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THE FEINHEIT METHOD:

A Phase-Independent Formulation of the Period-Luminosity Relation for Cepheids

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ABSTRACT

The tight correlation between color and luminosity during the cyclical variation of a Cepheid is calibrated and shown to be a direct means by which most of the phase-dependent temperature-induced amplitude of the variable can be transformed away. The resulting Feinheit function, $F = V - \alpha(B-V)$ gives rise to a random-phase period- luminosity relation with an *rms* scatter of less than 0.20 mag. A comparison shows that the blue Feinheit method for distance determinations method from single-phase observations is as accurate as near-infrared photometry but has the added advantage of being able to use panoramic detectors for the data acquisition. The Feinheit function is identified as the surface area variation of the Cepheid during its cycle.

Subject Headings: stars: Cepheids - stars: pulsation

I. Introduction

The observed correlation between the luminosity and period for Cepheids has been used for decades as a tool for probing the extragalactic distance scale. Some of the reasons for this, and the progress to date, are reviewed elsewhere (Madore 1985). However, it should be independently emphasized that unlike the majority of alternative extragalactic distance indicators, there exists a strong theoretical understanding of Cepheids, and so it seems likely, that without a major competitor on hand, Cepheids will continue to be used extensively to determine the distances to many more nearby galaxies. Accordingly, we should ask whether we are now using the Cepheids to best advantage. Given what we already know about the properties of Cepheids, can we plan our future observations more efficiently?

In what follows we take a new look at the documented properties of Cepheids. This particular reappraisal is motivated by the different perspective on the problem of the calibration of the cepheid period-luminosity relation provided by recent developments in the near-infrared photometry of Cepheids (McGonegal *et al.* 1982, 1983; McAlary *et al.* 1983; McAlary, and Madore 1984; McAlary, Madore, and Davis 1984; Madore *et al.* 1985; Visvanathan 1985; Welch *et al.* 1984). Set in the context of a wealth of existing photographic and photoelectric data from *optical* surveys of galactic and extragalactic Cepheids alike it can now be demonstrated that the advantages inherent in the infrared calibration of the cepheid period-luminosity relation were also already contained in the appropriate combination of optical observations.

Two physically distinct variations in surface properties lead to the luminosity and color changes observed in a single Cepheids during its cycle. (1) The star expands and contracts, and therefore the area of the star changes periodically. These radius-induced variations are geometric in nature, and so (except for small optical-depth effects) their contribution to the total light variation is essentially wavelength independent. (2) The surface temperature also varies. In this case the monochromatic surface brightness changes sensitively with temperature, and in turn is quite dependent on wavelength. For a given Cepheid, the temperature-induced surface brightness variations dominate the visual luminosity variations seen during a cycle; at longer wavelengths the radius variations dominate.

When considering variations of properties from Cepheid to Cepheid, as opposed to variations in one Cepheid through its cycle, the roles of temperature and radius are largely reversed. Although the visual luminosity variations through the cycle of one Cepheid are due primarily to temperature, the main driving force behind the luminosity differences between various Cepheids is the rapid increase of mean radius with increasing period. Accordingly, the statistical coupling of the period and the radius (by the constraints of the instability strip) is the physically dominant term contributing to the origin of the slope of the period-luminosity relation at almost all wavelengths. In the visual, temperature differences from Cepheid to Cepheid actually work against this luminosity projection of the period-radius relation, broadening it and flattening it, due to the statistical decrease in mean surface brightness (temperature) with period.

With knowledge of the above trends, one could then have predicted that in going from the optical to the near infrared, the observed period-luminosity relation should undergo two systematic changes. First, the observed width measured at fixed period should decrease with increased observing wavelength. And second, the slope of the period-luminosity relation should steepen, approaching the luminosity equivalent of the period-radius relation at long wavelengths. By eliminating a Cepheid's surface-brightness variation, many practical advantages pertaining to the distance scale can be realized. By decreasing the dispersion in the period-luminosity relation while simultaneously increasing the luminosity dependence on period one obtains a much more accurate distance indicator.

Moving to the infrared (McGonegal, *et al.* 1982) is one direct means of obtaining a low sensitivity to surface brightness, resulting in a direct luminosity measure of the star's surface area. Now we show how these very same advantages are inherent in conventional data and can be readily derived from photometry obtained in the blue/visual portion of the spectrum.

