left[\sec(\theta_0 + \theta) \sqrt{\sin^2(\theta_0 + \theta) + \tan^2 \lambda} \right] \quad (82b)$$

Finally,

$$\begin{aligned} X^* &= \sqrt{X^{*2} + Y^{*2}} \frac{\overline{OB}}{l} \\ &= R \left[\left(1 + \frac{h}{R} \right) \cos \tau - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \tau} \right] \frac{\sin \tau \sin \theta_0}{\sqrt{\sin^2 \theta_0 + \tan^2 \lambda}} \quad (83a) \end{aligned}$$

$$\begin{aligned} Y^* &= \sqrt{X^{*2} + Y^{*2}} \frac{\overline{BC}}{l} \\ &= -R \left[\left(1 + \frac{h}{R} \right) \cos \tau - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \tau} \right] \frac{\sin \tau \tan \lambda}{\sqrt{\sin^2 \theta_0 + \tan^2 \lambda}} \quad (83b) \end{aligned}$$

$$\begin{aligned}
X^{*'} &= \sqrt{X^{*2} + Y^{*2}} \frac{OB'}{l'} \\
&= R \left[\left(1 + \frac{h}{R}\right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right] \frac{\sin \xi \sin (\theta_o + \theta)}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}}
\end{aligned} \tag{83c}$$

$$\begin{aligned}
Y^{*'} &= \sqrt{X^{*2} + Y^{*2}} \frac{B'C'}{l'} \\
&= -R \left[\left(1 + \frac{h}{R}\right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right] \frac{\sin \xi \tan \lambda}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}}
\end{aligned} \tag{83d}$$

However, if one starts with (X^*, Y^*) given, then λ and θ_o would have to be computed. In that case,

$$\lambda = -\tan^{-1} \left[\frac{Y^*}{\left(R + h - \sqrt{R^2 - X^{*2} - Y^{*2}} \right) \sqrt{1 + \left(\frac{X^*}{R + h - \sqrt{R^2 - X^{*2} - Y^{*2}}} \right)^2}} \right] \tag{84a}$$

and

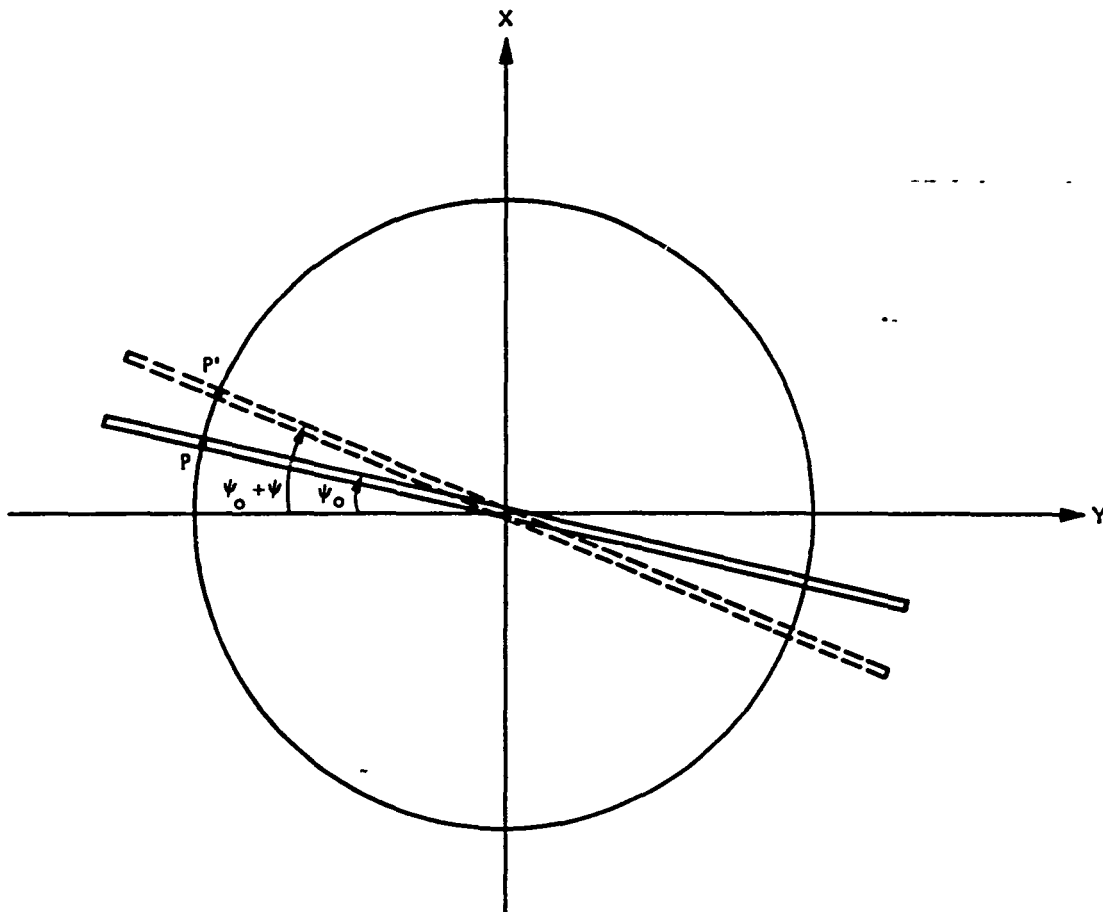
$$\theta_o = \tan^{-1} \left(\frac{X^*}{R + h - \sqrt{R^2 - X^{*2} - Y^{*2}}} \right) \tag{84b}$$

Once λ and θ_o are determined, τ and ξ can be computed using (82), and $X^{*'}$ and $Y^{*'}$ are determined using (83).

C.3 Geometric Errors Induced by Yaw Error

For a given yaw offset angle, ψ_o , and the view angle, λ , the ground point $P(X^*, Y^*)$ is located. A new point $P'(X^{*'}, Y^{*'})$ is found as yaw attitude error, ψ , is introduced (see Fig.24). The values for X^* , Y^* , $X^{*'}$, and $Y^{*'}$ can be computed as follows:

$$X^* = \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin \psi_o \tag{85a}$$



LEGEND:

ψ_0 = YAW OFFSET ANGLE

ψ = YAW ERROR

P = THE GROUND POINT CORRESPONDING TO VIEW ANGLE λ AND YAW OFFSET ψ_0

P' = THE GROUND POINT CORRESPONDING TO THE SAME λ AND ψ_0 AFTER YAW ERROR ψ IS INTRODUCED

Figure 24. Shift of Ground Point Induced by Yaw Error

$$Y^* = - \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos \psi_0 \quad (85b)$$

$$X^{*'} = \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin (\psi_0 + \psi) \quad (86a)$$

and

$$Y^{*'} = - \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos (\psi_0 + \psi) \quad (86b)$$

From Eqs. (85) and (86), we can obtain

$$\Delta X = X^* - X^{*'}$$

$$\Delta Y = Y^* - Y^{*'}$$

C.4 Geometric Errors Induced by Roll, Pitch, and Yaw Attitude Errors

C.4.1. The Shift of IS Line-of-Sight due to Attitude Error

Referring to Fig. 25, the shuttle is flying at an arbitrary attitude (ϕ, θ, ψ) . Consider the 3-2-1 sequence, i.e., the yaw, pitch, and roll rotation sequence. After yaw and pitch rotations, the Imaging Spectrometer line-of-sight will shift to \overline{AB} , where $B(X_1, Y_1)$ is its intersection with the XY-plane. The (X_1, Y_1) coordinates are determined as follows:

$$X_1 = X_0 + h \tan \theta \cos \psi \quad (87a)$$

$$Y_1 = Y_0 + h \tan \theta \sin \psi \quad (87b)$$

Through a roll angle rotation, point B will move to $C(X_2, Y_2)$ on the XY-plane,

where

$$X_2 = X_0 + h \tan \theta \cos \psi + \frac{h}{\cos \theta} \tan \phi \sin \psi \quad (88a)$$

$$Y_2 = Y_0 + h \tan \theta \sin \psi - \frac{h}{\cos \theta} \tan \phi \cos \psi \quad (88b)$$

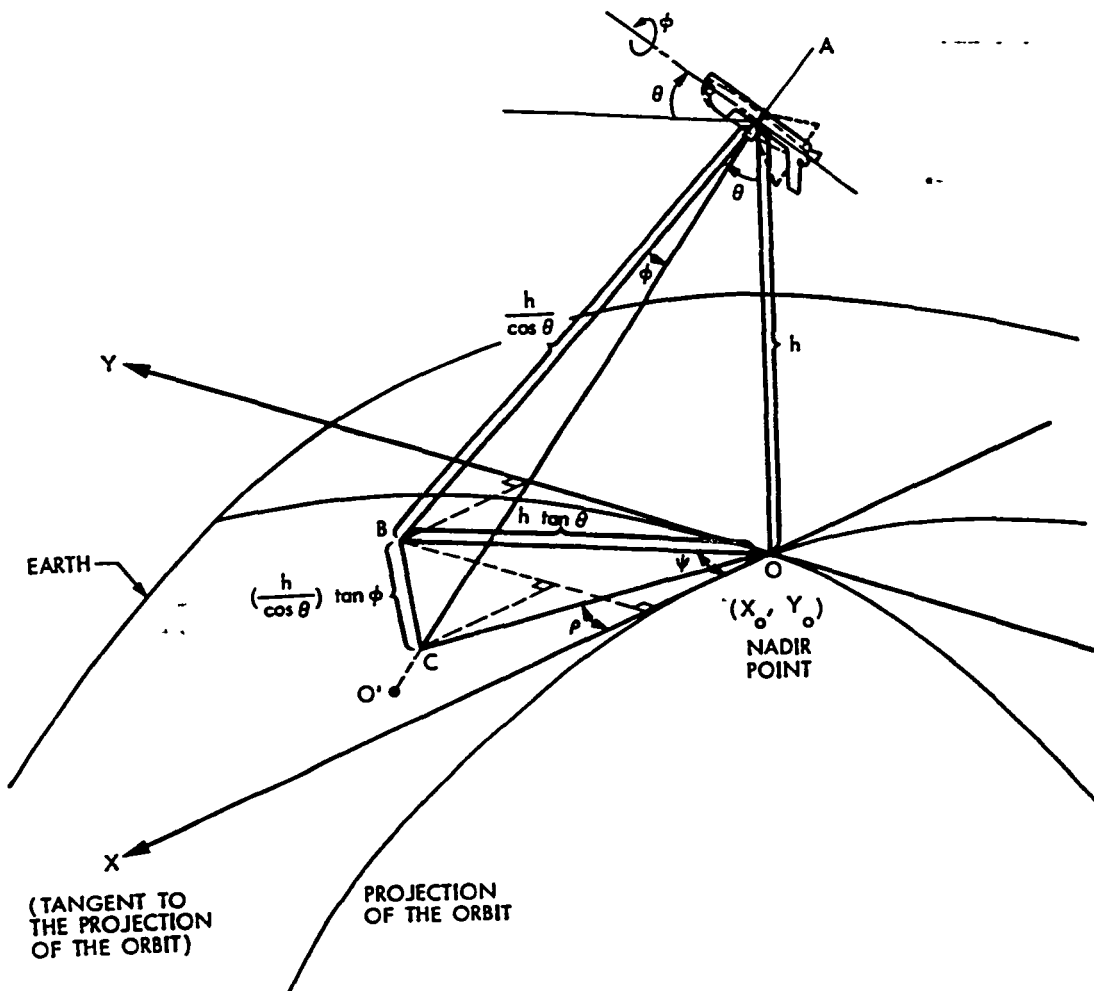


Figure 25. The Shift of Ground Projection of the IS Line-of-Sight due to Attitude Error

The interest here is to determine the coordinates of $O'(X,Y)$ corresponding to the final line-of-sight projected onto the ground. By taking a view normal to the ACO-plane, Figure 26 is obtained. The quantities ϵ , l , and ρ are obtained as follows,

$$\epsilon = \tan^{-1} \left(\frac{\sqrt{X_2^2 + Y_2^2}}{h} \right) \quad (89a)$$

$$l = R \sin \epsilon \left[\left(1 + \frac{h}{R} \right) \cos \epsilon - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \epsilon} \right] \quad (89b)$$

and

$$\rho = \sin^{-1} \left(\frac{Y_2}{\sqrt{X_2^2 + Y_2^2}} \right) = \cos^{-1} \left(\frac{X_2}{\sqrt{X_2^2 + Y_2^2}} \right) \quad (89c)$$

Hence, the coordinates of X and Y are,

$$X = l \cos \rho = R \sin \epsilon \left[\left(1 + \frac{h}{R} \right) \cos \epsilon - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \epsilon} \right] \frac{X_2}{\sqrt{X_2^2 + Y_2^2}} \quad (90a)$$

$$Y = l \sin \rho = R \sin \epsilon \left[\left(1 + \frac{h}{R} \right) \cos \epsilon - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \epsilon} \right] \frac{Y_2}{\sqrt{X_2^2 + Y_2^2}} \quad (90b)$$

where X_2 and Y_2 are given in Eq. (88).

Note that this derivation is general enough so that the formulae are valid if one replaces the attitude errors with errors plus offsets, i.e., replacing ϕ , θ , and ψ by $\phi_o + \phi$, $\theta_o + \theta$, and $\psi_o + \psi$, respectively.

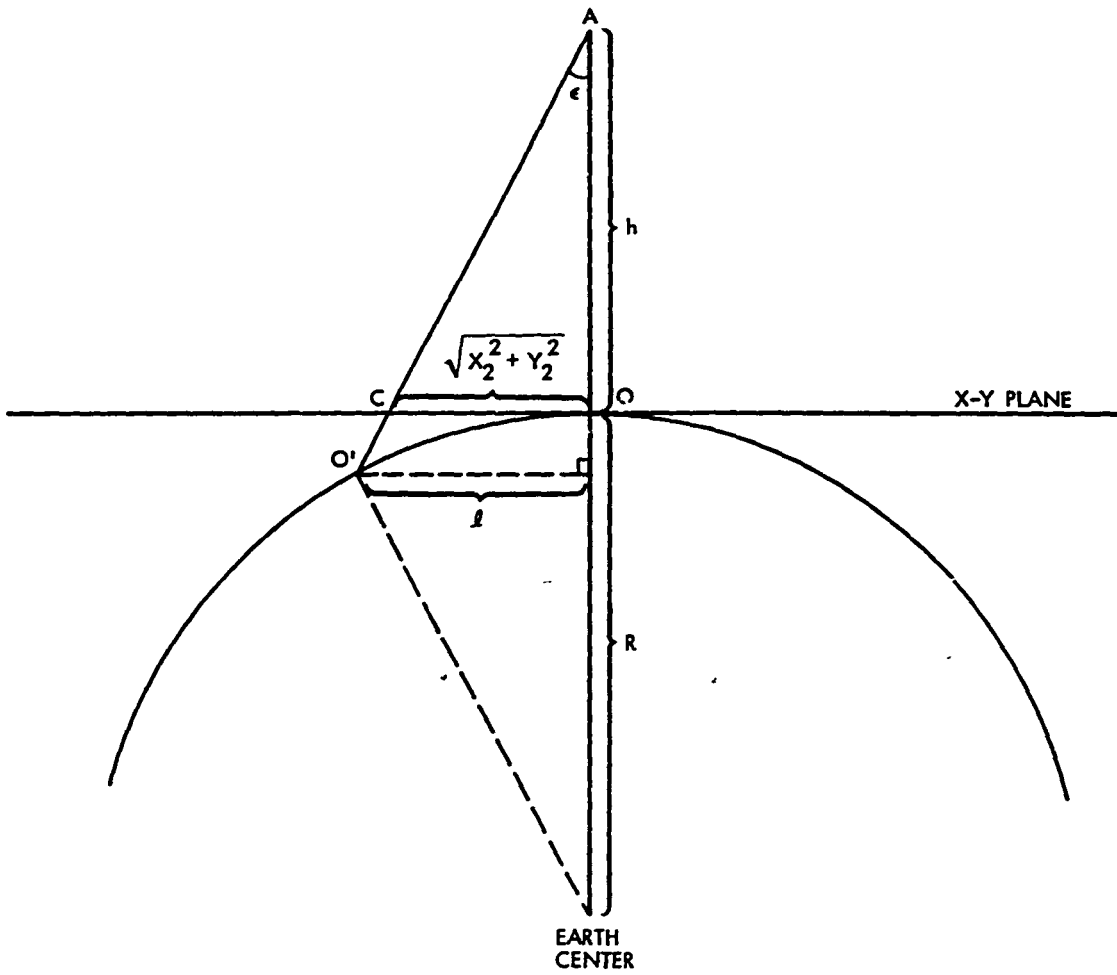


Figure 26. Side View of Figure 25



C.4.2. The Shift of Ground Image Due to Attitude Error

For simplicity, ϕ , θ , and ψ are used to represent the attitude angles as sums of attitude offsets and attitude errors. To fix the derivation, the yaw (ψ), pitch (θ), and roll (ϕ) Euler sequence is assumed.

In this subsection, the results of subsection C.4.1 are extended to cover the entire field of view of the IS slit. First consider the case of a nominal flight with zero attitude offset. Referring to Fig. 27, let O be the nadir point which is the origin of the XY-Frame and the X'Y'-Frame. The X'Y'-Frame is formed by rotating the XY-Frame through an angle ψ about the yaw axis. Let P be a point corresponding to a view angle λ (negative value shown in Fig. 27) on the Y-axis before any rotation occurs. After a ψ rotation, for the same view angle, the IS is sighting point P_1 . Similarly, after a pitch (θ) and then roll (ϕ) rotation, the points P_2 and then P_3 are sighted, respectively.

The coordinates of P_2 in the X'Y'-Frame are, from the results of subsections C.1 and C.2 and Fig. 23,

$$X'_2 = \frac{R \sin \theta \sin \xi_2 \left[\left(1 + \frac{h}{R}\right) \cos \xi_2 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_2} \right]}{\sqrt{\sin^2 \theta + \tan^2 \lambda}} \quad (91a)$$

$$Y'_2 = - \frac{R \tan \lambda \sin \xi_2 \left[\left(1 + \frac{h}{R}\right) \cos \xi_2 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_2} \right]}{\sqrt{\sin^2 \theta + \tan^2 \lambda}} \quad (91b)$$

where the angle ξ_2 is the angle ξ defined in Fig. 23 corresponding to point P_2 here, and

$$\xi_2 = \tan^{-1} \left(\sec \theta \sqrt{\sin^2 \theta + \tan^2 \lambda} \right) \quad (91c)$$

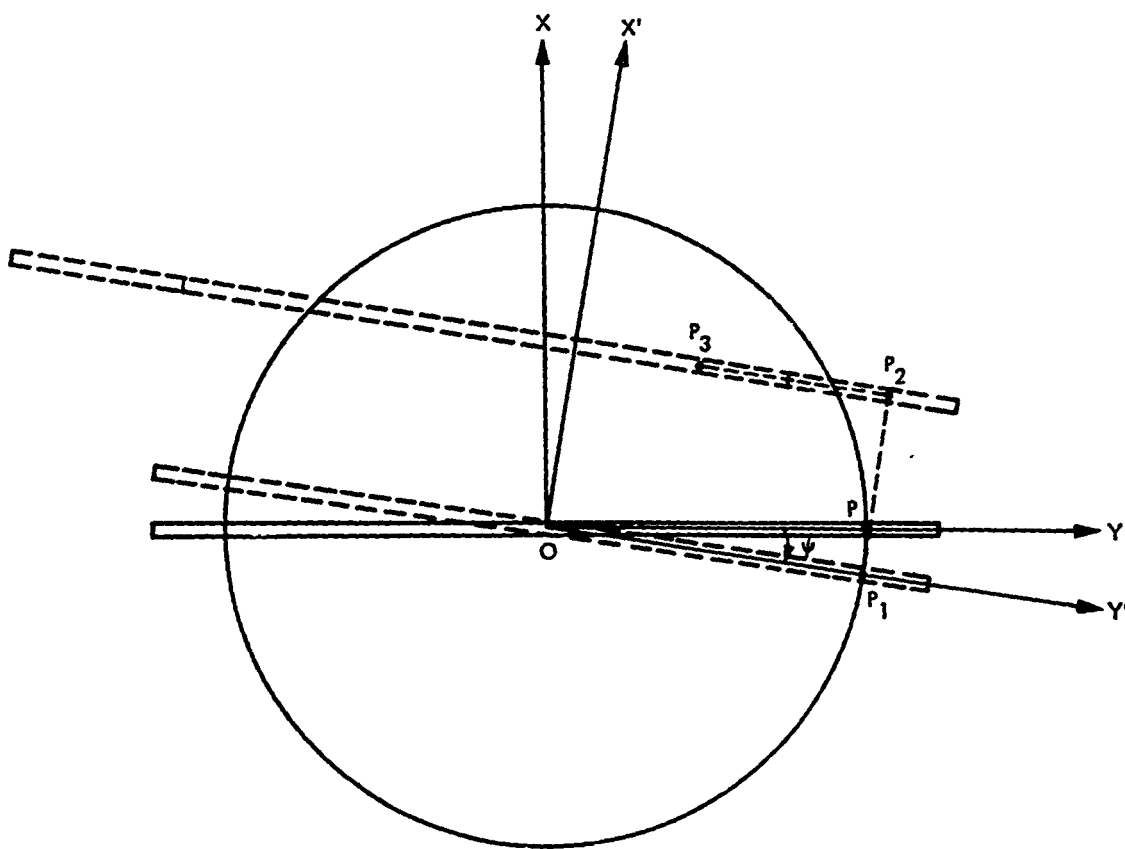


Figure 27. Shift of Ground Point Induced by Attitude Errors



The coordinates of P_3 in the $X'Y'$ -Frame are

$$X'_3 = \frac{R \sin \theta \sin \xi_3 \left[\left(1 + \frac{h}{R}\right) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}} \quad (92a)$$

$$Y'_3 = - \frac{R \tan (\lambda + \phi) \sin \xi_3 \left[\left(1 + \frac{h}{R}\right) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}} \quad (92b)$$

where ξ_3 is similarly defined as ξ_2 and

$$\xi_3 = \tan^{-1} \left(\sec \theta \sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)} \right) \quad (92c)$$

or, finally, in the XY -Frame

$$X_3 = X'_3 \cos \psi - Y'_3 \sin \psi \quad (93a)$$

$$Y_3 = X'_3 \sin \psi + Y'_3 \cos \psi \quad (93b)$$

where X'_3 and Y'_3 are given in (92).

The geometric error caused by the attitude errors is therefore,

$$\Delta X = X - X_3 \quad (94a)$$

$$\Delta Y = Y - Y_3 \quad (94b)$$

where

$$X = 0$$

$$Y = -R \sin \lambda \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right]$$

Now, consider the case of the shuttle (or the IS) flying at a nominal attitude $(\phi_0, \theta_0, \psi_0)$ with attitude uncertainties of (ϕ, θ, ψ) . Let $P_0(X_0, Y_0)$ be a ground point that is sighted by the IS with a view angle λ and attitude $(\phi_0, \theta_0, \psi_0)$. After (ϕ, θ, ψ) attitude rotations from the nominal, the ground point $P_3(X_3, Y_3)$ is sighted with the same view angle. In this case, the geometric error is

$$\Delta X = X_0 - X_3 \quad (95a)$$

$$\Delta Y = Y_0 - Y_3 \quad (95b)$$

where

$$X_0 = X'_0 \cos \psi_0 - Y'_0 \sin \psi_0 \quad (96a)$$

$$Y_0 = X'_0 \sin \psi_0 + Y'_0 \cos \psi_0 \quad (96b)$$

$$X_3 = X'_3 \cos (\psi_0 + \psi) - Y'_3 \sin (\psi_0 + \psi) \quad (96c)$$

$$Y_3 = X'_3 \sin (\psi_0 + \psi) + Y'_3 \cos (\psi_0 + \psi) \quad (96d)$$

and

$$X'_0 = \frac{R \sin \theta_0 \sin \xi_0 \left[\left(1 + \frac{h}{R}\right) \cos \xi_0 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_0} \right]}{\sqrt{\sin^2 \theta_0 + \tan^2 (\lambda + \phi_0)}} \quad (97a)$$

$$Y'_0 = - \frac{R \tan (\lambda + \phi_0) \sin \xi_0 \left[\left(1 + \frac{h}{R}\right) \cos \xi_0 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_0} \right]}{\sqrt{\sin^2 \theta_0 + \tan^2 (\lambda + \phi_0)}} \quad (97b)$$

where

$$\xi_0 = \tan^{-1} \left(\sec \theta_0 \sqrt{\sin^2 \theta_0 + \tan^2 (\lambda + \phi_0)} \right) \quad (97c)$$

and

$$X'_3 = \frac{R \sin (\theta_0 + \theta) \sin \xi_3 \left[\left(1 + \frac{h}{R}\right) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_0 + \theta) + \tan^2 (\lambda + \phi_0 + \phi)}} \quad (98a)$$

$$Y'_3 = - \frac{R \tan (\lambda + \phi_0 + \phi) \sin \xi_3 \left[\left(1 + \frac{h}{R}\right) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_0 + \theta) + \tan^2 (\lambda + \phi_0 + \phi)}} \quad (98b)$$

and where

$$\xi_3 = \tan^{-1} \left[\sec (\theta_0 + \theta) \sqrt{\sin^2 (\theta_0 + \theta) + \tan^2 (\lambda + \phi_0 + \phi)} \right] \quad (98c)$$



To include errors caused by altitude uncertainties, one can replace h by h_0 in Eq. (97), and h by $h_0 + \Delta h$ in Eq. (98), where h_0 is the nominal or estimated altitude and $h_0 + \Delta h$ is the actual altitude.

D. GEOMETRIC ERRORS INDUCED BY ATTITUDE RATE ERRORS

Attitude rates cause attitude angle changes which in turn cause geometric errors. To account for attitude rates, one integrates the rates to obtain the attitude angles, and then uses the formulae derived to obtain the corresponding time-varying geometric errors. The instantaneous attitude angles are:

$$\phi' = \phi_0 + \int_{t_0}^t \dot{\phi} d\tau \quad (99a)$$

$$\theta' = \theta_0 + \int_{t_0}^t \dot{\theta} d\tau \quad (99b)$$

$$\psi' = \psi_0 + \int_{t_0}^t \dot{\psi} d\tau \quad (99c)$$

E. GEOMETRIC ERRORS INDUCED BY MISALIGNMENT ERRORS

The misalignment errors can be incorporated in the attitude errors. Let $(\phi_b, \theta_b, \psi_b)$ be the attitude bias or misalignment, then the effective IS attitude angles will be

$$\phi' = \phi_0 + \phi_b + \phi \quad (100a)$$

$$\theta' = \theta_0 + \theta_b + \theta \quad (100b)$$

$$\psi' = \psi_0 + \psi_b + \psi \quad (100c)$$

The corresponding geometric errors can be determined using the formulae derived in subsection C.

F. GEOMETRIC ERRORS INDUCED BY EARTH ROTATION

The effect of earth rotation on images varies with the position of the Shuttle relative to the earth and the Shuttle orbital elements. In general, ground images become skewed as a result of earth rotation.

