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STATION KEEPING OF GEOSTATIONARY SATELLITES BY ELECTRIC PROPULSION

M. C. Eckstein

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16. Abstract

As various types of perturbations tend to drive a geostationary satellite away from its prescribed position, occasional orbit corrections have to be carried out by means of a suitable propulsion system. In future geostationary missions, low thrust electric propulsion is likely to be applied for station keeping because of considerable mass savings. In this paper a station keeping strategy for electric propulsion systems is developed. Both the unconstrained case and the case where thrust operation constraints are present are considered and tested by computer simulation of a realistic example.

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Station Keeping of Geostationary Satellites by Electric Propulsion /4*

Summary

As various types of perturbations tend to drive a geostationary satellite away from its prescribed position, occasional orbit corrections have to be carried out by means of a suitable propulsion system. In future geostationary missions, low thrust electric propulsion is likely to be applied for station keeping because of considerable mass savings. In this paper a station keeping strategy for electric propulsion systems is developed. Both the unconstrained case and the case where thrust operation constraints are present are considered and tested by computer simulation of a realistic example.

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1. Introduction

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The requirement, that a geostationary satellite must always be located vertically above a prescribed fixed point of the earth's equator, presupposes an equatorial and circular orbit whose rotational time coincides with the rotational duration of the earth. However, since an exact geostationary orbit cannot be realized because of unavoidable entry errors and the always acting orbit perturbances, the limits of allowable deviations from the prescribed position play an important role. Depending on the size of the tolerance window, the orbit must be corrected more or less frequently. The velocity increments needed for this purpose can be generated with the aid of the propulsion systems incorporated in the satellite. In general the latter are mounted such that thrusts are possible in directions tangential and normal to the orbit independent of one another.

From the requirement, on the one hand to maintain the limits of the tolerance window, and on the other hand to also save fuel, one can derive for a prescribed propulsion system the optimum correction strategy which specifies the thrust times. Here one obtains for the electrical propulsion systems planned to be deployed in the future to an increasing degree, considerable differences as compared to the today conventional chemical engines because of the low thrust level. The advantage of electrical propulsion lies in the considerable weight savings for missions of long duration [1], [2].

On the other hand, the low thrust level has a disadvantage in that numerous and long duration thrusts are necessary for station keeping, which under certain circumstances disturb the remaining mission operation. In addition the high energy requirement can lead to the fact that the already low thrust level and thrust times must be restricted if the limitedly available energy is needed at the same time for other mission purposes [3].

If no such constraints need to be taken into account, the required thrust times can be calculated analytically. But also in those cases, where constraints exist and are known for some time in advance, the optimal correction strategy can be found with the aid of known optimization methods. /8

Since the classical orbital elements for the geostationary case are in part undefined, it is useful to introduce another set of orbit parameters, which are to be designated as "geostationary elements" [4]:

$$
(1)\quad
\begin{aligned}
D &= (n-n_g)/n_g \\
h &= e\sin(\omega+\Omega) \\
l &= e\cos(\omega+\Omega) \\
p &= \sin\frac{i}{2}\sin\Omega \\
q &= \sin\frac{i}{2}\cos\Omega \\
\lambda &= M+\Omega+\omega-L \quad ,
\end{aligned}
$$

whereby

n	= average motion of the satellite
$a, e, i, \Omega, \omega, M$	= classical orbit elements
L	= right ascension of the geostationary required point
n_g	= angular velocity of the rotation of the earth
μ	= geocentric gravitation constant

The elements D, h, l, p, q, λ are also well defined for circular and equatorial orbits as dimensionless quantities and disappear altogether for undisturbed, exact geostationary orbits. They describe the motion of the satellite around the earth and thus also the deviation of its apparent position from the required point. D is the drift rate normalized with respect to the rotation of the earth; λ is the average deviation from the required point while the pairs (l,h) and (q,p) essentially represent the eccentricity- and inclination vector.

As a result of the perturbations the geostationary elements are functions of time which can be subdivided into secular and periodic terms. If one restricts himself to perturbances of the first order, one encounters primarily terms with periods of fractions /9 of a day, one moon orbit, and one year. Disturbance terms with longer periods, e.g. orbiting times of the apsides and the junctions of the lunar orbit can be treated during the station keeping problems as secular perturbations. On the other hand the short-periodical perturbations with periods below one day must be accepted as unavoidable deviations since, with the presently available propulsion systems, it is either ineffective or even impossible to control them.

The disturbances of the geostationary path are caused primarily by the three-axis nature of the earth potential, the gravitation from sun and moon, and by the solar radiation pressure. The analytical description can be simplified [5] because of the special properties of the geostationary orbit compared to the very complex general perturbation theory.

Figures 1 to 3 present examples for the time variation of the geostationary elements as the result of orbit perturbations. If one requires a tolerance window of $\pm 0.1^{\circ}$ in length and width, then - without taking into account the short-periodic effects - the north-south limits from an initially exact geostationary orbit are violated in this case after 33 days, the east-west limits already after about 20 days. These times are shortened still more if the unavailable short-periodic effects are taken into account.

2. Station keeping strategy

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In order to prevent the drifting of the satellite outside of the tolerance window, the orbit is corrected from time to time. This is done with the aid of a propulsion system which consists of several engines and which produce the necessary thrusts.

The specification of the thrust times for all engines taking into account the tolerance window and the requirement for minimum fuel consumption is the task of the station keeping strategy. As already mentioned in the introduction, it is neither possible nor desirable here to compensate for all appearing disturbance accelerations. Rather it makes much more sense to correct only the long-term acting effects from time to time while the short-period perturbations are taken into account on an overall basis by corresponding reduction of the window limits. Under "long-term" we understand here all secular effects and the periodic perturbations with periods of more than 30 days. Thus the orbit corrections to be made need to be calculated only from the disturbance effects which, for instance, are shown in figures 1 to 3 as solid curves.

The conventional station keeping makes use of a type of "wait-and-see strategy": By means of continuous orbit determination one follows, e.g., the position of points (q,p) in figure 3 long enough until it reaches the tolerance limit - reduced by the value of the short-period disturbances - and then corrects elements q and p by a thrust directed northward or southward in such a way that (q,p) is displaced to the opposite-lying edge of the window.

Similar methods apply for the east-west corrections where the conditions are somewhat more complicated only because the four elements D, h, l, λ cannot be corrected independently of one another. If the perturbations of the eccentricity are only relatively small, it suffices to reverse by means of a tangential thrust the algebraic sign of the drift rate D. With correct dosing the point (λ,D) then describes a parabola which utilizes only a part of the allowable tolerance window, while the remainder is taken up by the eccentricity effects which were corrected only partially or not at all [6]. /11

However, the narrow tolerance windows of future geostationary satellites as well as the considerable perturbations caused by radiation pressure on the large solar paddles will also require an extensive correction of the eccentricity. In these cases at least two separate tangential thrusts are necessary for a complete east-west correction.

For the example shown in figures 1 to 3 a north-south correction would have to be made, according to this strategy, about every 60 days and an east-west correction every 40 days.

The relatively large north-south thrusts include, as the result of unavailable orientation errors under certain circumstances a noticeable non-calculable east-west component, which makes premature east-west corrections necessary. Therefore the station keeping strategy is usually arranged such that a few days after every north-south correction an east-west correction follows [7].

The conventional "wait-and-see strategy" cannot be used for station keeping with electrical engines because the thrust level, lower by a factor of about 1000, is not sufficient to achieve the necessary inclination changes of 0.2° with a single thrust. This could only be done in numerous small steps and would take too much time since the perturbation effects are counteracting. If one assumes that the TV satellite weighing 1058 kg and planned for 1983 is propelled daily for 6 hours with 16.4 mN, one would require about 170 days for the desired north-south correction.

When using electrical propulsion systems it makes no sense therefore, to wait until the window limits have been violated, but rather to correct the orbit by small amounts in intervals as short as possible. For example, one could carry out the north-south station keeping in such a way that the point (q,p) is brought back [8] daily to the origin (0,0). However, here one must consider the small value of the changes of the orbit plane, caused by the disturbances, of only about 0.002° per day, which under certain circumstances can be found by orbit determinations just barely or only with considerable errors. Thus by the use of such a "one-day strategy" one encounters the danger to correct along with increased fuel consumption deviations which in reality do not exist, but which are only simulated by orbit determination errors. /12

In order to avoid this as far as possible, one can, to be sure, retain the daily corrections, but specify them each time only in accordance with larger time intervals.

The orbit is determined continuously and by taking into account the daily corrections. If a cycle of duration T has passed, the duration of the next cycle and the thrust times are specified such that the target orbit is reached nominally at the end of the new cycle. The corrections to be made for the elements ΔE_i are obtained from the initial values E_i^{o} determined from the orbit determination and the disturbance effects δE_i during the next cycle.

$$\Delta E_i = E_i^z - E_i^o - \delta E_i \quad , \tag{2}$$

whereby E_i^z are the elements of the target orbit.

The thrust times can be calculated from T and from the ΔE_i's with the aid of formulas which will be derived in the next section.

Such a long-duration strategy has the following advantages:

- Observations from a longer duration timespan are available which make possible a more accurate orbit determination.

- The deviations from the required orbit have grown as the result of the orbit model- and execution errors after time T to a more distinctly recognizable value.

- The time T can be selected appropriately from the requirement whereby the magnitude of the various errors and mission-engineering reasons can play a role.

3. Calculation of the thrust times for unconstrained station keeping /1

For a tolerance window of the order of magnitude of $\pm 0.1^{\circ}$ all orbit corrections to be considered can be considered as differentially small and those perturbation equations can be used for their determination which assume for the geostationary orbit elements the following simplified form [4]:

$$
\begin{aligned}
\dot{D} &= -\frac{3}{V}\, b_T \\
\dot{h} &= \frac{1}{V}\,(-b_R \cos L + 2 b_T \sin L) \\
\dot{i} &= \frac{1}{V}\,(b_R \sin L + 2 b_T \cos L) \\
\dot{p} &= \frac{1}{2V}\, b_N \sin L \\
\dot{q} &= \frac{1}{2V}\, b_N \cos L \\
\dot{\lambda}_0 &= -\frac{2}{V}\, b_R \quad .
\end{aligned}
$$

Here $V = 3.074647$ km/s is the orbit velocity.