II. The Color-Magnitude Diagram

The periodic trajectory of a Cepheid in the $(B, B\text{-}V)$ color-magnitude diagram is illustrated in Figure 1. The data for the example star, *X Cyg*, are taken from Moffett and Barnes (1984). The Cepheid executes a loop in a clockwise direction, reaching maximum luminosity approximately at its bluest color and passing through minimum light when it is reddest. While the temperature variation in this specific case results in a color amplitude of 0.65 mag in $(B\text{-}V)$ and a luminosity amplitude of 1.60 mag in B, the radius variation is intrinsically quite small. Moreover, the radius variation is out of phase by about 90 degrees with respect to the surface brightness variations, causing an opening of the loop in Figure 1, but not significantly affecting the luminosity or color amplitude.

To first order then, the visual light variation is strongly coupled to the color variation such that at any phase the instantaneous magnitude V_φ can be considered to be a linear function of the instantaneous color $(B\text{-}V)_\varphi$, such that $V_\varphi = \alpha\,(B\text{-}V)_\varphi + \gamma$. To this we can add a residual term R_φ which accounts for the small radius variations occurring out of phase with the color.

The above relation can be rewritten in a more suggestive and useful form,

$$F = V_\varphi - \alpha\,(B\text{-}V)_\varphi = \gamma,$$

where F, the Feinheit function, is independent of the phase of observation (ignoring for the moment the small radius variation R_φ). Graphically, Figure 1 illustrates the origin and magnitude of the small amplitude ΔF. This residual variation is the minimized projection of the loop seen along its major axis.

Another representation of similar data further illustrating the nature of the Feinheit function is shown in Figure 2. The upper curve shows the V luminosity variation as a function of phase; the middle curve shows the $(B\text{-}V)$ color variation as a function of phase. Note the similarity of the two curves, again emphasizing how strongly coupled the visual luminosity is to the temperature. The lower panel shows the residual variations in F as a function of phase. The opening of the loop found in Figure 1 is now se n as a periodic cycloidal variation shifted by about 90 degrees with respect to the color curve. In this projection, F bears all of the characteristics typical of the radius variation, independently known to be of this fc m from radial velocity curve studies.

In decoupling the radius-induced luminosity from the temperature-induced luminosity variations we have, using visual data, now matched the advantages inherent in the infrared. The instantaneous values of F are never more than a few tenths of a magnitude away from the mean, and as such provide a fast, economical and accurate means of measuring the luminosities of Cepheids.

III. Period Dependence of the Loops

In empirically evaluating the coefficient α in $F = V_\varphi - \alpha\ (B\text{-}V)_\varphi$, the tabulation of Schaltenbrand and Tammann (1971) was used. Rather than replotting the original data as in Figure 1, a good estimate of α can be had from $\Delta V / \Delta\ (B\text{-}V)$ which conveniently, are individually tabulated by those authors. Since there is no *a priori* reason to expect that α will be the same for all Cepheids, the individual slopes were plotted as a function of log P. Only the brightest stars (with $V < 10.0$ mag) with well determined amplitudes were considered; and stars known to be binaries were omitted. The resulting relation is shown in Figure 3, where a clear trend with period is evident. The data are adequately fit by $\alpha = -0.3 \log P + 2.1$. For the majority of Cepheids then, α falls in the range 1.5 to 2.0. No residual trend with amplitude was found for the scatter in Figure 3.

The change in α as a function of period emphasizes that there is a differing sensitivity of the visual surface brightness to the observed color, either as a function of the mean surface gravity or more likely as a function of mean surface temperature, both of which are statistically coupled to the period. The same effect is probably manifesting itself in the period dependence of the "constant" zero point of the period- luminosity-color relation presented by Coulson, Caldwell and Gieren (1985). Furthermore from a study of seventeen Cepheids with good photometry and radial velocities Ivanov (1981) finds that the slope of his surface brightness-color relationship is also dependent upon period to the same degree and sense as reported here, ranging from 2.1 at short periods to 1.6 at long periods.

IV. The Period-Luminosity Relation

We now test the formulation of the blue Feinheit method for an ensemble of points using data obtained for a sample of Cepheids in the Small Magellanic Cloud (Gascoigne and Kron 1965). The observations plotted in Figure 4 and 5 are the first photoelectric observations obtained by these authors for each of the stars. The periods were known in advance from photographic studies but the observations are essentially random samplings of the Cepheids' light curves. In principle, the data could have been obtained in one night of observing.