Referring to Fig. 28. Let O be the nadir point at which the latitude is ζ . Let ω_e be the spin rate of the earth, where $\omega_e = \frac{2\pi}{86400} = 7.2722 \times 10^{-5}$ rad/sec. Let V_e be the linear velocity of the earth at O ,

$$V_e = \omega_e R \cos \zeta \quad (101)$$

Let Γ be the orbital inclination at the equator, and let Λ be the orbital inclination to the local meridian at O . From Fig. 29 and Appendix B, one has the following spherical geometrical relation,

$$\cos \Gamma = \cos \zeta \sin \Lambda$$

or

$$\sin \Lambda = \frac{\cos \Gamma}{\cos \zeta} \quad (102a)$$

and

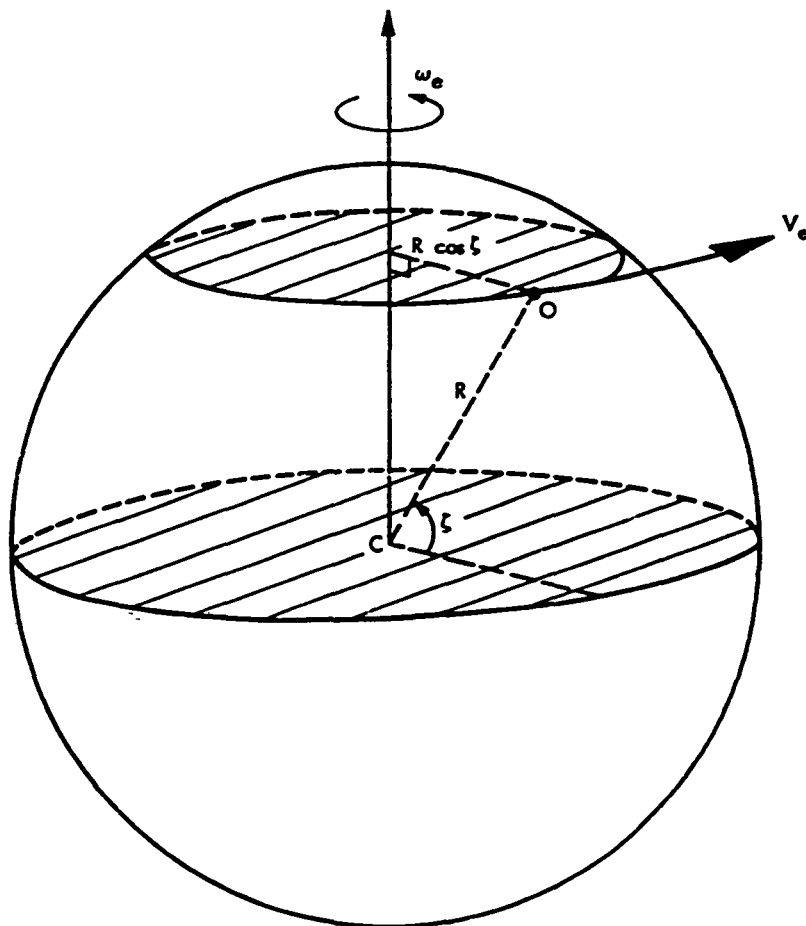
$$\cos \Lambda = \sqrt{1 - \sin^2 \Lambda} = \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2} \quad (102b)$$

From Fig. 29 again, Λ is also the angle between the linear velocity vector V_e and the Y-axis. Hence, velocity components V_{eX} and V_{eY} are, using (102),

$$V_{eX} = V_e \sin \Lambda = \frac{V_e \cos \Gamma}{\cos \zeta} \quad (103a)$$

and

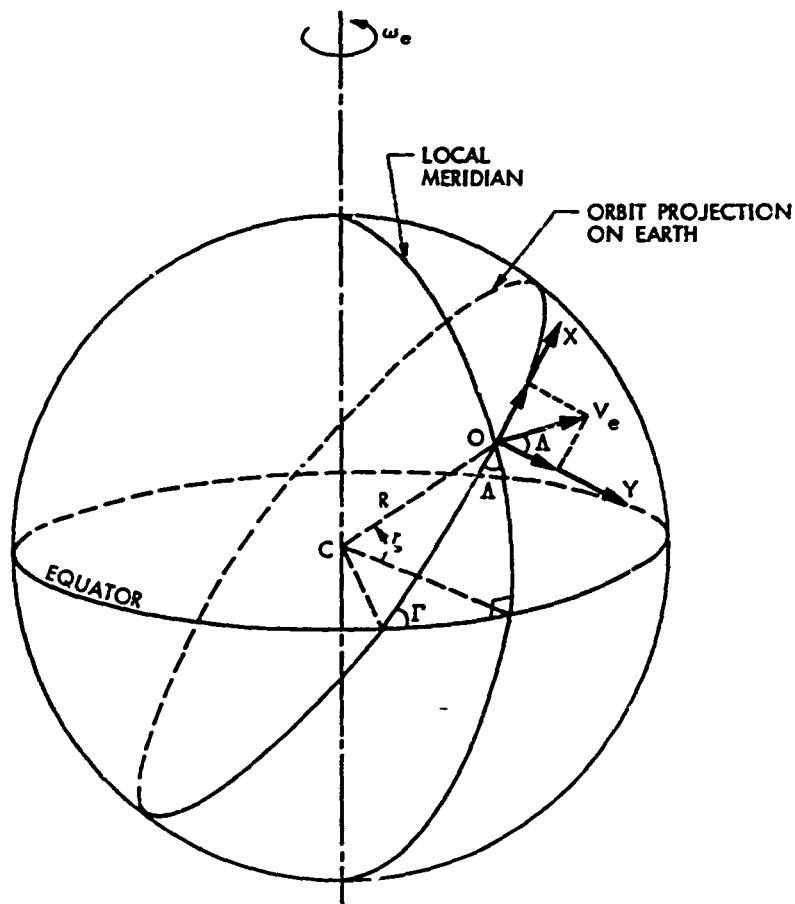
$$V_{eY} = V_e \cos \Lambda = V_e \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2} \quad (103b)$$



LEGEND:

- ω_e = ANGULAR VELOCITY OF EARTH SPIN
- ζ = LATITUDE AT POINT O
- V_e = LINEAR VELOCITY OF THE EARTH AT POINT O

Figure 28. Linear Velocity of the Earth at the Nadir Point



(a) INTRACK AND CROSSTRACK COMPONENTS OF EARTH LINEAR VELOCITY VECTOR: AT THE NADIR POINT

(b) ORBIT INCLINATION TO THE LOCAL MERIDIAN (Δ), degrees

ORBITAL INCLINATION (degrees)	NORTH LATITUDE, degrees				
	0	20	40	60	80
0	90	-	-	-	-
18.5	71.5	-	-	-	-
40	50	54.61	90	-	-
60	30	32.15	40.75	90	-
85	5	5.32	6.53	10.04	30.13
90	0	0	0	0	0

Figure 29. Earth Spherical Geometry

During the time interval from t_1 to t_2 , the ground point has moved from $P(X, Y)$ to $P'(X', Y')$, the corresponding position changes are,

$$\Delta X = \int_{t_1}^{t_2} v_{eX} dt = \int_{t_1}^{t_2} \frac{V_e \cos \Gamma}{\cos \zeta} dt \quad (104a)$$

$$\Delta Y = \int_{t_1}^{t_2} v_{eY} dt = \int_{t_1}^{t_2} V_e \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2} dt \quad (104b)$$

Note that for small time intervals, ζ may be considered constant and (104) may be approximated as

$$\Delta X \cong \frac{\cos \Gamma}{\cos \zeta} V_e (t_2 - t_1) \quad (105a)$$

$$\Delta Y \cong \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta}\right)^2} V_e (t_2 - t_1) \quad (105b)$$

and

$$X' = X + \Delta X \quad (106a)$$

$$Y' = Y + \Delta Y \quad (106b)$$

G. NUMERICAL RESULTS: GEOMETRIC ERRORS AND ERROR SENSITIVITIES

Numerical results of error sensitivities and geometric errors due to various direct errors have been generated. These data are summarized in Tables 5-10 and discussed in the following subsections.

G.1 Geometric Errors Due to Altitude Uncertainties

Table 5 shows the geometric error for a 1 km altitude change and error sensitivity caused by altitude uncertainty. The nominal altitude of 400 km was used to generate these data. This result is also plotted in Fig. 30 as a function of view angles for altitude changes ranging from 1 km to 4 km. As indicated in this figure, the geometric errors are quite linear with the view angles and the altitude errors for the range shown.

For the Imaging Spectrometer illustrated in Fig. 4, the view angles are limited by the field of view of $\pm .825^\circ$. Using the various expected on-orbit navigation accuracies shown in Table 4, and the sensitivity data of Table 5, the corresponding geometric error can be determined. For instance, from Table 4, the 3σ altitude uncertainty, with TDRSS in the tracking system for the 150 nmi. orbit, if the small unmodeled perturbation is used, is 800 feet (or ~ 244 m) and with the STDN system and large unmodeled perturbation and 105 nmi. orbit, the 3σ altitude uncertainty is 8000 feet (or ~ 2440 m). From Table 5, the corresponding 3σ geometric errors for the largest view angle ($.825^\circ$) are 3.51 m and 35.1 m, respectively. Note that the nominal pixel size is 30 m.

G.2 Geometric Errors Due to Roll Uncertainties

The geometric errors for view angles ranging from -9° to $+9^\circ$ due to a roll error of 1° are tabulated in Table 6. The error sensitivity at the nominal attitude is also tabulated in Table 6. By comparing the errors for 1° and the sensitivity for the same view angle, it is found that the differences are .2% or less. That is, for small angles, the error sensitivity data can be used to compute errors. Fig. 31 shows the plots of geometric errors for roll errors of up to 5° . Unlike the errors corresponding to altitude uncertainties, these curves are relatively flat. The geometric error is about 7 km per 1° of roll error.

It is noted from Fig. 31, that for small roll errors, the geometric errors are nearly symmetrical with the view angles. For larger roll angles, the geometric errors are greater for view angles that have the same sign of ϕ . This is more visible from Fig. 32 which shows the plots of geometric errors corresponding to roll errors of 1° and 2° from a roll offset of 20° .

G.3 Geometric Errors Due to Pitch Uncertainties

Table 7 shows the error sensitivity and the geometric errors corresponding to the 1° pitch error for the view angles of up to $+9^\circ$. The geometric errors caused by pitch errors are about the same in amplitude as those caused by roll errors. Fig. 33 shows that pitch curves are rather flat, unlike the corresponding roll curves. Figs. 34 and 35 show the 1° and 2° pitch uncertainty induced geometric error curves with 22.5° and 45° pitch offsets, respectively. These curves, especially those in Fig. 35, are more similar to the roll curves in shape.

G.4 Geometric Errors Due to Yaw Uncertainties

Geometric errors due to rotations about the yaw or the local vertical axis (assuming zero offsets) are tabulated in Table 8 for 1° and plotted in Fig. 36 for up to 5° . Linearity applies for both the view angles and the yaw angles for the ranges covered here. Unlike roll and pitch errors, yaw errors induce much smaller geometric errors.

G.5 Geometric Errors Due to Attitude Uncertainties

Table 9 shows the geometric errors due to 1° error in each of the roll, pitch, and yaw axes. The error magnitude for a given view angle is approximately the vector sum of the roll and pitch induced errors.

Fig. 37 shows the plots of geometric errors caused by up to 5° roll, pitch, and yaw errors. Fig. 38 shows two curves, one corresponding to 1° pitch error and the other, 1° yaw error, with the roll offset of 20° .

Comparing pitch curve with that of Fig. 33, the 20° roll offset has no noticeable effects on the geometric errors. On the other hand, the effects on the yaw curve are quite pronounced, as a large bias of approximately 2.5 km results due to the 20° roll.

Fig. 39 shows the 1° roll and 1° yaw curves for a pitch offset of 22.5° . Fig. 40 shows the same curves for 45° pitch offset.

With the large angular pitch offsets, the yaw curves appear quite different from those without pitch offset, as noted in Figs. 36, 39, and 40. With these offsets, yaw-induced error curves look quite similar to those induced by roll errors in shape. By offsetting the pitch angle to 45° , the yaw-induced geometric errors increase to about sevenfold from zero pitch offset, and more than twofold over the 22.5° pitch offset.

The roll-induced geometric errors were less drastically affected by pitch angular offsets. For instance, for 22.5° pitch offset, the error has increased about 7% from that with zero offset, and about 46% for 45° pitch offset.

G. 6 Ground Point Shift Due to Shuttle Motion and Earth Rotation

The earth rotation effect on the shift of the ground image varies with the latitude of the shuttle, and the combined effects of the earth rotation and shuttle motion vary with the orbital inclination in addition to the latitude. Table 10 shows the tabulation with latitude ranging from 0° to 80° . The largest earth rotation effect occurs at the equator, .46 km in one second, and diminishes to .08 km in one second at the latitude of 80° as shown in Fig. 41.

The combined effects are dominated by the motion of the shuttle since at 400 km altitude the projected ground speed is much greater than the earth rotation speed. In a one-second period, the image will shift more than 7 km for any latitude.

Table 5. Geometric Errors and Error Sensitivity
Due to Altitude Uncertainties

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

View Angle, deg.	0	1	2	3	4	5	6	7	8	9
Geometric Error Induced by 1 km Altitude Error, km	0	0.01746	0.03492	0.05242	0.06995	0.08753	0.1052	0.1229	0.1407	0.1586
Geometric Error Sensitivity, km/km	0	0.01746	0.03492	0.05242	0.06995	0.08753	0.1052	0.1229	0.1407	0.1586



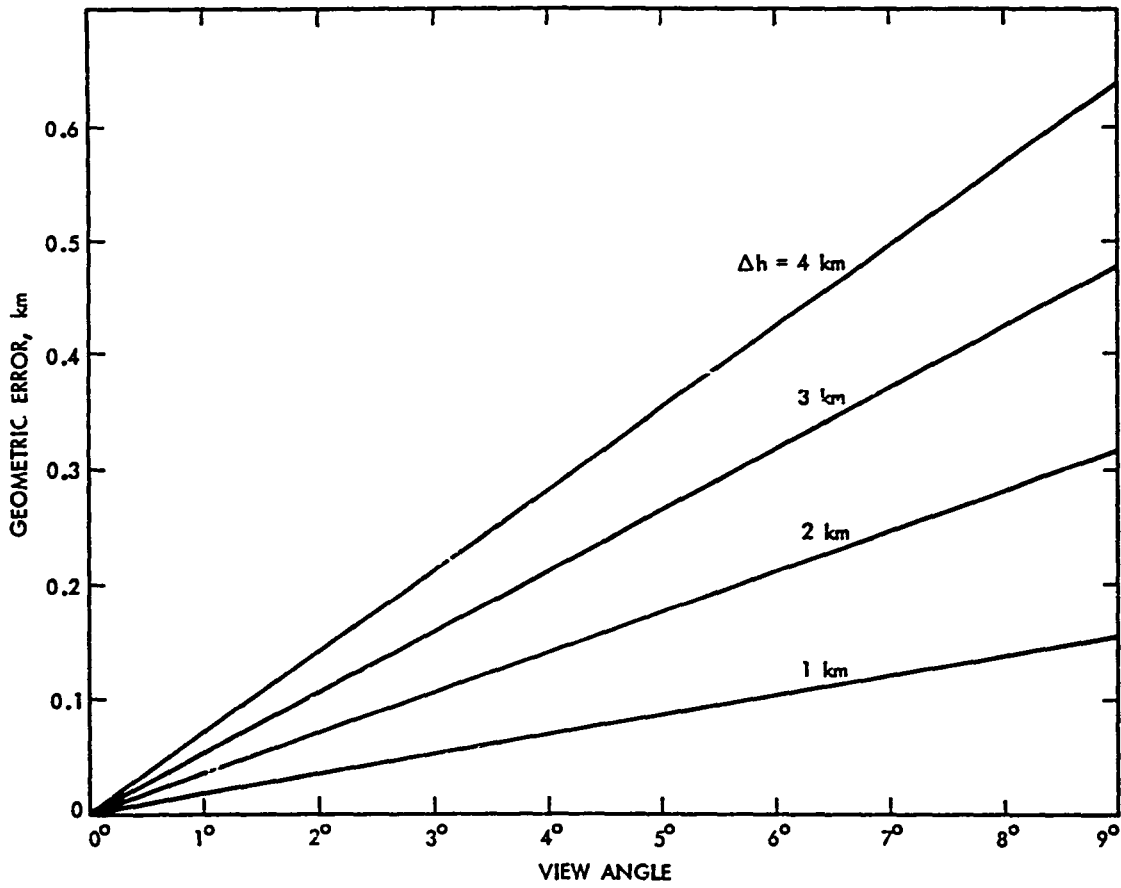


Figure 30. Geometric Error due to Altitude Uncertainties (Δh)

Table 6. Geometric Errors and Error Sensitivity
Due to Roll Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

View Angle, deg.	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Geometric Error Induced by +1° Roll Error, km	7.153	7.114	7.081	7.052	7.029	7.010	6.996	6.987	6.982	6.982
Geometric Error Sensitivity, km/deg.	7.173	7.133	7.097	7.066	7.040	7.019	7.002	6.991	6.984	6.981
View Angle, deg.	1	2	3	4	5	6	7	8	9	
Geometric Error Induced by +1° Roll Error, km	6.987	6.996	7.010	7.029	7.052	7.081	7.114	7.153	7.196	
Geometric Error Sensitivity, km/deg.	6.984	6.991	7.002	7.019	7.040	7.066	7.097	7.133	7.173	

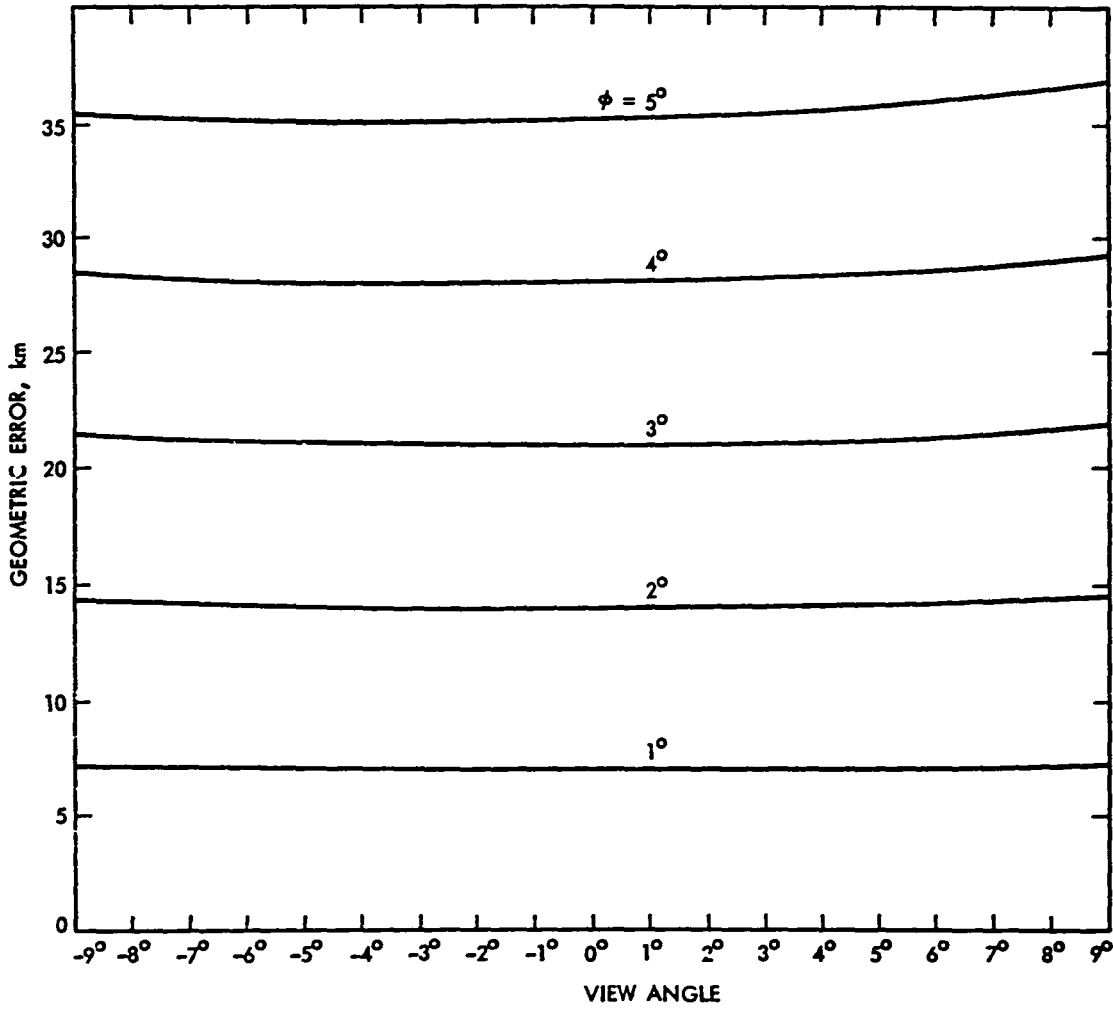


Figure 31. Geometric Error due to Roll Attitude Error (ϕ)

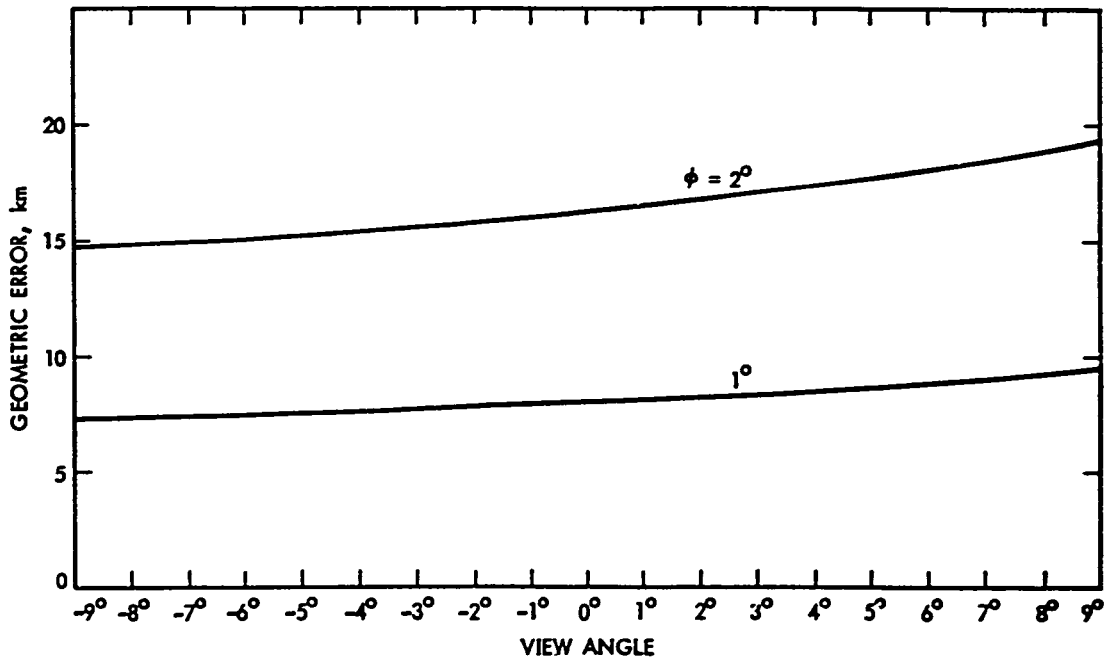


Figure 32. Geometric Error due to Roll Uncertainty (ϕ) About 20° Roll Offset

Table 7. Geometric Errors and Error Sensitivity
Due to Pitch Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

View Angle, deg.	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Geometric Error Induced by 1° Pitch Error, km	6.988	6.986	6.985	6.985	6.984	6.983	6.983	6.982	6.982	6.982
Geometric Error Sensitivity, km/deg.	6.987	6.986	6.985	6.984	6.983	6.982	6.982	6.982	6.981	6.981
View Angle, deg.	1	2	3	4	5	6	7	8	9	
Geometric Error Induced by 1° Pitch Error, km	6.982	6.982	6.983	6.983	6.984	6.985	6.985	6.986	6.986	
Geometric Error Sensitivity, km/deg.	6.981	6.982	6.982	6.982	6.983	6.984	6.985	6.986	6.987	

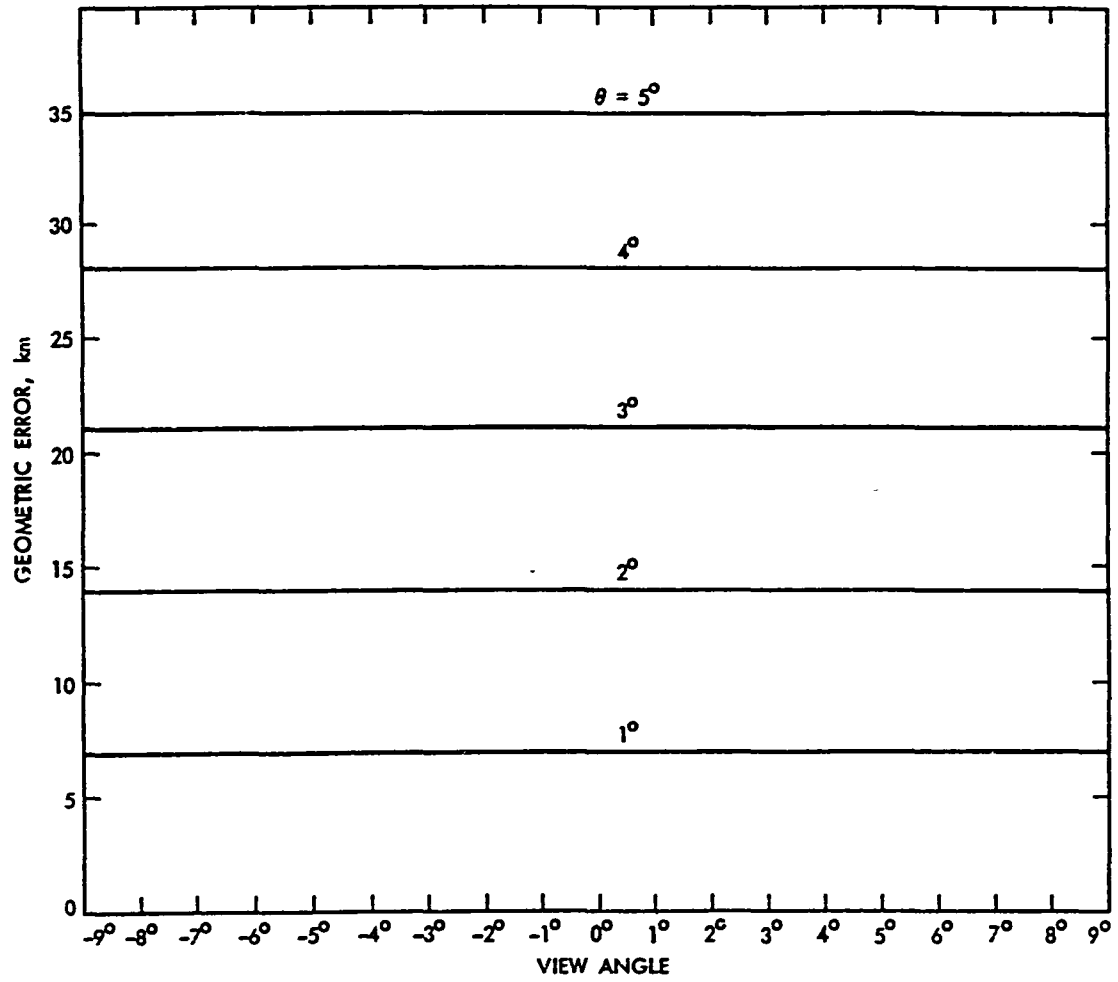


Figure 33. Geometric Error due to Pitch Attitude Error (θ)

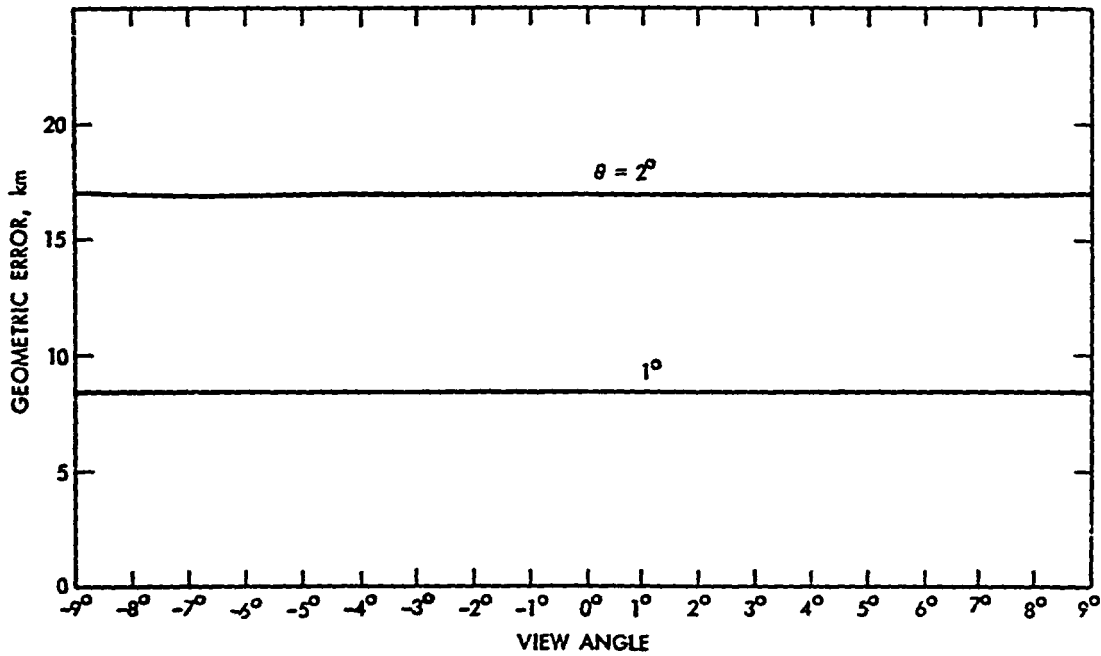


Figure 34. Geometric Error due to Pitch Uncertainty (θ) About 22.5° Pitch Offset

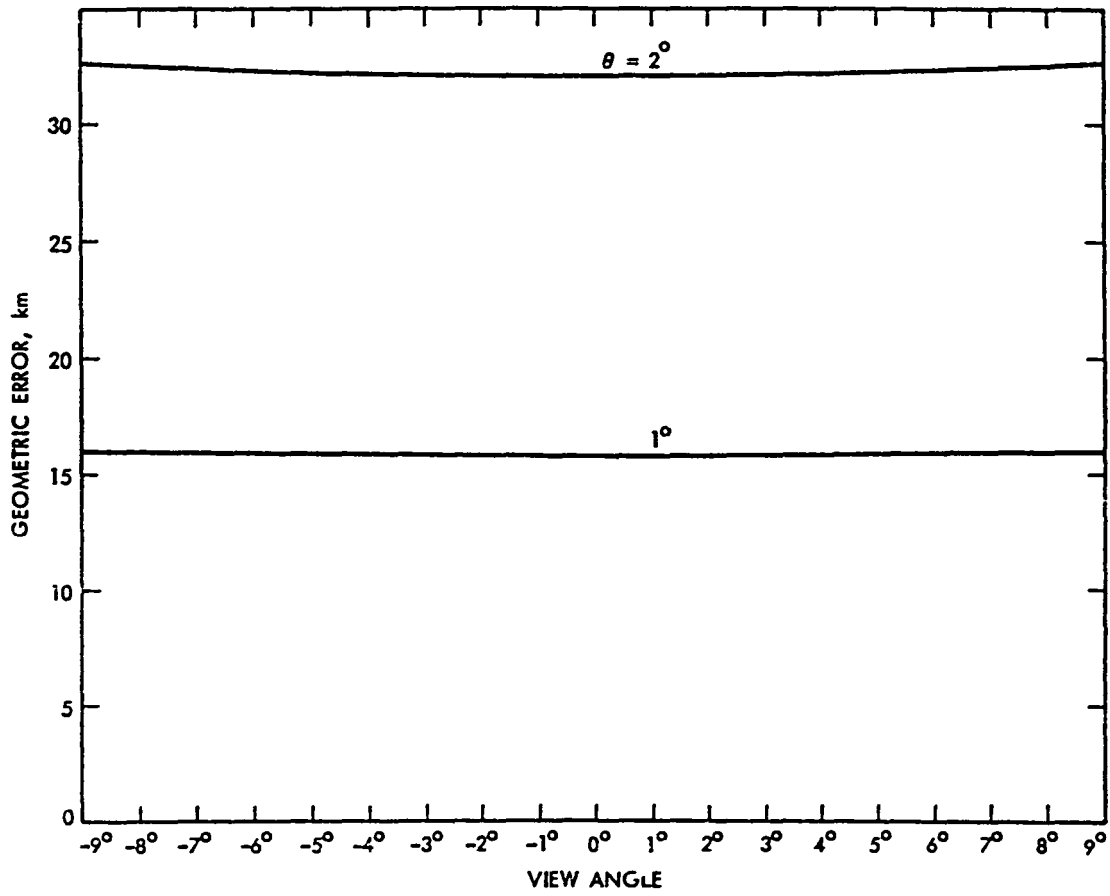


Figure 35. Geometric Error due to Pitch Uncertainty (θ) About 45° Pitch Offset

Table 8. Geometric Errors and Error Sensitivity
Due to Yaw Attitude Error

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km

View Angle, deg.	0	1	2	3	4	5	6	7	8	9
Geometric Error Induced by 1 Yaw Error, km	0	0.1219	0.2438	0.3659	0.4883	0.6109	0.7340	0.8576	0.9818	1.107
Geometric Error Sensitivity, km/deg.	0	0.1219	0.2438	0.3659	0.4883	0.6109	0.7340	0.8576	0.9818	1.107

