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Formulation and Implementation of Nonstationary Adaptive Estimation Algorithm With Applications to Air-Data Reconstruction

Stephen A. Whitmore

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Stephen A. Whitmore
Ames Research Center, Dryden Flight Research Facility, Edwards, California

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FORMULATION AND IMPLEMENTATION OF NONSTATIONARY ADAPTIVE ESTIMATION ALGORITHM WITH APPLICATIONS TO AIR-DATA RECONSTRUCTION

Stephen A. Whitmore

NASA Ames Research Center
Dryden Flight Research Facility
Edwards, California

ABSTRACT

The dynamics model and data sources used herein to perform air-data reconstruction are discussed, and a brief discussion of the Kalman filter is included. The discussion indicates the need for adaptive determination of the noise statistics of the process. The filter innovations are presented as a means of developing the adaptive criterion, which is based on the estimation of the true mean and covariance of the filter innovations. A method for the numerical approximation of the mean and covariance innovations is presented.

The algorithm as developed is applied to air-data reconstruction for the space shuttle, and data obtained from the third landing are presented. To verify the performance of the adaptive algorithm, the reconstruction is also performed using a constant covariance Kalman filter. The results of the reconstructions are compared, and the adaptive algorithm exhibits better performance.

SYMBOLS

A	kinematic acceleration of aircraft
b	white, Gaussian distributed unity covariance error in measured system response
C	system geometry matrix, $L \times M$ dimensioned matrix
e	filter innovations vector, $L \times 1$ dimensioned
F	state-noise gain vector, $M \times 1$ dimensioned
FF^T	discrete-time state-noise covariance, $M \times M$ dimensioned
f	rolloff frequency of low-pass filter used in dynamics equation
G	measurement noise vector, $L \times 1$ dimensioned
GG^T	measurement noise covariance matrix, $L \times L$ dimensioned
k	quantity corresponding to kth sample
L	dimensions of measurement vector
M	dimensions of state vector
n	nonstationary disturbance to state dynamics
P	filter error covariance matrix, $M \times M$ dimensioned
t	continuous-time index
V	groundspeed state used in dynamics equation
Vw	airspeed state used in dynamics equation
W	windspeed state used in dynamics equation
X	position state used in dynamics equation
x	state vector, M dimensioned
z	measurement vector, L dimensioned
ϕ	discrete transition matrix, $M \times M$ dimensioned

Mathematical operators:

COV( )	covariance of quantity ( )
d/dt( )	derivative of ( ) with respect to time
E( )	expectation of quantity ( )
S( )	sample mean of quantity ( )
$( )^T$	transpose of ( )
$(\hat{\ })$	quantity estimated by filter

INTRODUCTION

Accurate estimates of the air data of high-speed aircraft are essential to successfully perform flight research and developmental tasks. Recent expansions of operational flight envelopes have necessitated the development of advanced reconstruction techniques to obtain these estimates. Many of these techniques use a Kalman filter to merge several data sources to enhance the result. Most of these methods assume that random disturbances in the aircraft dynamics can be accounted for by assuming constant filter statistics which are assumed to be known *a priori*. Unfortunately, there is often little physical evidence by which one can determine the statistics of these random disturbances. In many cases the disturbances are nonstationary; in other words, in such cases the random disturbances in the aircraft dynamics can only be described by time-varying statistics. These cases include such unsteady regimes as flight in turbulence, high-angle-of-attack flight, and aerostructural buffet. In such cases, the statistics of random disturbances are often selected purely by guesswork. The so-called "optimal" results of such filters are optimal only in the sense that an error criterion is minimized.

For nonstationary cases, the estimation scheme must allow for the adaptive estimation of the statistics that describe random disturbances in the aircraft dynamics as part of the filter loop. This paper presents such an algorithm. The algorithm, although primarily intended for applications to air-data reconstruction, has applications to a variety of other fields. Because the space shuttle is known to encounter several or all of the above-mentioned unsteady flight conditions during its reentry, space shuttle reentry data are used to verify the resulting algorithm.