Figure 4 shows the *B* photoelectric data plotted as a function of period. Because the observations are random-phase the width of the period-luminosity relation is quite large. In addition to the intrinsic width of the relation, known to be about 1.2 mag at *B* for time-averaged observations, the plot has the additional scatter induced by the additive effects of the amplitude of each Cepheid seen superimposed to various degrees on top of the mean relation. Since typical amplitudes of Cepheids at *B* are known to be similar in magnitude to the intrinsic width of the instability strip itself we should expect the random-phase period-luminosity relation to be about twice as wide as the time-averaged period-luminosity relation. That this is indeed the case is illustrated in Figure 4. The solid lines mark the maximum extent of the random-phase period-luminosity relation which has an observed width of 2 magnitudes. The broken lines inside and parallel to that relation show the expected width for the time-averaged relation.

Figure 5 shows the same data now transformed to the phase-independent Feinheit function. The full width of this randomly sampled period-luminosity relation has dropped by a factor of three with respect to the blue, confirming our expectations based on the observations of one star through its cycle. Furthermore the slope of the *F*-log *P* relation is noticeably steeper than the *B*-

log P relation. Were one simply interested in having an economical means of determining modern distances to galaxies in which Cepheids were already detected (as has been the case for the Magellanic Clouds and several other Local Group galaxies for some years now) then one night's worth of observing, regardless of the period range of the Cepheids involved, would suffice.

This situation is highly reminiscent of the advantages claimed for observing Cepheids in the near infrared, where the scatter in the random-phase H-band observations was also shown (McGonegal *et al.* 1982) to be less than the time-averaged B observations. In fact the two methods are in the end equivalent, with F and the infrared each primarily responding to radius (i.e. surface area) variations. In both cases the only variable remaining during the cycle is the geometric term; F is by construction designed to transform the temperature terms away, the infrared by its very nature is insensitive to temperature-induced variations. Radius is the only physical variable remaining and common to both observations. A cursory comparison of the form, phase shift and amplitude of F and the infrared light curves for individual Cepheids as a function of phase shows that the two are basically indistinguishible from that predicted by the radius variations alone.

V. Discussion and Conclusions

Do any of the other advantages found in the infrared accrue to the Feinheit method? Radiation observed at $H(1.6\mu)$ is known to suffer about three times less extinction due to interstellar dust as compared to V. That is, $A_H \sim E(B\text{-}V)$ or $H = H_0 + E(B\text{-}V)$. Because of the form of F it too, implicitly cancels some of the effects of interstellar extinction. For example, if $F = V - 2(B\text{-}V)$, then

$$F = V_0 + A_V - 2(B\text{-}V)_0 - 2E(B\text{-}V)$$

$$= F_0 + E(B\text{-}V)$$

Accordingly, F and H have similarly low sensitivities to reddening.

Although they are harder to quantify, it is certainly true that for both the infrared and for F the effects of atmospheric line blanketting and the effects of contamination due to main sequence companions are greatly reduced relative to the blue. Again, the infrared is intrinsically insensitive to these effects, while for F the sense of each effect is correlated in luminosity and color such that F partially cancels their influence. Atmospheric line blanketting is notoriously wavelength dependent with heavy crowding of lines favoring the blue and ultraviolet portions of the spectrum. Accordingly stars with low metal abundance are bluer and brighter than their high-metallicity counterparts. This effect combines fortuitously in the same sense as the temperature and surface brightness combine; therefore, to first order metallicity effects are compensated for in the Feinheit magnitude. Similarly, if a blue companion contaminates the Cepheid's photometry it will make the variable appear brighter and bluer, again precisely the type of correlation F tends to cancel out.

In summary then, the Feinheit function is a means of explicitly minimizing the surface-brightness variations of a Cepheid during its cycle, leaving only the small residual radius variations. By combining the magnitude and color to optimize this cancelation several other effects are simultaneously compensated for. As in the infrared, the Feinheit method accounts in part for reddening, metallicity differences and contamination from blue companions. However, there is one added advantage that F has over the infrared. Since F uses optical data it can employ existing panoramic detectors to acquire new photometry. At the moment infrared observations at faint magnitudes are limited to single-channel devices using aperture techniques. For Cepheids in distant galaxies crowding effects have set a limit on the accuracy of near-infrared stellar photometry (Freedman, Grieve and Madore 1985; Madore *et al.* 1985). With CCDs and profile-fitting reduction techniques, seeing limited photometry can be done in the optical and be a full order of magnitude less susceptible to crowding and confusion problems as compared to H-band photometry done through a five arcseond aperture, say.