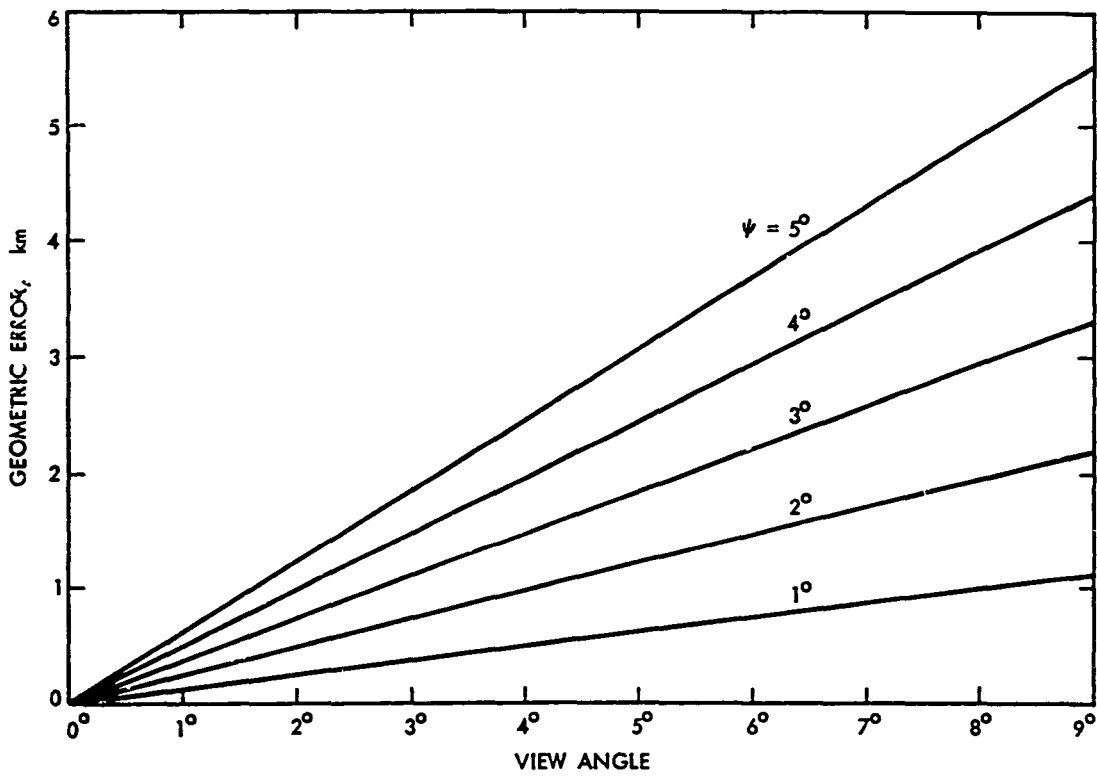


Figure 36. Geometric Error due to Yaw Attitude Error (ψ)

Table 9. Geometric Errors Induced by
Yaw, Pitch, and Roll Errors

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Altitude = 400 km
- Yaw Error = +1°
- Pitch Error = +1°
- Roll Error = +1°

View Angle, deg.	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Error, km	9.245	9.296	9.353	9.414	9.480	9.551	9.625	9.704	9.788	9.875
View Angle, deg.	1	2	3	4	5	6	7	8	9	
Error, km	9.966	10.06	10.16	10.27	10.37	10.48	10.60	10.72	10.85	

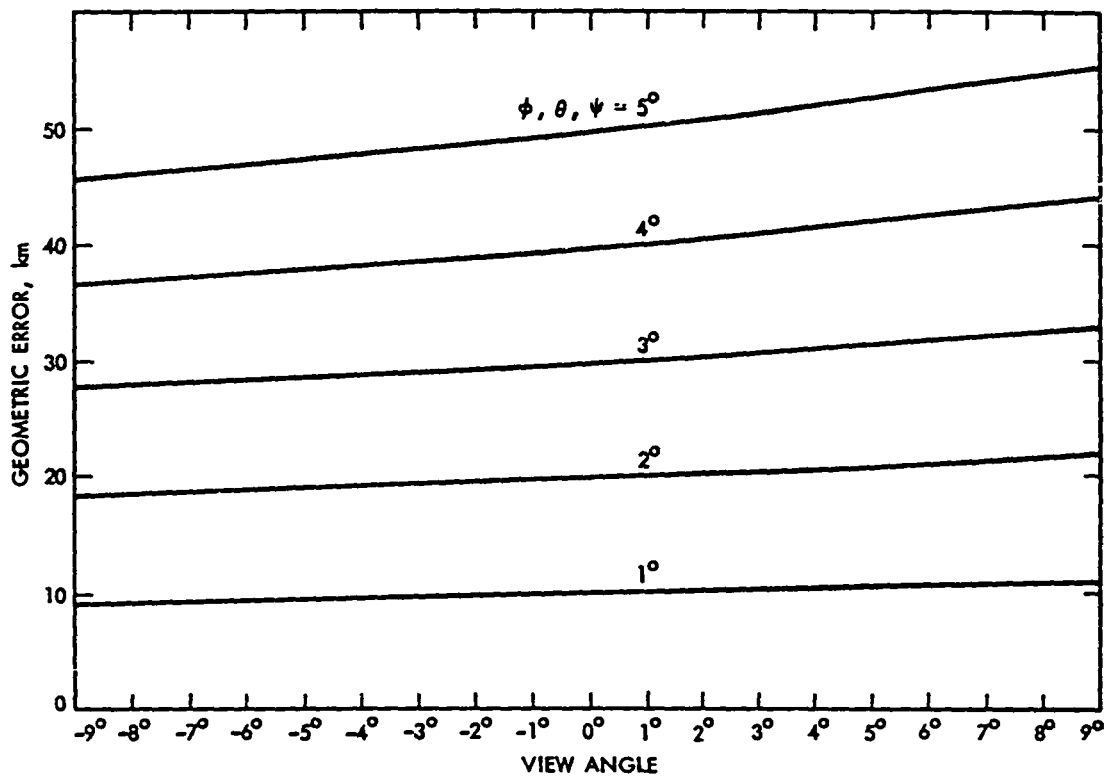


Figure 37. Geometric Error due to Roll (ϕ), Pitch (θ), and Yaw (ψ) Errors

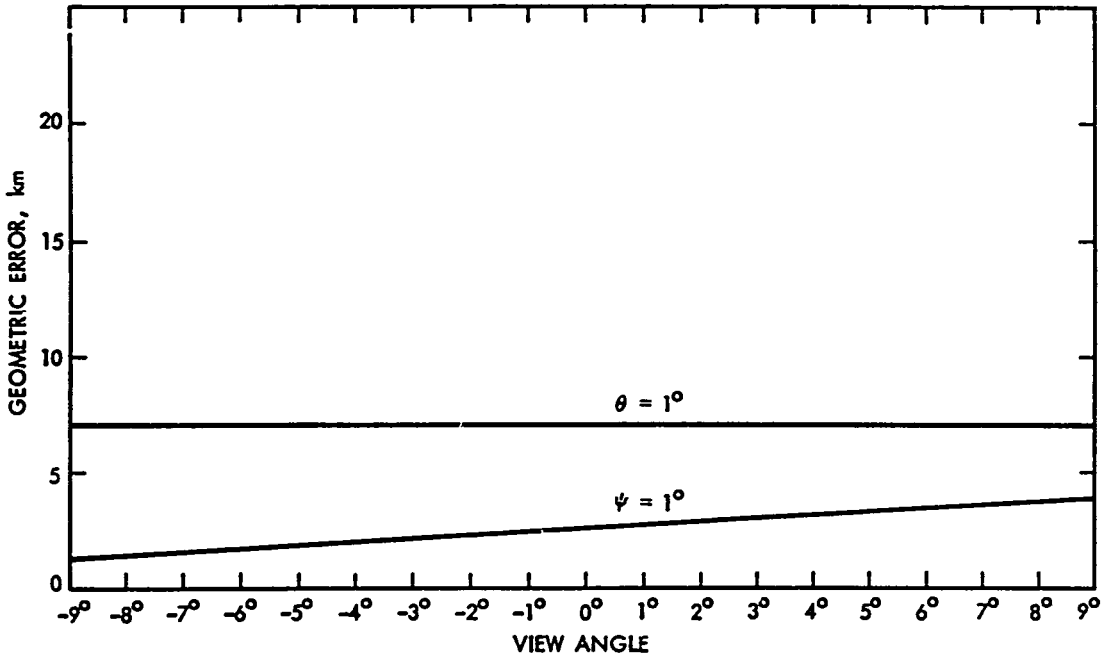


Figure 38. Geometric Error due to Pitch (θ) and Yaw (ψ) Uncertainties About 20° Roll Offset

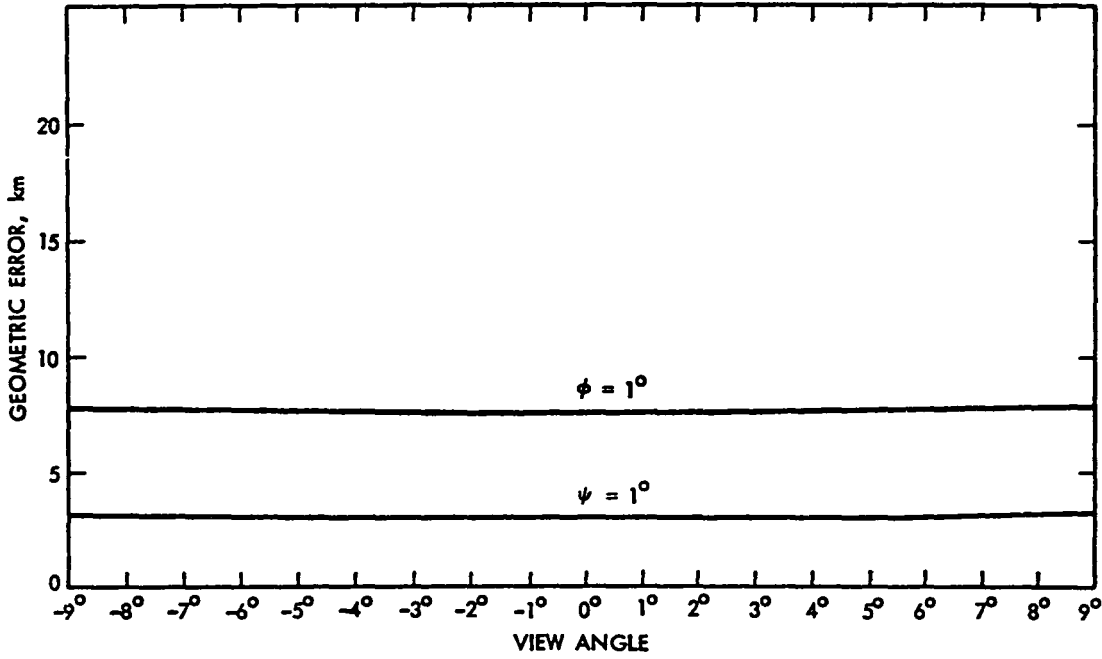


Figure 39. Geometric Error due to Roll (ϕ) and Yaw (ψ) Uncertainties About 22.5° Pitch Offset

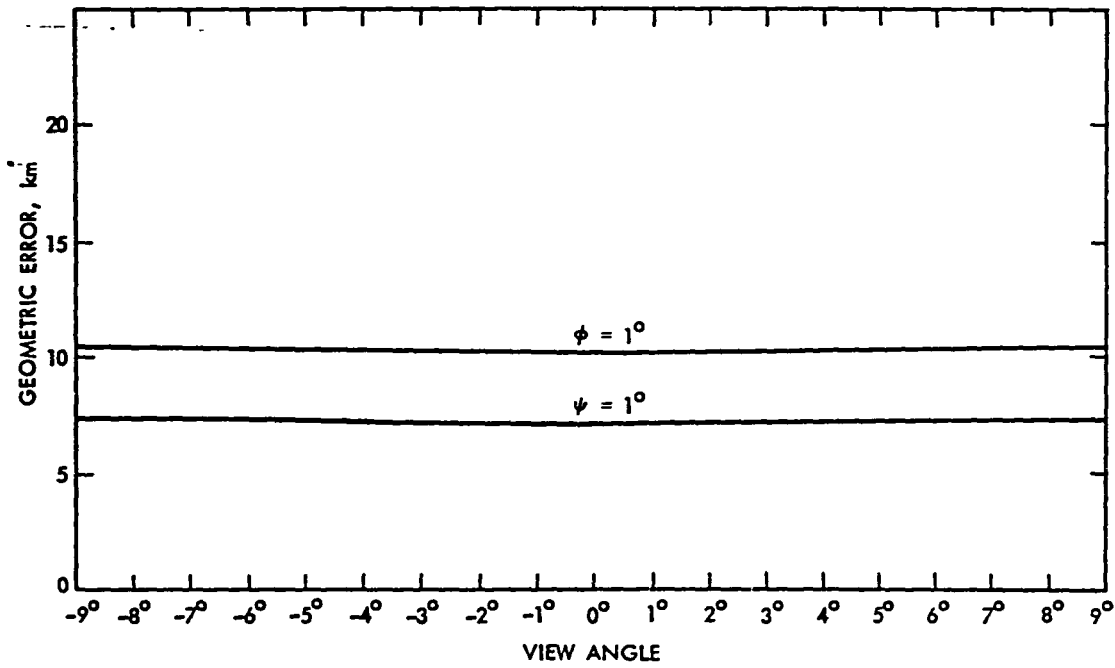


Figure 40. Geometric Error due to Roll (ϕ) and Yaw (ψ) Uncertainties About 45° Pitch Offset

Table 10. Ground Point Shift Induced by Shuttle Motion and Earth Rotation

- Nominal Earth Radius = 6356.785 km (Polar)
- Nominal Orbit Inclination = 85°
- Nominal Shuttle Ground Speed = 7.202 km/sec

Latitude, deg.	0	20	40	60	80
Ground Point Shift Velocity due to Earth Rotation, km/sec	0.4623	0.4344	0.3541	0.2311	0.08027
Ground Point Shift Velocity Relative to Nadir Point, km/sec	7.177	7.175	7.171	7.166	7.162
Angle of Velocity Vector with Y-axis, deg.	-86.32	-86.54	-87.19	-88.18	-89.44
Shift in 0.1 sec, km	0.7177	0.7175	0.7171	0.7166	0.7162

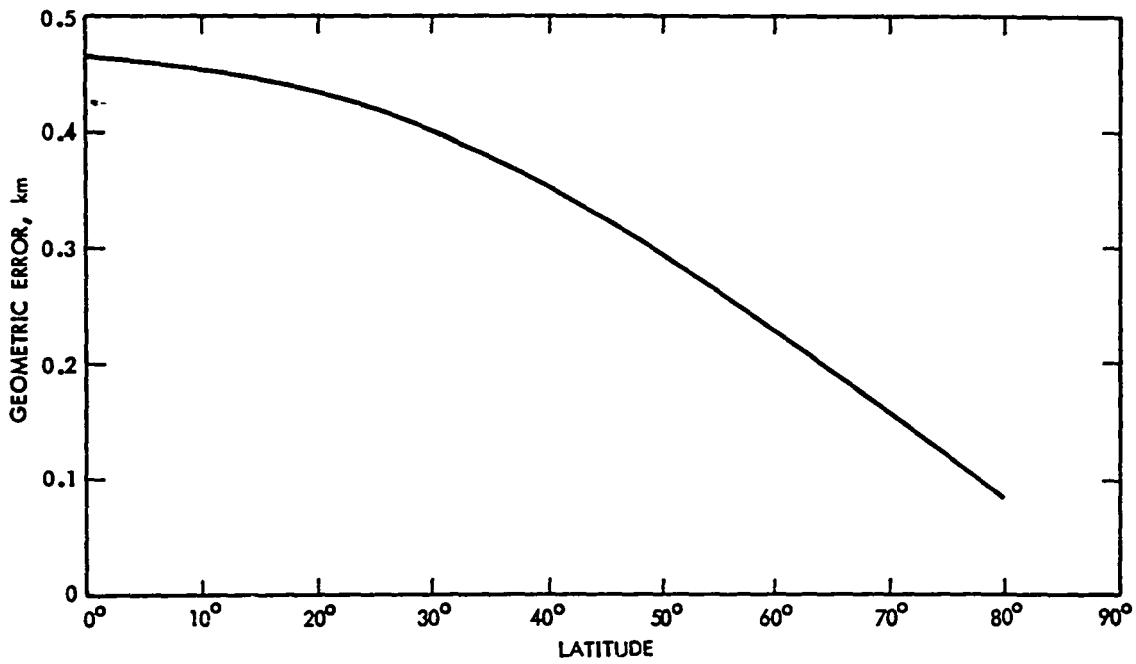


Figure 41. Geometric Error Caused by Earth Rotation for the Time Period of 1 Second

H. SUMMARY OF MAJOR FINDINGS

In this section geometric errors caused by the "direct error sources" are analyzed. Equations that map the direct errors onto the geometric errors are derived in Subsections B through G. Extensive illustrations of geometry are used to assist the derivations. Direct error effects included here are those of pointing errors and angular rates, ephemeris prediction errors including altitude, in-track, and cross-track uncertainties, earth rotation and curvature, and shuttle and instrument misalignment. For quick reference the key error mapping functions are listed in Appendix A. A program list is given in Appendix E.

The numerical results for both the geometric errors and error sensitivities are tabulated in Tables 5 through 10 and also plotted in Figs. 30 through 41. To illustrate the effects of view angles and linearity, an extended range of $\pm 9^\circ$ is used. For the push broom imaging spectrometer studied here, the field of view is limited by $\pm 8.25^\circ$ (see Fig. 4). The following are the specific findings resulting from further analysis:

1. The effects of earth curvature are very small for the applications here. For instance, the linear displacement of a 10 km arc is 9.9999...km. Fig. 42 illustrates several cases of earth curvature effects.

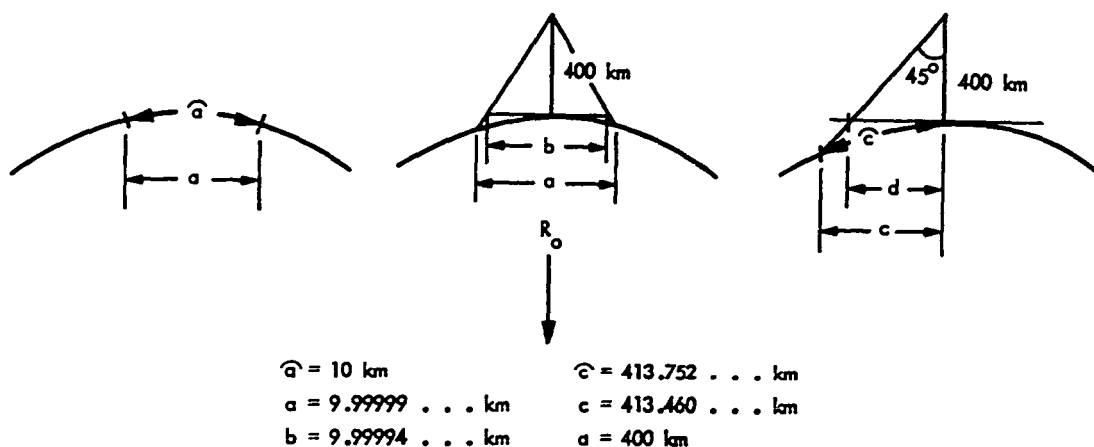


Figure 42. Effects of Earth Curvature

2. Altitude uncertainties cause only moderate geometric errors. Table 11 shows that the worst 1 σ geometric error is 11.71 m and the corresponding rate is .133 m/sec. The worst case associates with the STDN system and large unmodeled perturbation and 200 km orbit. The least 1 σ errors are .22 m and .0032 m/sec corresponding to the TDRSS system and small unmodeled perturbation and 300 km orbit. The effects of other navigation uncertainties such as downtrack and crosstrack position errors are not small, however. Table 12 shows the 3 σ uncertainties in feet. Downtrack and crosstrack position errors are mapped directly to the geometric errors.

3. Geometric errors caused by altitude uncertainties are, within the range of interest here, proportional to the view angle and altitude error (see Table 5 and Fig. 30).

4. The effects of roll and pitch attitude errors are relatively large compared with those of altitude errors and yaw attitude errors. Table 13 shows that, for nominal flight conditions, the error sensitivity is approximately 1.94 m/arc sec, or ~ 7000 m/degree. The yaw sensitivity is very small, from 0 for 0 $^\circ$ view angle to .0279 m/arc sec for maximum IS view angle $\pm 825^\circ$.

The effects of attitude errors increase significantly for large attitude offsets. For instance, for 20 $^\circ$ roll offset (side looking), the roll sensitivity increases to 2.28 m/arc sec from 1.94 and yaw sensitivity increases to .75 m/arc sec from .0279. The pitch sensitivity, in this case, is nearly unaffected. By offsetting the pitch angle to 45 $^\circ$ (forward looking), the roll sensitivity increases to 2.83 m/arc sec, the pitch to 4.36 m/arc sec, and the yaw to 1.97 m/arc sec. Table 13 shows the error sensitivities and 1 $^\circ$ geometric errors for many combinations of interest.

5. Without attitude offsets and within the IS field of view, the geometric errors are almost proportional to roll errors (Fig.31), pitch errors (Fig.33), and yaw errors (Fig.36), and are independent of the view angle except for the yaw cases (for which the error is proportional to the view angle). For the cases of large roll or pitch angular offset, most of the properties change only slightly (see Figs. 32, 34, 35, 38, 39, and 40).

6. The performance of the Imaging Spectrometer is limited by the shuttle Inertial Measuring Unit accuracy, the shuttle reference misalignment, and the misalignment between the shuttle and the Imaging Spectrometer, unless means of error reduction, such as using ground control points and a precision point mount between the shuttle and the IS instruments, are employed.

On top of these uncertainties, the shuttle deadband is another source of error that can cause gross geometric errors. This problem, of course, can be resolved by using a precision point mount.

Table 14 shows geometric errors caused by the IMU/shuttle misalignment, IMU resolution, IMU noise, rate gyro drift, and combined shuttle/IS misalignment, for nominal attitude and attitude offsets. Single axis geometric errors corresponding to the combined shuttle/IS misalignment are 169 m, 198 m, and 379 m, respectively, for nominal, 20° roll offset, and 45° pitch offset, for instance. Other cases are shown in detail in Table 14.

7. Earth rotation causes images to shift toward the direction of rotation. The magnitude of this shift depends on the latitude of the object (see Fig. 41). For instance, at the equator, the object moves approximately 462 m in 1 second, while at 60° latitude, it moves only 231 m in one second.

Table 11. Geometric Errors Due to Expected On-orbit Navigation Uncertainty

Navigation Tracking System	Unmodeled Perturbation	Minimum Perigee, km	1 σ Altitude Error, m	Max. Geom. Error in FOV, m	1 σ Radial Velocity, m/s	Max. Geom. Error in FOV, m/s
STDN	Nominal	200	508	7.32	3.87	.056
TDRSS		200	366	5.27	2.25	.032
STDN	Small	200	305	4.39	2.50	.036
		300	122	1.76	.59	.0085
TDRSS		200	183	2.63	1.12	.016
		300	81	1.17	.22	.0032
STDN	Large	200	813	11.71	9.27	.133
TDRSS		200	610	8.78	5.69	.082
STDN - Spaceflight Tracking and Data Network TDRSS - Tracking Data Relay Satellite System						

Table 12. Expected On-Orbit Navigation Accuracies (Ref. 12)

Navigation Tracking System	Unmodeled Perturbation	Minimum Perigee, n.mi.	3 σ Position, Feet			3 σ Velocity, FTS		
			Radial	Down Track	Cross Track	Radial	Down Track	Cross Track
STDN	Nominal	105	5,000	34,500	5,000	38.4	5.9	9.8
TDRSS		105	3,600	18,500	4,000	22.1	3.3	4.0
STDN	Small	105	3,000	22,000	1,500	24.6	3.3	2.0
		150	1,200	5,500	1,000	5.8	1.3	1.8
TDRSS		105	1,800	10,000	2,000	11.0	1.8	2.9
		150	800	2,000	1,500	2.2	0.9	2.1
STDN	Large	105	8,000	80,000	5,000	91.2	8.0	8.0
TDRSS		105	6,000	50,000	5,000	56.0	6.2	7.8

NOTE:

The correlation between downtrack position and radial velocity is -0.95.
The correlation between radial position and downtrack velocity is -0.80.

STDN - Spaceflight Tracking and Data Network
TDRSS - Tracking Data Relay Satellite System

Table 13. Geometric Error Sensitivity

Nominal Orbit: 400 km Circular

Error Source	Attitude Offset	Error Sensitivity, m/arc sec		Geometric Error for 1° Angular Error, m
		For View Angle = 0°	For View Angle = ±.825°	
Roll angle	No offset	1.939	1.940	6,987
Pitch angle		1.939	1.939	6,982
Yaw angle		0	.0279	101
Roll angle	20° roll offset	2.25	2.28 2.22	8,200
Pitch angle		1.94	1.94	7,000
Yaw angle		.72	.75 .70	2,683
Roll angle	22.5° pitch offset	2.11	2.11	7,600
Pitch angle		2.31	2.31	8,300
Yaw angle		.83	.83	3,000
Roll angle	45° pitch offset	2.82	2.93	10,200
Pitch angle		4.33	4.36	15,700
Yaw angle		1.94	1.97	7,100
Altitude uncertainty		0	.0144 m/m	14.4 m per 1 km altitude change max in FOV

Table 14. Geometric Error Induced by Shuttle and Imaging Spectrometer Measurement Uncertainties

Nominal Orbit: 400 km Circular

Source	Attitude Offset	Per Axis, arc · sec	Per Axis Geometric Error, m	2 Axes,† Geometric Error, m
IMU/Shuttle Misalignment	No Offset	82	159	225
IMU Resolution*		20	39	55
IMU Noise		20	39	55
Rate Gyro		60 arc sec/sec	116 m/s	165 m/s
Combined Shuttle/IS Misalignment		87	169	239
IMU/Shuttle Misalignment	20° Roll Offset	82	187	245
IMU Resolution		20	46	60
IMU Noise		20	46	60
Rate Gyro		60 arc sec/sec	137 m/s	180 m/s
Combined Shuttle/IS Misalignment		87	198	260
IMU/Shuttle Misalignment	45° Pitch Offset	82	358	426
IMU Resolution		20	87	104
IMU Noise		20	87	104
Rate Gyro		60 arc sec/sec	262 m/s	312 m/s
Combined Shuttle/IS Misalignment		87	379	452

† Combined roll and pitch axes.

V. GROUND PATTERNS AND IMAGE DISTORTIONS

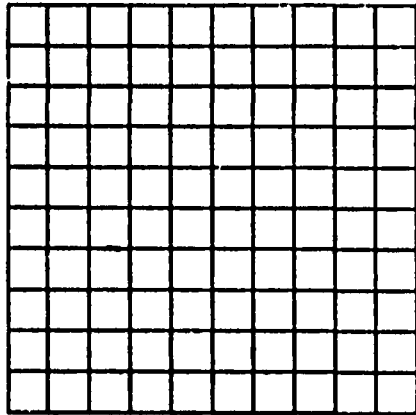
The extent that direct errors and earth geometric properties affect the performance of the Shuttle Imaging Spectrometer are analyzed and extensively illustrated with charts and tables in section IV. The advantages of tables and charts are that they carry precise and specific information which is invaluable for design and performance prediction. However, it is difficult to relate this information directly to images of ground objects by human brains. To compensate for this and to make direct observation of how errors in the system affect ground images, selected ground patterns as seen through this optical system are studied here.

The patterns that are readily generated and, most importantly, suitable for exhibiting imaging distortions are square grid patterns.