ESTIMATION CONCEPT

The estimation problem is essentially one of complementary filtering. Data from four independent measurement sources are merged by means of the filtering algorithm to give an enhanced result. The data sources include high-, medium-, and low-frequency data. The high-frequency data are provided by a strapdown linear accelerometer package; the medium-frequency data are provided by a pair of pneumatic hemispherical air-data sensors and C-band radar tracking; and the low-frequency data are provided by a meteorological analysis of the atmospheric conditions. The air data are complementary to and blended with the meteorological data; the acceleration data are complementary to and blended

with the tracking data. These are in turn combined to give enhanced estimates of the aircraft position, groundspeed, windspeed, and airspeed. The resulting estimates possess characteristics of all four measurement sources. A schematic of this concept is presented in Fig. 1. Each of the data sources is described in detail in the INFORMATION SOURCES section.

INFORMATION SOURCES

Tracking Data

Radar tracking data are obtained from a skin track using an FPS-16, C-band, high-range tracking system (Ref. 1). Provided are highly accurate, medium-frequency measurements of the aircraft's range, azimuth, and elevation relative to the radar site. The tracking data were recorded at 20 Hz and interpolated to 25 Hz.

Meteorological Data

Steady-state meteorologically derived wind and barometric data are used to provide accurate but very low frequency information concerning the dynamics of the atmosphere along the reentry flightpath. The data are obtained by a series of weather balloons launched at various times and locations along the anticipated flightpath. The raw data thus obtained were corrected for diurnal and spatial variations (Ref. 2). Examination of time and altitude variations in the data gives indications of both the steady-state magnitude and the turbulence in the winds aloft. The meteorological data were interpolated to 25 Hz using radar position data.

Air-Data Measurements

The air-data measurements are obtained from a pair of side-mounted hemispherical pneumatic sensor (Ref. 2). In addition to sensing turbulence and compression caused by the local flow field, the sensors are subject to pneumatic lag. These factors are manifested as nonstationary disturbances. The hemispherical sensor data are recorded at 12.5 Hz and interpolated to 25 Hz.

Strapdown Linear Accelerometer Data

High-frequency data concerning the aircraft's inertial dynamics are provided by an onboard, strapdown, linear accelerometer package (Ref. 2). The accelerometer package, intended for aerodynamic coefficient identification, provides very high frequency measurements. The package, however, is not inertial quality and is subject to significant bias errors. Thus resulting data cannot be integrated open loop. The strapdown data were rotated to earth-relative topodetic coordinates before use in the filter. The direction cosines used in performing the rotation were obtained directly from the shuttle's inertial measurement unit (Ref. 3). After rotation, the acceleration of gravity as a function of altitude was added to the vertical component. The strapdown data are recorded at 176.8 Hz and decimated to 25 Hz.

FILTER EQUATIONS

The discrete-time form of the Kalman filter was used as the starting point for developing the algorithm. The filter as mechanized consists of three parallel filters, one for each topodetic axis component. The topodetic axis system (Fig. 2) is defined so as to have its x-axis directed northward with respect to the local horizon, the y-axis directed eastward, and the z-axis directed toward the center of the earth. Each component filter was assumed to have four states: aircraft position, groundspeed, windspeed, and true airspeed. The dynamics equations, although continuous time, are easily discretized by transition matrix integration. The matrix equation chosen to describe the process dynamics is

$$
d/dt \begin{bmatrix} X \\ V \\ W \\ V_w \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -f & -f & f \\ 0 & -f & -f & f \end{bmatrix} \begin{bmatrix} X \\ V \\ W \\ V_w \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} A \quad (1a)
$$

$$
+ F(t) \, n(t)
$$

The relationships of the states to the measured data are

$$
\begin{bmatrix} X \\ V_w \\ W \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 1/2 \\ 0 & -1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} X \\ V \\ W \\ V_w \end{bmatrix} \quad (1b)
$$

$$
+ G(t) \, b(t) c
$$

In Eqs. (1a) and (1b), the parameter f represents a first-order low-pass filter rolloff frequency. This parameter affects only the windspeed and airspeed components of the filter. The low-pass filter terms are chosen so as to insure that the resulting windspeed and airspeed terms will be smooth.

The function $n(t)$ represents a nonstationary random disturbance in the process at time t. It is assumed that the model of Eq. (1a) adequately accounts for all systematic aircraft dynamics; as a result, $n(t)$ is assumed to be zero mean. The vector $F(t)$, a four-element vector, is used to describe the relative effect of random disturbances occurring at time t on each of the states. Since $n(t)$ is assumed to be zero mean, the vector product of F with itself is assumed to approximate the covariance of the random disturbances at time t. Consequently, it is the vector product of F with itself that must be identified by means of the adaptive criterion developed in the ADAPTATION CRITERION section.