Finally it should be noted that the Feinheit method itself is not restricted for use on BV data alone. Any combination of filters can be used to cancel the temperature effects on the luminosity. For CCD's whose operating efficiency peaks around the R-band it would be reasonable to reformulate F for that natural system where $F = R - \beta(V-R)$, or $F = I - \gamma(R-I)$ would be obvious alternatives. However, if the only physics contributing to F is the radius variation, the amplitude observed for F should be independent of the combination of filters employed.

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Captions to Figures:

Figure 1: Individual observations of the Cepheid *X Cyg*, from the compilation of Moffett and Barnes (1984), plotted in the *B-(B-V)* color-magnitude plane. The loop is traversed cyclically with time in a clockwise direction, projecting a variation in luminosity which is tightly correlated with (and causally linked to, the color variation. The minimum projected variation with phase, the Feinheit function, is seen looking down along the major axis of the loop. In this case the cyclical variation in F is about four times smaller than the variation seen in B

Figure 2: Phase plots for photoelectric data from Moffett and Barnes (1984) for the Cepheid *RR Lac*. The upper two curves are the V and *(B-V)* light and color curves, respectively. Note the similarity of the shapes of the two curves. By appropriately scaling the color data and subtracting them from the light curve, the lower Feinheit function is found. Cancellation is not complete; but a residual variation amounting to only a few tenths of a magnitude is left. Its variation is cyclical in shape and shifted by about a quarter of a cycle with respect to the color curve.

Figure 3: The period dependence of the slope of the loop for cepheid trajectories in the *V-(B-V)* color-magnitude diagram, as estimated from the ratio of amplitudes tabulated in Schaltenbrand and Tammann (1971). The solid line through the data has the form -0.3 log P + 2.1.

Figure 4: Period-luminosity relations for Small Magellanic Cloud Cepheids based on random-phase photoelectric observations. Solid lines mark the width of this empirical relation (2.0 mag), the inner broken lines show the width of the same

relation (1.2 mag) when time-averaged data are used. Compare with Figure 5.

Figure 5: The same random-phase data as in Figure 4 but now transformed to the Feinheit magnitude. The width of the relation is three times narrower than the corresponding B relation and is even a factor of two tighter than the time-averaged B relation. Note the steeper slope of the F-log P relation in comparison to the B-log P relation.