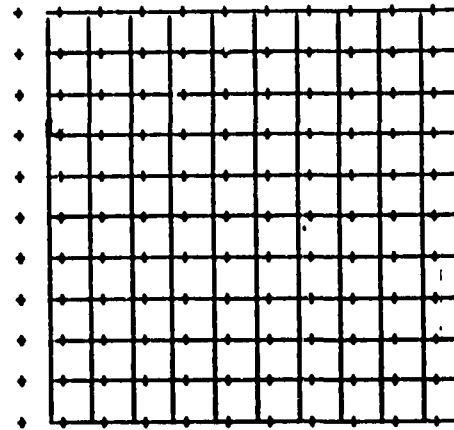
To cover a large ground that is comparable to the field of view of the imaging spectrometer, a 10 km field is selected and the field is evenly divided into 10 segments of 1 km each. Since the imaging spectrometer employs a push broom principle, each field is then 10 km wide (cross track) and 30 m "long" (along track). However, to see a large ground area, a 10 km x 10 km field is employed with the along track grids tagged with time. Considering a perfectly spherical earth, an imaginary 10 x 10 grid is painted on the ground. As the satellite flies through this region, in the direction from the bottom to the top of the paper, a push broom camera should see a pattern just like that of Fig. 43(a). This pattern is referred to as the nominal ground pattern.

Fig. 43 shows how the ground pattern changes when simple attitude error occurs without considering the effect of Earth rotation. To show the effect of errors, the solid pattern (actual) is overlaid with the "+" pattern (the undistorted nominal pattern). Fig. 43(b) shows that when a 0.1° roll attitude error occurs, the IS instrument points 0.1° left of the object field and the image on

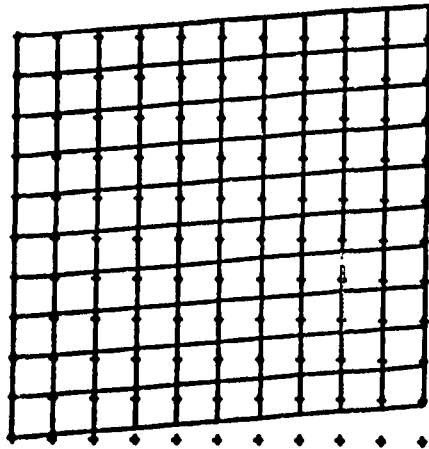
(a) NOMINAL GROUND PATTERN



(b) WITH 0.1° ROLL ERROR



(c) WITH 5° YAW ERROR



(d) WITH 0.1° PITCH ERROR

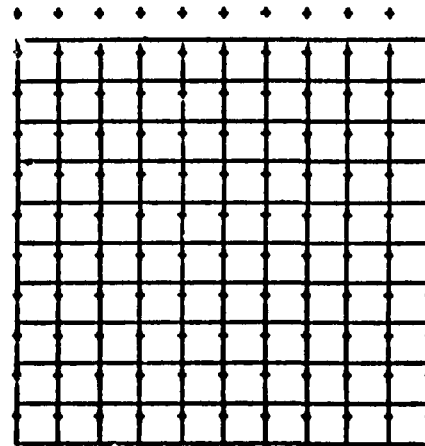


Fig. 43. Ground Pattern Distortions With No Earth Rotation Effect

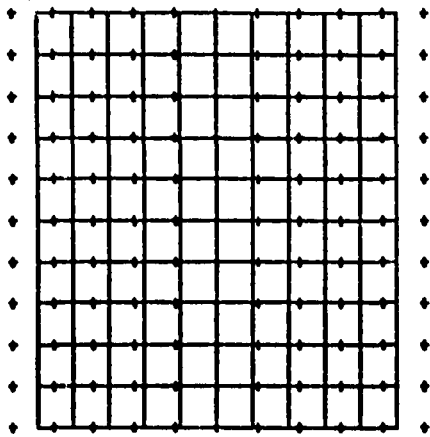
the "film" has shifted to the right; therefore, the solid pattern has shifted to the right of the "+" pattern. The same explanation applies also to the 0.1° pitch error as shown in Fig. 43(d), except that, for pitch, the pattern image has shifted to the "-along track" direction. A 5° yaw error (clockwise rotation) will make the pattern appear skewed as shown in Fig. 43(c).

Fig. 44 shows the effects of simple attitude errors with 20° roll offset. With the large roll angle, the nominal pattern itself has suffered cross track distortions, i.e., the pattern image appears shrunk cross-trackwise as shown in Fig. 44(a). In Fig. 44(b), (c), and (d), the "+" patterns represent the distorted nominal patterns due to roll offset alone and the solid patterns represent those due to attitude errors on top of roll offset. The distortions are similar to those found in Fig. 43 except that with the 20° roll offset the images appear narrower.

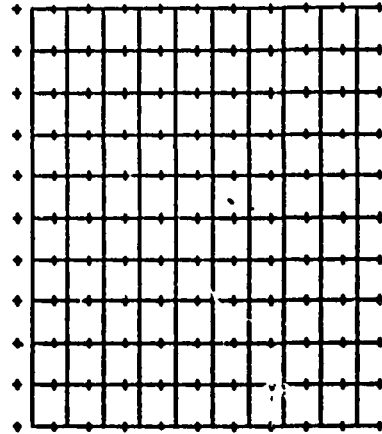
The effects of simple attitude errors with 45° pitch offset are shown in Fig. 45. The effect of pitch offset is similar to roll offset, i.e., the nominal pattern image has shrunk cross-trackwise. One might expect also along-track shrinkage. The reason that there is no along-track shrinkage is because of the push-broom effect. Shrinkage will occur for a frame camera, however. It is noted that the pattern images associated with the 45° pitch offset are narrower than those of the 20° roll offset. This is due to the fact that 20° is a lot less than 45° .

The effects of constant attitude rate errors are shown in Fig. 46. Rate errors cause accumulation of attitude errors, i.e., the attitude errors grow with time t . Take the 0.1 deg/sec rate error for the roll axis for instance, the roll error is $0.1 t$; whereas, the 0.1° roll error (Fig. 43(b)) at $t = 0$ is 0; therefore, the "+" pattern image coincides with the solid one at a very

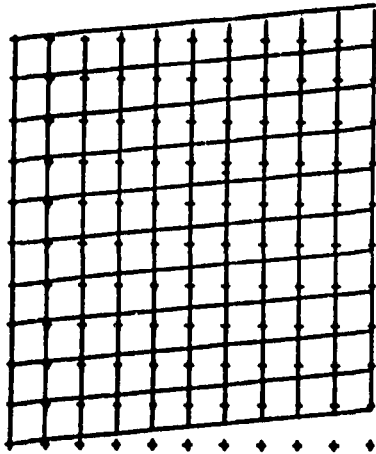
(a) NOMINAL GROUND PATTERN



(b) WITH 0.05° ROLL ERROR



(c) WITH 5° YAW ERROR



(d) WITH 0.1° PITCH ERROR

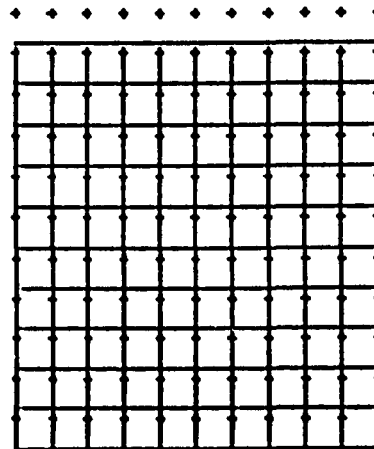
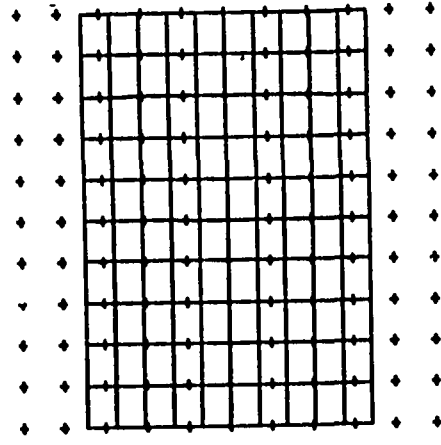


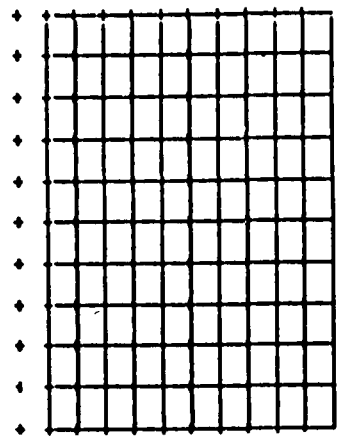
Fig. 44. Ground Pattern Distortions With 20° Roll Offset and No Earth Rotation Effect

M. S. RAJ

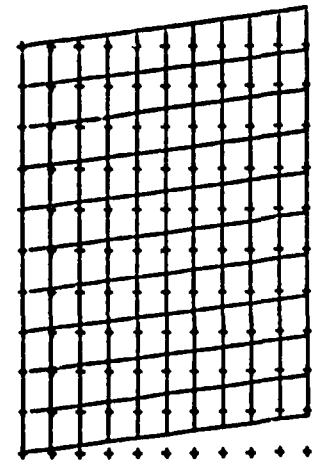
(a) NOMINAL GROUND PATTERN



(b) WITH 0.1° ROLL ERROR



(c) WITH 5° YAW ERROR



(d) WITH 1° PITCH ERROR

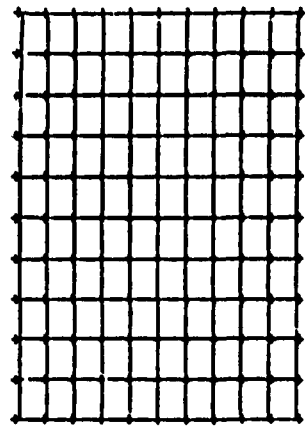
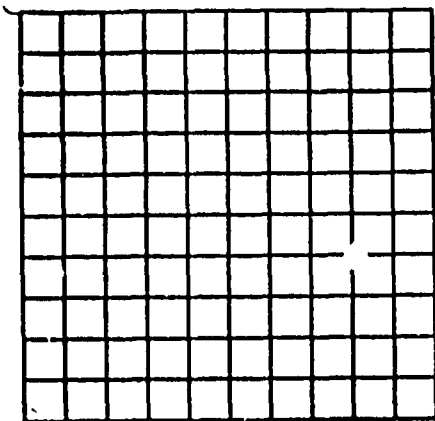
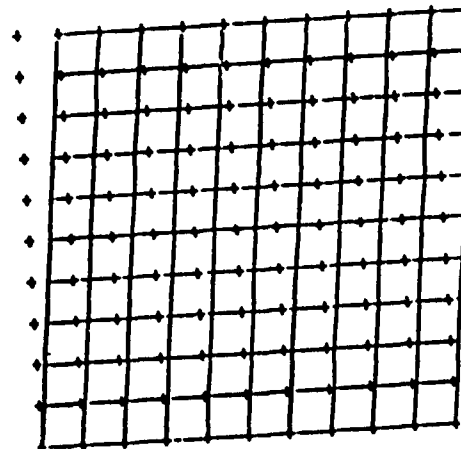


Fig. 45. Ground Pattern Distortions With 45° Pitch Offset and No Earth Rotation Effect

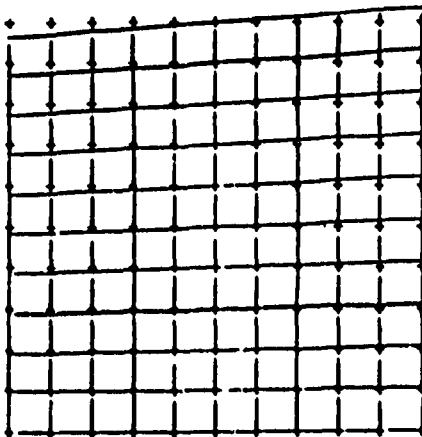
(a) NOMINAL GROUND PATTERN



(b) WITH 0.1°t ROLL ERROR



(c) WITH 3°t YAW ERROR



(d) WITH 0.1°t PITCH ERROR

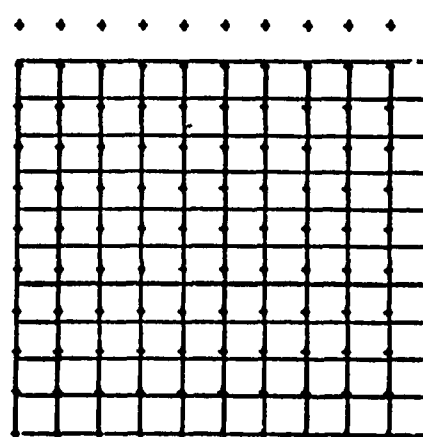


Fig. 46. Ground Pattern Distortions With Constant Rotation Rate and No Earth Rotation Effect

small t . Shown in Fig. 46(b) is the solid pattern shift to the right at a constant slope with time, which is quite different from Fig. 43(b). Similar comment applies to Fig. 46(c) and (d).

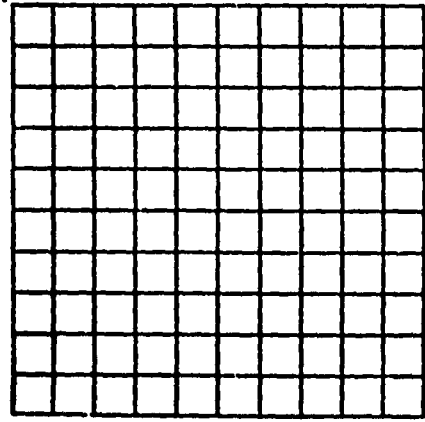
The effects of sinusoidal attitude rates on the pattern images are shown in Fig. 47. For instance, the sinusoidal rate causing a $0.2 \sin (4.51378t)$ degree roll error will make the image appear as the shape "S", as shown in Fig. 47(b). The same function applied to the pitch axis will cause along-track distortions with densely packed grid lines followed by loosely packed ones, etc., as illustrated in Fig. 47(d).

Altitude change causes images to vary in size. Again, it affects the cross-track much more than the along-track due to the push-broom principle. As shown in Fig. 48, the solid image has shrunk cross-trackwise but expanded along-trackwise. The former is due to the altitude increase of 40 km which has no amplification effect on push-broom camera; the latter is due to the fact that, at higher altitude, the orbital rate is decreased, i.e., it will take a longer period of time to fly through the same ground area.

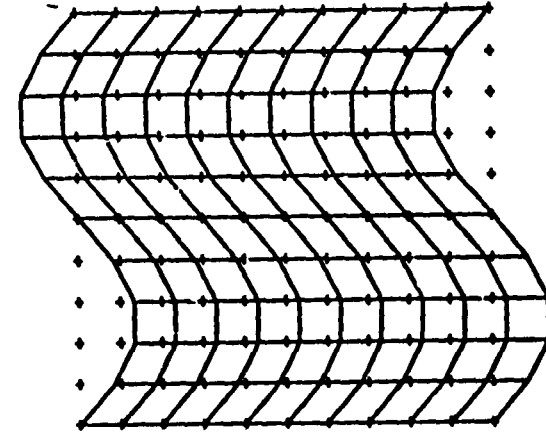
Finally, the effects of earth rotation are shown in Fig. 49. Earth rotation will cause the image to skew toward the direction of earth rotation. As discussed in section IV, the effect of the image shift is most pronounced when the satellite flies through the equator, and reduces as the latitude increases. Earth rotation will not cause along-track distortions.

Computer programs that are used to generate these patterns and distortions are listed in Appendix F.

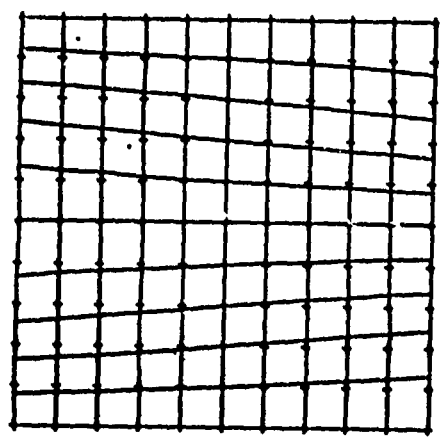
(a) NOMINAL GROUND PATTERN



(b) WITH $0.2^\circ \sin(4.51378t)$ ROLL ERROR



(c) WITH $5^\circ \sin(4.51378t)$ YAW ERROR



(d) WITH $0.1^\circ \sin(4.51378t)$ PITCH ERROR

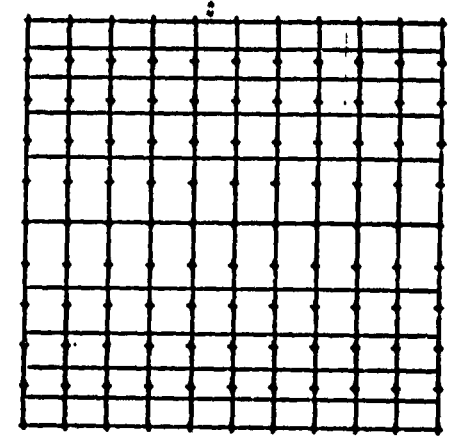
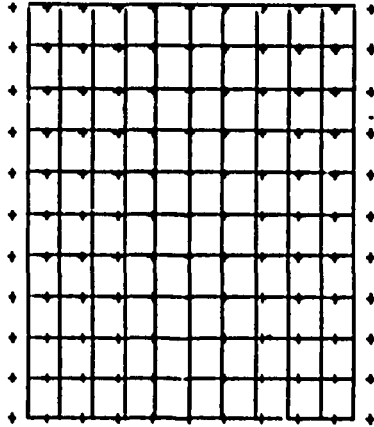
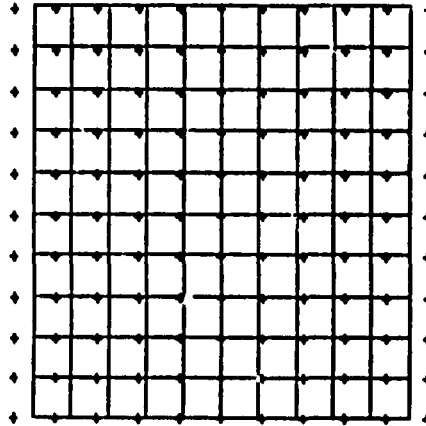


Fig. 47. Ground Pattern Distortions With Sinusoidal Rotation Rate and No Earth Rotation Effect

(a) NOMINAL GROUND PATTERN
WITH 20° ROLL OFFSET



(b) NOMINAL GROUND PATTERN
WITH NO OFFSETS



(c) NOMINAL GROUND PATTERN
WITH 45° PITCH OFFSET

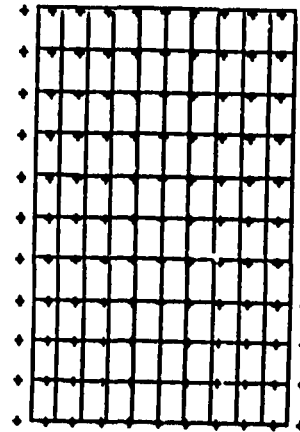
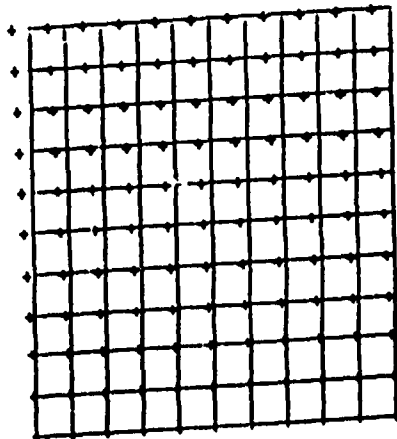
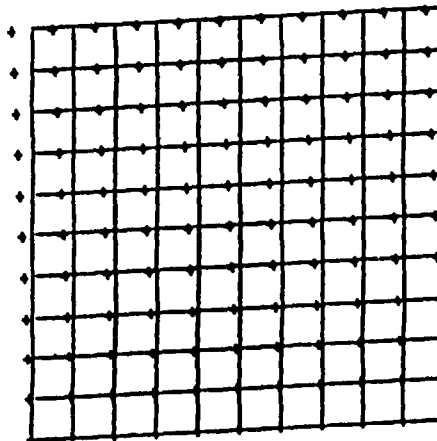


Fig. 48. Ground Pattern Distortions With Effect of Altitude Change (increase) of 40 km and No Earth Rotation Effect

(a) NOMINAL GROUND PATTERN
WITH 20° ROLL OFFSET



(b) NOMINAL GROUND PATTERN
WITH NO OFFSETS



(c) NOMINAL GROUND PATTERN
WITH 45° PITCH OFFSET

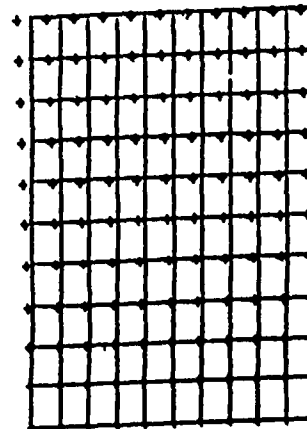


Fig. 49. Ground Pattern Distortions With Effect of Earth Rotation for 40° Latitude
and Orbit Inclination of 85°

VI. CONCLUSIONS

1. The effects of earth curvature are very small for the application here (see Fig. 42).
2. Altitude uncertainties cause only moderate geometric errors. The worst 1σ geometric errors are 11.71 m in position and 0.133 m/sec in rate with STDN and large unmodeled perturbations at 200 km orbit. The performance improves with TDRSS. For the 300 km orbit and with small unmodeled perturbations, the 1σ geometric errors will reduce to 0.22 m and 0.0032 m/sec (see Table 11).
3. The effects of other navigation errors are significantly greater. The 1σ downtrack errors range from 203 m (300 km orbit) to 8128 m (200 km orbit); and those for the cross track are 152 m to 508 m (see Table 12).
4. The effects of roll and pitch attitude errors are relatively large compared with, for instance, those caused by yaw errors and altitude uncertainties. The error sensitivity is 1.94 m/arc sec or approximately 7000 m/degree. The yaw sensitivity is 0 for a 0° view angle and 0.028 m/arc sec for the maximum view angle of $\pm 0.825^\circ$ (see Table 12).
5. The error sensitivities of attitude errors increase significantly for large attitude offsets. For 20° side looking, the sensitivity increases to 2.28 m/arc sec for roll errors and to 0.75 m/arc sec for yaw errors, and the pitch error sensitivity is almost unaffected. For the 45° forward looking case, the sensitivities for the pitch, roll, and yaw errors increase to 2.83, 4.36, and 1.97 m/arc sec, respectively (see Table 13).
6. The performance of the Shuttle Imaging Spectrometer is limited by the Shuttle IMU (Inertial Measuring Unit) accuracy, instrument misalignment, Shuttle RCS (Reaction Control Subsystem) deadband, etc. unless some means of error reduc-

tion are employed. For instance, ground control points may be used to reduce navigation prediction errors; and precision point mounts, such as AGS (ASPS⁺ Gimbal System) and IPS (Instrument Pointing System), may be used to reduce the attitude errors. The single axis geometric errors due to the combined Shuttle/IS misalignment, for instance, for normal nadir pointing, 20° side looking, and 45° forward looking are 169 m, 198 m, and 379 m, respectively (refer to Table 14).

7. Earth rotation causes shifts of images toward the direction of rotation. The magnitude of these shifts depends on the latitude of the object. For instance, at the equator the object moves approximately 462 m in one second (for 400 km orbit), and at 60° latitude it moves only 231 m in one second.
8. Distorted images for selected ground patterns provide revealing information of the effects of system errors to the images of ground objects.

⁺ Annular Suspension Pointing System

REFERENCES

1. "LANDSAT D Position Determination and Correction Study," Final Report prepared for NASA Goddard Space Flight Center by General Electric-Space Division, GE DOC #76SDS4277, NASA CR-144829, 22 Nov. 1976.
2. "Shuttle Imaging Spectrometer Experiment, Vol. I: Investigation and Technical Plan," Proposal submitted to NASA, JPL internal document D-755, July 18, 1983.
3. Wellman, J.B., Breckinridge, J.B., Kupferman, P.N., and Salazar, R., "Imaging Spectrometer: An Advanced Multispectral Imaging Concept," Vol. I, Digest of 1982 International Geoscience and Remote Sensing Symposium, Munich, West Germany, June 1-4, 1982.
4. Shuttle Operational Data Book, JSC-08934, Vol. II, Rev. B, August 1980; Amendment 46, January 6, 1982.
5. Space Shuttle System Payload Accomodations, JSC-07700, Vol. XIV, Rev. G, September 26, 1980, Change No. 35, August 21, 1981.
6. "Three-Plate On-Orbit Aerodynamic Torque Model," SSFS Doc. Series AER037, JSC-12604 (LEC-10294) Johnson Space Center, Houston, TX, March 1977.
7. "General Consideration of Control Disturbances for Large Space Structures," Final Report for JPL, contract #955345, prepared by Lockheed Missiles and Space Co., Sunnyvale, CA, January 18, 1980.
8. Space Transportation System User Handbook, N77-82346, NASA, Washington, D.C., June 1977.
9. "LANDSAT D Position Determination and Correction Study," Final Report prepared for NASA Goddard Space Flight Center by General Electric-Space Division, CR-144829, November 22, 1978.
10. Space Shuttle Orbiter Flight Test, Level C, Functional Subsystem Software Requirements, Guidance, Navigation, and Control, Part C, Flight Control, Part 2, SD 76-SH-0009 Book 2B, September 15, 1980.
11. Sackett, L.L., and Kirchway, C.B., "Dynamic Interaction of the Shuttle On-Orbit Flight Control System with Deployed Flexible Payload," Paper #82-1535, AIAA Guidance and Control Conference, San Diego, CA, August 9-11, 1982.
12. Osborne, D., Update of JSC-07700, Vol. XIV, July 1982.
13. Wertz, J.R., Spacecraft Attitude Determination and Control, D. Reidel Publishing Co., Dordrecht, Holland, pp. 730-731, 1978.

APPENDICES

APPENDIX A

GEOMETRIC ERROR MAPPING FUNCTION TABLE

Part A: Given the attitude offsets and a view angle, find the coordinates of the corresponding ground points before and after the errors are introduced.

Error Sources	Geometric Error Mapping Function	Comments
Altitude Error, Δh	<p>At nominal altitude h, a view angle λ is given which sights a ground point $P(O, Y_o)$. After elevating Δh, the same view angle will sight another ground point $P'(O, Y'_o)$. The following equations are for obtaining Y_o and Y'_o:</p> $Y_o = -R \sin \lambda \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right]$ $Y'_o = -R \sin \lambda \left[\left(1 + \frac{h + \Delta h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R}\right)^2 \sin^2 \lambda} \right]$	<p>R = Earth Radius</p>
Yaw Error, ψ	<p>Given: a yaw offset ψ_o and a view angle λ which sights a ground point $P(X^*, Y^*)$. The same view angle will sight another ground point $P'(X^{*'}, Y^{*'})$ after yaw error ψ is introduced. The following equations are for obtaining X^*, Y^*, $X^{*'}$, and $Y^{*'}$:</p> $X^* = \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin \psi_o$ $Y^* = - \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos \psi_o$	<p>R = Earth Radius h = Nominal Altitude</p>

Error Sources	Geometric Error Mapping Function	Comments
	$X^{*'} = \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin (\psi_0 + \psi)$ $Y^{*'} = - \left[\left(1 + \frac{h}{R}\right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos (\psi_0 + \psi)$	
Roll Error, ϕ	<p>Given: a roll offset ϕ_0 and a view angle λ which sights a ground point P(0, Y*). The same view angle will sight another ground point P'(0, Y*') after roll error ϕ is introduced. The following equations are for obtaining Y* and Y*':</p> $Y^* = -R \sin (\lambda + \phi_0) \left[\left(1 + \frac{h}{R}\right) \cos (\lambda + \phi_0) - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 (\lambda + \phi_0)} \right]$ $Y^{*'} = -R \sin (\lambda + \phi_0 + \phi) \left[\left(1 + \frac{h}{R}\right) \cos (\lambda + \phi_0 + \phi) - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 (\lambda + \phi_0 + \phi)} \right]$	<p>R = Earth Radius h = Nominal Altitude</p>
Pitch Error, θ	<p>Given: a pitch offset θ_0 and a view angle λ which sights a ground point P(X*, Y*). The same view angle will sight another ground point P'(X*', Y*') after pitch error θ is introduced. The following equations are for obtaining X*, Y*, X*', and Y*':</p> $X^* = R \left[\left(1 + \frac{h}{R}\right) \cos \tau - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \tau} \right] \frac{\sin \tau \sin \theta_0}{\sqrt{\sin^2 \theta_0 + \tan^2 \lambda}}$ $Y^* = -R \left[\left(1 + \frac{h}{R}\right) \cos \tau - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \tau} \right] \frac{\sin \tau \tan \lambda}{\sqrt{\sin^2 \theta_0 + \tan^2 \lambda}}$	<p>R = Earth Radius h = Nominal Altitude</p> <p>For τ and ξ, refer to Fig. 23.</p>

Error Sources	Geometric Error Mapping Function	Comments
	$X^{*'} = R \left[\left(1 + \frac{h}{R}\right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right] \frac{\sin \xi \sin (\theta_o + \theta)}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}}$ $Y^{*'} = -R \left[\left(1 + \frac{h}{R}\right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right] \frac{\sin \xi \tan \lambda}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda}}$ $\tau = \tan^{-1} \left(\sec \theta_o \sqrt{\sin^2 \theta_o + \tan^2 \lambda} \right)$ $\xi = \tan^{-1} \left[\sec (\theta_o + \theta) \sqrt{\sin^2 (\theta_o + \theta) + \tan^2 \lambda} \right]$	
117 Yaw Error ψ , Followed by pitch error θ , then roll error ϕ	Given a yaw offset ψ_o , followed by a pitch offset θ_o , then a roll offset ϕ_o . A view angle λ is also given which sights a ground point $P_o(X_o, Y_o)$. After the errors ψ , θ , and ϕ are introduced, the same view angle will sight another ground point $P_3(X_3, Y_3)$. The following equations are for obtaining X_o , Y_o , X_3 , and Y_3 : $X_o' = \frac{R \sin \theta_o \sin \xi_o \left[\left(1 + \frac{h}{R}\right) \cos \xi_o - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_o} \right]}{\sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)}}$ $Y_o' = - \frac{R \tan (\lambda + \phi_o) \sin \xi_o \left[\left(1 + \frac{h}{R}\right) \cos \xi_o - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi_o} \right]}{\sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)}}$	R = Earth Radius h = Nominal Altitude For ξ_o and ξ_3 , refer to Fig. 23.