Since only perturbations of 1. order are of interest, we introduced on the right-hand side the values of the exact geostationary orbit. The quantities b_R, b_T, b_N are the disturbance acceleration components in radial, tangential, and normal direction with respect to the orbit. They can also be interpreted as the acceleration caused by one engine or by a combination of several engines. If one assumes these to be constant and considers that the length L increases linearly with time, one obtains the orbit correction resulting from K thrusts by integration:

$$(4)\qquad \Delta D = -\frac{3}{V}\sum_{k=1}^{K} b_{Tk}\tau_k$$

$$(5)\qquad \begin{aligned} \Delta h &= \frac{2}{Vn}\sum_{k=1}^{K}(-b_{Rk}\cos L_k + 2b_{Tk}\sin L_k)\sin\tfrac{1}{2}n\tau_k \\ \Delta l &= \frac{2}{Vn}\sum_{k=1}^{K}(b_{Rk}\sin L_k + 2b_{Tk}\cos L_k)\sin\tfrac{1}{2}n\tau_k \end{aligned}$$ /14

$$(6)\qquad \begin{aligned} \Delta p &= \frac{1}{Vn}\sum_{k=1}^{K} b_{Nk}\sin L_k \sin\tfrac{1}{2}n\tau_k \\ \Delta q &= \frac{1}{Vn}\sum_{k=1}^{K} b_{Nk}\cos L_k \sin\tfrac{1}{2}n\tau_k \end{aligned}$$

$$(7)\qquad \Delta\lambda_o = -\frac{2}{V}\sum_{k=1}^{K} b_{Rk}\tau_k \quad .$$

Here b_{Rk}, b_{Tk}, b_{Nk} are the acceleration components, τ_k the duration and L_k the arithmetic mean from the length at the beginning and at the end of the k^{th} thrust. $\Delta\lambda_o$ is the correction for the average epoch deviation. The correction of the mean deviation at a time T_E after the expiration of all thrusts is obtained as the result of k drift rate changes as

$$(8)\qquad \Delta\lambda = \Delta\lambda_o - \frac{3}{V}\sum_{k=1}^{K} b_{Tk}\tau_k(L_E - L_k) \quad ,$$

where L_E denotes the length associated with T_E.

On the left-hand side of equations (4) to (8) one finds the desired corrections while the desired maneuver parameters L_k and τ_k appear on the right-hand side, from which one can determine the starting times according to the relation

$$(9) \qquad \begin{matrix} t_{2k-1} \\ t_{2k} \end{matrix} = t_o + \frac{1}{n} \left[L_k - \lambda_E - \Theta_G(t_o) \right] \mp \frac{1}{2}\tau_k \, .$$

With t_o we denote the starting time of the cycle, with λ_E the eastern length of the geostationary required point, and with θ_G the sidereal time of the meridian of Greenwich. On the left side the odd indices apply for the starting times and the even ones for the shut-off times.

The values for b_{Rk}, b_{Tk}, and b_{Nk} are fixed by the propulsion system and the mass of the satellite which, with good approximation, can be assumed to be constant.

The fuel requirement is proportional to the total velocity increment /15

$$(10) \qquad \Delta V = \sum_{k=1}^{K} b_k \tau_k$$

The b_k values are composed of the amount of the acceleration generated by the individual engines, which take part in the k^{th} thrust. Since the engines can frequently not be aligned radially, tangentially, or normally but rather must be set at a given angle (figure 5), only the projections of the thrusts on these principal directions are effective so that the value of the acceleration acting on the orbit is $\sqrt{b_{Rk}^2 + b_{Tk}^2 + b_{Nk}^2} \leqq b_k$.

From the system of equations (4) to (8) one can derive the following:

- $K \geq 3$ is a necessary prerequisite for the correction of all 6 orbit elements. For an unequivocal solution for τ_k and L_k at $K > 3$ additional criteria must be used, such as the minimization of the fuel consumption.
- One can dispense with radial acceleration components since all corrections achievable thereby can be obtained also - and more effectively - with the tangential components.
- Since equation (6) contains only the b_{Nk} values and the remaining equations only the b_{Tk} (and b_{Rk}) the north-south corrections ($\Delta p, \Delta q$) can be carried out independent of the east-west corrections (ΔD, Δh, Δl, $\Delta \lambda$), insofar as the propulsion system can also generate the north-south or east-west thrusts independently of one another.
- For short thrust times the sine $\frac{1}{2} n\tau$ can be replaced by the arguments so that on the right-hand side only the velocity increments $\Delta V_{Tk} = b_{Tk}\tau_k$ etc. still appear, such as is the case for pulse-shaped corrections.

North-south corrections:

The corrections of the orbits are determined solely by equations (6). Under the assumption of equal normal components of the thrust vectors, that is $b_{Nk} = b_N =$ constant, one can show that the following solution is optimal from a fuel consumption standpoint: /16

$$\text{(11)} \qquad \begin{aligned} L_k &= \arctan \frac{\Delta p}{\Delta q} + 2(k-1)\pi \qquad k=1,\ldots,K \\ \tau_k &= \frac{2}{n} \arcsin \left(\frac{Vn}{K|b_N|} \sqrt{\Delta p^2+\Delta q^2}\right) = \tau_1 \end{aligned}$$

whereby

$$-\frac{\pi}{2} \leq L_1 \leq \frac{\pi}{2} \quad \text{for} \quad b_N > 0 \quad ,$$

$$+\frac{\pi}{2} \leq L_1 \leq \frac{3}{2}\pi \quad \text{for} \quad b_N < 0 \quad .$$

If thrusts of the same value are possible also in opposite directions, then $b_{Nk} = (-1)^{k-1} b_N$ and

$$(12) \qquad L_k = \arctan \frac{\Delta p}{\Delta q} + (k-1)\pi \qquad k=1,\ldots,K$$

with

$$-\frac{\pi}{2} < L_1 < \frac{\pi}{2}$$

Equations (11) and (12) state that for a prescribed north-south correction by means of K thrusts one must carry out each time at a given length position or two oppositely-lying length positions of the satellite thrusts of equal duration in its orbit, if the fuel consumption is to be minimized at the same time. If the point (q,p) is to be transposed in the direction to the origin (0,0), that is $(\Delta q, \Delta p) = \alpha(q,p)$, then according to equations (12) and (1) the L_k values coincide with the junction of the initial orbit.

The value of the normal component of the velocity increment

$$(13) \qquad \Delta V_N = \sum_{k=1}^{K} |b_{Nk}| \tau_k = \frac{2K|b_N|}{n} \arcsin \frac{Vn}{K|L_N|} \sqrt{\Delta p^2 + \Delta q^2}$$

decreases for an increasing number of thrusts K with the limiting value $\Delta V_N^{\infty} = 2V\sqrt{\Delta p^2+\Delta q^2}$ for $K \to \infty$, as one can recognize by the example of figure 4. Thus one should carry out in principle as many thrusts as possible and thus as short as possible. However, /17
with the relatively high thrust level of a chemical engine ($\approx$ 10 N) one obtains for a 1000 kg satellite mass the optimum limiting value

practically already with a single thrust. On the other hand the lower thrust level of electrical engines can lead to the fact that for $K < K_{crit}$

$$(14) \qquad K b_N < V n \sqrt{\Delta p^2 + \Delta q^2} = K_{krit} \cdot b_N$$

no solution exists. This constraint plays a deciding role in the design of the station keeping strategy for electrical propulsion systems.

East-west corrections:

The correction of the longitudinal deviations from the geostationary required point is determined by equations (4), (5), and (8), whereby the radial acceleration components are set as $b_{Rk} = 0$, since all orbit changes can be carried out more effectively with the tangential components b_{Tk}. At least $K = 2$ thrusts are necessary to satisfy all four conditions. The general solution of the transcendental system is not possible in closed form, but one can reach the goal iteratively if one uses for the first step the approximation

$$(15) \qquad \sin\frac{1}{2}n\tau_k \approx \frac{1}{2}n\tau_k \,.$$

This simplification can be justified by the fact that the east-west corrections necessary for station keeping require only a fraction of the velocity requirement for the north-south corrections. Thus the burning times also become correspondingly shorter and mostly lie below 4 hours, whereby the deviation between the sine and the argument is about 5%.

If one first considers the case $K = 2$, then the equations (4), (5), and (8) can be transformed after the introduction of

$$(16) \qquad v_k = \frac{b_{Tk}\tau_k}{V} \quad , \qquad \Delta L = L_2 - L_1$$

as follows: /18

$$
(17) \quad
\begin{aligned}
v_1+v_2 &= -\frac{1}{3}\Delta D \\
v_1^2+2v_1v_2\cos\Delta L+v_2^2 &= \frac{1}{4}(\Delta h^2+\Delta l^2) \\
\frac{v_1\sin L_1+v_2\sin(L_1+\Delta L)}{v\ \cos L\ +v_2\cos(L_1+\Delta L)} &= \frac{\Delta h}{\Delta l} \\
(L_E-L_1)v_1+(L_E-L_1-\Delta L)v_2 &= -\frac{1}{3}\Delta\lambda\ .
\end{aligned}
$$

The solution of the first two equations (17) furnishes the values for v_1 and v_2 as a function of the still undetermined parameters ΔL, whereby L_1 is obtained from the 3. equation. If one has found for a specific value ΔL^o the associated values of $v_1{}^o$, $v_2{}^o$, $L_1{}^o$, then one also has

$$
(18) \quad v_1 = v_1^o, \quad v_2 = v_2^o, \quad L_1 = L_1^o+2m_1\pi, \quad \Delta L = \Delta L^o+2m_2\pi
$$

solutions which, however, generally do not satisfy the 4. equation. However, this can be obtained by a variation of ΔL^o by iteration. Thus one obtains a 2-parameter group of solutions which, however, do not necessarily exist for all pairs of numbers m_1, m_2.

The tangential velocity increment is

$$
(19) \quad \Delta V_T = \begin{cases} -\frac{V}{3}|\Delta D| & \text{for } v_1v_2 > 0 \\ V\sqrt{\frac{1}{4}(\Delta h^2+\Delta l^2)-\frac{1}{18}\Delta D^2(1+\cos\Delta L)}/\sin\frac{1}{2}\Delta L & \text{for } v_1v_2 < 0 \end{cases}
$$

and becomes a minimum for an opposite sign of v_k for $\Delta L^o = \pi$. Therefore one should be able to select from the multitude of solutions with oppositely directed thrusts those with the smallest value of $\pi - \Delta L^{o}$. On the other hand, all solutions with thrusts in the same direction require the same amount of fuel since the selection must be made in accordance with other criteria.

For moderate accuracy requirements it is not absolutely necessary to satisfy all 4 equations (17). Thus it is customary, for instance, for conventional east-west station keeping to satisfy only the first 3 conditions with $\Delta L^{o} = \pi$, that is, optimally from a fuel requirement, while the last requirement is satisfied only approximately by a suitable choice of m_1 and m_2. Vice-versa one can also proceed in a manner where one solves the last 3 equations (17) with $\Delta L^{o} = \pi$ and select from the resulting amount of solutions that solution which least violates the drift requirements. /19

Such incomplete corrections can be designated as differential Hohmann transitions. They have the advantage that they are always possible and in addition are optimal from a fuel consumption standpoint. The approximation becomes the better, the larger one is able to select the values of m_1 and m_2 which, however, is limited essentially by L_E or by the time T_E available for orbit correction.