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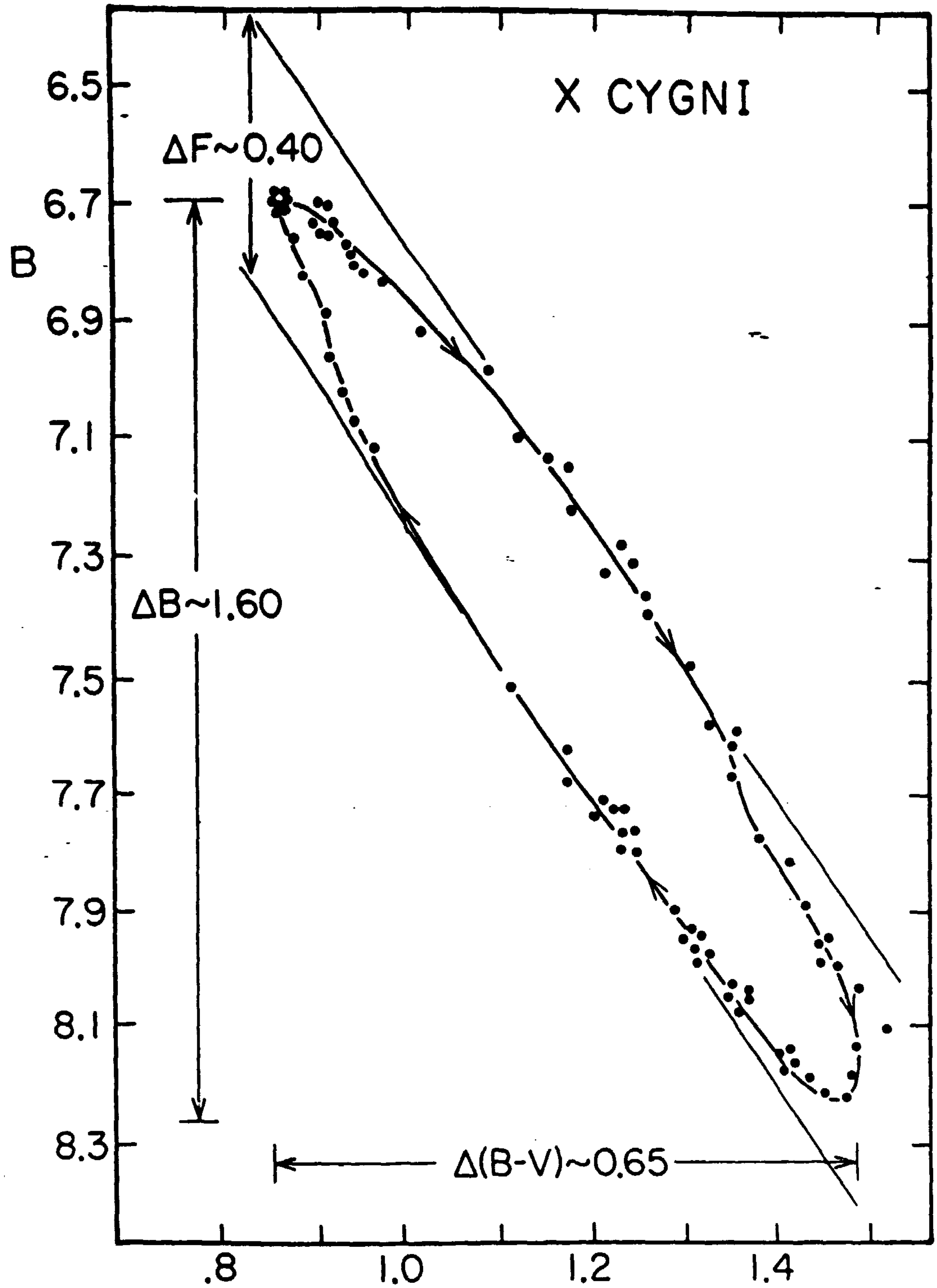

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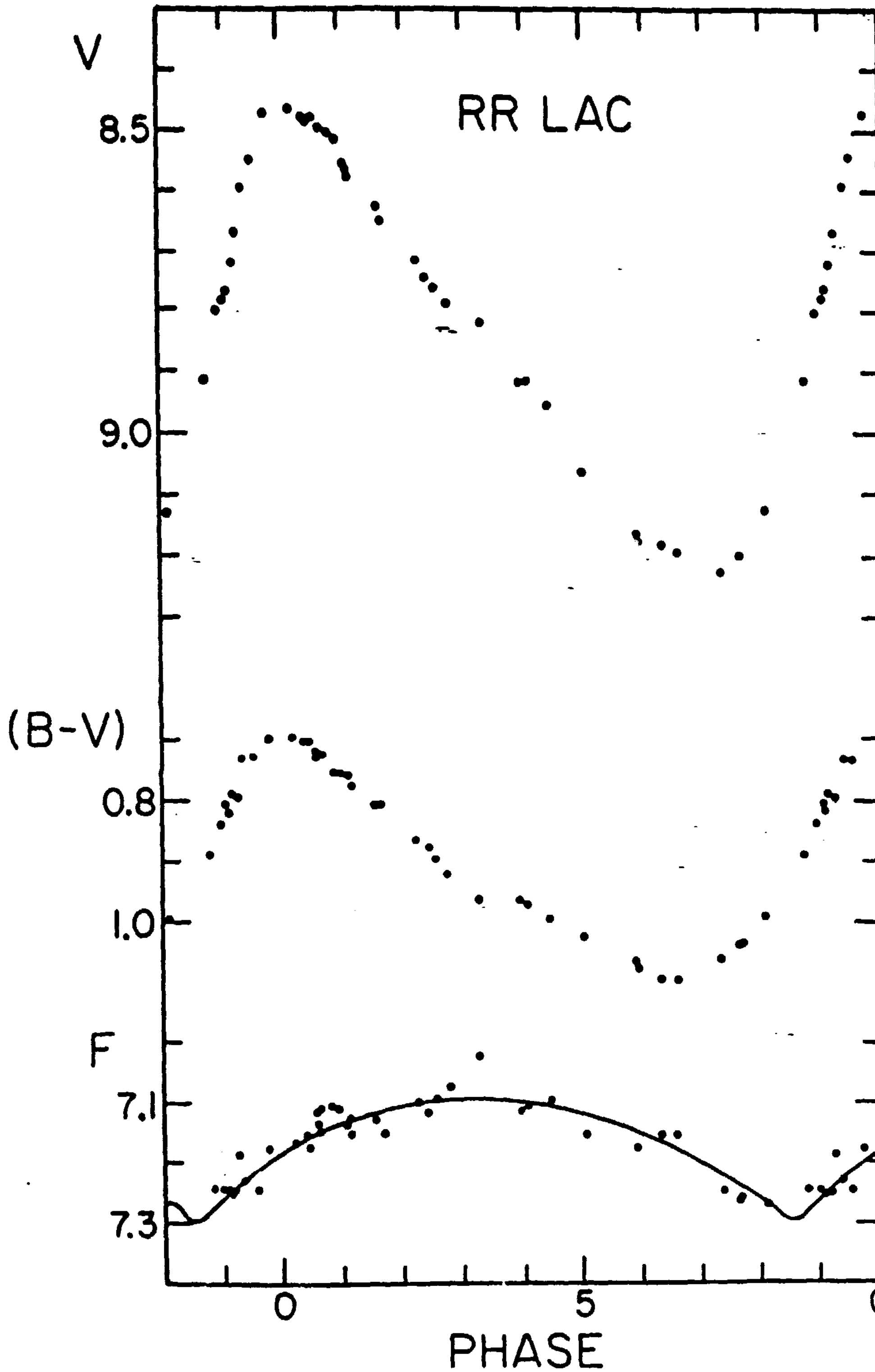