Error Sources	Geometric Error Mapping Function	Comments
	$X'_3 = \frac{R \sin (\theta_o + \theta) \sin \xi_3 \left[\left(1 + \frac{h}{R} \right) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 (\lambda + \phi_o + \phi)}}$ $Y'_3 = - \frac{R \tan (\lambda + \phi_o + \phi) \sin \xi_3 \left[\left(1 + \frac{h}{R} \right) \cos \xi_3 - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \xi_3} \right]}{\sqrt{\sin^2 (\theta_o + \theta) + \tan^2 (\lambda + \phi_o + \phi)}}$ $\xi_o = \tan^{-1} \left(\sec \theta_o \sqrt{\sin^2 \theta_o + \tan^2 (\lambda + \phi_o)} \right)$ $\xi_3 = \tan^{-1} \left[\sec (\theta_o + \theta) \sqrt{\sin^2 (\theta_o + \theta) + \tan^2 (\lambda + \phi_o + \phi)} \right]$ $X_o = X'_o \cos \psi_o - Y'_o \sin \psi_o$ $Y_o = X'_o \sin \psi_o + Y'_o \cos \psi_o$ $X_3 = X'_3 \cos (\psi_o + \psi) - Y'_3 \sin (\psi_o + \psi)$ $Y_3 = X'_3 \sin (\psi_o + \psi) + Y'_3 \cos (\psi_o + \psi)$	

Part B: Given the coordinates of a ground point, find the attitude offsets, the corresponding view angle and the coordinates of the point with the same view angle after the errors are introduced.

Error Sources	Geometric Error Mapping Function	Comments
Altitude Error, Δh	<p>At nominal altitude h, a ground point $P(X_o, Y_o)$ corresponding to view angle λ is given. After elevating Δh, the same view angle λ will aim at another ground point $P'(X'_o, Y'_o)$. The following equations are for obtaining X'_o and Y'_o:</p> $\lambda = \tan^{-1} \left(\frac{\sqrt{X_o^2 + Y_o^2}}{R + h - \sqrt{R^2 - X_o^2 - Y_o^2}} \right)$ $X'_o = \frac{RX_o \sin \lambda}{\sqrt{X_o^2 + Y_o^2}} \left[\left(1 + \frac{h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right]$ $Y'_o = \frac{RY_o \sin \lambda}{\sqrt{X_o^2 + Y_o^2}} \left[\left(1 + \frac{h + \Delta h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h + \Delta h}{R} \right)^2 \sin^2 \lambda} \right]$	R = Earth Radius

Error Sources	Geometric Error Mapping Function	Comments
Roll Error, ϕ	<p>A ground point P(0, Y*) corresponding to roll offset ϕ_o and view angle λ is given. After roll error ϕ is introduced, the same view angle λ will aim at another ground point P'(0, Y*'). The following equations are for obtaining Y*':</p> $\lambda + \phi_o = -\text{sgn}(Y^*) \tan^{-1} \left(\frac{ Y^* }{R + h - \sqrt{R^2 - Y^{*2}}} \right)$ $Y^{*'} = -R \sin' (\lambda + \phi_o + \phi) \left[\left(1 + \frac{h}{R} \right) \cos (\lambda + \phi_o + \phi) - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 (\lambda + \phi_o + \phi)} \right]$	<p>R = Earth Radius h = Nominal Altitude</p>

Error Sources	Geometric Error Mapping Function	Comments
Pitch Error, θ	<p>A ground point P(X^*, Y^*) corresponding to pitch offset θ_o and view angle λ is given. After pitch error θ is introduced, the same view angle λ will aim at another ground point P'(X^{*}, Y^{*}). The following equations are for obtaining X^{*} and Y^{*}:</p> $\lambda = \tan^{-1} \left[\frac{Y^*}{\left(R + h - \sqrt{R^2 - X^{*2} - Y^{*2}} \right) \left(\sqrt{1 + \left(\frac{X^*}{R + h - \sqrt{R^2 - X^{*2} - Y^{*2}}} \right)^2} \right)} \right]$ $\theta_o = \tan^{-1} \left[\frac{X^*}{R + h - \sqrt{R^2 - X^{*2} - Y^{*2}}} \right]$ $\xi = \tan^{-1} \left[\sec(\theta_o + \theta) \sqrt{\sin^2(\theta_o + \theta) + \tan^2 \lambda} \right]$ $X^{*} = \frac{R \sin(\theta_o + \theta) \sin \xi \left[\left(1 + \frac{h}{R} \right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \xi} \right]}{\sqrt{\sin^2(\theta_o + \theta) + \tan^2 \lambda}}$ $Y^{*} = - \frac{R \tan \lambda \sin \xi \left[\left(1 + \frac{h}{R} \right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \xi} \right]}{\sqrt{\sin^2(\theta_o + \theta) + \tan^2 \lambda}}$	<p>R = Earth Radius h = Nominal Altitude For ξ, refer to Fig.23.</p>

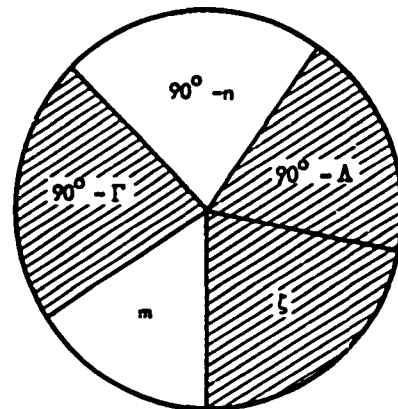
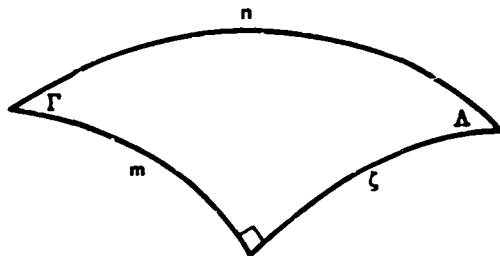
Error Sources	Geometric Error Mapping Function	Comments
Yaw Error, ψ	<p>A ground point $P(X^*, Y^*)$ corresponding to yaw offset ψ_0 and a certain view angle is given. After yaw error ψ is introduced, the same view angle will aim at another ground point $P'(X^{*'}, Y^{*'})$. The following equations are for obtaining $X^{*'}$ and $Y^{*'}$:</p> $\psi_0 = -\tan^{-1} \left(\frac{X^*}{Y^*} \right)$ $\lambda = -\text{sgn}(Y^*) \tan^{-1} \left(\frac{\sqrt{X^{*2} + Y^{*2}}}{R + h - \sqrt{R^2 - X^{*2} - Y^{*2}}} \right)$ $X^{*' } = \left[\left(1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right] R \sin \lambda \sin (\psi_0 + \psi)$ $Y^{*' } = - \left[\left(1 + \frac{h}{R} \right) \cos \lambda - \sqrt{1 - \left(1 + \frac{h}{R} \right)^2 \sin^2 \lambda} \right] R \sin \lambda \cos (\psi_0 + \psi)$	<p>R = Earth Radius h = Nominal Altitude</p>
Earth Rotation	<p>Let Γ be the shuttle orbital inclination at the equator, and ζ be the latitude of the nadir point. A given point $P(X, Y)$, after time period Δt, will move to $P'(X', Y')$. The following equations are for obtaining X' and Y':</p> $V_e = \omega R \cos \zeta$ $\omega = \frac{2\pi}{86,400} \text{ radians/sec}$ $X' = X + \frac{\cos \Gamma}{\cos \zeta} V_e \Delta t$ $Y' = Y - \sqrt{1 - \left(\frac{\cos \Gamma}{\cos \zeta} \right)^2} V_e \Delta t$	<p>V_e = Linear Velocity of Ground Point at Latitude ζ ω = Angular Velocity of Earth Rotation Δt is assumed small.</p>

Error Sources	Geometric Error Mapping Function	Comments
Yaw Error ψ followed by Pitch Error θ then Roll Error ϕ	<p>A ground point $P(0, Y^*)$ corresponding to view angle λ (with no attitude offset) is given. A yaw error ψ is introduced first, then a pitch error θ, finally a roll error ϕ. The same view angle λ will then aim at another ground point $P'(X^{*'}, Y^{*'})$. The following equations are for obtaining $X^{*'}$ and $Y^{*'}$:</p> $\lambda = -\text{sgn}(Y^*) \tan^{-1} \left(\frac{ Y^* }{R + h - \sqrt{R^2 - Y^{*2}}} \right)$ $\xi = \tan^{-1} \left[\sec \theta \sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)} \right]$ $X = \frac{R \sin \theta \sin \xi \left[\left(1 + \frac{h}{R}\right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right]}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}$ $Y = \frac{-R \tan (\lambda + \theta) \sin \xi \left[\left(1 + \frac{h}{R}\right) \cos \xi - \sqrt{1 - \left(1 + \frac{h}{R}\right)^2 \sin^2 \xi} \right]}{\sqrt{\sin^2 \theta + \tan^2 (\lambda + \phi)}}$ $X^{*' } = X \cos \psi - Y \sin \psi$ $Y^{*' } = X \sin \psi + Y \cos \psi$	<p>R = Earth Radius h = Nominal Altitude For ξ, refer to Fig. 23.</p>

APPENDIX B

NAPIER'S RULES FOR RIGHT SPHERICAL TRIANGLES*

A right spherical triangle has five variable parts. If these components and their complements (complement of $\Gamma \equiv 90^\circ - \Gamma$) are arranged in a circle, as illustrated below



then, the following relationships hold between the five components in the circle:

The sine of any component equals the product of either:

1. The tangents of the adjacent components, or
2. The cosines of the opposite components

For our case, from 2:

$$\sin (90^\circ - \Gamma) = \cos \zeta \cos (90^\circ - A)$$

i.e.,

$$\cos \Gamma = \cos \zeta \sin A$$

*See Ref. 13

APPENDIX C

COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER POINTING ERROR

POWER SPECTRAL DENSITIES FOR CONFIGURATION A -- PAYLOAD-BAY NADIR POINTING

```
ELT WED-10/20/82-10:23:28-(52,)  
12537*IS(1).SRNA/PROG(52)  
1:PROGRAM IMAGING SPECTROMETER SHUTTLE RIGID MOUNT - A  
2:INITIAL  
3:VARIABLE T=0.0  
4:   INTEGER FNUM,NPOINT  
5:  
6:  
7:COMMENT INITIALIZE SOME PARAMETERS  
8:   CONSTANT NPOINT=641  
9:   CONSTANT FNUM=39  
10:  CALL FOPEN(FNUM)  
11:  TFINAL=1.0*NPOINT-1.0  
12:  CONSTANT FACTOR=0.0  
13:  CONSTANT SCFAC=1.0E+5  
14:  SCFAC2=SCFAC*SCFAC  
15:  SCF2DB=10.0*ALOG10(SCFAC2)  
16:  
17:COMMENT SET VARIOUS MATHEMATICAL CONSTANTS  
18:   PI=3.14159265  
19:   ASRCC=PI/(180.0*3600.0)  
20:   RSHZCC=1.0/(2.0*PI)  
21:  
22:COMMENT DEFINE MOMENTS OF INERTIA  
23:   CONSTANT IX=1.38E+6  
24:   CONSTANT IY=1.00E+7  
25:   CONSTANT IZ=1.05E+7  
26:   IX2=IX*IX  
27:   IY2=IY*IY  
28:   IZ2=IZ*IZ  
29:   IXYZ=IX*IY*IZ  
30:   IXYZ2=IXYZ*IXYZ  
31:  
32:COMMENT DEFINE W0  
33:   CONSTANT W0=0.0011315  
34:   W02=W0*W0  
35:   W04=W02*W02  
36:  
37:COMMENT DEFINE INITIAL ANGLES AND RATES  
38:   CONSTANT TH0=0.0  
39:   CONSTANT PH0=0.0  
40:   CONSTANT PS0=0.0  
41:   CONSTANT TH0D=0.0  
42:   CONSTANT PH0D=0.0  
43:   CONSTANT PS0D=0.0  
44:   TH02=TH0*TH0  
45:   TH0D2=TH0D*TH0D  
46:   PH0D2=PH0D*PH0D  
47:   PH0D2=PH0D*PH0D  
48:   PS0D2=PS0D*PS0D  
49:   PS0D2=PS0D*PS0D  
50:  
51:COMMENT DEFINE Q'S  
52:   CONSTANT QX=2.014E-6  
53:   CONSTANT QY=1.251E-3  
54:   CONSTANT QZ=5.438E-4  
55:  
56:COMMENT DEFINE RATE FILTER PARAMETERS  
57:   CONSTANT WD=0.2513  
58:   CONSTANT ZETA=0.8  
59:   WD2=WD*WD  
60:   WD4=WD2*WD2  
61:   ZD2=ZETA*ZETA  
62:  
63:COMMENT DEFINE VARIOUS CONSTANTS  
64:   CONSTANT R=8.55E-10
```

```

65:      CONSTANT SIGBO=87.32
66:      SIGBO=SIGBO*ASRCC
67:      SIGBO2=SIGBO*SIGBO
68:      CONSTANT TAU=0.023
69:      TAU2=TAU*TAU
70:      DELW=1.0
71:
72: COMMENT DEFINE W CONSTANTS AND INITIALIZE W
73:      CONSTANT WLO=6.28318531E-7
74:      CONSTANT WFACT=1.029200527
75:      W=WLO/WFACT
76:
77: END
78: DYNAMIC
79: CINTERVAL CI=1.0
80:
81: COMMENT COMPUTE W
82:      W=W*WFACT
83:      PROCEDURAL (DELW=W)
84:      DELW=0.0
85:      IF (W.LE.0.) DELW=1.0
86:      END
87:      W#2=W*RS#ZCC
88:      FREQ=ALOG10(W#Z)
89:      W2=W*W
90:      W4=W2*W2
91:
92: COMMENT COMPUTE HW2 AND DW2
93:      HW2=1.0/(TAU2*W2+1.0)
94:      HDW2=WD4*W2/((W2-WJ2)**2+4.0*ZD2*WD2*W2)
95:      DW2=IX*IZ*W4-WO2*(IX*IZ+IY2+3.0 IY=IZ-3.0*IZL)*W2
96: COMMENT DW2=DW2+4.0*W04*(IY-IX)*(IY-IZ)
97:      DW2=DW2/SCFAC
98:      DW2=DW2*DW2
99:
100: COMMENT COMPUTE F'S
101: COMMENT FTHW2=HW2*SCFAC2/((IY*W2+3.0*W02*(IZ-IX))**2)
102: COMMENT FRW2=(IZ*W2-WO2*(IY-IX))**2*HW2/DW2
103: COMMENT FYW2=(IX*W2+4.0*W02*(IZ-IY))**2*HW2/DW2
104:      FTHW2=HW2*SCFAC2/((IY*W2)**2)
105:      FRW2=(IZ*W2)**2*HW2/DW2
106:      FRYW2=W02*IXYZ2*W2*HW2/DW2
107:      FYW2=(IX*W2)**2*HW2/DW2
108:
109: COMMENT COMPUTE PSD'S
110:
111: COMMENT PITCH PSD COMPUTATIONS
112:      PPW1=FTHW2*IY2*THOD2
113:      PPW2=FTHW2*IY2*THO2*W2
114:      PPW3=FTHW2*OY
115:      PPW=R+SIGBO2*DELW*(PPW1+PPW2+PPW3)/SCFAC2
116:      PPWD=PPW*HDW2
117:
118: COMMENT ROLL PSD COMPUTATIONS
119:      PRW1=FRW2*IX2*PHIOD2
120:      PRW2=FRW2*IX2*PHIO2*W2
121:      PRW3=FRW2*W02*IXYZ2*PSIO2
122:      PRW4=FRW2*OX
123:      PRW5=FRYW2*IZ2*PSIOD2
124:      PRW6=FRYW2*IZ2*PSIO2*W2
125:      PRW7=FRYW2*W02*IXYZ2*PHIO2
126:      PRW8=FRYW2*OZ
127:      RS=R+SIGBO2*DELW
128:      PRW=RS*(PRW1+PRW2+PRW3+PRW4+PRW5+PRW6+PRW7+PRW8)/SCFAC2
129:      PRWD=PRW*HDW2
130:

```

```

131:COMMENT YAW PSD COMPUTATIONS
132:    PYW1=FYW2*I2Z*PSIOD2
133:    PYW2=FYW2*I2Z*PSIO2*M2
134:    PYW3=FYW2*W02*IXYZZ*PHIO2
135:    PYW4=FYW2*Q2
136:    PYW5=FRYW2*IX2*PHIOD2
137:    PYW6=FRYW2*IX2*P-IIO2*W2
138:    PYW7=FRYW2*W02*IXYZZ*PSIO2
139:    PYW8=FRYW2*OX
140:    PYW=RS+(PYW1+PYW2+PYW3+PYW4+PYW5+PYW6+PYW7+PYW8)/SCFAC2
141:    PYWD=PYW*HDWZ
142:
143:
144:COMMENT PREPARE VARIABLES FOR OUTPUT
145:    RSIG=P+SIGB02*DELW
146:    RSIGDB=10.0*ALOG10(RSIG)+FACTOR
147:    PPW3DB=10.0*ALOG10(PPW3)-SCF2DB+FACTOR
148:    PPWDB=10.0*ALOG10(PPW)+FACTOR
149:    PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
150:    PRW8DB=10.0*ALOG10(PRW8)-SCF2DB+FACTOR
151:    PRWDB=10.0*ALOG10(PRW)+FACTOR
152:    PYW4DB=10.0*ALOG10(PYW4)-SCF2DB+FACTOR
153:    PYW8DB=10.0*ALOG10(PYW8)-SCF2DB+FACTOR
154:    PYWDB=10.0*ALOG10(PYW)+FACTOR
155:
156:    PPWddb=10.0*ALOG10(PPWd)+FACTOR
157:    PRWddb=10.0*ALOG10(PRWd)+FACTOR
158:    PYWddb=10.0*ALOG10(PYWd)+FACTOR
159:
160:COMMENT SAVE NUMBERS IN FILE
161:    CALL FSAVE(FREQ,PPWDB,PRWDB,PYWDB,FNUM)
162:
163:
164:    TERMT(T,GE,TFINAL)
165:DERIVATIVE
166:ALGORITHM IALG=3
167:    GO=INTEG(1.0,0.0)
168:END
169:END
170:TERMINAL
171:
172:COMMENT CLOSE FILE
173:    CALL FCLOSE(FNUM)
174:END
175:END
EOF:175

```

```
0: )ELT WED-10/20/82-10:30:02-(3, )
12537*IS(1).SRMA/FSUBS(3)
1: 0FOR, IS FF.FOPEN
2:     SUBROUTINE FOPEN(N)
3:     REWIND N
4:     RETURN
5:     END
6: 0FOR, IS FF.FSAVE
7:     SUBROUTINE FSAVE(W,P1,P2,P3,N)
8:     WRITE(N,100)W,P1,P2,P3
9:     100 FORMAT(4G14.8)
10:    RETURN
11:    END
12: 0FOR, IS FF.FCLOSE
13:    SUBROUTINE FCLOSE(N)
14:    ENDFILE N
15:    RETURN
16:    END
EOF:16
```

APPENDIX D

COMPUTER PROGRAM FOR SHUTTLE IMAGING SPECTROMETER POINTING ERROR

POWER SPECTRAL DENSITIES FOR CONFIGURATION B -- NOSE-DOWN NADIR POINTING

```
ELT WED-10/20/82-10:36:41-(14,)  
12537*IS(1).SRMB/PROG(14)  
1:PROGRAM IMAGING SPECTROMETER SHUTTLE RIGID MOUNT - B  
2:INITIAL  
3:VARIABLE T=0.0  
4:   INTEGER FNUM,NPOINT  
5:  
6:  
7:COMMENT INITIALIZE SOME PARAMETERS  
8:   CONSTANT NPOINT=641  
9:   CONSTANT FNUM=39  
10:  CALL FOPEN(FNUM)  
11:  TFINAL=1.0#NPOINT-1.0  
12:  CONSTANT FACTOR=0.0  
13:  CONSTANT SCFAC=1.0E+10  
14:  SCFAC2=SCFAC*SCFAC  
15:  SCF2DB=10.0#ALOG10(SCFAC2)  
16:  
17:COMMENT SET VARIOUS MATHEMATICAL CONSTANTS  
18:  PI=3.14159265  
19:  ASRCC=PI/(180.0#3600.0)  
20:  RSHZCC=1.C/(2.0#PI)  
21:  
22:COMMENT DEFINE MOMENTS OF INERTIA  
23:  CONSTANT IX=1.00E+7  
24:  CONSTANT IY=1.05E+7  
25:  CONSTANT IZ=1.38E+6  
26:  IX2=IX*IX  
27:  IY2=IY*IY  
28:  IZ2=IZ*IZ  
29:  IYZ=IX*IY+IZ  
30:  IXY2=IXYZ*IXYZ  
31:  
32:COMMENT DEFINE  $\omega$   
33:  CONSTANT  $\omega_0$ =0.0011315  
34:   $\omega_02$ = $\omega_0$ # $\omega_0$   
35:   $\omega_04$ = $\omega_02$ # $\omega_02$   
36:  
37:COMMENT DEFINE INITIAL ANGLES AND RATES  
38:  CONSTANT THO=0.0  
39:  CONSTANT PHIO=0.0  
40:  CONSTANT PSIO=0.0  
41:  CONSTANT THOD=0.0  
42:  CONSTANT PHIOD=0.0  
43:  CONSTANT PSIOD=0.0  
44:  THO2=THO*THO  
45:  THOD2=THOD*THOD  
46:  PHIO2=PHIO*PHIO  
47:  PHIOD2=PHIOD*PHIOD  
48:  PSIO2=PSIO*PSIO  
49:  PSIOD2=PSIOD*PSIOD  
50:  
51:COMMENT DEFINE  $Q$ 'S  
52:  CONSTANT  $Q_X$ =1.970E-4  
53:  CONSTANT  $Q_Y$ =1.286E-3  
54:  CONSTANT  $Q_Z$ =6.320E-4  
55:  
56:COMMENT DEFINE RATE FILTER PARAMETERS  
57:  CONSTANT  $\omega_D$ =0.2513  
58:  CONSTANT ZETAD=0.8  
59:   $\omega_D2$ = $\omega_D$ # $\omega_D$   
60:   $\omega_D4$ = $\omega_D2$ # $\omega_D2$   
61:  ZD2=ZETAD*ZETAD  
62:  
63:COMMENT DEFINE VARIOUS CONSTANTS  
64:  CONSTANT R=8.55E-10
```

```

65:      CONSTANT SIGBO=87.32
66:      SIGBO=SIGBO*ASRCC
67:      SIGBO2=SIGBO*SIGBO
68:      CONSTANT TAU=0.023
69:      TAU2=TAU*TAU
70:      DELW=1.0
71:
72: COMMENT DEFINE W CONSTANTS AND INITIALIZE W
73:      CONSTANT WLO=6.28318531E-7
74:      CONSTANT WFACT=1.029200527
75:      W=WLO/WFACT
76:
77: END
78: DYNAMIC
79: CINTERVAL CI=1.0
80:
81: COMMENT COMPUTE W
82:      W=W*WFACT
83:      PROCEDURAL (DELW=W)
84:      DELW=0.0
85:      IF (W.LE.O.) DELW=1.0
86:      END
87:      W4=W*W*W*W
88:      FREQ=ALOG10(W4)
89:      W2=W*W
90:      W4=W2*W2
91:
92: COMMENT COMPUTE H42 AND DW2
93:      H42=1.0/(TAU2*W2+1.0)
94:      HDW2=H42*W2/((W2-WD2)**2+4.0*W2*WD2)
95:      DW2=IX*I2*W4-WO2*(IX*I2+IY*2+3.0*IY*I2-3.0*I2*Z2)*W2
96:      DW2=DW2+4.0*WO4*(IY-IX)*(IY-I2)
97:      DW2=DW2/SCFAC
98:      DW2=DW2*DW2
99:
100: COMMENT COMPUTE F'S
101:      FTHW2=H42*SCFAC2/((IY*W2+3.0*WO2*(I2-IX))**2)
102:      FRW2=(I2*W2-WO2*(IY-IX))**2*H42/DW2
103:      FYW2=(IX*W2+4.0*WO2*(I2-IY))**2*H42/DW2
104:      FRYW2=WO2*IY*Z2*W2*H42/DW2
105:
106: COMMENT COMPUTE PSD'S
107:
108: COMMENT PITCH PSD COMPUTATIONS
109:      PPW1=FTHW2*IY2*THO2
110:      PPW2=FTHW2*IY2*THO2*W2
111:      PPW3=FTHW2*QY
112:      PPW=R+SIGBO2*DELW+(PPW1+PPW2+PPW3)/SCFAC2
113:      PPWD=PPW*HDW2
114:
115: COMMENT ROLL PSD COMPUTATIONS
116:      PRW1=FRW2*IX2*PHIO2
117:      PRW2=FRW2*IX2*PHIO2*W2
118:      PRW3=FRW2*W2*IXYZ*PSIO2
119:      PRW4=FRW2*OX
120:      PRW5=FRYW2*I2*PSIO2
121:      PRW6=FRYW2*I2*PSIO2*W2
122:      PRW7=FRYW2*WO2*IXYZ*PHIO2
123:      PRW8=FRYW2*GZ
124:      RS=R+SIGBO2*DELW
125:      PRW=RS+(PRW1+PRW2+PRW3+PRW4+PRW5+PRW6+PRW7+PRW8)/SCFAC2
126:      PRWD=PRW*HDW2
127:
128: COMMENT YAW PSD COMPUTATIONS
129:      PYW1=FYW2*I2*PSIO2
130:      PYW2=FYW2*I2*PSIO2*W2

```



```

131:      PYW3=FW2*W02*IXYZ2*PHIO2
132:      PYW4=FW2*02
133:      PYW5=FRYW2*IX2*PHIO2
134:      PYW6=FRYW2*IX2*PHIO2*W2
135:      PYW7=FRYW2*W02*IXYZ2*PSIO2
136:      PYW8=FRYW2*0X
137:      PYW=RS+(PYW1+PYW2+PYW3+PYW4+PYW5+PYW6+PYW7+PYW8)/SCFAC2
138:      PYWD=PYW*HDWZ
139:
140:
141: COMMENT PPEARE VARIABLES FOR OUTPUT
142:      RSIG=R+SIGB02*DELM
143:      RSIGDB=10.0*ALOG10(RSIG)+FACTOR
144:      PPW3DB=10.0*ALOG10(PPW3)-SCF2DB+FACTOR
145:      PPWDB=10.0*ALOG10(PPW)+FACTOR
146:      PRW4DB=10.0*ALOG10(PRW4)-SCF2DB+FACTOR
147:      PRW8DB=10.0*ALOG10(PRW8)-SCF2DB+FACTOR
148:      PRWDB=10.0*ALOG10(PRW)+FACTOR
149:      PYW4DB=10.0*ALOG10(PYW4)-SCF2DB+FACTOR
150:      PYW8DB=10.0*ALOG10(PYW8)-SCF2DB+FACTOR
151:      PYWDB=10.0*ALOG10(PYW)+FACTOR
152:
153:      PPWDBB=10.0*ALOG10(PPWD)+FACTOR
154:      PRWDBB=10.0*ALOG10(PRWD)+FACTOR
155:      PYWDBB=10.0*ALOG10(PYWD)+FACTOR
156:
157: COMMENT SAVE NUMBERS IN FILE
158:      CALL FSAVE(FREQ,PPWDB,PRWDB,PYWDB,FNUM)
159:
160:
161:      TERM(T.GE.TFINAL)
162: DERIVATIVE
163: ALGORITHM IALG=3
164:      QQ=INTEG(1.0,0.0)
165: END
166: END
167: TERMINAL
168:
169: COMMENT CLOSE FILE
170:      CALL FCLOSE(FNUM)
171: END
172: END
EOF:172

```

```
ELT MED-10/20/82-10:30:42-(G,)
12537#IS(1).SRMB/FSUBS(0)
  1: @FOR, IS FF.FOPEN
  2:   SUBROUTINE FOPEN(N)
  3:   REWIND N
  4:   RETURN
  5:   END
  6: @FOR, IS FF.FSAVE
  7:   SUBROUTINE FSAVE(N,P1,P2,P3,N)
  8:   WRITE(N,100)N,P1,P2,P3
  9:   100 FORMAT(4G14.8)
 10:   RETURN
 11:   END
 12: @FOR, IS FF.FCLOSE
 13:   SUBROUTINE FCLOSE(N)
 14:   ENDFILE N
 15:   RETURN
 16:   END
EOF:16
```