The function $b(t)$ represents an error in the measurement vector at time t. This quantity is assumed to be locally stationary (that is, it has statistics that vary slowly with time), and it has a nonzero mean. The vector $G(t)$, a three-element vector, is used to describe the respective amplitudes of the errors in each measurement at time t. The elements of $G(t)$ are essentially measurement weighting factors. As a part of the adaptive algorithm presented in the ESTIMATION OF INNOVATIONS MEAN section, the bias errors in the measured data are estimated and compensated for. For this reason, the vector product of G(t) with itself is assumed to approximate the covariance of the measurement errors at time t.

The first element of $G(t)$ is used to weight the errors that are expected to occur in the radar-derived position data. Radar position errors are known to become significant for elevation angles of less than 10°. For this reason the first element of G(t) is prescribed to decrease linearly with increasing elevation angle until elevation angle reaches 10°. At elevation angles greater than 10°, the first element of G(t) is held constant. For elevation angles less than 0°, the value of the first element of G(t) is fixed at 1. A schematic of this weighting scheme is presented in Fig. 3(a).

The second element of $G(t)$ is used to weight the expected errors in the hemispherical sensor-derived airspeed data. Hemispherical sensor data are known to become poor at high angles of attack. Consequently, for angles of attack greater than 10°, the second element of G(t) is prescribed to increase linearly with angle of attack. For angles of attack less than 10°, the weighting is held constant at 1. A schematic of this weighting scheme is depicted in Fig. (3b).

The third element of $G(t)$ is used to weight the expected errors in the meteorologically derived windspeed data. Because little precise information is available regarding the accuracy of these measurments, these data are considered to be equally accurate throughout the flight envelope. For this reason, the third element of G(t) is held constant at 1.

ADAPTATION CRITERION

As mentioned previously, the statistics of the random disturbances in the process dynamics are assumed to vary as a function of time. For this reason, we must develop an adaptive criterion by which these statistics can be estimated. The adaptive scheme discussed in this paper uses information provided by the filter error vector. The error vector is defined as the difference between the measured system response and the expected system response. This vector, e_{k+1}, is also called the filter innovations, which is the name used henceforth. The statistics of the filter innovations tell a great deal about how well the filter model is performing. Because of the assumptions used in deriving the discrete Kalman filter, the innovations should be a purely white, Gaussian-distributed, zero-mean sequence. If such is the case, then one can be fairly confident that the filter model is correct and that the resulting estimates are close to optimal (Ref. 4). If this is not the case, then the parameters of the filter model are in error and information provided by the innovations can be used to drive the assumed model toward the correct model. This process is depicted in Fig. 4.

ESTIMATION OF INNOVATIONS MEAN

The Kalman filter is an unbiased estimator; consequently, for unbiased measurements, the mean of the innovations should be exactly zero (Ref. 5). If, however, there are biases in the measurements, the mean of the innovations will equal the expected bias error in the measurements. Measurement bias errors do not usually change rapidly with time, and for this reason the true mean can be approximated by sampled statistics. The sample mean can be allowed to change slowly with time by taking the time average over only a fairly local region. The time-averaged estimate of the bias error $S(e_k)$, can be recursively subtracted from the measurement vector to form a transformed measurement vector that is nearly unbiased. Using this transformed vector, the estimation algorithm proceeds as in the standard Kalman filter.

ESTIMATION OF THE INNOVATIONS COVARIANCE

The negation of the approximate biases from the measurement vector allows for considerable simplification of the adaptive process; the problem reduces to one of estimating the mean square of the innovations. This quantity is estimated in the following manner: The actual system response is given to be related to the system states by some geometry matrix, C, and an error term, b(t); that is,

$$z_k = Cx_k + G_k b_k \qquad (2a)$$

Therefore, the expected system response is given by

$$z_k = Cx_k \qquad (2b)$$

As a result (remembering that biases have been removed), the innovations covariance is described by