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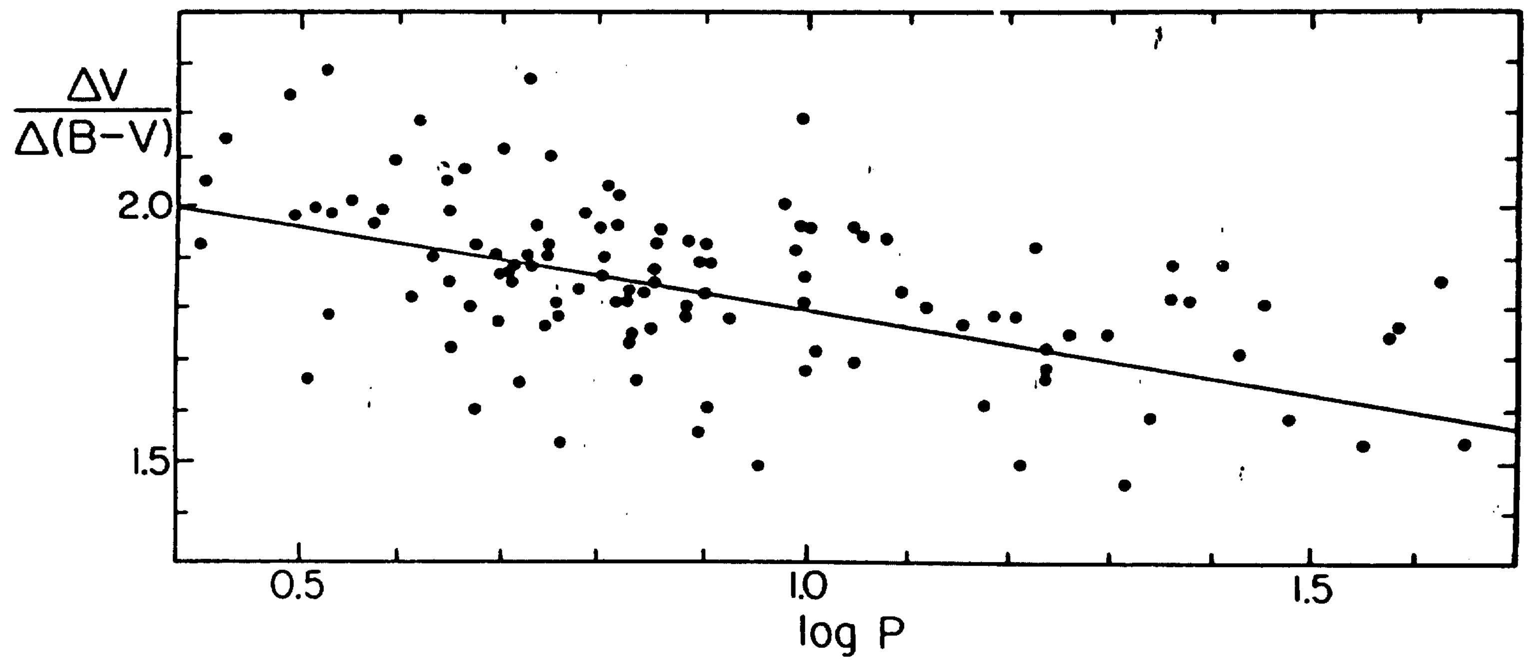