Complete and at the same time fuel-optimal east-west corrections can be achieved by the addition of a third thrust.

With $b_{RK} = 0$ and

$$(20) \qquad L_k = L^{o} + m_k \pi \qquad k = 1,2,3$$

one obtains from equation (5) by multiplication by $\cos L^{o}$ and sine L^{o} at first

$$(21) \qquad \Delta h \cos L^{o} - \Delta l \sin L^{o} = 0$$

and

$$(22) \qquad \tan L^{o} = \frac{\Delta h}{\Delta l} \quad .$$

Furthermore one obtains as an expansion of equation (17) the linear system of equations

$$(23)\qquad \begin{pmatrix} 1 & 1 & 1 \\ (-1)^{m_1} & (-1)^{m_2} & (-1)^{m_3} \\ L_E-L_1 & L_E-L_2 & L_E-L_3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3}\,\Delta D \\ \frac{1}{2}\sqrt{\Delta h^2+\Delta l^2} \\ -\frac{1}{3}\,\Delta\lambda \end{pmatrix}$$

which can be solved for v_k for all combinations of m_k, for which the determinant does not vanish. Thus one obtains a 3-parameter group /20 of solutions. The criterion of the smallest velocity increment here is not sufficient for an equivocal determination of m_k since many fuel-equal solutions result. As an additional criterion one can then use, for instance, the maximum longitudinal deviation $|\lambda_{max}|$ which appears up to time T_E.

Naturally the presented method can be expanded for a greater number $K > 3$ of thrusts, whereby the number of the possible combinations of the m_k values is also increased. However, the thus resulting amount of solutions is limited by the time T_E, since per revolution of the satellite at most 2 east-west thrusts come into consideration.

All solutions thus obtained are only approximate because of the assumption (15). The accuracy can be increased by iteration if one increases on the right-hand side of equation (17) or (23) the expression $\frac{1}{2}\sqrt{\Delta h^2+\Delta l^2}$ by the term

$$(24)\qquad \sum_{k=1}^{K} (-1)^{m_k}\, v_k \left[1 - \frac{\sin\frac{1}{2}nV\frac{v_k}{b_{Tk}}}{\frac{1}{2}nV\frac{v_k}{b_{Tk}}}\right]$$

which for the 0^{th} iteration is set equal to zero and subsequently calculated with the values v_k which were obtained from the preceding iteration.

Since the burning times τ_k are shortened with increasing value of K, the 0^{th} iteration must be the more accurate the larger one selects the number of thrusts K. Even the fuel consumption simultaneously becoming lower would suggest that one choose K as large as possible. However, since the fuel required for the east-west correction is already relatively small, there thus results only a minor savings which, because of the increased expenditures during calculation and the carrying-out of numerous thrusts would pay only in special cases.

4. Optimum station keeping during operational limitations /21

The station keeping strategies and methods for calculating the thrust times, mentioned in the previous sections, are only useful if the calculated thrusts can be carried out without limitations. However, if one must figure that for mission-engineering reasons the propulsion system cannot be deployed or only to a reduced extent at certain times, the thrust strategy must be determined in another way.

In the following we assume that such limitations are known in advance at least for a correction cycle of duration $T_E - T_o$ and that they are such that within prescribed time intervals no thrusts or only reduced thrusts are possible. Then the problem to achieve in spite of such limitations a station keeping as accurate as possible with minimum fuel consumption becomes an optimization problem which can be formulated as follows:

Given:

T_o, T_E	starting- and final time of a correction cycle
E_i^o	initial values of the geostationary elements
E_1^z	target values of geostationary elements at the end of the cycle
δE_i	element changes during the time interval (T_o, T_E) as the result of long duration disturbances
K	number of the intended thrusts
T_{2k-1}, T_{2k}	lower and upper limits of the time intervals during which one thrust each can occur. $(k = 1, 2, \ldots, K)$
b_{Rk}, b_{Tk}, b_{Nk}	radial, tangential, and normal acceleration components of k^{th} thrusts $(k = 1, 2, \ldots, K)$
b_k	sum of the acceleration values of the engines taking part in the k^{th} thrust. $(k = 1, \ldots, K)$
t_{2k-1}^o, t_{2k}^o	estimated values for the starting- and shut-off times $(k = 1, 2, \ldots, K)$

We seek the following: /22

t_{2k-1}, t_{2k}	starting- and shut-off times of the thrust so that the secondary conditions are satisfied and that the cost function is minimized $(k = 1, 2, \ldots K)$.

The secondary conditions are:

$$T_{2k-1} \leqq t_{2k-1} \leqq t_{2k} \leqq T_{2k} \quad k=1,2,\ldots,K \quad . \tag{25}$$

The cost function is defined by the following expression:

$$(26) \qquad \Phi = \sum_{i=1}^{6} G_i (E_i - E_i^z)^2 + G_7 \Delta V \quad ,$$

where G_i are suitably chosen masses, E_i the geostationary elements at the end of the correction cycle and

$$(27) \qquad \Delta V = \sum_{k=1}^{K} b_k (t_{2k} - t_{2k-1})$$

the total velocity increment proportional to the fuel consumption.

The E_i values are defined by

$$(28) \qquad E_i = E_i^o + \delta E_i + \Delta E_i \qquad\qquad (i=1,\ldots,5)$$

or

$$E_6 = E_6^o + E_1^o n (T_E - T_o) + \delta E_6 + \Delta E_6$$

and by equations (4) - (9) as functions of the t_{2k-1}, t_{2k}. The long-term disturbances δE_i may be approximated within a correction cycle by linear or δE_6 by quadratic functions.

Naturally there are many other possibilities to define the cost function and the secondary conditions. Thus it would suggest itself, for instance, to introduce the exact attainment of the target orbit with $E_i - E_i^z = 0$ as secondary conditions and merely to minimize ΔV. However, such a method would fail in those cases where the target orbit cannot be achieved at all because of operational restrictions.

The formulation chosen here has the following advantages: /23

o There always exists a solution.

- All secondary conditions are linear in the variables t_{2k-1}, t_{2k}.
- The cost function can be differentiated everywhere.
- The minimization of the cost function leads to a compromise solution between station keeping accuracy and fuel requirement, which can be displaced by a suitable choice of the weighting into the one or the other direction.
- By setting the corresponding weight functions equal to zero one can limit the method to pure north-south or east-west station keeping.

The above-formulated optimization problem can be solved with the aid of the one of the numerous optimization methods found in the literature, for which completed computer programs are also available.

The estimated values t_{2k-1}^{o}, t_{2k}^{o} are obtained from the analytical solution of the unrestricted problem according to the method of section 3.

5. Application example /24

In this section we shall determine the optimum station keeping strategy for a randomly selected example. Here we make use of a combination of the analytical procedures according to section 3 and the optimization methods according to section 4. First the unrestricted problem is solved. From the many solutions we select one which is affected the least by the operational constraints. It provides the estimated values for the following optimization method "Generalized Reduced Gradient Method" [9], which finally provides the desired switching times for the restricted problem.

The geostationary elements of the initial orbit without short-term effects should be on 1/1/1983, Oh WZ:

$$D = -5.664826 \cdot 10^{-5}$$

$$h = l = p = q = \lambda = 0.$$

The value of D is chosen such that with the secular perturbance of λ it attains the value of zero.

The target orbit should be equal to the starting orbit and should be reached after 10 days so that the following disturbances occurring during the 10 days must be compensated by orbit corrections.

$$\Delta D = -11.33 \cdot 10^{-6} \qquad \Delta p = 268.44 \cdot 10^{-6}$$

$$\Delta h = 18.21 \cdot 10^{-6} \qquad \Delta q = -69.37 \cdot 10^{-6}$$

$$\Delta l = 59.30 \cdot 10^{-6} \qquad \Delta \lambda = 272.79 \cdot 10^{-6}$$

These values were calculated according to [5] for a satellite in a 19° west required position with 1058 kg mass and 35 m^2 effective cross-section.

Let the propulsion system, which is planned for the operational TV-satellite missions, consist of 4 electrical engines with 0.01 N thrust each which are attached to the satellites in the sky directions NE, NW, SW, SE. Thus one pair of engines must always be started simultaneously in order to achieve thrusts in north-south or east-west direction (figure 5). Since thrusts can be produced toward the north as well as toward the south, two north-south corrections are possibly daily. /25

The results shown in figures 6 to 14 were obtained with the aid of a computer program which consists of 2 parts. In the 1. part we programmed essentially the equations (9) to (24) from which the solutions for the unrestricted problem are obtained. Then we searched according to the criteria of a minimum fuel consumption, lowest longitudinal deviation, and lowest losses resulting from the prohibited times automatically for the most favorable solution and modified the constraints accordingly. The satellite data, the acceleration vectors produced by the propulsion system, and the type of east-west corrections (number of thrusts, complete or incomplete corrections) as well as the operational constraints are fixed by the input while the target orbit is defined by a subprogram which can be exchanged at any time.

In the second part we call out the subprogram GRGA after preparation of the input data, whose results are used to simulate the time-related course of the geostationary elements. The optimization can be carried out in succession for several weightings. The number of the weightings and the weighting factors are input.

The solution of the unrestricted problem in accordance with the method described in section 3 is shown in figure 6. The 20 thrusts for the north-south corrections last daily 2-times 1.8 hours and take place shortly after midnight or during the early afternoon (world-time).

Of the 3 planned east-west thrusts one with only ≈0.3 hours duration occurs on the evening of the fifth day and two with about equal duration (≈0.9 or 0.8 hours) on the morning and evening of the seventh day. The selection from the large number of possible /26 solutions was made in accordance with the criteria of minimum fuel consumption requirements and maximum longitudinal deviation as small as possible during the course of the 10 days.

If now this solution is limited by the fact that the engines cannot be turned on during certain times, then the optimization method defined in the 4. section is being used.

At first we again select from the large number of solutions for the east-west corrections of the unrestricted problems the most favorable one, whereby an additional criterion states that the selected solution must be affected as little as possible by the constraints. Then the thrust times colliding with the prohibited times are shortened appropriately or eliminated entirely. The remaining thrust times serve as estimated values for the optimization method GRGA for the solution of the restricted problem.

The weighting factors G_i in equation (26) were chosen such that the cost function has a minimum for all elements for deviations of about 10^{-6}. The prohibited times assumed for this example and the resulting thrust times are plotted in figure 7. Because of the omission of several north-south thrusts the remaining thrust times are correspondingly longer than shown in figure 6. Even the east-west thrusts are positioned differently because another solution of the unrestricted problem, not affected by the prohibition times, was used to form the estimated values.