APPENDIX E

GEOMETRIC ERROR ANALYSIS PROGRAM LISTINGS

```

100 D 004552 DDLTA      0000 R 000263 DPITCH      0000 R 000156 DP8A      0000 R 000067 (
100 R 000064 DP8I2      0000 R 000244 DROLL      0000 D 002262 DBXY      0000 R 000170 (
100 R 000125 DTETA1     0000 R 000130 DTETA2      0000 R 000213 DTHETA     0000 D 003214 (
100 D 001240 DY         0000 R 000225 DYAW      0000 D 022676 ERR       0000 D 023102 (
100 D 013626 FI        0000 D 000352 FI1       0000 D 000360 FI2       0000 R 000000 (
100 I 030711 J         0000 I 030706 K         0000 D 000302 LAMDA     0000 D 000626 (
100 D 001530 PDXY      0000 D 013640 PDY       0000 R 030710 PI        0000 D 017166 (
100 D 004252 PSI       0000 D 001776 PSXY      0000 D 003264 PXY       0000 D 000742 (
100 D 004266 RA        0000 D 006520 RMO      0000 D 010254 RMO1     0000 D 010436 (
100 D 026522 R01      0000 D 002522 S         0000 R 000047 SDM      0000 D 013350 (
100 D 013166 SHFT      0000 D 022532 SLAMDA     0000 D 015576 SQURT     0000 D 010620 (
100 D 027614 SQU1     0000 D 002236 SXY       0000 D 003240 T         0000 D 022502 (
100 R 000052 TDM2      0000 D 007314 TETA1     0000 D 007322 TETA2     0000 D 014732 (
100 D 007526 T1I2     0000 D 007330 T1I      0000 D 007336 T1I2     0000 D 003144 (
100 D 015552 T1        0000 D 025372 T1I      0000 D 005130 T2         0000 D 023206 (
100 D 022750 VP        0000 D 022736 VPX       0000 D 022734 V8         0000 D 022550 (
100 D 014756 X1        0000 R 000075 X0         0000 D 000326 XY       0000 R 000133 (
100 D 012074 X2        0000 D 001530 X22      0000 D 017224 X3         0000 D 000674 (
100 D 014136 Y1        0000 D 005062 Y0         0000 D 023140 Y0         0000 D 011346 (
100 D 011712 Y22      0000 D 020316 Y3

```

```

1* C THIS PROGRAM COMPUTES THE ERROR SENSITIVITIES AND GEOMETRIC
2* C ERRORS OF GROUND POINTS INDUCED BY ALTITUDE,ROLL,YAW AND
3* C PITCH ERRORS AS WELL AS EARTH ROTATION AND SHUTTLE MOTION
4*
5*
6*
7*
8*
9*

```

```

10* PARAMETER NLAMDA=10,NLOMDA=10,NDH=8,NFI=3,NPSI=3,NTETA=3
11* S,NDT=3,NDETA=5,NDFI=5,NDP8A=10,NDTHE=10,NCOM=15,N8DH=2
12* REAL N0R,DM(NDH),DLAMDA(NLAMDA),DLOMDA(NLOMDA),SDM(N8DH),
13* STDM1,TDM2,DFI1(NFI),DFI2(NFI),DPSI1(NPSI),DPSI2(NPSI),
14* SDPSI(NPSI),DBETA(NPSI),XO(NLOMDA),OFI(NDFI),
15* SDTETA1(NTETA),DTETA2(NTETA),XO(NLOMDA),DP8A(NDP8A)
16* S,DT(NDT),DTAU,DETA(NDETA),DALFA(NDP8A),DTHETA(NDTHE)
17* S,DYAW(NCOM),DROLL(NCOM),DPITCH(NCOM)
18* DOUBLE PRECISION LAMDA(NLAMDA),XY(NLAMDA),FI1(NFI),FI2(NFI)
19* S,DXY(NDH,NLAMDA),LOMDA(NLOMDA),Y(NLOMDA),PY(NDFI,NLOMDA)
20* S,DY(NDFI,NLOMDA),PDXY(NDH,NLAMDA),PSXY(NLAMDA,NDH),
21* SBXY(NLAMDA),DBXY(NLAMDA,NDH),S(NLAMDA,NDH),BY1(NFI,NLOMDA),
22* STXY1(NLAMDA),TXY2(NLAMDA),DTXY(NLAMDA),T(NLAMDA),PXY(NDH,NLAMDA)
23* SA),BY2(NFI,NLOMDA),DBY(NFI,NLOMDA),B(NFI,NLOMDA),PSI(NPSI),
24* SBETA(NPSI),RA(NLAMDA),DELTA(NPSI,NLAMDA),D(NPSI,NLAMDA),
25* SP8A(NDP8A),ALFA(NDP8A),DLTA(NDP8A,NLAMDA),YO(NLOMDA),
26* STZ(NDTHE,NLOMDA),DEL(NDTHE,NLOMDA),RHO(NDTHE,NLOMDA)
27* S,TETA1(NTETA),TETA2(NTETA),T1I(NTETA),T1I2(NTETA),
28* ST1I1(NTETA,NLOMDA),T1I2(NTETA,NLOMDA),DEL1(NTETA,NLOMDA),
29* SDEL2(NTETA,NLOMDA),RMO1(NTETA,NLOMDA),RMO2(NTETA,NLOMDA),
30* SSQURT1(NTETA,NLOMDA),SQURT2(NTETA,NLOMDA),X1I(NTETA,NLOMDA),
31* SY1I(NTETA,NLOMDA),X22(NTETA,NLOMDA),Y22(NTETA,NLOMDA)
32* DOUBLE PRECISION X2(NCOM,NLOMDA),
33* SBHFT(NTETA,NLOMDA),SEN(NTETA,NLOMDA),YAW(NCOM),ROLL(NCOM),
34* SFI(NDFI),PDY(NDFI,NLOMDA),YI(NDTHE,NLOMDA),THETA(NDTHE),
35* SXI(NDTHE,NLOMDA),T1(NDTHE),SQURT(NDTHE,NLOMDA),
36* SBHF(NDTHE,NLOMDA),PITCH(NCOM),

```

ORIGINAL PAGE IS
OF POOR QUALITY

```
34*      SX3(NCOM,NLONDA),Y3(NCOM,NLONDA)
35*      S,DIFF(NCOM,NLONDA),TAU,OMEGA,ETA(NDETA):
36*      SV(NCETA),SLUMDA(NDETA),CLUMDA(NDETA),VX(NDETA),VY(NDETA):
37*      SDEX(NDETA,NDT),DEY(NDETA,NDT),ERR(NDETA,NDT),VB,VFX(NDETA),
38*      SVF(NDETA),ANGL(NDETA),DANGL(NDETA),DELX(NDETA,NDT)
39*      S,JEJY(NDETA,NDT),ERRJ(NDETA,NDT),Y0(NLONDA)
40*      S,T33(NCOM,NLONDA),DE1(NCOM,NLONDA),T11(NCOM):
41*      SY2(NCOM,NLONDA),RO1(NCOM,NLONDA),SOUT1(NCOM,NLONDA)
42*      DATA (OH(K),K=1,NOM)/1.,1.,2.,3.,3.,4.,4.,/
43*      DATA H,R,DYAU/400.,6350.,785.,85./
44*      DATA (DFI(K),K=1,NDFI)/18.,19.,20.,21.,22./
45*      DATA (DPBA(K),K=1,NDPBA)/.5,1.,1.5,2.,2.5,3.,3.5,4.,4.5,5./
46*      DATA (DTMETH(K),K=1,NDTME)/20.5,21.5,22.5,23.5,24.5,25.,26.,
47*      845.,46.,47./
48*      DATA (DYAW(K),K=1,NCOM)/0.,0.,0.,1.,1.,1.,2.,2.,2.,2.,2.,2.,
49*      80.,0.,1.,1.,1./
50*      DATA (DPITC(K),K=1,NCOM)/0.,1.,1.,1.,0.,0.,22.5,22.5,22.5,22.5,
51*      822.5,45.,45.,45.,45./
52*      DATA (DROLL(K),K=1,NCOM)/20.,20.,20.,20.,0.,1.,1.,0.,0.,0.
53*      80.,1.,1.,1.,0.,0./
54*      DATA (DLAMDA(I),I=1,NLAMDA)/0.,1.,2.,3.,4.,5.,6.,7.,8.,9./
55*      DATA (DLOMDA(I),I=1,NLONDA)/-9.,-8.,-7.,-6.,-5.,-4.,-3.,-2.,
56*      8-1.,0.,1.,2.,3.,4.,5.,6.,7.,8.,9./
57*      DATA (SDM(I),I=1,NSDM)/.01,.02/
58*      DATA TDM1,TDM2/.01,=.01/
59*      DATA (DFI1(K),K=1,NFI), (DFI2(K),K=1,NFI)/.01,.02,.05,=.01,
60*      8-.02,=.05/
61*      DATA (DPBI1(K),K=1,NPBI), (DPBI2(K),K=1,NPBI)/.01,.02,.05,
62*      8-.01,=.02,=.05/
63*      DATA (DTETA1(K),K=1,NTETA), (DTETA2(K),K=1,NTETA)/.01,.02,
64*      8.05,=.01,=.02,=.05/
65*      DATA (XO(K),K=1,NLONDA)/1900./
66*      DATA (XO(K),K=1,NLONDA)/1900./
67*      DATA (DT(K),K=1,NDT)/1.,1.,01/
68*      DATA (DETA(K),K=1,NDETA)/0.,20.,40.,60.,80./
69*      PI=3.14159265
70*
71*
72*      DO 100 K=1,NOM
73*      DO 90 I=1,NLAMDA
74*      LAMDA(I)=DLAMDA(I)*PI/180.
75*      XY(I)=R*((1+H/R)*DCOS(LAMDA(I))-DSQRT(1-(1+H/R)*(1+H/R)*
76*      8DSIN(LAMDA(I))*DSIN(LAMDA(I)))*DSIN(LAMDA(I))
77*      PXY(K,I)=R*((1+(H+OH(K))/R)*DCOS(LAMDA(I))-DSQRT(1-(1+(H+OH(K))/
78*      8)/R)*(1+(H+OH(K))/R)*DSIN(LAMDA(I))*DSIN(LAMDA(I)))*DSIN
79*      S(LAMDA(I))
80*      DXY(K,I)=PXY(K,I)-XY(I)
81*      IF(DXY(K,I).EQ.0.) GO TO 80
82*      PDXY(K,I)=DABS(DXY(K,I))*100./DABS(XY(I))
83*      GO TO 90
84*      80 PDXY(K,I)=0.
85*      90 CONTINUE
86*      100 CONTINUE
87*
88*
89*      DO 102 J=1,NSDM
90*      DO 101 I=1,NLAMDA
```

```

910      LAMDA(I)=DLAMDA(I)*PI/180.
920      SXY(I)=R*((1+H/R)*DCOS(LAMDA(I))-DSQRT(1-(1+H/R)*(1+H/R)*
930      S(SIN(LAMDA(I))*DSIN(LAMDA(I))))*DSIN(LAMDA(I))
940      PSXY(I,J)=R*((1+(H+SDH(J))/R)*DCOS(LAMDA(I))-DSQRT(1-(1+(H+SDH(J))
950      S(1/R)*(1+(H+SDH(J))/R)*DSIN(LAMDA(I))*DSIN(LAMDA(I))))*DSIN(
960      SLAMDA(I))
970      DS=(I,J)=PSXY(I,J)-SXY(I)
980      S(I,J)=DSXY(I,J)/SDH(J)
990
1000     101 CONTINUE
1010     102 CONTINUE
1020
1030     DO 103 I=1,NLAMDA
1040     LAMDA(I)=DLAMDA(I)*PI/180.
1050     TXY1(I)=R*((1+(H+TDH1)/R)*DCOS(LAMDA(I))-DSQRT(1-(1+(H+TDH1)/R)*
1060     S(1+(H+TDH1)/R)*DSIN(LAMDA(I))*DSIN(LAMDA(I))))*DSIN(LAMDA(I))
1070     TXY2(I)=R*((1+(H+TDH2)/R)*DCOS(LAMDA(I))-DSQRT(1-(1+(H+TDH2)/R)*
1080     S(1+(H+TDH2)/R)*DSIN(LAMDA(I))*DSIN(LAMDA(I))))*DSIN(LAMDA(I))
1090     DTXY(I)=TXY1(I)-TXY2(I)
1100     T(I)=DTXY(I)/(TDH1-TDH2)
1110     103 CONTINUE
1120
1130
1140     DO 111 K=1,NDFI
1150     FI(K)=DFI(K)*PI/180.
1160     DO 110 J=1,NLOMDA
1170     LOMDA(J)=OLOMDA(J)*PI/180.
1180     Y(J)=R*((1+H/R)*DCOS(LOMDA(J))-DSQRT(1-(1+H/R)*(1+H/R)*
1190     S(SIN(LOMDA(J))*DSIN(LOMDA(J))))*DSIN(LOMDA(J))
1200     PY(K,J)=R*((1+H/R)*DCOS(LOMDA(J)+FI(K))-DSQRT(1-(1+H/R)*(1+H/R)*
1210     S(SIN(LOMDA(J)+FI(K))*DSIN(LOMDA(J)+FI(K))))*DSIN(LOMDA(J)+FI(K))
1220     DY(K,J)=PY(K,J)-Y(J)
1230     IF(Y(J).EQ.0.) GO TO 109
1240     PDY(K,J)=DABS(DY(K,J))*100./DABS(Y(J))
1250     GO TO 110
1260     109 PDY(K,J)=10.0100
1270     110 CONTINUE
1280     111 CONTINUE
1290
1300
1310     DO 130 K=1,NFI
1320     FI1(K)=DFI1(K)*PI/180.
1330     FI2(K)=DFI2(K)*PI/180.
1340     DO 120 J=1,NLOMDA
1350     LOMDA(J)=OLOMDA(J)*PI/180.
1360     BY1(K,J)=R*((1+H/R)*DCOS(LOMDA(J)+FI1(K))-DSQRT(1-(1+H/R)*
1370     S(1+H/R)*DSIN(LOMDA(J)+FI1(K))*DSIN(LOMDA(J)+FI1(K))))*
1380     S(SIN(LOMDA(J)+FI1(K)))
1390     BY2(K,J)=R*((1+H/R)*DCOS(LOMDA(J)+FI2(K))-DSQRT(1-(1+H/R)*
1400     S(1+H/R)*DSIN(LOMDA(J)+FI2(K))*DSIN(LOMDA(J)+FI2(K))))*
1410     S(SIN(LOMDA(J)+FI2(K)))
1420     DBY(K,J)=BY1(K,J)-BY2(K,J)
1430     B(K,J)=DBY(K,J)/(DFI1(K)-DFI2(K))
1440     120 CONTINUE
1450     130 CONTINUE
1460
1470

```

```

148*      DO 132 K=1,NPBA
149*      DALFA(K)=(180.-DPBA(K))/2.
150*      PSA(K)=DPBA(K)*PI/180.
151*      ALFA(K)=DALFA(K)*PI/180.
152*      DO 131 J=1,NLMDA
153*      LAMDA(J)=DLAMDA(J)*PI/180.
154*      RA(J)=R*((1+H/R)*DCOS(LAMDA(J)))-DSQRT(1-(1+H/R)*(1+H/R)*
155*      S*DSIN(LAMDA(J))*DSIN(LAMDA(J)))*DSIN(LAMDA(J))
156*      DELTA(K,J)=(RA(J)*DSIN(PSA(K)))/DSIN(ALFA(K))
157*      131 CONTINUE
158*      132 CONTINUE
159*
160*
161*      DO 136 K=1,NPSI
162*      OPSI(K)=DPSI1(K)-DPSI2(K)
163*      OBETA(K)=(180.-DPSI(K))/2.
164*      PSI(K)=DPSI(K)*PI/180.
165*      BETA(K)=OBETA(K)*PI/180.
166*      DO 135 J=1,NLMDA
167*      LAMDA(J)=DLAMDA(J)*PI/180.
168*      RA(J)=R*((1+H/R)*DCOS(LAMDA(J)))-DSQRT(1-(1+H/R)*(1+H/R)*
169*      S*DSIN(LAMDA(J))*DSIN(LAMDA(J)))*DSIN(LAMDA(J))
170*      DELTA(K,J)=(RA(J)*DSIN(PSI(K)))/DSIN(BETA(K))
171*      O(K,J)=DELTA(K,J)/DPSI(K)
172*      135 CONTINUE
173*      136 CONTINUE
174*
175*
176*      DO 206 K=1,NDTHE
177*      THETA(K)=DTHETA(K)*PI/180.
178*      DO 205 J=1,NLMDA
179*      LOMDA(J)=DLOMDA(J)*PI/180.
180*      YO(J)=R*((1+H/R)*DCOS(LOMDA(J)))-DSQRT(1-(1+H/R)*(1+H/R)*DSIN
181*      S(LOMDA(J))*DSIN(LOMDA(J)))*DSIN(LOMDA(J))
182*      T1(K)=M*DTAN(THETA(K))
183*      T2(K,J)=-DABS(H/DCOS(THETA(K)))*DTAN(LOMDA(J))
184*      DEL(K,J)=DSQRT(T1(K)*T1(K)+T2(K,J)*T2(K,J))
185*      RHO(K,J)=DATAN(DEL(K,J)/H)
186*      SQURT(K,J)=R*((1+H/R)*DCOS(RHO(K,J)))-DSQRT(1-(1+H/R)*(1+H/R)*DSIN
187*      S(RHO(K,J))*DSIN(RHO(K,J)))*DSIN(RHO(K,J))
188*      XI(K,J)=(SQURT(K,J)*T1(K))/DEL(K,J)
189*      YI(K,J)=(SQURT(K,J)*T2(K,J))/DEL(K,J)
190*      SHF(K,J)=DSQRT(XI(K,J)*XI(K,J)+YI(K,J)*YI(K,J))*{YI(K,J)-YO(J)}
191*      205 CONTINUE
192*      206 CONTINUE
193*
194*
195*      DO 208 K=1,NTETA
196*      TETA1(K)=DTETA1(K)*PI/180.
197*      TETA2(K)=DTETA2(K)*PI/180.
198*      DO 207 J=1,NLMDA
199*      LOMDA(J)=DLOMDA(J)*PI/180.
200*      T11(K)=M*DTAN(TETA1(K))
201*      T12(K)=M*DTAN(TETA2(K))
202*      T111(K,J)=-DABS(H/DCOS(TETA1(K)))*DTAN(LOMDA(J))
203*      T112(K,J)=-DABS(H/DCOS(TETA2(K)))*DTAN(LOMDA(J))
204*      DEL1(K,J)=DSQRT(T11(K)*T11(K)+T111(K,J)*T111(K,J))

```

ORIGINAL PAGE IS
OF POOR QUALITY

```
205* DEL2(K,J)=DSQRT(TI2(K)*TI2(K)+TII2(K,J)*TII2(K,J))
206* RHO1(K,J)=DATAN(DEL1(K,J)/H)
207* RHO2(K,J)=DATAN(DEL2(K,J)/H)
208* SQURT1(K,J)=R*((1+H/R)*DCOS(RHO1(K,J))-DSQRT(1-(1+H/R)*
209* S((1+H/R)*DSIN(RHO1(K,J))*DSIN(RHO1(K,J))))*DSIN(RHO1(K,J))
210* SQURT2(K,J)=R*((1+H/R)*DCOS(RHO2(K,J))-DSQRT(1-(1+H/R)*
211* S((1+H/R)*DSIN(RHO2(K,J))*DSIN(RHO2(K,J))))*DSIN(RHO2(K,J))
212* X1(K,J)=(SQURT1(K,J)*T11(K))/DEL1(K,J)
213* Y1(K,J)=(SQURT1(K,J)*T11(K))/DEL1(K,J)
214* X2(K,J)=(SQURT2(K,J)*T12(K))/DEL2(K,J)
215* Y2(K,J)=(SQURT2(K,J)*T12(K))/DEL2(K,J)
216* SHFT(K,J)=DSQRT((X1(K,J)-X2(K,J))*(Y1(K,J)-Y2(K,J))+
217* S((Y1(K,J)-Y2(K,J))*(X1(K,J)-X2(K,J))))
218* SEN(K,J)=SHFT(K,J)/(DTETA1(K)-DTETA2(K))
219*
220* 207 CONTINUE
221* 208 CONTINUE
222*
223* DO 211 K=1,NCOM
224* YAW(K)=OYAW(K)*PI/180.
225* ROLL(K)=OROLL(K)*PI/180.
226* PITCH(K)=OPITCH(K)*PI/180.
227* DO 210 J=1,NLONDA
228* LONDA(J)=DLONDA(J)*PI/180.
229* Y0(J)=R*((1+H/R)*DCOS(LONDA(J))-DSQRT(1-(1+H/R)*
230* S((1+H/R)*DSIN(LONDA(J))*DSIN(LONDA(J))))*DSIN(LONDA(J))
231* T11(K)=H*DTAN(PITCH(K))
232* T33(K,J)=DABS(H/DCOS(PITCH(K)))*DTAN(LONDA(J)+ROLL(K))
233* DE1(K,J)=DSQRT(T11(K)*T11(K)+T33(K,J)*T33(K,J))
234* RO1(K,J)=DATAN(DE1(K,J)/H)
235* SQUT1(K,J)=R*((1+H/R)*DCOS(RO1(K,J))-DSQRT(1-(1+H/R)*
236* S((1+H/R)*DSIN(RO1(K,J))*DSIN(RO1(K,J))))*DSIN(RO1(K,J))
237* X2(K,J)=(SQUT1(K,J)*T11(K))/DE1(K,J)
238* Y2(K,J)=(SQUT1(K,J)*T33(K,J))/DE1(K,J)
239* X3(K,J)=X2(K,J)*DCOS(YAW(K))-Y2(K,J)*DSIN(YAW(K))
240* Y3(K,J)=X2(K,J)*DSIN(YAW(K))+Y2(K,J)*DCOS(YAW(K))
241* DIFF(K,J)=DSQRT((X3(K,J)-X0(J))*(X3(K,J)-X0(J))+
242* S((Y3(K,J)-Y0(J))))
243*
244* 210 CONTINUE
245* 211 CONTINUE
246*
247* TAU=DTAU*PI/180.
248* OMEGA=2.*PI/86400.
249* DO 220 K=1,NDETA
250* ETA(K)=JETA(K)*PI/180.
251* V(K)=OMEGA*R*DCOS(ETA(K))
252* SLUMDA(K)=DCOS(TAU)/DCOS(ETA(K))
253* CLUMDA(K)=DSQRT(1-SLUMDA(K)*SLUMDA(K))
254* VX(K)=V(K)*SLUMDA(K)
255* VY(K)=V(K)*CLUMDA(K)
256* DO 215 J=1,NDT
257* DEX(K,J)=VX(K)*DT(J)
258* DEY(K,J)=VY(K)*DT(J)
259* ERR(K,J)=DSQRT(DEX(K,J)*DEX(K,J)+DEY(K,J)*DEY(K,J))
260*
261* 215 CONTINUE
262* 220 CONTINUE
```

```

262*
263*
264*      VS=2.*PI*R/5545.484
265*      TAU=DTAU*PI/180.
266*      OMEGA=2.*PI/86400.
267*      DO 222 K=1,NDETA
268*      ETA(K)=DETA(K)*PI/180.
269*      V(K)=OMEGA*R*DCOS(ETA(K))
270*      SLUMDA(K)=DCOS(TAU)/DCOS(ETA(K))
271*      CLUMDA(K)=DSQRT(1-SLUMDA(K)*SLUMDA(K))
272*      VX(K)=V(K)*SLUMDA(K)
273*      VY(K)=V(K)*CLUMDA(K)
274*      VPX(K)=VX(K)-VS
275*      VP(K)=DSQRT(VPX(K)*VPX(K)+VY(K)*VY(K))
276*      ANGL(K)=DATAN(VPX(K)/VY(K))
277*      DANGL(K)=ANGL(K)*180./PI
278*      CO 221 J=1,NOT
279*      DELX(K,J)=VPX(K)*DT(J)
280*      DELY(K,J)=VY(K)*DT(J)
281*      ERRO(K,J)=DSQRT(DELX(K,J)*DELX(K,J)+DELY(K,J)*DELY(K,J))
282*      221 CONTINUE
283*      222 CONTINUE
284*
285*
286*      DO 237 I=1,NDM
287*      WRITE(6,236) R,M,DM(K),(DLAMDA(I),I=1,NLAMDA),(XY(I),I=1,NLAMDA
288*      SA),(PXV(K,I),I=1,NLAMDA),(DXY(K,I),I=1,NLAMDA),(PDXY(K,I),I=1,
289*      NLAMDA)
290*      236 FORMAT(1H1,'***** GEOMETRIC ERROR INDUCED BY:
291*      S1 ALTITUDE ERROR *****',//////,
292*      S1 EARTH RADIUS R =',F9.4,'KM',////,' NOMINAL ALTITUDE',
293*      S1 M=',F4.0,'KM',////,' ALTITUDE ERROR DM =',F3.0,'KM',
294*      S1'//////', V1E1,'5X,'ANGLE',//,' DLAMDA(DEGREES)',
295*      S1'2X,F2.0,9(9X,F2.0),////,' NOMINAL LOCATION (KM)',10D11.4,
296*      S1'////', S1HIFTED LOCATION (KM)',10D11.4,////,'7X,'ERROR',6X,
297*      S1'(KM)',10D11.4,////,'4X,'ABS(ERRO)',3X,'(S)',2X,F5.3,
298*      S1'8X,F5.3,8(8X,F5.3))
299*      237 CONTINUE
300*
301*
302*      DO 239 J=1,NSDM
303*      WRITE(6,238) R,M,SDM(J),(DLAMDA(I),I=1,NLAMDA),(SXY(I),I=
304*      S1,NLAMDA),(PSXY(I,J),I=1,NLAMDA),(DSXY(I,J),I=1,NLAMDA),(S(I,J)
305*      S,I=1,NLAMDA)
306*      238 FORMAT(1H1,'***** GEOMETRIC ERROR SENSITIVITY'
307*      S1 ' WITH RESPECT TO ALTITUDE ERROR *****',//////
308*      S1' EARTH RADIUS R =',F9.4,'KM',////,' NOMINAL ALTITUDE M =',
309*      S1'F4.0,'KM',////,' ALTITUDE ERROR SDM =',F4.2,'KM',//////,' V1E1
310*      S1'5X,'ANGLE',//,' DLAMDA(DEGREES)',12X,F2.0,9(9X,F2.0),////,
311*      S1' NOMINAL LOCATION (KM)',10D11.4,////,' S1HIFTED LOCATION (KM)',
312*      S1'10D11.4,////,'7X,'ERROR',6X,'(KM)',10D11.4,////,'6X,'ERROR',//
313*      S1' SENSITIVITY (KM/KM)',10D11.4)
314*      239 CONTINUE
315*
316*
317*      WRITE(6,240) R,M,TDM1,TDM2,(DLAMDA(I),I=1,NLAMDA),(TXY1(I),I=
318*      S1,NLAMDA),(TXY2(I),I=1,NLAMDA),(DTXY(I),I=1,NLAMDA),(T(I),I=

```


ORIGINAL PAGE 13
OF FOUR QUALITY

```

3190      $1,NLMDA)
3200 240 FORMAT(1M1,'***** GEOMETRIC ERROR SENSITIVITY'
3210 $' WITH RESPECT TO ALTITUDE ERROR *****',
3220 $'//////////',EARTH RADIUS R =',F9.4,'KM',///,'NOMINAL'
3230 $' ALTITUDE H =',F8.0,'KM',///,'ALTITUDE ERROR 1 TDM1 =',
3240 $F8.2,'KM',///,'ALTITUDE ERROR 2 TDM2 =',F5.2,'KM',
3250 $'//////////',VIEW',5X,'ANGLE',/,',DLMDA(DEGREES)',12X,
3260 $F2.0,9(9X,F2.0),///,'SHIFTED LOCATION',/,',DUE TO ERROR'
3270 $' 1 (KM)',10D11.4,///,'SHIFTED LOCATION',/,',DUE TO'
3280 $' ERROR 2 (KM)',10D11.4,///,'DIFFERENCE (KM)',
3290 $10D11.4,///,'ERROR',/,',SENSITIVITY (KM/KM)',10
3300 $D11.4)
3310
3320
3330      DO 246 K=1,NOFI
3340      WRITE(6,245) R,H,DFI(K),(DLMDA(J),J=1,10),(Y(J),J=1,10),
3350      $(PY(K,J),J=1,10),(DY(K,J),J=1,10),(PDY(K,J),J=1,10),(DLMDA(J),
3360      $J=1,NLMDA),(Y(J),J=1,NLMDA),(PY(K,J),J=1,NLMDA),(DY(K,J),
3370      $J=1,NLMDA),(PDY(K,J),J=1,NLMDA)
3380 245 FORMAT(1M1,'***** GEOMETRIC ERROR INDUCED BY'
3390 $' ROLL ERROR *****',//////////,
3400 $' EARTH RADIUS R =',F9.4,'KM',///,'NOMINAL ALTITUDE H =',
3410 $F8.0,'KM',///,'ROLL ERROR DFI =',F8.0,'DEGREE',//////////,
3420 $' VIEW',5X,'ANGLE',/,',DLMDA(DEGREES)',12X,F3.0,9(9X,F3.0),
3430 $'//////////',NOMINAL LOCATION (KM)',10D11.4,///,'SHIFTED'
3440 $' LOCATION (KM)',10D11.4,///,'7X',ERROR',6X,'(KM)',10D11.4,
3450 $'//////////',ABS(ERROR)',5X,'(X)',F10.4,8F11.4,3X,F5.2,
3460 $'//////////',VIEW',5X,'ANGLE',/,',DLMDA(DEGREES)',13X,F2.0,
3470 $8(9X,F2.0),///,'NOMINAL LOCATION (KM)',9D11.4,///,'SHIFTED'
3480 $' LOCATION (KM)',9D11.4,///,'7X',ERROR',6X,'(KM)',9D11.4,
3490 $'//////////',ABS(ERROR)',5X,'(X)',F10.4,8F11.4)
3500 246 CONTINUE
3510
3520
3530      DO 300 K=1,NFI
3540      WRITE(6,250) R,H,DFI1(K),DFI2(K),(DLMDA J),J=1,10),(BY1(K
3550      $,J),J=1,10),(BY2(K,J),J=1,10),(DBY(K,J), J=1,10),(B(K,J),J=
3560      $1,10),(DLMDA(J),J=1,NLMDA),(BY1(K,J),J=1,NLMDA),(BY2
3570      $(K,J),J=1,NLMDA),(DBY(K,J),J=1,NLMDA),(B(K,J),J=1,NLMD
3580      $DA)
3590 250 FORMAT(1M1,'***** GEOMETRIC ERROR SENSITIVI'
3600 $' TY WITH RESPECT TO ROLL ERROR ** *****',//////////
3610 $',EARTH RADIUS R =',F9.4,'KM',///,'NOMINAL ALTITUDE H =',
3620 $F8.0,'KM',///,'ROLL ERROR 1 DFI1 =',F5.2,'DEGREE',//////////,
3630 $' ROLL ERROR 2 DFI2 =',F5.2,'DEGREE',//////////',VIEW',5X,
3640 $'ANGLE',/,',DLMDA(DEGREES)',12X,F3.0,9(9X,F3.0),///,
3650 $'SHIFTED LOCATION',/,',DUE TO ERROR 1 (KM)',10D11.4
3660 $'//////////',SHIFTED LOCATION',/,',DUE TO ERROR 2 (KM)',
3670 $10D11.4,///,'DIFFERENCE (KM)',10D11.4,///,'6X',
3680 $'ERROR',/,',SENSITIVITY(KM/DEG)',10D11.4,//////////,
3690 $'VIEW',5X,'ANGLE',/,',DLMDA(DEGREES)',12X,F3.0,8(8X,
3700 $F3.0),///,'SHIFTED LOCATION',/,',DUE TO ERROR 1 (KM)',
3710 $9D11.4,///,'SHIFTED LOCATION',/,',DUE TO ERROR 2 (KM)',
3720 $9D11.4,///,'DIFFERENCE (KM)',9D11.4,///,'6X',
3730 $'ERROR',/,',SENSITIVITY(KM/DEG)',9D11.4)
3740 300 CONTINUE
3750