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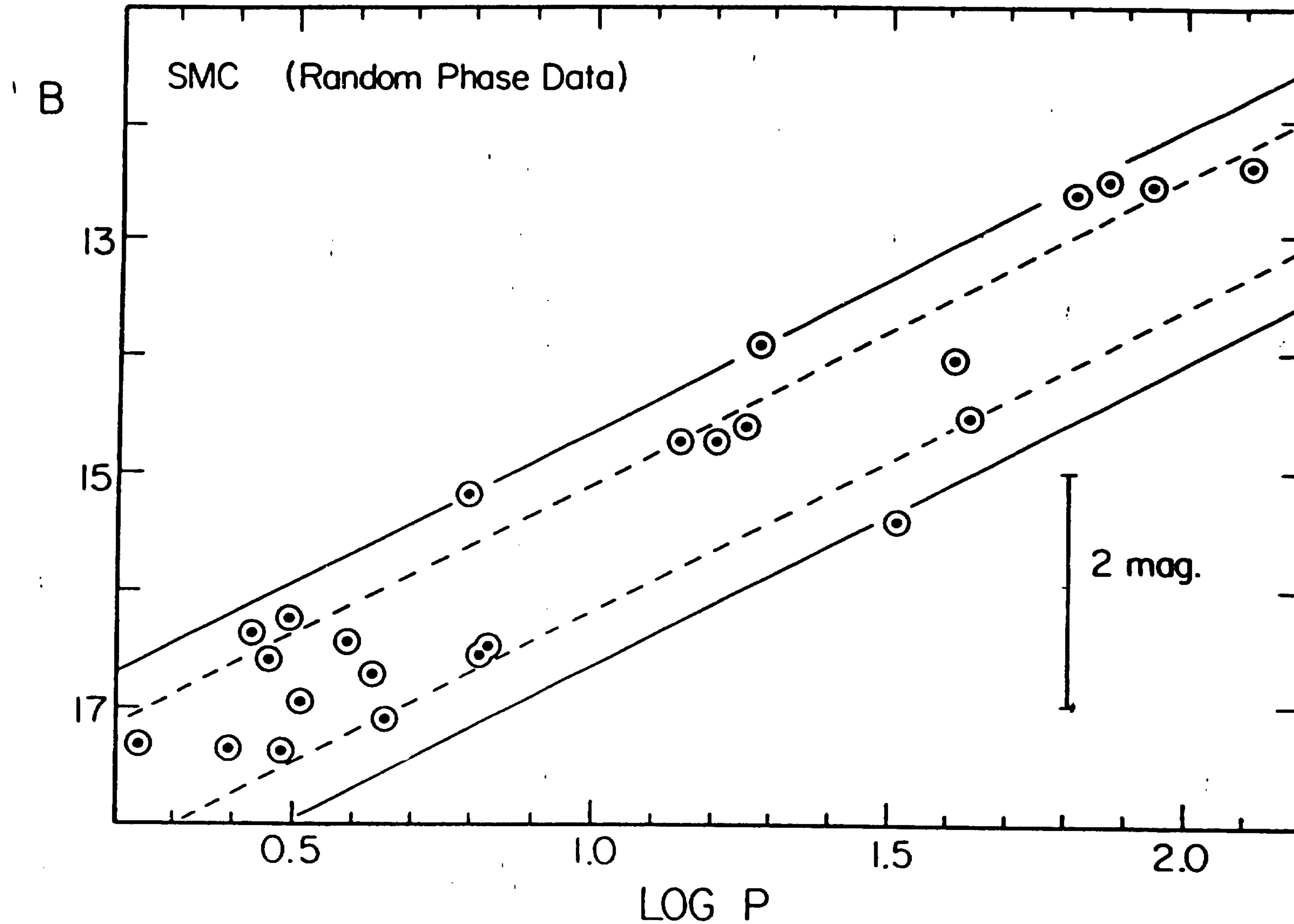