In figures 8 to 13 the time course of the geostationary elements during the 10-day station keeping cycle is simulated for 4 different cases. Perturbances with periods below 1 month are not taken into account.

As expected the station keeping is most accurate for the unrestricted case while the optimization leads to satisfactory results also for the restricted case. However, if one pursues in spite of the constraints the strategy of the unrestricted problem, one obtains as the result of the omissions considerable deviations from the required orbit, which under certain circumstances can be greater than without any station keeping. This is to be feared, /27
for example, whenever an important east-west thrust is omitted (figure 13).

The total velocity increment for the 10 days of station keeping is increased as the result of the operational constraints only insignificantly by 3.5% from 2.57 m/s to 2.66 m/s.

For figure 14 we simulated the motion of the satellites with all perturbances and corrections in order to demonstrate the effect of position keeping with electrical propulsion engines. Because of the close sequence of the orbit correction only a very small part of the available tolerance window is used. However, it must be pointed out that neither orbit model errors nor execution errors nor orbit determination errors were taken into account in the simulation.

6. References /28

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[7] R. Metzger. Stationkeeping for Symphonie. IAF XXVth Congress, Amsterdam, Sept. 30 - October 5, 1974, Paper 74-019, MBB Report UR-245-74.

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[9] J. Abadie. Generalized Reduced Gradient Method: Le Code GRGA. /29
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7. Figures

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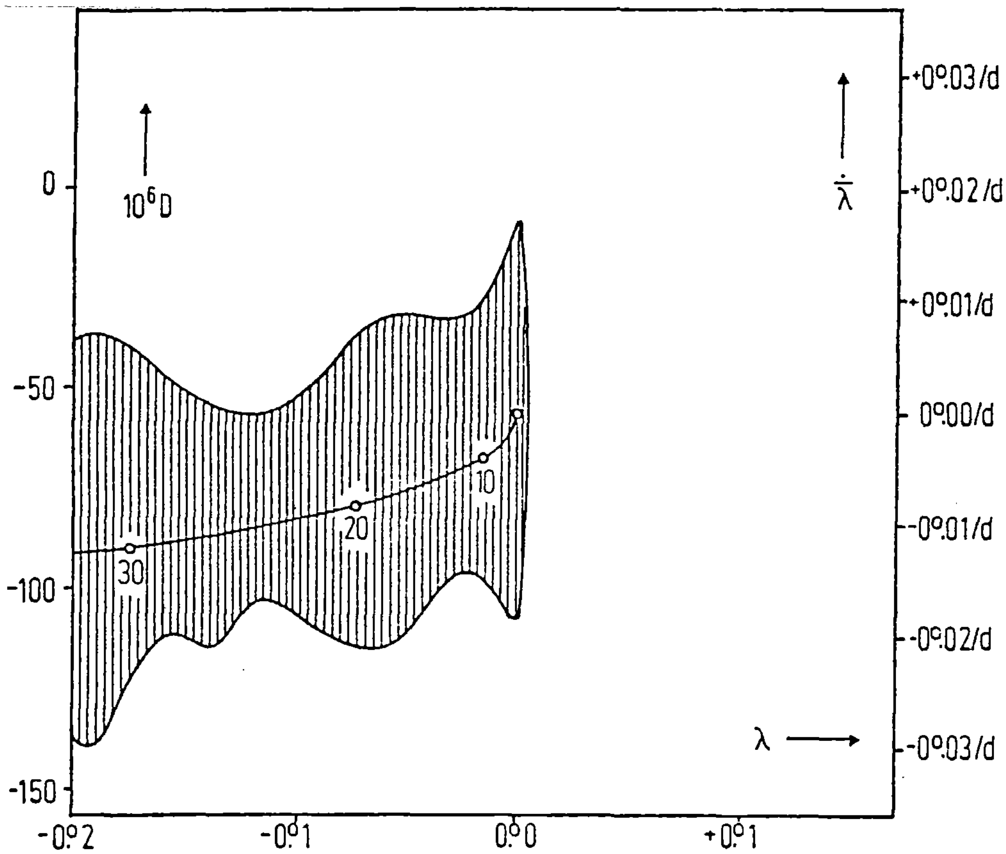

Figure 1: Perturbances of the drift rate D and the longitudinal deviation λ of a geostationary satellite in 19° west geographical longitude, starting on 1/1/1983. The mass was assumed to be 1058 kg, the effective cross-section 35 m^2. The parabolic curve shows the long duration course with time markings in days, the cross-hatched area denotes the variation width of the perturbances with periods under one month. The left ordinate scale gives the values of the drift rate D defined in equation (1) while on the right the to-be-observed daily longitudinal change $\dot{\lambda} = Dn + \dot{\lambda}_0$ is plotted. Inspite of the favorable longitudinal position removed a few degrees from an extreme of the earth potential the longitudinal deviation reaches already after about 3 weeks the value of 0.1°, which is often demanded as tolerance limit for future geostationary satellites.

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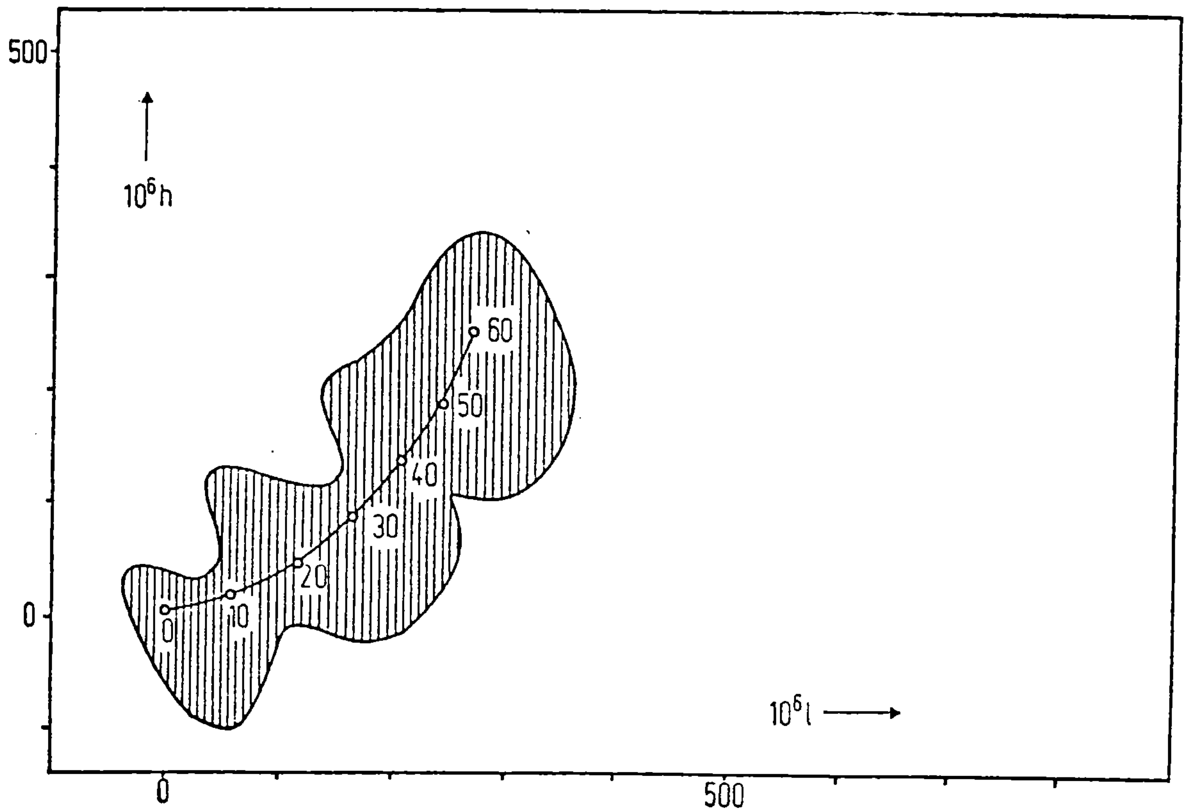
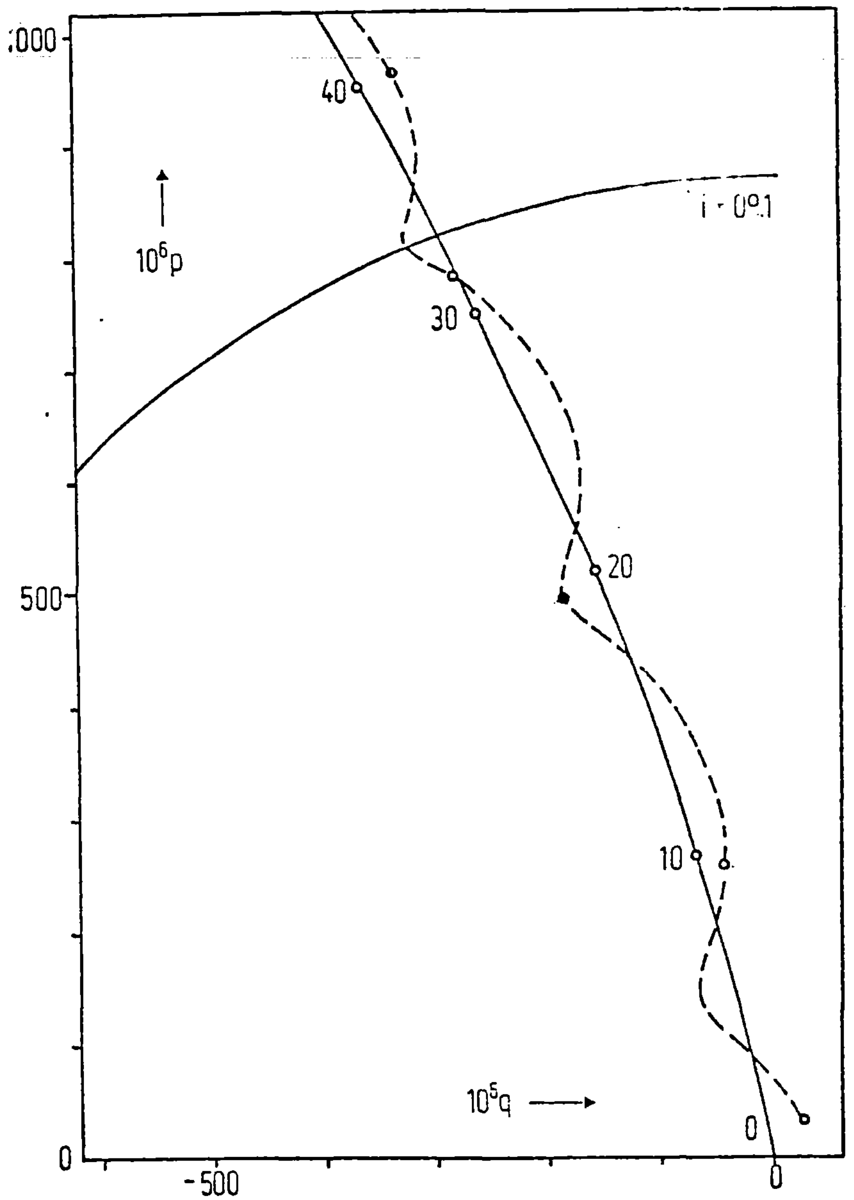


<u>Figure 2</u>: Perturbances of the eccentricity vector (l,h) of a geostationary satellite weighing 1058 kg with 35 m^2 effective cross-section in 19^o western longitude, starting on 1/1/1983. The curve with the time markings denotes the long-term orbit changes while the cross-hatched area represents the variation width of the perturbances with periods under 1 month. After 50 days the eccentricity attains the value of about $3 \cdot 10^{-4}$ which corresponds to a swing with 0.034^o longitudinal amplitude.