```


ORIGINAL FILE
OF POOR QUALITY

```
433* 460 FORMAT(1M1,'***** GEOMETRIC ERROR SENSITIVI  
434* SITY WITH RESPECT TO PITCH ERROR *****',/////  
435* S' EARTH RADIUS R =,F9.4,'KM',////, 'NOMINAL ALTITUDE H =,  
436* SF4.0,'KM',////, 'PITCH ERROR 1 DTETA1 =,F5.2,'DEGREE',///  
437* S' PITCH ERROR 2 DTETA2 =,F5.2,'DEGREE',////, 'VIEW',5X,  
438* S' ANGLE',/, 'DLOMDA(DEGREES)',12X,F3.0,9(8X,F3.0),///  
439* S' SHIFTED X COORD.,',/, 'DUE TO ERROR 1 (KM)',10D11.4,///  
440* S' SHIFTED Y COORD.,',/, 'DUE TO ERROR 1 (KM)',10D11.4,///  
441* S' SHIFTED X COORD.,',/, 'DUE TO ERROR 2 (KM)',10D11.4,///  
442* S' SHIFTED Y COORD.,',/, 'DUE TO ERROR 2 (KM)',10D11.4,///  
443* S' DIFFERENCE (KM)',10D11.4,/,6X,'ERROR',/, 'SENSI  
444* S' TIVITY(KM/DEG)',10D11.4,////, 'VIEW',5X,'ANGLE',/  
445* S' DLOMDA(DEGREES)',12X,F3.0,8(8X,F3.0),///  
446* S' SHIFTED X COORD.,',/, 'DUE TO ERROR 1 (KM)',9D11.4,///  
447* S' SHIFTED Y COORD.,',/, 'DUE TO ERROR 1 (KM)',9D11.4,///  
448* S' SHIFTED X COORD.,',/, 'DUE TO ERROR 2 (KM)',9D11.4,///  
449* S' SHIFTED Y COORD.,',/, 'DUE TO ERROR 2 (KM)',9D11.4,///  
450* S' DIFFERENCE (KM)',9D11.4,/,6X,'ERROR',/, 'SENSI  
451* S' TIVITY(KM/DEG)',9D11.4)  
452* 500 CONTINUE  
453*  
454*  
455* DO 601 K=1,NCOM  
456* WRITE(6,600) R,H,DYAW(K),DPITCH(K),DROLL(K),(DLOMDA(J),J=1,10),  
457* S(XO(J),J=1,10),(YO(J),J=1,10),(X3(K,J),J=1,10),(Y3(K,J),J=1,10),  
458* S(DIFF(K,J),J=1,10),(DLOMDA(J),J=1,NLQMDA),(XO(J),J=1,NLQMDA)  
459* S,(YO(J),J=1,NLQMDA),(X3(K,J),J=1,NLQMDA),(Y3(K,J),J=1,NLQMDA)  
460* S,(DIFF(K,J),J=1,NLQMDA)  
461* 600 FORMAT(1M1,'***** GEOMETRIC ERROR INDUCED BY'  
462* S' YAW,PITCH AND ROLL ERRORS *****',/////  
463* S' EARTH RADIUS R =,F9.4,'KM',////, 'NOMINAL ALTITUDE H =,  
464* SF4.0,'KM',////, 'YAW ERROR DYAW =,F4.1,'DEGREE',///  
465* S' PITCH ERROR DPITCH =,F4.1,'DEGREE',////, 'ROLL ERROR'  
466* S' DROLL =,F4.1,'DEGREE',////, 'VIEW',5X,'ANGLE',/, 'DLOMDA'  
467* S'(DEGREES)',12X,F3.0,9(8X,F3.0),////, 'NOMINAL X COORD. (KM)'  
468* S',7X,F2.0,9(9X,F2.0),////, 'NOMINAL Y COORD. (KM)',10D11.4,  
469* S'////, 'SHIFTED X COORD. (KM)',10D11.4,////, 'SHIFTED Y COORD. '  
470* S'(KM)',10D11.4,/,7X,'ERROR',6X,'(KM)',10D11.4,////,  
471* S' VIEW',5X,'ANGLE',/, 'DLOMDA(DEGREES)',12X,F3.0,8(8X,F3.0),  
472* S'////, 'NOMINAL X COORD. (KM)',7X,F2.0,8(9X,F2.0),///  
473* S' NOMINAL Y COORD. (KM)',9D11.4,////, 'SHIFTED X COORD. (KM)'  
474* S',9D11.4,/,7X,'SHIFTED Y COORD. (KM)',9D11.4,/,7X,'ERROR',  
475* S',6X,'(KM)',9D11.4)  
476* 601 CONTINUE  
477*  
478*  
479* DO 705 K=1,NDETA  
480* WRITE(6,700) R,DTAU,DETA(K),V(K),(DT(J),J=1,NDT),(DEX(K,J),  
481* S(J=1,NDT),(DEY(K,J),J=1,NDT),(ERR(K,J),J=1,NDT)  
482* 700 FORMAT(1M1,'***** GEOMETRIC ERROR INDUCED BY'  
483* S' EARTH ROTATION *****',////, 'EARTH'  
484* S' RADIUS =,F9.4,'KM',////, 'ORBIT INCLINATION =,F6.0,  
485* S' DEGREE',////, 'LATITUDE =,F4.0,'DEGREE',////,  
486* S' GROUND POINT SHIFT VELOCITY DUE TO EARTH ROTATION =,  
487* S' 3010.4,'KM/SEC',////,  
488* S' TIME INTERVAL (SEC)',3F15.2,////, 'SHIFT IN',/  
489* S' X DIRECTION (KM)',3X,3D15.4,////, 'SHIFT IN',/
```

```

490*      S1  Y DIRECTION      (KM)'0.3X,3D15.4,////,' TOTAL SHIFT'
491*      S1  (KM)'0.3X,3D15.4)
492*      705 CONTINUE
493*
494*
495*      DO 707 KM=1,NDETA
496*      WRITE(0,706) R,DTAU,DETA(K),V(K),VS,VP(K),DANGL(K),(DT(J),
497*      SJ=1,NDT),(DELX(K,J),J=1,NDT),(DELY(K,J),J=1,NDT),(ERRR(K,J),
498*      SJ=1,NDT)
499*      706 FORMAT(1H1,'***** GROUND POINT SHIFT INDUCED'
500*      S1 BY SHUTTLE MOTION AND EARTH ROTATION *****'
501*      S,//////,' EARTH RADIUS =',F9.4,'KM',//////,' ORBIT'
502*      S1 INCLINATION =',F4.0,'DEGREE',//////,' LATITUDE =',F4.0,
503*      S1 DEGREE',//////,' GROUND POINT SHIFT VELOCITY DUE TO EARTH'
504*      S1 ROTATION =',D10.4,'KM/SEC',//////,' SHUTTLE GROUND SPEED =',
505*      SD10.4,'KM/SEC',//////,' GROUND POINT SHIFT VELOCITY RELATIVE'
506*      S1 TO NADIR POINT =',D10.2,
507*      S1 KM/SEC',//////,' ANGLE OF VELOCITY VECTOR WITH Y-AXIS =',
508*      SD10.4,'DEGREE',//////,' TIME INTERVAL (SEC)',F15.2,//////
509*      S1 SHIFT IN',/,,' X DIRECTION (KM)'0.3X,3D15.4,//////
510*      S1 SHIFT IN',/,,' Y DIRECTION (KM)'0.3X,3D15.4,//////
511*      S1 TOTAL SHIFT (KM)'0.3X,3D15.4)
512*      707 CONTINUE
513*
514*
515*      STOP
516*      END

```

41 CTP1.134 SUP8110.789

.A SHUTEXF.IMAGE/ABS
283-JB 04/27/83 10131121

2 SUP8.736 CPU8.090 IOW.302 CC-ERR.433

ORIGINAL PAGE IS
OF POOR QUALITY.

***** GEOMETRIC ERROR INDUCED BY ALTITUDE ERROR *****

EARTH RADIUS R 6350.7850KM

NOMINAL ALTITUDE M 4000.0KM

ALTITUDE ERROR DM = 1.0KM

VIEW ANGLE PLANDA(DEGREES)	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
NOMINAL LOCATION (KM)	.0000	.6982+001	.1397+002	.2094+002	.2740+002	.3300+002	.4200+002	.4916+002	.5625+002	.6341+002
SHIFTED LOCATION (KM)	.0000	.7000+001	.1400+002	.2102+002	.2804+002	.3506+002	.4210+002	.4926+002	.5637+002	.6350+002
ERROR (KM)	.0000	.1740+001	.3492+001	.5243+001	.6995+001	.8753+001	.1052+000	.1229+000	.1407+000	.1586+000
ABS(ERROR) (%)	.000	.250	.250	.250	.250	.250	.250	.250	.250	.250

***** GEOMETRIC ERROR SENSITIVITY WITH RESPECT TO ALTITUDE ERROR *****

EARTH RADIUS R 6350.7850KM

NOMINAL ALTITUDE M 4000.0KM

ALTITUDE ERROR 1 DM1 = 0.01KM

ALTITUDE ERROR 2 DM2 = 0.01KM

VIEW ANGLE PLANDA(DEGREES)	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
SHIFTED LOCATION DUE TO ERROR 1 (KM)	.0000	.6982+001	.1397+002	.2097+002	.2708+002	.3300+002	.4200+002	.4918+002	.5625+002	.6341+002
SHIFTED LOCATION DUE TO ERROR 2 (KM)	.0000	.6982+001	.1397+002	.2096+002	.2707+002	.3300+002	.4200+002	.4918+002	.5625+002	.6341+002
DIFFERENCE (KM)	.0000	.3491+003	.6089+003	.1048+002	.1309+002	.1751+002	.2100+002	.2458+002	.2814+002	.3173+002
ERROR SENSITIVITY (KM/KM)	.0000	.1740+001	.3492+001	.5243+001	.6995+001	.8753+001	.1052+000	.1229+000	.1407+000	.1586+000

ORIGINAL PAGE IS
OF POOR QUALITY

***** GEOMETRIC ERROR INDUCED BY PULL ERROR *****

EARTH RADIUS R 6356.7856KM

NOMINAL ALTITUDE H 8000.KM

PULL ERROR OP1 01.DEGREE

VIEW DLOMOA(DEGREES)	ANGLE DLOMOA(DEGREES)	-9.	-8.	-7.	-6.	-5.	-4.	-3.	-2.	-1.	0.
NOMINAL LOCATION (KM)		.63400002	.56250002	.49100002	.42000002	.35000002	.27900002	.20900002	.13970002	.07020001	.0000
SHIFTED LOCATION (KM)		.56250002	.49140002	.42000002	.35000002	.27900002	.20900002	.13970002	.07020001	.0000	.00000001
ERROR (KM)		-.07153001	-.07114001	-.07081001	-.07052001	-.07029001	-.07010001	-.06990001	-.06970001	-.06952001	-.06932001
ABS(ERROR)	(K)	11.2009	12.0071	14.0103	16.0709	20.0002	25.0004	33.3704	50.0107	100.0000	0000

VIEW DLOMOA(DEGREES)	ANGLE DLOMOA(DEGREES)	1.	2.	3.	4.	5.	6.	7.	8.	9.
NOMINAL LOCATION (KM)		.00902001	.01397002	.02090002	.02790002	.03500002	.04200002	.04914002	.05625002	.06340002
SHIFTED LOCATION (KM)		.01397002	.02090002	.02790002	.03500002	.04200002	.04914002	.05625002	.06340002	.07000002
ERROR (KM)		-.00907001	-.00900001	-.00710001	-.00720001	-.00520001	-.00510001	-.00310001	-.003153001	-.00100001
ABS(ERROR)	(K)	100.0007	50.0005	33.00373	25.1295	20.1075	16.0005	14.0070	12.7153	11.0000

***** GEOMETRIC ERROR SENSITIVITY WITH RESPECT TO PULL ERROR *****

EARTH RADIUS R 6356.7856KM

NOMINAL ALTITUDE H 8000.KM

PULL ERROR 1 OP11 0 .01DEGREE

PULL ERROR 2 OP12 0 .01DEGREE

VIEW DLOMOA(DEGREES)	ANGLE DLOMOA(DEGREES)	-9.	-8.	-7.	-6.	-5.	-4.	-3.	-2.	-1.	0.
SHIFTED LOCATION DUE TO ERROR 1 (KM)		.03330002	.56100002	.49070002	.41990002	.34930002	.27900002	.20890002	.13900002	.07020001	.00010001
SHIFTED LOCATION DUE TO ERROR 2 (KM)		.03400002	.56320002	.49210002	.42130002	.35070002	.28050002	.21030002	.14000002	.07020001	.00010001
DIFFERENCE (KM)		-.00070000	-.00220000	-.00140000	-.00140000	-.00140000	-.00150000	-.00160000	-.00160000	-.00160000	-.00160000
ERROR SENSITIVITY(KM/DEG)		-.07173001	-.07133001	-.07097001	-.07060001	-.07020001	-.07010001	-.07002001	-.06991001	-.06980001	-.06961001

VIEW DLOMOA(DEGREES)	ANGLE DLOMOA(DEGREES)	1.	2.	3.	4.	5.	6.	7.	8.	9.
SHIFTED LOCATION DUE TO ERROR 1 (KM)		-.07052001	-.14040002	-.21030002	-.28090002	-.35070002	-.42130002	-.49210002	-.56320002	-.63400002
SHIFTED LOCATION DUE TO ERROR 2 (KM)		-.00912001	-.13000002	-.20890002	-.27900002	-.34930002	-.41990002	-.49070002	-.56100002	-.63330002
DIFFERENCE (KM)		-.01397000	-.01398000	-.01000000	-.01000000	-.01000000	-.01000000	-.01000000	-.01000000	-.01000000
ERROR SENSITIVITY(KM/DEG)		-.00000001	-.00010001	-.00020001	-.00100001	-.00100001	-.00100001	-.00100001	-.00100001	-.00100001

***** GEOMETRIC ERROR INDUCED BY PITCH ERROR *****

ORIGINAL PAGE IS
OF POOR QUALITY

EARTH RADIUS R = 6356.7858m

NOMINAL ALTITUDE H = 4000m

PITCH ERROR BYETA = 0.1 DEGREE

VIEW ANGLE DLOMBA(DEGREES)	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
NOMINAL X COORD. (MM)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
NOMINAL Y COORD. (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED X COORD. (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED Y COORD. (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
ERROR (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

VIEW ANGLE DLOMBA(DEGREES)	1.	2.	3.	4.	5.	6.	7.	8.	9.
NOMINAL X COORD. (MM)	0.	0.	0.	0.	0.	0.	0.	0.	0.
NOMINAL Y COORD. (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED X COORD. (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED Y COORD. (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
ERROR (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

***** GEOMETRIC ERROR SENSITIVITY WITH RESPECT TO PITCH ERROR *****

EARTH RADIUS R = 6356.7858m

NOMINAL ALTITUDE H = 4000m

PITCH ERROR 1 DETA1 = 0.01 DEGREE

PITCH ERROR 2 DETA2 = 0.01 DEGREE

VIEW ANGLE DLOMBA(DEGREES)	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
SHIFTED X COORD. DUE TO ERROR 1 (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED Y COORD. DUE TO ERROR 1 (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED X COORD. DUE TO ERROR 2 (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED Y COORD. DUE TO ERROR 2 (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DIFFERENCE (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
ERROR SENSITIVITY(MM/DEG)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

VIEW ANGLE DLOMBA(DEGREES)	1.	2.	3.	4.	5.	6.	7.	8.	9.
SHIFTED X COORD. DUE TO ERROR 1 (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED Y COORD. DUE TO ERROR 1 (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED X COORD. DUE TO ERROR 2 (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SHIFTED Y COORD. DUE TO ERROR 2 (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
DIFFERENCE (MM)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
ERROR SENSITIVITY(MM/DEG)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

***** GEOMETRIC ERROR INDUCED BY YAW ERROR *****

EARTH RADIUS R 6356.765KM
 NOMINAL ALTITUDE H 8400.0M
 YAW ERROR DPBA 01.0DEGREE

VIEW ANGLE DLANDA(DEGREES)	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
ERROR (KM)	.0000	.1219+000	.2436+000	.3653+000	.4869+000	.6086+000	.7303+000	.8520+000	.9737+000	.1195+001

***** GEOMETRIC ERROR SENSITIVITY WITH RESPECT TO YAW ERROR *****

EARTH RADIUS R 6356.765KM
 NOMINAL ALTITUDE H 8400.0M
 YAW ERROR 1 DPB11 0 .01DEGREE
 YAW ERROR 2 DPB12 0 -.01DEGREE

VIEW ANGLE DLANDA(DEGREES)	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
TOTAL SHIFT (KM)	.0000	.2437+002	.4874+002	.7311+002	.9748+002	.1222+001	.1404+001	.1715+001	.1999+001	.2219+00
ERROR SENSITIVITY(KM/DEG)	.0000	.1219+000	.2436+000	.3653+000	.4869+000	.6086+000	.7303+000	.8520+000	.9737+000	.1195+000

***** COMMUTIC ERROR INDUCED BY VARIOUS FACTORS *****

EARTH RADIUS R 6350.7850M

NOMINAL ALTITUDE H 6000.M

VAN ERROR STAN 01.DEGREE

PITCH ERROR DPITCH 01.DEGREE

ROLL ERROR DROLL 01.DEGREE

VIEW ANGLE DLOMB(DEGREES)	-9.	-8.	-7.	-6.	-5.	-4.	-3.	-2.	-1.	0.
NOMINAL X COORD. (MM)	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
NOMINAL Y COORD. (MM)	.0340002	.5025002	.0910002	.0200002	.3500002	.2790002	.2090002	.1397002	.0692001	.0000
SHIFTED X COORD. (MM)	.0005001	.0120001	.0250001	.0372001	.0490001	.0610001	.0730001	.0850001	.0981001	.7182001
SHIFTED Y COORD. (MM)	.5037002	.0920002	.0210002	.3510002	.2010002	.2100002	.1000002	.7105001	.1230000	.0005001
ERROR (MM)	.0205001	.0290001	.0355001	.0410001	.0480001	.0550001	.0625001	.0700001	.0787001	.0870001

VIEW ANGLE DLOMB(DEGREES)	1.	2.	3.	4.	5.	6.	7.	8.	9.
NOMINAL X COORD. (MM)	0.	0.	0.	0.	0.	0.	0.	0.	0.
NOMINAL Y COORD. (MM)	.0092001	.1197002	.0700002	.0270002	.3500002	.0200002	.0900002	.5025002	.0340002
SHIFTED X COORD. (MM)	.7225001	.7347001	.7070001	.7590001	.7717001	.7801001	.7900001	.8002001	.8219001
SHIFTED Y COORD. (MM)	.01305002	.0200002	.02705002	.0300002	.0393002	.0001002	.5013002	.0320002	.0700002
ERROR (MM)	.0900001	.1000002	.1010002	.1020002	.1030002	.1040002	.1050002	.1060002	.1070002

***** GROUND POINT SHIFT INDUCED BY SHUTTLE MOTION AND EARTH ROTATION *****

EARTH RADIUS R 6350.7850M

ORBIT INCLINATION = 85.DEGREE

LATITUDE = 0.DEGREE

GROUND POINT SHIFT VELOCITY DUE TO EARTH ROTATION = .0023000MM/SEC

SHUTTLE GROUND SPEED = .7202001MM/SEC

GROUND POINT SHIFT VELOCITY RELATIVE TO NAUTIC POINT = .7177001MM/SEC

ANGLE OF VELOCITY VECTOR WITH VERTICAL = .0032000206RADIANS

TIME INTERVAL (SEC)	1.00	.10	.01
SHIFT IN X DIRECTION (MM)	.7102001	.7102000	.7102001
SHIFT IN Y DIRECTION (MM)	.0005000	.0005001	.0005002
TOTAL SHIFT (MM)	.7177001	.7177000	.7177001

GROUND POINT SHIFT INDUCED BY SHUTTLE MOTION AND EARTH ROTATION

EARTH RADIUS = 6350.785 KM

ORBIT INCLINATION = 89. DEGREE

LATITUDE = 20. DEGREE

GROUND POINT SHIFT VELOCITY DUE TO EARTH ROTATION = .4344000 KM/SEC

SHUTTLE GROUND SPEED = .7202001 KM/SEC

GROUND POINT SHIFT VELOCITY RELATIVE TO NADIR POINT = .7175001 KM/SEC

ANGLE OF VELOCITY VECTOR WITH Y-AXIS = -.8050002 DEGREE

TIME INTERVAL (SEC)	1.00	.10	.01
SHIFT IN X DIRECTION (KM)	-.7102001	-.7102000	-.7102001
SHIFT IN Y DIRECTION (KM)	.4325000	.4325001	.4325002
TOTAL SHIFT (KM)	.7175001	.7175000	.7175001

GROUND POINT SHIFT INDUCED BY SHUTTLE MOTION AND EARTH ROTATION

EARTH RADIUS = 6350.785 KM

ORBIT INCLINATION = 89. DEGREE

LATITUDE = 40. DEGREE

GROUND POINT SHIFT VELOCITY DUE TO EARTH ROTATION = .3541000 KM/SEC

SHUTTLE GROUND SPEED = .7202001 KM/SEC

GROUND POINT SHIFT VELOCITY RELATIVE TO NADIR POINT = .7171001 KM/SEC

ANGLE OF VELOCITY VECTOR WITH Y-AXIS = -.8719002 DEGREE

TIME INTERVAL (SEC)	1.00	.10	.01
SHIFT IN X DIRECTION (KM)	-.7102001	-.7102000	-.7102001
SHIFT IN Y DIRECTION (KM)	.3510000	.3510001	.3510002
TOTAL SHIFT (KM)	.7171001	.7171000	.7171001

***** GROUND POINT SHIFT INDUCED BY SHUTTLE MOTION AND EARTH ROTATION *****

EARTH RADIUS = 6356.7650 KM

ORBIT INCLINATION = 85.0 DEGREE

LATITUDE = 00.0 DEGREE

GROUND POINT SHIFT VELOCITY DUE TO EARTH ROTATION = .2311000 KM/SEC

SHUTTLE GROUND SPEED = .7202001 KM/SEC

GROUND POINT SHIFT VELOCITY RELATIVE TO NADIR POINT = .7100001 KM/SEC

ANGLE OF VELOCITY VECTOR WITH Y-AXIS = .0010002 DEGREE

TIME INTERVAL (SEC)		1.00	.10	.01
SHIFT IN X DIRECTION (KM)		-.7102001	-.7102000	-.7102001
SHIFT IN Y DIRECTION (KM)		.2270000	.2270001	.2270002
TOTAL SHIFT (KM)		.7100001	.7100000	.7100001

***** GROUND POINT SHIFT INDUCED BY SHUTTLE MOTION AND EARTH ROTATION *****

EARTH RADIUS = 6356.7650 KM

ORBIT INCLINATION = 85.0 DEGREE

LATITUDE = 00.0 DEGREE

GROUND POINT SHIFT VELOCITY DUE TO EARTH ROTATION = .0027001 KM/SEC

SHUTTLE GROUND SPEED = .7202001 KM/SEC

GROUND POINT SHIFT VELOCITY RELATIVE TO NADIR POINT = .7102000 KM/SEC

ANGLE OF VELOCITY VECTOR WITH Y-AXIS = .0000002 DEGREE

TIME INTERVAL (SEC)		1.00	.10	.01
SHIFT IN X DIRECTION (KM)		-.7102001	-.7102000	-.7102001
SHIFT IN Y DIRECTION (KM)		.0003001	.0003002	.0003003
TOTAL SHIFT (KM)		.7102001	.7102000	.7102001

ORIGINAL PAGE IS OF POOR QUALITY

APPENDIX F

GROUND PATTERN AND IMAGING

DISTORTIONS GENERATION PROGRAM LISTINGS

0000 I 006610 IM	0000 I 006021 IM1	0000 I 005706 IPT	0000 I 0
0000 I 005712 J	0000 I 006045 J00	0000 I 005742 J6	0000 I 0
0000 I 006011 JM	0000 I 006022 J91	0000 I 005723 JR	0000 I 0
0000 I 005714 L	0000 R 000000 LAMDA	0000 I 003030 LFLAG	0000 I 0
0000 R 000003 LHMDA	0000 R 000002 LYMDA	0000 I 001747 MFLAG	0000 I 0
0000 I 005673 NGX	0000 I 005674 NGY	0000 I 006050 MLIN	0000 I 0
0000 I 006065 NLIN3	0000 I 006070 NLIN4	0000 I 006073 MLIN5	0000 I 0
0000 I 006047 NTIC	0000 I 006053 NTIC1	0000 I 006056 NTIC2	0000 I 0
0000 I 006072 NTIC5	0000 I 006061 NTIC6	0000 I 006075 NTIC7	0000 I 0
0000 R 006024 MEGA	0000 R 005646 PD	0000 R 005631 PI	0000 R 0
0000 R 005630 R	0000 R 005767 RA	0000 R 006037 RAA	0000 R 0
0000 R 005751 RHO1	0000 R 006000 ROLL	0000 R 005656 RS	0000 R 0
0000 R 005752 SCURT1	0000 R 006023 TAU	0000 R 005743 TEMTA	0000 R 0
0000 R 006052 TIC1	0000 R 006055 TIC2	0000 R 006063 TIC3	0000 R 0
0000 R 006060 TIC6	0000 R 006074 TIC7	0000 R 005730 T1	0000 R 0
0000 R 005747 T21	0000 R 006026 V	0000 R 005636 VS	0000 R 0
0000 R 006034 VTY	0000 R 006031 VX	0000 R 006032 VY	0000 R 0
0000 R 005740 V9	0000 R 006040 XBB	0000 R 005756 XB1	0000 R 0
0000 R 005664 XCENT	0000 R 005757 XC1	0000 R 005220 XDB	0000 R 0
0000 R 003262 XF	0000 R 006012 XG	0000 R 004610 XG1	0000 R 0
0000 R 005754 XI1	0000 R 005770 XJ	0000 R 006005 XK	0000 R 0
0000 R 005675 XLEFT	0000 R 005667 XLEM	0000 R 006017 XL1	0000 R 0
0000 R 005707 XNOM	0000 R 000202 XP	0000 R 002140 XP1	0000 R 0
0000 R 003221 XP4	0000 R 003015 XP5	0000 R 005172 XP6	0000 R 0
0000 R 005661 YSTA	0000 R 006001 XSTAO	0000 R 005774 XT	0000 R 0
0000 R 005720 XZ	0000 R 000421 Y	0000 R 005725 YA	0000 R 0
0000 R 005651 YAWD	0000 R 005652 YAWS	0000 R 005745 YA1	0000 R 0
0000 R 005700 Y80T	0000 R 005755 Y81	0000 R 005703 YC	0000 R 0
0000 R 005760 YC1	0000 R 005764 YD	0000 R 005411 YDD	0000 R 0
0000 R 003453 YF	0000 R 001003 YG	0000 R 005001 YG1	0000 R 0
0000 R 005753 YI1	0000 R 005771 YJ	0000 R 006002 YK	0000 R 0
0000 R 005670 YLEN	0000 R 006020 YL1	0000 R 001365 YM	0000 R 0
0000 R 000215 YP	0000 R 002153 YP1	0000 R 002201 YP2	0000 R 0
0000 R 003247 YP5	0000 R 005205 YP6	0000 R 005615 YP7	0000 R 0
0000 R 005644 YSTAP	0000 R 005645 YSTAP1	0000 R 005634 YSTAR	0000 R 0
0000 R 005701 YTOP	0000 R 005713 YW	0000 R 000200 YY	0000 R 0

```

100 1* C DECLARE PARAMETERS
101 2* PARAMETER NLINES=11,NDT=400,NLAMDA=460,NLPMDA=310,
101 3* SNLYMDA=450,NLHMDA=480
103 4*
103 5* REAL LAMDA,LFPDA,LYMDA,LRMDA
104 6* INTGER FLAG,AFLAG
105 7*
105 9* C DIMENSION OF VARIABLES
105 9* DIMENSION XX(2),YY(2),XP(NLINES),YP(NLINES),X(NLINES,NLINES)
105 10* S,Y(NLINES,NLINES),XG(NLINES,NLINES),YG(NLINES,NLINES),
105 11* SX(NLINES,NLINES),YM(NLINES,NLINES),NFLAG(NLINES,NLINES),
105 12* SFLAG(NLINES,NLINES),XP1(NLINES),YP1(NLINES)
105 13* S,XP2(NLINES),YP2(NLINES),YM(NLINES,NLINES),YH(NLINES,NLINES)
105 14* SXFLAG(NLINES,NLINES),YP3(NLINES),YP3(NLINES),XPS(NLINES),
105 15* SFLAG(NLINES,NLINES),YP4(NLINES),YP4(NLINES),YPS(NLINES),
105 16* S,F(NLINES,NLINES),YF(NLINES,NLINES),LFLAG(NLINES,NLINES),
105 17* S,M(NLINES,NLINES),YM(NLINES,NLINES),NFLAG1(NLINES,NLINES),