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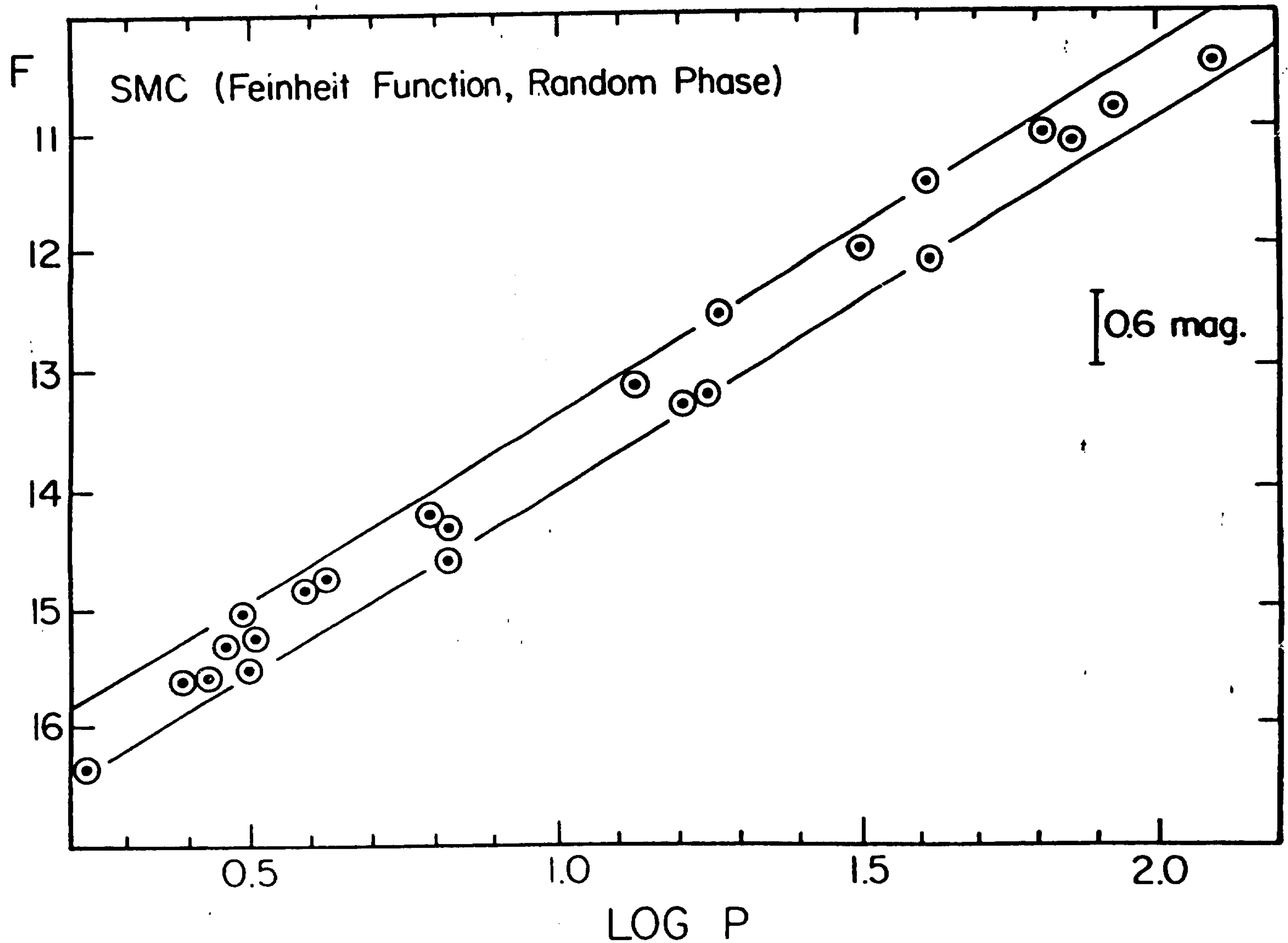

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