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<u>Figure 3</u>: Perturbances of the elements q and p of a geostationary satellite weighing 1058 kg with 35 m^2 effective cross-section at 19° western longitude, starting on 1/1/1983. The solid curve shows the long-term changes while the dashed curve also contains perturbances with periods under 1 month. The 14 day period of a perturbance member originating from the moon stands out clearly while all other short-period terms are unnoticeably small. Here the tolerance limit of 0.1° inclination is exceeded after about 33 days.

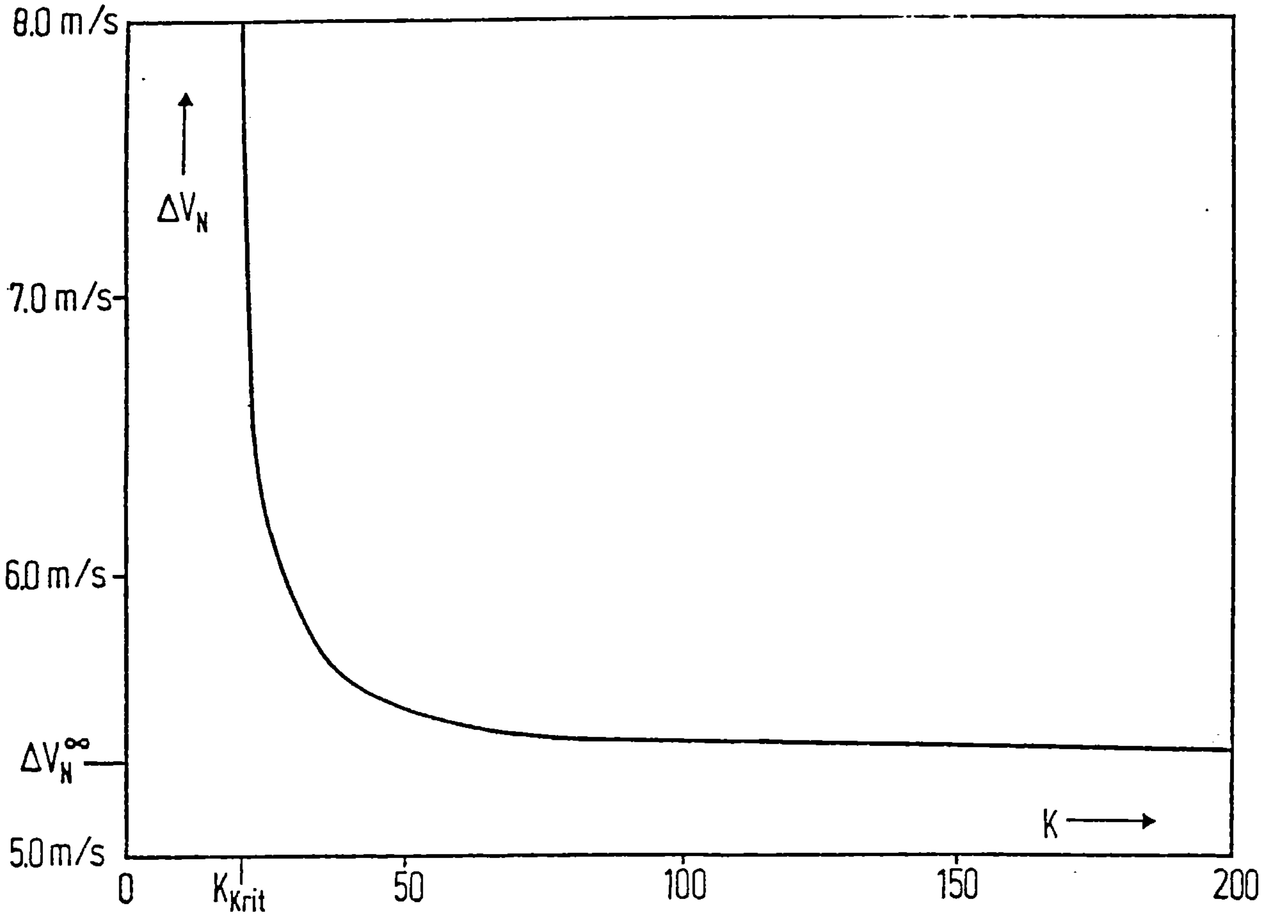

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Figure 4: The velocity increment ΔV_N necessary for an inclination correction of 0.1° as function of the number of thrusts K for a north-south acceleration $b_N = 10^{-8}$ km/s. The limiting value ΔV_N^∞ lies near 5.366 m/s. The required correction cannot be achieved with fewer than 20 thrusts.

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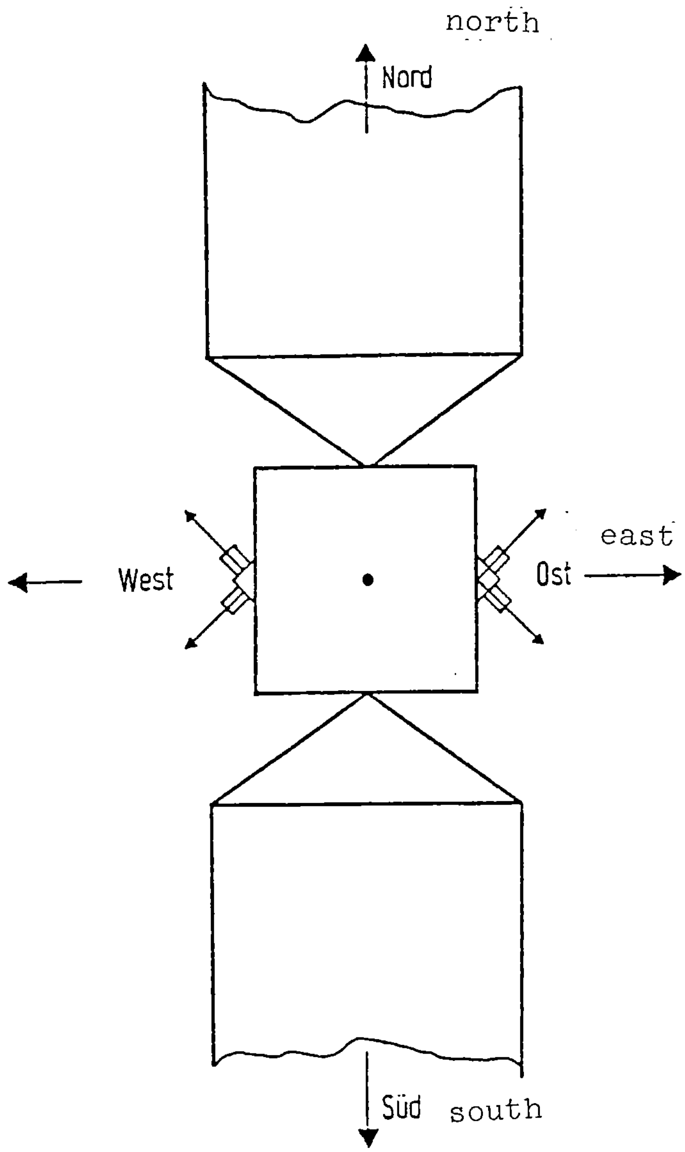

Figure 5: The electrical propulsion system, consisting of 4 engines, planned for the operational TV-satellite missions. The axes of the engines must be inclined toward the north-south direction in order to avoid contamination of the solar paddles. Thrusts in all 4 compass directions can be achieved with the arrangement shown by paired engine starts.

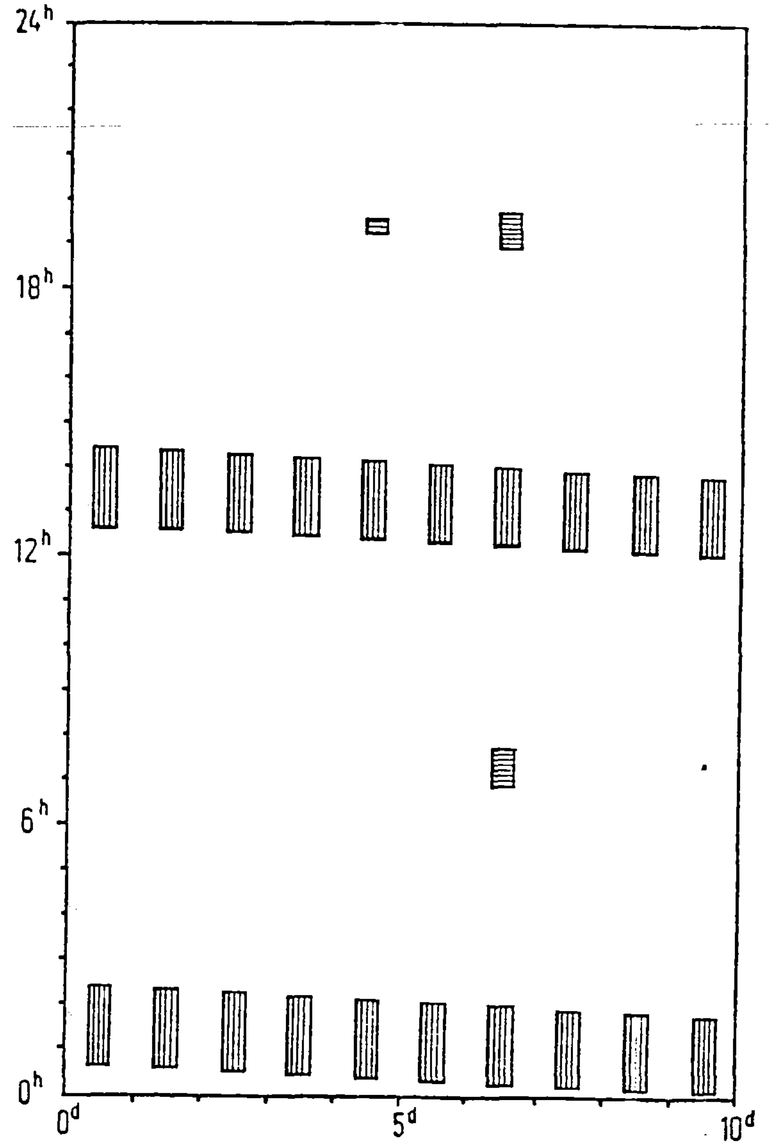

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Figure 6: Thrust times for station keeping of a geostationary satellite weighing 1058 kg with 35 m^2 effective cross-section at 19° western longitude from January 1-10, 1983. It was assumed that the propulsion system shown in fig. 5 can be operated without restrictions.