```

ORIGINAL PAGE IS
OF POOR QUALITY

```

00105 18* S(G1(NLINES,NLINES),YG1(NLINES,NLINES),XP6(NLINES),
00105 19* SYP6(NLINES),XDD(NLINES,NLINES),YDD(NLINES,NLINES),
00105 20* SAFLAG(NLINES,NLINES),XP7(NLINES),YP7(NLINES)
00106 21*
00106 22* DATA R,PI,F,D,YSTAR,DT,VS,H,DD,DS/6356.7853,141592654,
00106 23* S1000000.,400.,-5.5.,0C4.7,1399,440.,.0039.,3/
00121 24* DATA EPS1,DET,YSTAP,YSTAP1,PD,PS,FLAG/.02,1.,
00121 25* S-5.,-434.7.,.0055.,.85,2/
00131 26* DATA YSTAY,YAWD,YAWS/-5.,.0037.,.825/
00135 27* DATA DROLL,YSTAO,RD,RS/B.,-5.0.,.0042,1./
00142 28* DATA DTAU,DETA,XSTA,YSTA,VS4/85.,.40.,.0.,-5.,.7,202/
00150 29*
00150 30* C INITIALIZE THE PLOTTER
00150 31* CALL PGNPLT
00151 32*
00151 33* C CENTER PLCT
00151 34* XCENY=6.0
00152 35* YCENT=1.0
00153 36* CALL ORIGIN(XCENT,YCENT)
00154 37*
00154 38* C DEFINE THE FORM OF THE PLOT
00154 39* PTYPE='L INLIN'
00155 40* XLEN=P.0
00156 41* YLEN=P.0
00157 42* CALL PLFORM(PTYPE,XLEN,YLEN)
00160 43*
00160 44* C SCALE THE PLOT
00160 45* XX(1)=-E
00161 46* XX(2)=B
00162 47* YY(1)=-B
00163 48* YY(2)=B
00164 49* NXX=2
00165 50* NYY=2
00166 51* NGX=-1
00167 52* NGY=-1
00170 53* CALL PLSCALE(XX,NXX,NGX,YY,NYY,NGY)
00171 54*
00171 55* C TO START PLOTTING
00171 56* CALL PLGRAF
00172 57*
00172 58* C DEFINE GRID LIMITS
00172 59* XLEFT=-5
00173 60* XRIGHT=5
00174 61* XDEL=(XRIGHT-XLEFT)/(NLINES-1)
00175 62* YBOT=-5
00176 63* YTOP=5
00177 64* YDEL=(YTOP-YBOT)/(NLINES-1)
00200 65*
00200 66* C CONSTRUCT DATA
00200 67* YC=YTOP
00201 68* DO 150 INCR=1,NLINES
00201 69* XC=XLEFT
00205 70* DO 100 IPT=1,NLINES
00210 71* YNOM=R*SIN(YC/R)
00210 72* XD=(XC-XNCRS)*F
00210 73* C ANOM=XC-XD
00211 74* C YNOM=YC

```

```

0212      75•          X(IHOR,IPT)=XLOW
0213      76•          Y(IHOR,IPT)=YLOW
0214      77•          XC=XC+YDEL
0215      78•          100 CONTINUE
0217      79•          YC=YC-YDEL
0220      80•          150 CONTINUE
0222      81•          IF(FLAG.EQ.1) GO TO 200
0224      82•          IF(FLAG.EQ.2) GO TO 208
0226      83•          IF(FLAG.EQ.3) GO TO 231
0230      84•          IF(FLAG.EQ.4) GO TO 251
0232      85•          GO TO 261
0233      86•
0233      87•          200 CONTINUE
0234      88•          DO 202 I=1,NLINES
0237      89•          DO 201 J=1,NLINES
0242      93•          MFLAG(I,J)=0
0243      91•          201 CONTINUE
0245      92•          202 CONTINUE
0247      93•          YW=YSTAR
0250      94•          DO 206 L=1,NDT
0253      95•          DLAMDA=DS
0254      96•          DO 205 K=1,NLAMDA
0257      97•          LAMDA=DLAMDA*PI/180.
0260      98•          XW=-R*((1+H/R)*COS(LAMDA)-SQRT(1-(1+H/R)*(1+H/R)*
0260      99•          SSIN(LAMDA)+SIN(LAMDA)))+SIN(LAMDA)
0261      100•         DO 90 I=1,NLINES
0264      101•         DO 70 J=1,NLINES
0267      102•         ZC=YW-X(I,J)
0270      103•         YZ=YJ-Y(I,J)
0271      104•         IR=I
0272      105•         JR=J
0273      106•         IF(ABS(YZ).LE.EPSI.AND.ABS(YZ).LE.EPSI) GO TO 204
0275      107•         70 CONTINUE
0277      108•         80 CONTINUE
0301      109•         GO TO 99
0302      110•         204 MFLAG(IR,JR)=1
0303      111•         XF(IR,JR)=D*(-TAN(LAMDA))
0304      112•         YF(IR,JR)=YSTAR*(L-1)+.0289
0305      113•         WRITE(6,95) X,YJ,IR,JR,DLAMDA,XF(IR,JR),YF(IR,JR)
0316      114•         95 FORMAT(/,2F10.5,2I4,2F10.5)
0317      115•         99 DLAMDA=DLAMDA-DB
0320      116•         205 CONTINUE
0322      117•         YJ=YSTAR+L*DT*VS
0323      118•         206 CONTINUE
0325      119•         GO TO 299
0326      120•
0326      121•         208 CONTINUE
0327      122•         DO 2 I=1,NLINES
0332      123•         DO 1 J=1,NLINES
0335      124•         MFLAG(I,J)=0
0336      125•         1 CONTINUE
0340      126•         2 CONTINUE
0342      127•         DTHETA=C.
0343      128•         YA=YSTAR
0344      129•         DO 217 L=1,NDT
0347      130•         THETA=DTHETA+PI/180.
0350      131•         DLAMDA=PS

```

ORIGINAL PAGE IS
OF POOR QUALITY

```
0351 132* DO 216 K=1,NLPMDA
0354 133* LPMDA=DLPMDA*PI/180.
0355 134* T1=D*TAN(THETA)
0356 135* T2=-ABS(D/COS(THETA))*TAN(LPMDA)
0357 136* DEL=SQRT(T1*T1+T2*T2)
0360 137* RHO=ATAN(DEL/D)
0361 138* SQURT=R*((1+D/R)*COS(RHO)-SQRT(1-(1+D/R)*(1+D/R)*SIN
0361 139* (RHO)*SIN(RHO)))/DEL
0362 140* YI=(SQURT-T1)/DEL
0363 141* XI=(SQURT-T2)/DEL
0364 142* YB=YI+YA
0365 143* XB=XI
0366 144* DO 22 I=1,NLINES
0371 145* DO 11 J=1,NLINES
0374 146* XC=XB-X(I,J)
0375 147* YC=YB-Y(I,J)
0376 148* IG=I
0377 149* JG=J
0400 150* IF(ABS(XC).LE.EPSI.AND.ABS(YC).LE.EPSI) GO TO 210
0402 151* 11 CONTINUE
0404 152* 22 CONTINUE
0406 153* GO TO 215
0407 154* 210 NFLAG(IG,JG)=1
0410 155* XG(IG,JG)=D*(-TAN(LPMDA))
0411 156* YG(IG,JG)=YBOT+(L-1)*.0288
0412 157* WRITE(6,212) XB,YB,IG,JG,DLPMDA,XG(IG,JG),YG(IG,JG)
0423 158* 212 FORMAT(/,2F10.5,2I4,3F10.5)
0424 159* 215 DLPMDA=DLPMDA-PI
0425 160* 216 CONTINUE
0427 161* YA=YSTAP+L*.0288
0430 162* DTHETA=L/2500.
0431 163* 217 CONTINUE
0433 164* GO TO 299

0434 165*
0434 166* DO 220 I=1,NLINES
0437 167* DO 219 J=1,NLINES
0442 168* NFLAG1(I,J)=0
0443 169* 219 CONTINUE
0445 170* 220 CONTINUE
0447 171* THETA=DTHETA*PI/180.
0450 172* ET=DET*PI/180.
0451 173* YA1=YSTAP1
0452 174* DO 227 L=1,NDT
0455 175* DLPMDA=PI
0456 176* DO 226 K=1,NLPMDA
0461 177* LPMDA=DLPMDA*PI/180.
0462 178* T11=D*TAN(THETA+ET)
0463 179* T21=-ABS(D/COS(THETA+ET))*TAN(LPMDA)
0464 180* DEL1=SQRT(T11*T11+T21*T21)
0465 181* RHO1=ATAN(DEL1/D)
0466 182* SQURT1=R*((1+D/R)*COS(RHO1)-SQRT(1-(1+D/R)*(1+D/R)*
0466 183* SIN(RHO1)*SIN(RHO1)))/DEL1
0467 184* YI1=(SQURT1-T11)/DEL1
0470 185* XI1=(SQURT1-T21)/DEL1
0471 186* YB1=YI1+YA1
0472 187* XB1=XI1
```

```

0473 193*      DO 222 I=1,NLINES
0476 189*      DO 221 J=1,NLINES
0501 190*      XCI=XBI-Y(I,J)
0502 191*      YCI=YBI-Y(I,J)
0503 192*      IG1=I
0504 193*      JG1=J
0505 194*      IF(ABS(XCI).LE.EPSI.AND.ABS(YCI).LE.EPSI) GO TO 223
0507 195*      221 CONTINUE
0511 196*      222 CONTINUE
0513 197*      GO TO 225
0514 198*      223 XFLAG1(IG1,JG1)=1
0515 199*      XG1(IG1,JG1)=D*(-TAN(LPMDA))
0516 200*      YG1(IG1,JG1)=YSTAP1*(L-1)*.0288*Y11
0517 201*      WRITE(6,224) YB1,YBI,IG1,JG1,DLPMDA,XG1(IG1,JG1),
0517 202*      SYG1(IG1,JG1)
0530 203*      224 FORMAT(/,2F10.5,2I4,3F10.5)
0531 204*      225 DLPMDA=DLPMDA-PD
0532 205*      226 CONTINUE
0534 206*      YAI=YSTAP1*L*.0288
0535 207*      227 CONTINUE
0537 208*      GO TO 805
0540 209*
0540 210*      231 CONTINUE
0541 211*      DO 4 I=1,NLINES
0544 212*      DO 3 J=1,NLINES
0547 213*      KFLAG(I,J)=0
0550 214*      3 CONTINUE
0552 215*      4 CONTINUE
0554 216*      DYAW=0.
0555 217*      YD=YSTAY
0556 218*      DO 240 L=1,NDT
0561 219*      YAA=DYAB*PI/180.
0562 220*      DLYMDA=YAWS
0563 221*      DO 238 K=1,NLYMDA
0566 222*      LYMDA=DLYMDA*PI/180.
0567 223*      RA=-R*((1+D/R)*COS(LYMDA)-SQRT(1-(1+D/R)*(1+D/R)*SIN(LYMDA)
0567 224*      *SIN(LYMDA)))*SIN(LYMDA)
0570 225*      XJ=RA*CGS(YAW)
0571 226*      YJ=-RA*SIA(YAW)
0572 227*      XE=XJ
0573 228*      YE=YJ+YD
0574 229*      DO 44 I=1,NLINES
0577 230*      DO 33 J=1,NLINES
0602 231*      XT=XE-Y(I,J)
0603 232*      YT=YE-Y(I,J)
0604 233*      IM=I
0605 234*      JM=J
0606 235*      IF(ABS(XT).LE.EPSI.AND.ABS(YT).LE.EPSI) GO TO 233
0610 236*      33 CONTINUE
0612 237*      44 CONTINUE
0614 238*      GO TO 235
0615 239*      233 KFLAG(IM,JM)=1
0616 240*      XM(IM,JM)=D*(-TAN(LYMDA))
0617 241*      YM(IM,JM)=YROT*(L-1)*.0288
0620 242*      WRITE(6,234) XE,YE,IM,JM,DLYMDA,XM(IM,JM),YM(IM,JM)
0631 243*      234 FORMAT(/,2F10.5,2I4,3F10.5)
0632 244*      235 DLYMDA=DLYMDA-YAWD

```


ORIGINAL PAGE IS
OF POOR QUALITY

```
0633 145* 238 CONTINUE
0635 246* YD=YSTAY+L*.0289
0636 247* DYAM=5.*SIN((L*PI)/174.)
0637 248* 240 CONTINUE
0641 249* GO TO 299

0642 250*
0642 251* 241 CONTINUE
0643 252* DO 6 I=1,NLINES
0646 253* DO 5 J=1,MLINES
0651 254* LFLAG(I,J)=0
0652 255* 5 CONTINUE
0654 256* 6 CONTINUE
0656 257* ROLL=DROLL*PI/180.
0657 258* XSTAO=R*((1+D/R)*COS(ROLL)-SQRT(1-(1+D/R)*(1+D/R)*SIN(ROLL)
0657 259* S*SIN(ROLL)))*SIN(ROLL)
0660 260* YK=YSTAO
0661 261* DO 250 L=1,NDT
0664 262* DLRMDA=RS
0665 263* DO 245 K=1,MLRMDA
0670 264* LRMDA=DLRMDA*PI/180.
0671 265* DIS=-R*((1+D/R)*COS(LRMDA+ROLL)-SQRT(1-(1+D/R)*(1+D/R)*SIN
0671 266* S(LRMDA+ROLL)*SIN(LRMDA+ROLL)))*SIN(LRMDA+ROLL)
0672 267* XK=DIS*XSTAO
0673 268* DO 66 I=1,NLINES
0676 269* DO 55 J=1,MLINES
0701 270* XL=XK-X(I,J)
0702 271* YL=YK-Y(I,J)
0703 272* IM=I
0704 273* JM=J
0705 274* IF(ABS(XL).LE.EPSI.AND.ABS(YL).LE.EPSI) GO TO 242
0707 275* 55 CONTINUE
0711 276* 66 CONTINUE
0713 277* GO TO 244
0714 278* 242 LFLAG(IM,JM)=1
0715 279* XM(IM,JM)=D*(-TAN(LRMDA))
0716 280* YM(IM,JM)=YSTAO*(L-1)*.0289
0717 281* WRITE(6,243) XK,YK,IM,JM,DLRMDA,XM(IM,JM),YM(IM,JM)
0730 282* 243 FORMAT(/,2F10.5,2I4,3F10.5)
0731 283* 244 DLRMDA=DLRMDA-RD
0732 284* 245 CONTINUE
0734 285* YK=YSTAO+L*.0289
0735 286* 250 CONTINUE

0737 287*
0737 288* 251 CONTINUE
0740 289* DO 9 I=1,NLINES
0743 290* DO 7 J=1,MLINES
0746 291* LFLAG1(I,J)=0
0747 292* 7 CONTINUE
0751 293* 8 CONTINUE
0753 294* ROLL=DROLL*PI/180.
0754 295* DER=0.
0755 296* XSTAO=R*((1+D/R)*COS(ROLL)-SQRT(1-(1+D/R)*(1+D/R)*SIN(ROLL)
0755 297* S*SIN(ROLL)))*SIN(ROLL)
0756 298* YK1=YSTAO
0757 299* DO 260 L=1,NDT
0762 300* ER=DER*PI/180.
```

```

0763 301*          DLRMDA=RS
0764 302*          DO 255 K=1,NLRMDA
0767 303*          LRMDA=DLRMDA*PI/180.
0770 304*          DIS1=-R*((1+D/R)*COS(LRMDA+ROLL+ER)-SQRT(1-(1+D/R)*(1+D/R)
0770 305*          S+SIN(LRMDA+ROLL+ER)*SIN(LRMDA+ROLL+ER)))*SIN(LRMDA+ROLL+ER)
0771 306*          XK1=DIS1*XSTAO
0772 307*          DO 99 I=1,NLINES
0775 308*          DO 77 J=1,NLINES
1000 309*          XLI=XK1-Y(I,J)
1001 310*          YLI=YK1-Y(I,J)
1002 311*          IM1=I
1003 312*          JM1=J
1004 313*          IF(ABS(XLI).LE.EPSI.AND.ABS(YLI).LE.EPSI) GO TO 252
1006 314*          77 CONTINUE
1010 315*          89 CONTINUE
1012- 316*          GO TO 254
1013 317*          252 LFLAG1(I,J)=1
1014 318*          XM1(IM1,JM1)=D*(-TAN(LRMDA))
1015 319*          YM1(IM1,JM1)=YBCT*(L-1)*.0288
1016 320*          WRITE(6,253) XK1,YK1,IM1,JM1,DLRMDA,XM1(IM1,JM1),
1016 321*          SYM1(IM1,JM1)
1027 322*          253 FORMAT(/,2F10.5,2I4,3F10.5)
1030 323*          254 DLRMDA=DLRMDA-RD
1031 324*          255 CONTINUE
1033 325*          YK1=YSTAC*L*.0238
1034 326*          DER=D*.2*SIN((L*PI)/179.)
1035 327*          260 CONTINUE
1037 328*          GO TO 299
1040 329*
1040 330*          261 CONTINUE
1041 331*          DO 263 I=1,NLINES
1044 332*          DO 262 J=1,NLINES
1047 333*          AFLAG(I,J)=0
1050 334*          262 CONTINUE
1052 335*          263 CONTINUE
1054 336*          TAU=DTAU*FI/180.
1055 337*          OMEGA=2.*FI/26400.
1056 338*          ETA=DETA*PI/180.
1057 339*          V=OMEGA*R*COS(ETA)
1060 340*          SLUMDA=C*S(TAU)/COS(ETA)
1061 341*          CLUMDA=SQRT(1-SLUMDA*SLUMDA)
1062 342*          VX=V*CLUMDA
1063 343*          VY=V*SLUMDA
1064 344*          VTX=-VX
1065 345*          VTY=VS4-VY
1066 346*          XAA=XSTA
1067 347*          YAA=YSTA
1070 348*          DO 270 L=1,NDT
1073 349*          OLYMDA=YALS
1074 350*          DO 269 F=1,NLYMDA
1077 351*          LYMDA=OLYMDA*PI/180.
1100 352*          XAA=-K*((1+D/R)*COS(LYMDA)-SQRT(1-(1+D/R)*(1+D/R)*SIN
1100 353*          S(LYMDA)*SIN(LYMDA)))*SIN(LYMDA)
1101 354*          XBA=XAA+FAA
1102 355*          YBA=YAA
1103 356*          DO 265 I=1,NLINES
1106 357*          DO 264 J=1,NLINES

```

ORIGINAL PAGE IS
OF POOR QUALITY

```
1111 355.      XCC=YHB-Y(I,J)
1112 359.      YCC=YHB-Y(I,J)
1113 360.      IDD=I
1114 361.      JDD=J
1115 362.      IF(ABS(YCC).LE.EPSI.AND.ABS(YCC).LE.EPSI) GO TO 266
1117 363.      264 CONTINUE
1121 364.      265 CONTINUE
1123 365.      GO TO 268
1124 366.      266 AFLAG(I,J)=1
1125 367.      ADD(IDD,JDD)=D*(-TAN(LYMDA))
1126 368.      YDD(IDD,JDD)=YSTA*(L-1)*.028E
1127 369.      WRITE(6,267) XBB,YHB,IDD,JDD,DLYMDA,XDD(IDD,JDD),
1127 370.      SYDD(IDD,JDD)
1140 371.      267 FORMAT(/,2F10.5,2I4,3F10.5)
1141 372.      268 DLYMDA=DLYMDA-DD
1142 373.      269 CONTINUE
1144 374.      AAA=XSTA+L*DT*VT
1145 375.      YAA=YSTA+L*DT*VT
1146 376.      270 CONTINUE
1150 377.
1150 378.      C      PLOT HORIZONTAL LINES
1150 379.
1150 380.      299 CONTINUE
1151 381.      TIC=0
1152 382.      NTIC=-1
1153 383.      NLIN=NLINES
1154 384.      DO 400 IHOR=1,NLINES
1157 385.      DO 300 IPT=1,NLINES
1162 386.      XP(IPT)=X(IHOR,IPT)
1163 387.      YP(IPT)=Y(IHOR,IPT)
1164 388.      300 CONTINUE
1166 389.      CALL PLCURV(XP,YP,NLIN,NTIC,TIC)
1167 390.      400 CONTINUE
1171 391.
1171 392.      C      PLOT VERTICAL LINES
1171 393.      DO 500 IVERT=1,NLINES
1174 394.      CALL PLCURV(Y(1,IVER),Y(1,IVER),NLIN,NTIC,TIC)
1175 395.      500 CONTINUE
1177 396.      IF(FLAG.EQ.1) GO TO 655
1201 397.      IF(FLAG.EQ.2) GO TO 805
1203 398.      IF(FLAG.EQ.3) GO TO 901
1205 399.      IF(FLAG.EQ.4) GO TO 009
1207 400.      GO TO 921
1210 401.
1210 402.      C      PLOT HORIZONTAL LINES
1210 403.
1210 404.      655 CONTINUE
1211 405.      TIC1=0
1212 406.      NTIC1=0
1213 407.      NLIN1=NLINES
1214 408.      DO 750 IHOR=1,NLINES
1217 409.      DO 700 IPT=1,NLINES
1222 410.      XP1(IPT)=XP(IHOR,IPT)
1223 411.      YP1(IPT)=YP(IHOR,IPT)
1224 412.      700 CONTINUE
1226 413.      CALL PLCURV(XP1,YP1,NLIN1,NTIC1,TIC1)
1227 414.      750 CONTINUE
```

```

1231 415•
1231 416• C PLOT VERTICAL LINES
1231 417• DO 800 IVERT=1,NLINES
1234 418• CALL PLCURV(XF(1,IVERT),YF(1,IVERT),NLIN1,NTIC1,TIC1)
1235 419• 800 CONTINUE
1237 420• GO TO 9999
1240 421•
1240 422• C PLOT HORIZONTAL LINES
1240 423• 805 CONTINUE
1241 424• TIC2=0
1242 425• NTIC2=0
1243 426• NLIN2=NLINES
1244 427• DO 850 INOR=1,NLINES
1247 428• DO 820 IPT=1,NLINES
1252 429• XP2(IPT)=YG(INOR,IPT)
1253 430• YP2(IPT)=YG(INOR,IPT)
1254 431• 820 CONTINUE
1256 432• CALL PLCURV(YP2,YP2,NLIN2,NTIC2,TIC2)
1257 433• 850 CONTINUE
1261 434•
1261 435• C PLOT VERTICAL LINES
1261 436• DO 851 IVERT=1,NLINES
1264 437• CALL PLCURV(YG(1,IVERT),YG(1,IVERT),NLIN2,NTIC2,TIC2)
1265 438• 851 CONTINUE
1267 439• GO TO 9999

1270 440•
1270 441• C PLOT HORIZONTAL LINES
1270 442• TIC6=0
1271 443• NTIC6=0
1272 444• NLIN6=NLINES
1273 445• DO 854 INCR=1,NLINES
1276 446• DO 853 IPT=1,NLINES
1301 447• XP6(IPT)=YG1(INOR,IPT)
1302 448• YP6(IPT)=YG1(INOR,IPT)
1303 449• 853 CONTINUE
1305 450• CALL PLCURV(XP6,YP6,NLIN6,NTIC6,TIC6)
1306 451• 854 CONTINUE
1310 452•
1310 453• C PLOT VERTICAL LINES
1310 454• DO 855 IVERT=1,NLINES
1313 455• CALL PLCURV(YG1(1,IVERT),YG1(1,IVERT),NLIN6,NTIC6,TIC6)
1314 456• 855 CONTINUE
1316 457• GO TO 9999
1317 458•
1317 459• C PLOT HORIZONTAL LINES
1317 460• 901 CONTINUE
1320 461• TIC3=0
1321 462• NTIC3=0
1322 463• NLIN3=NLINES
1323 464• DO 905 INCR=1,NLINES
1326 465• DO 903 IPT=1,NLINES
1331 466• XP3(IPT)=YH(INOR,IPT)
1332 467• YP3(IPT)=YH(INOR,IPT)
1333 468• 903 CONTINUE
1335 469• CALL PLCURV(YP3,YP3,NLIN3,NTIC3,TIC3)
1336 470• 905 CONTINUE

```

ORIGINAL PAGE IS
OF POOR QUALITY

```

11340 471*
11340 472* C PLOT VERTICAL LINES
11340 473* DO 907 IVERT=1,NLINES
11343 474* CALL PLCURV(XM(1,IVERT),YM(1,IVERT),NLIN3,NTIC3,TIC3)
11344 475* 907 CONTINUE
11346 476* GO TO 9999
11347 477*
11347 478* C PLOT HORIZONTAL LINES
11347 479* 911 CONTINUE
..
11350 480* TIC4=***
11351 481* NTIC4=-1
11352 482* NLIN4=NLINES
11353 483* DO 915 IHOR=1,NLINES
11356 484* DO 913 IPT=1,NLINES
11361 485* (P4(IPT)=YM(IHOR,IPT))
11362 486* YP4(IPT)=YM(IHOR,IPT)
11363 487* 913 CONTINUE
11365 488* CALL PLCURV(XP4,YP4,NLIN4,NTIC4,TIC4)
11366 489* 915 CONTINUE
11370 490*
11370 491* C PLOT VERTICAL LINES
11370 492* DO 917 IVERT=1,NLINES
11373 493* CALL PLCURV(XM(1,IVERT),YM(1,IVERT),NLIN4,NTIC4,TIC4)
11374 494* 917 CONTINUE
11374 495*
11376 496* C PLOT HORIZONTAL LINES
11376 497* 909 CONTINUE
11377 498* TIC5=0
1400 499* NTIC5=0
1401 500* NLIN5=NLINES
1402 501* DO 918 IHOR=1,NLINES
1405 502* DO 918 IPT=1,NLINES
1410 503* (P5(IPT)=YM(IHOR,IPT))
1411 504* YP5(IPT)=YM(IHOR,IPT)
1412 505* 918 CONTINUE
1414 506* CALL PLCURV(XP5,YP5,NLIN5,NTIC5,TIC5)
1415 507* 919 CONTINUE
1417 508*
1417 509* C PLOT VERTICAL LINES
1417 510* DO 920 IVERT=1,NLINES
1422 511* CALL PLCURV(XM(1,IVERT),YM(1,IVERT),NLIN5,NTIC5,TIC5)
1423 512* 920 CONTINUE
1425 513* GO TO 9999
1426 514*
1426 515* C PLOT HORIZONTAL LINES
1426 516* 921 CONTINUE
1427 517* TIC7=0
1430 518* NTIC7=0
1431 519* NLIN7=NLINES
1432 520* DO 923 IHOR=1,NLINES
1435 521* DO 922 IPT=1,NLINES
1440 522* (P7(IPT)=YD(IHOR,IPT))
1441 523* YP7(IPT)=YD(IHOR,IPT)
1442 524* 922 CONTINUE
1444 525* CALL PLCURV(XP7,YP7,NLIN7,NTIC7,TIC7)
1445 526* 923 CONTINUE

```

```

1447 527*
1447 528* C PLOT VERTICAL LINES
1447 529* DO 924 IVERT=1,NLINES
1452 530* CALL PLCURV(YDD(1,IVERT),YDD(1,IVERT),MLIN7,NTIC7,TIC7)
1453 531* 924 CONTINUE
1455 532*
1455 533* C TO FINISH PLOTTING
1455 534* 9999 CALL ENDPLT
1456 535*
1456 536* * STOP
1457 537* END

```

```

ND FLR:
PU:1.399 CTP:.115 SUPS:9.131

```

```

DELETE,A SHUTEVP,IMAGE-1/ABS
URPUR 2-3-JB 05/19/63 11:56:41

```

```

TP=.333 SUP=.737 CPU=.008 IO=.302 CC-ER=.433

```

End of Document