The vertically cross-hatched fields represent the north-south thrusts whose daily times are pushed back somewhat from day to day and which during one year would move away by a whole day. The combination of 3 east-west thrusts was selected in accordance with the criteria of the minimum velocity increment and the minimum longitudinal deviation during the cycle.

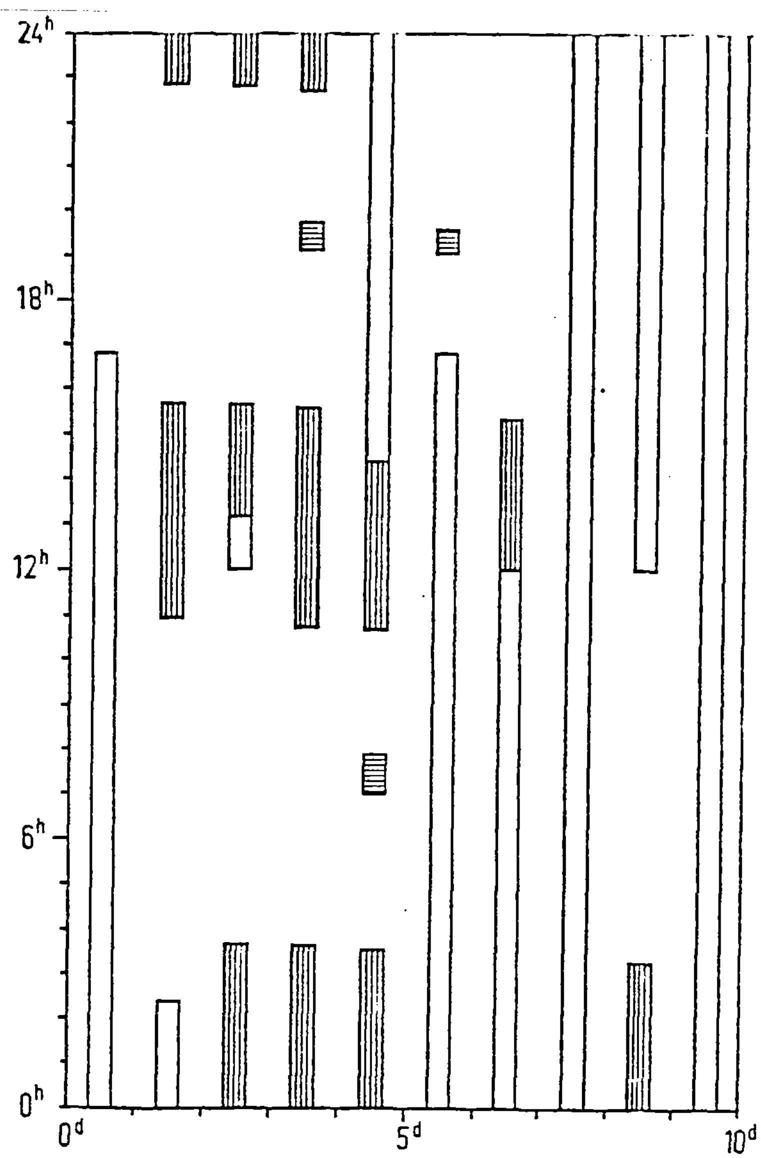


Figure 7: Thrust times for station keeping of a geostationary satellite as in figure 6, but by taking into account prohibited times which restrict their operation. They were fixed arbitrarily and are plotted as open fields. Since the combination of east-west thrusts shown in figure 6 must be voided because of the prohibited times, we selected a solution which, although somewhat less favorable with respect to longitudinal deviation, was not affected by the constraints. Compared to the unconstrained case the fuel consumption is increased by about 3.5%.

Figures 8 - 13: Simulation of the time course of the 6 geostationary elements during the 10 day station-keeping cycle without taking into account perturbance effects with periods under one month. /37

curves 1: shape without station keeping

curves 2: shape for unconstrained station keeping

curves 3: shape for operational constraints not included in calculations

curves 4: shape for included operational constraints and optimized station-keeping strategy.

Figure 8:

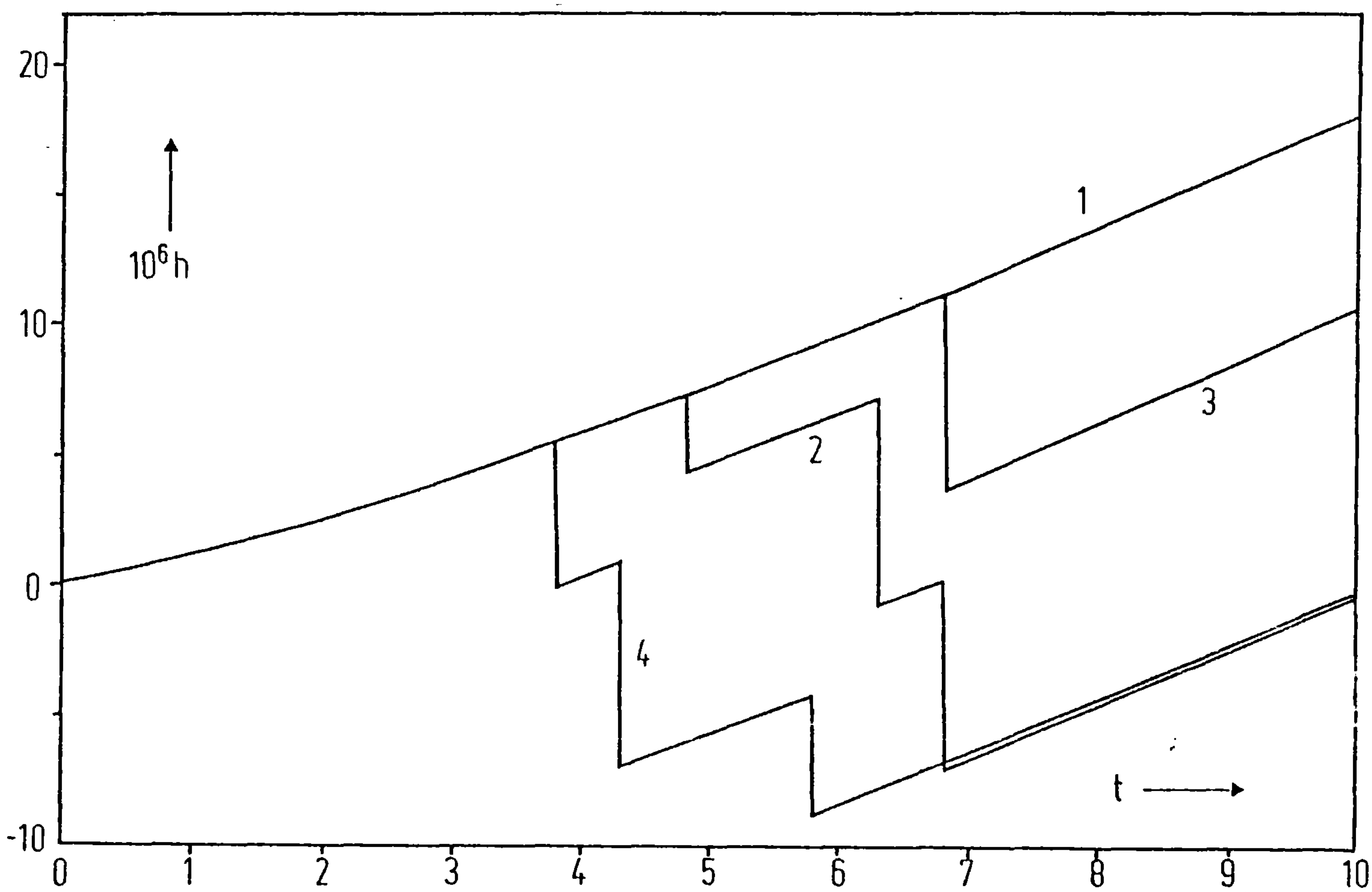


Figure 9:

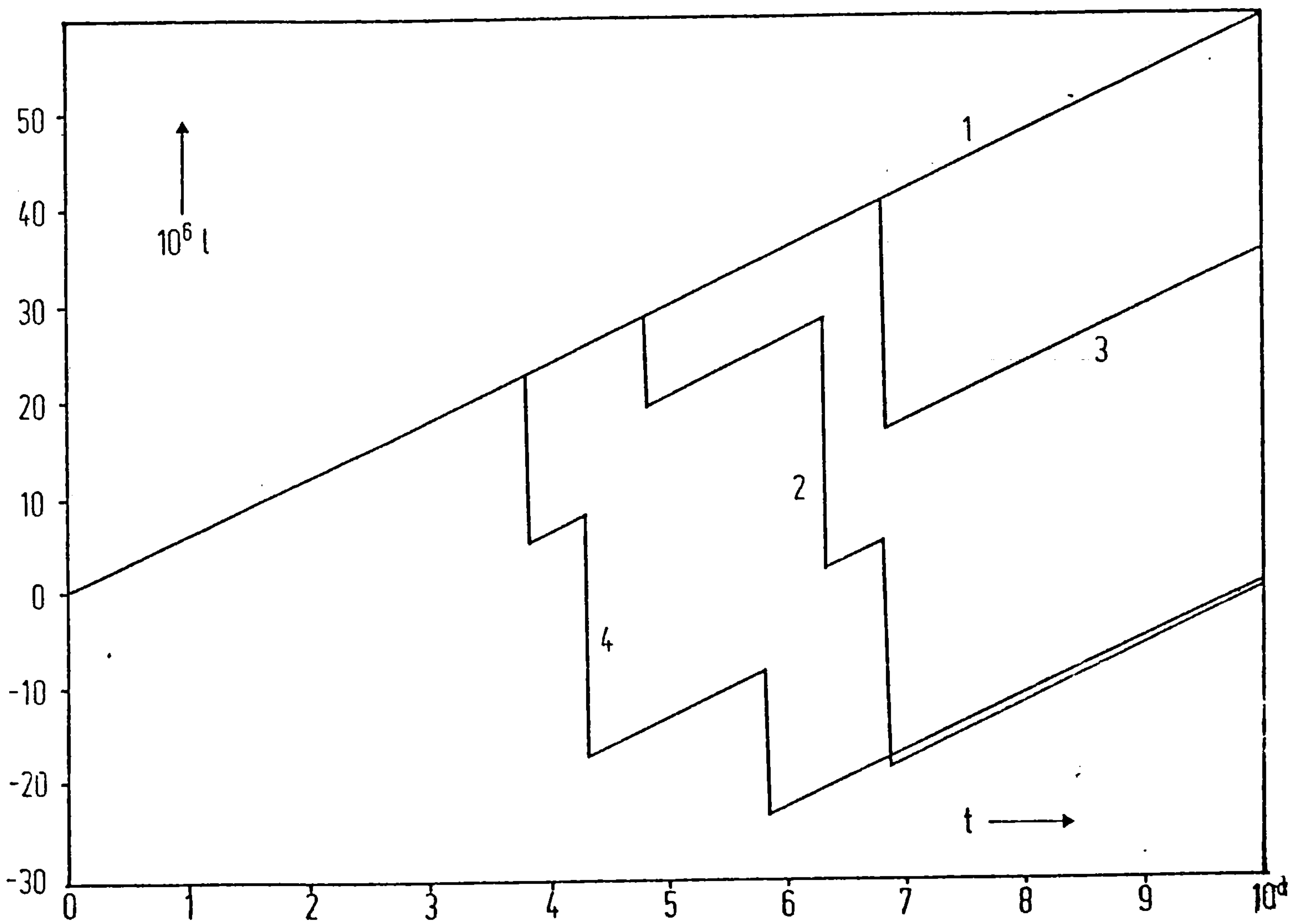


Figure 10:

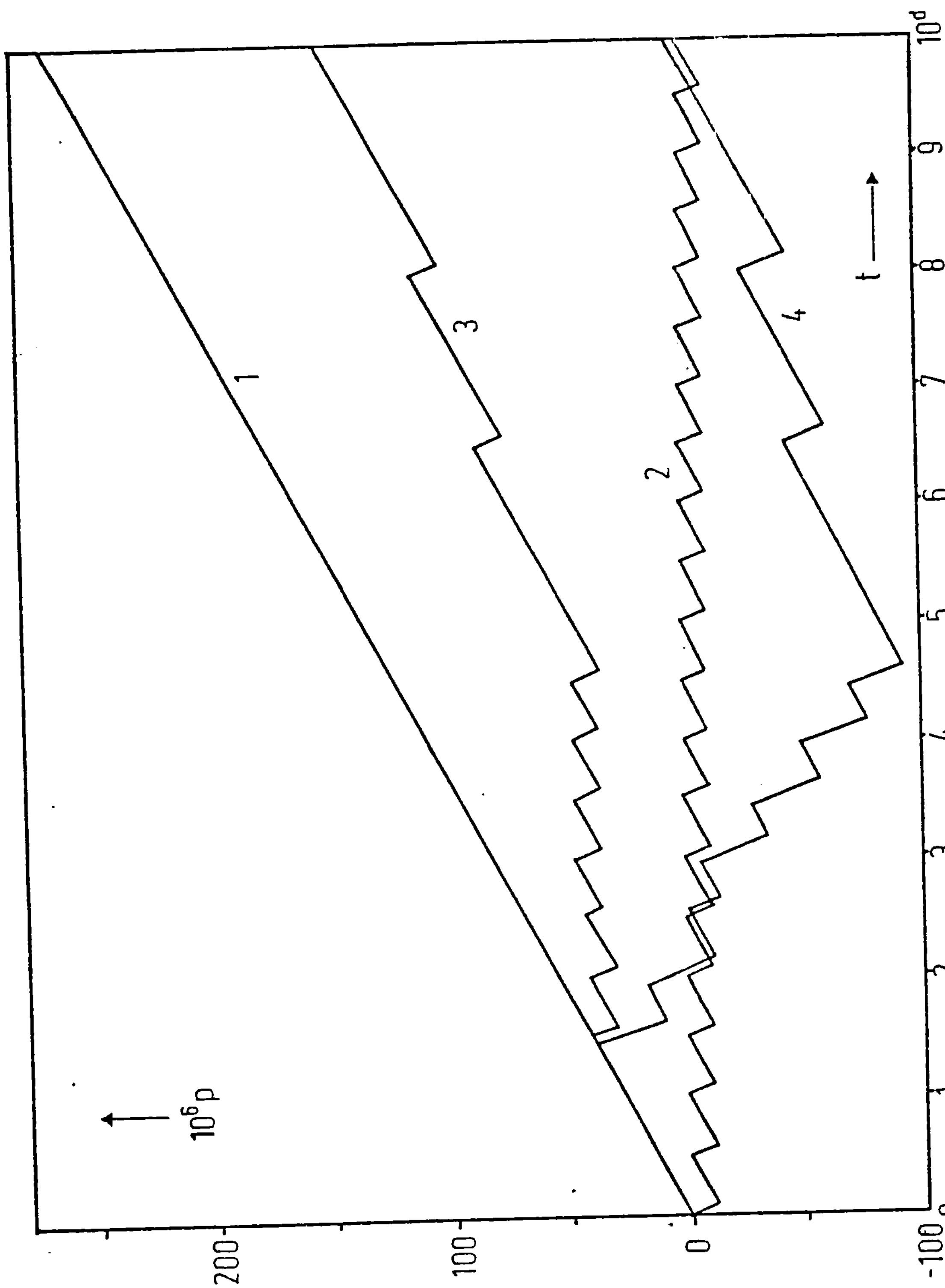


Figure 11:

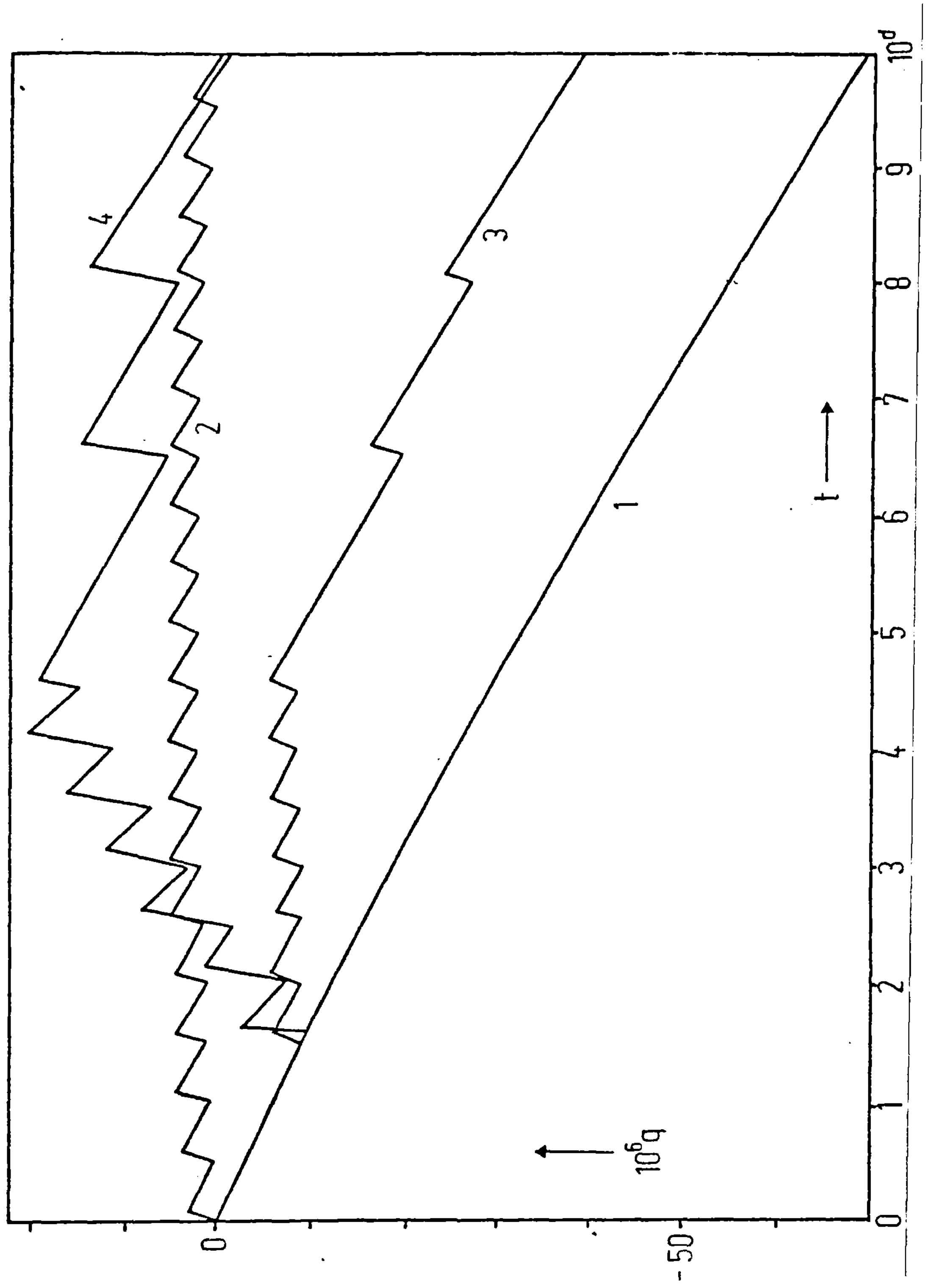


Figure 12:

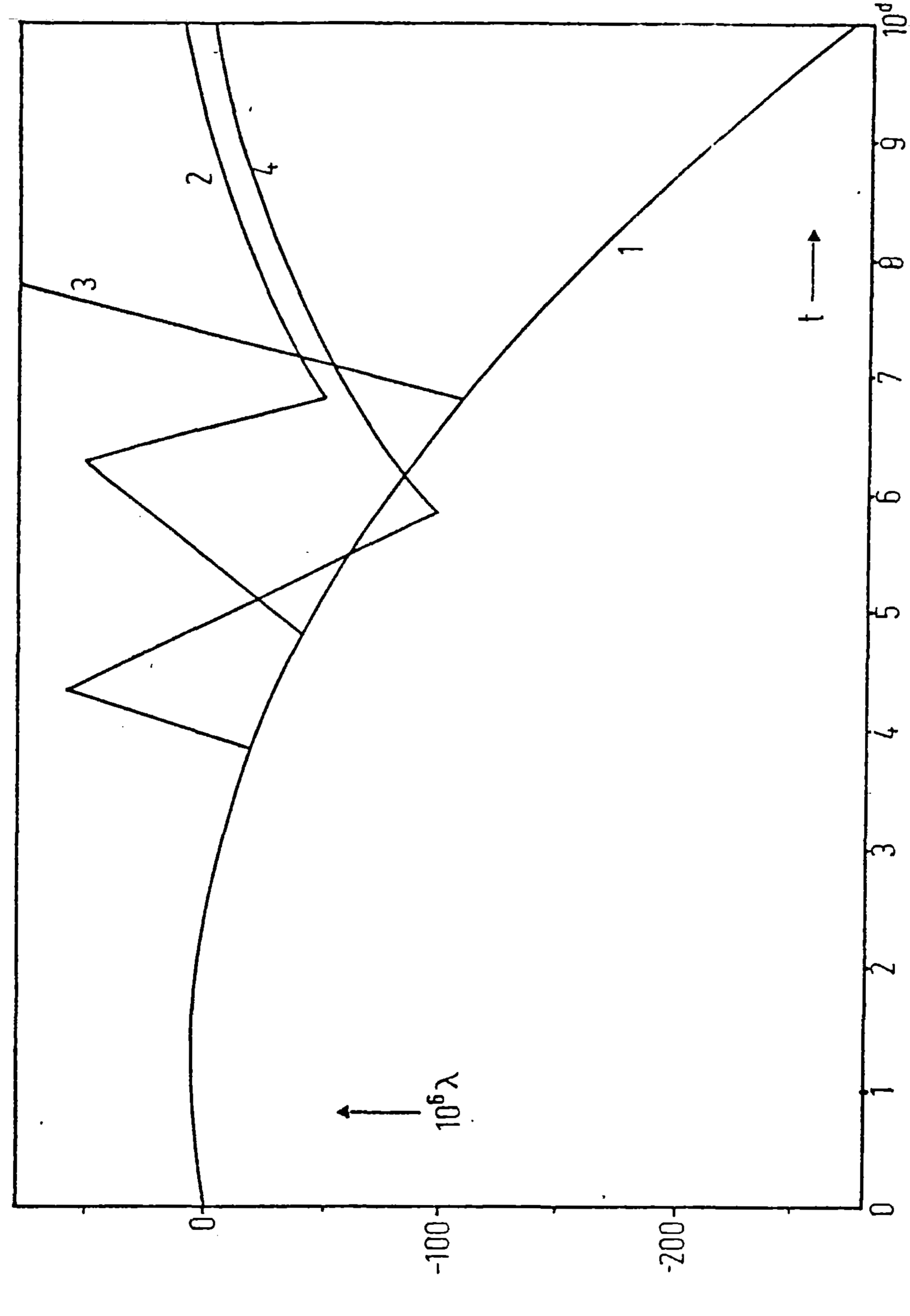
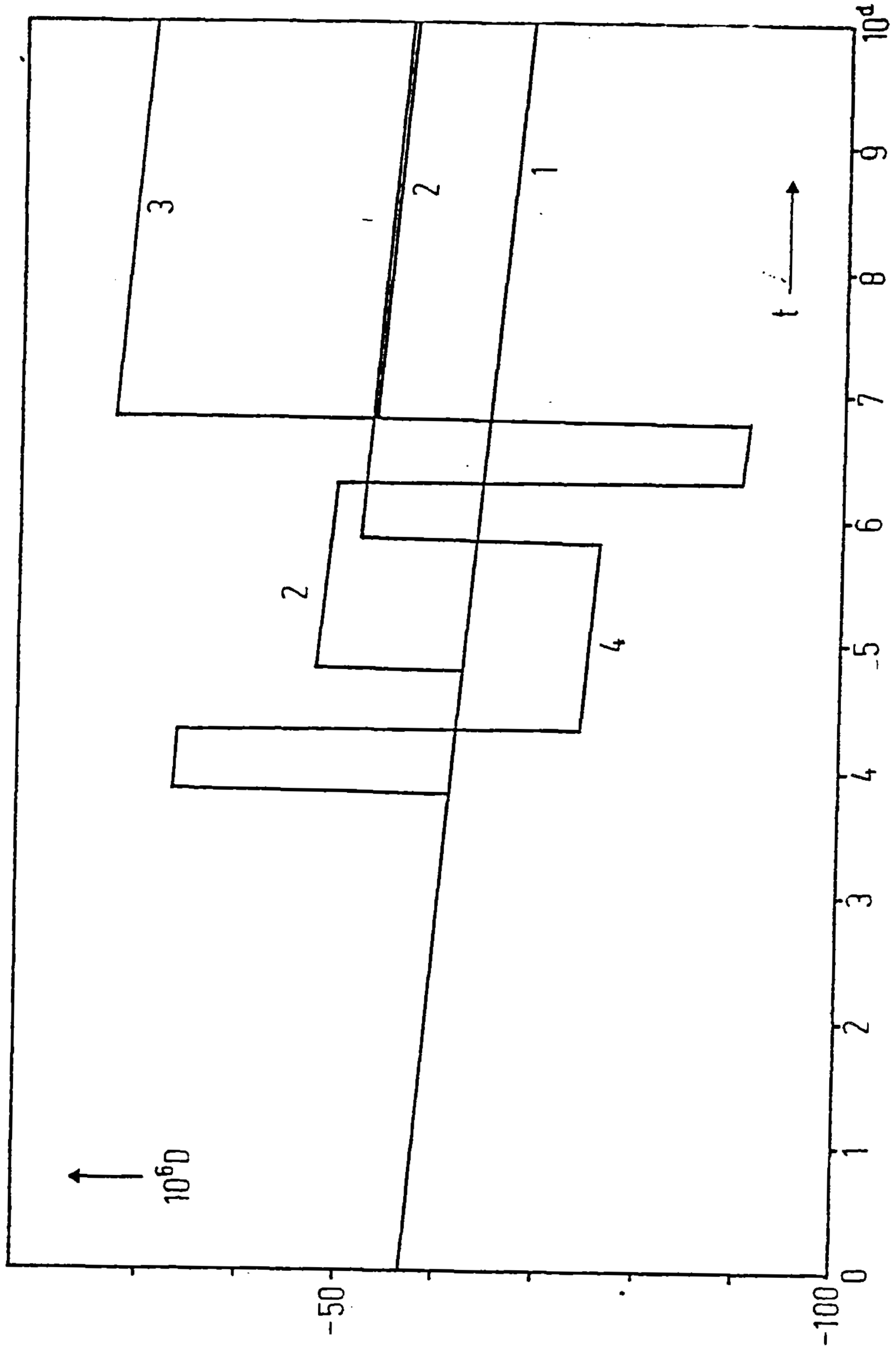
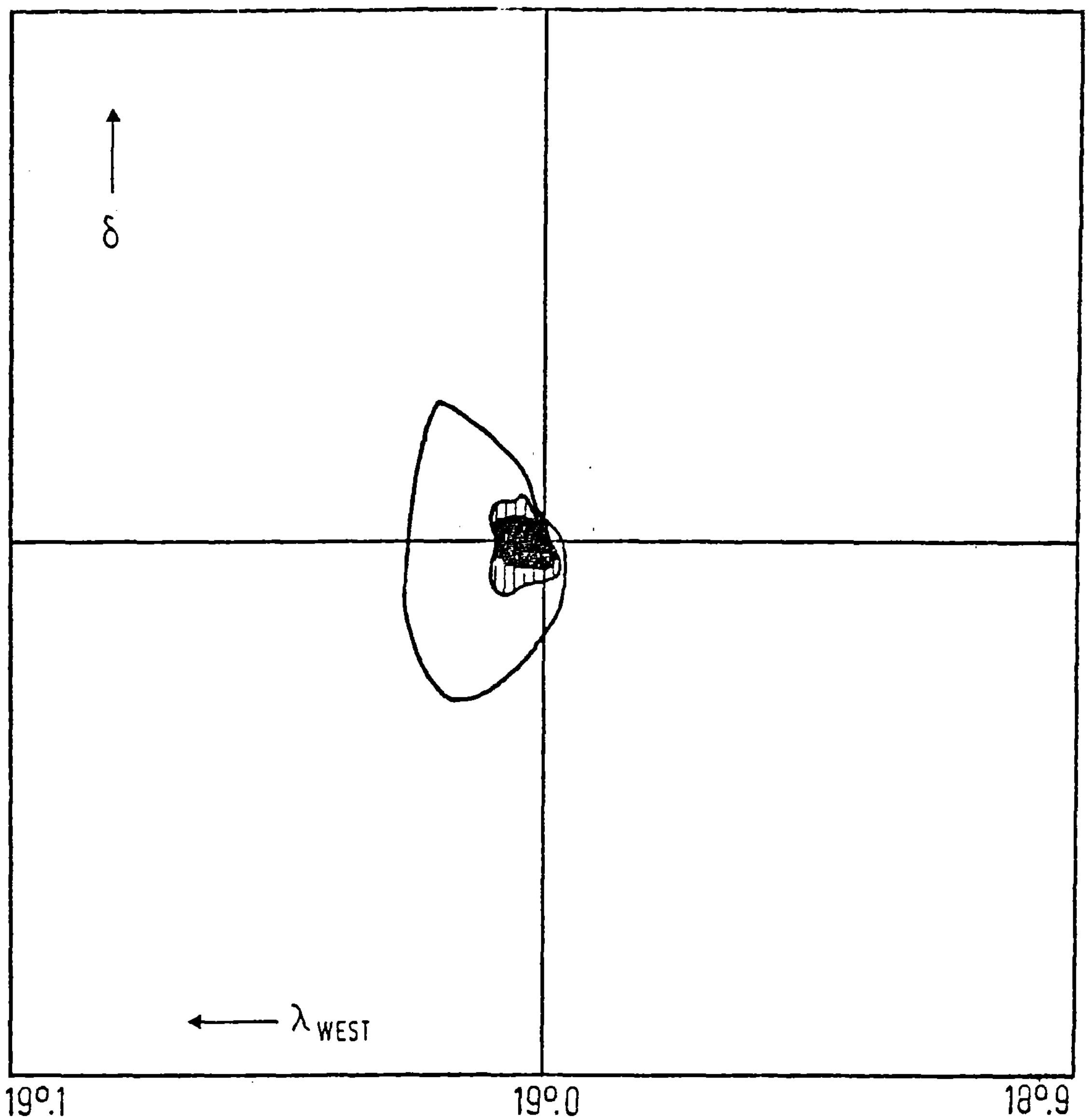


Figure 13:





<u>Figure 14</u>: The area occupied in a tolerance window of $\pm$ 0.1° at 19° west as the result of longitude and latitude satellite movements during the 10 day cycle, taking all perturbances into account.

dark region:	for unrestricted station keeping
cross-hatched area:	for optimized station keeping with constraints
empty area:	without station keeping.