$$COV(e_k) = E(z_k - Cx_{k/k-1})(z_k - Cx_{k/k-1})^T \qquad (3a)$$

Substituting in Eq. (2a) for z gives

$$COV(e_k) = E[C(x_k - x_{k/k-1}) + G_k n_k][C(x_k - x_{k/k-1}) + G_k n_k]^T$$

Now, since the measurement noise is assumed to be white and all biases have been previously removed, the previous expression reduces to

$$COV(e_k) = CE(x_k - x_{k/k-1})(x_k - x_{k/k-1})^T C^T + GG_k^T$$

From the definition for the filter error covariance matrix,

$$P_{k/k-1} = E(x_k - x_{k/k-1})(x_k - x_{k/k-1})^T$$

we get

$$COV(e_k) = CP_{k/k-1} C^T + GG_k^T \qquad (3b)$$

The quantity COV e_k is computed as a part of the gain expression in the standard implementation of the discrete Kalman filter. Conveniently, using the previously stated assumptions, the covariance of the innovation is available for use with no additional computational expense.

ADAPTIVE COMPUTATION OF THE STATE-NOISE COVARIANCE

For the standard formulation of the discrete Kalman filter, the state-noise covariance extrapolation step is given by

$$P_{k+1/k} = \Phi P_{k/k} \Phi^T + FF_k^T \qquad (4a)$$

where FF_k^T is the state-noise covariance and Φ is the system transition matrix. Substituting Eq. (4a) into Eq. (3b) and rearranging gives

$$CFF_k^T C^T = COV(e_k) - C(\Phi P_{k-1/k-1} \Phi^T)C - GG_k^T \qquad (4b)$$

If the geometry matrix, C_k, is square and non-singular, we can solve for the state-noise covariance matrix directly. If the number of measurements does not equal the number of states, we can expand the matrix equation to form a set of L scalar equations in M(M − 1) unknowns, where L is the number of measurements and M is the number of states. We then must assume that the off-diagonal elements of the covariance matrix are insignificant, and the covariance matrix is essentially diagonal. This allows us to solve directly for L of the diagonal elements in the state-noise covariance matrix. In the case of Eq. (1), we can solve directly for the first, second, and fourth diagonal elements; the remaining element can be solved for by considering that by definition

$$Vw = V + W \qquad (4c)$$

from which we reason that the third diagonal element is simply the sum of the second and fourth diagonal elements. Equation (4b) can now be implemented as a part of the filter loop to give a closed-loop estimate of the covariance of random disturbances in the aircraft dynamics.

PRESENTATION OF ADAPTIVE TECHNIQUE

The adaptive algorithm is presented in schematic form in Fig. 5. The portion of Fig. 5 that lies outside the dashed line represents the process dynamics; the portion that lies inside the dashed line represents the estimation loop. The arrows depict the flow of information through the filter. The state-noise covariance is computed by means of Eq. 4 as a part of the recursion of the filter.

APPLICATION OF ALGORITHM TO TRAJECTORY RECONSTRUCTION PROBLEM

As a verification of its validity, the adaptive algorithm is now applied to the problem of air-data reconstruction. Data obtained from the third space

shuttle reentry (STS-3) are chosen to illustrate the problem. The STS-3 landing at White Sands, New Mexico, occurred on a worst-case day. Extremely high winds and moderate-to-severe turbulence were known to exist. Examination of landing-day rawinsonde and Jimsphere balloon data provided by the Air Force Flight Test Center at Edwards AFB indicated that jetstream velocities were measured in excess of 150 knots. The Jimsphere balloon was observed to have rise-rate oscillations that varied from 0 to 10 ft/sec. These wind conditions extended throughout the troposphere and the lower stratosphere. Because of these conditions, the vertical wind component was subject to greater random variation than has been experienced during any of the other space shuttle reentries. The conditions are suggestive of mountain gravity wave activity. Analytical solutions documented in Ref. 2 also suggest that this is the case. Such conditions are known to produce highly nonstationary disturbances (Refs. 2 and 6).

The air-data reconstruction was performed using the adaptive algorithm with arbitrary initial covariances assumed for the state disturbances. The measurement error covariances were chosen based upon the prescribed methods mentioned earlier. To verify the effectiveness of the adaptive algorithm, the air-data reconstruction was also performed using a standard implementation of the discrete Kalman filter. The standard implementation was performed using the same initial covariances as above; however, in this case both the state disturbance and measurement error covariances were held constant.

The results of the adaptive case are presented in Fig. 6. Presented are comparisons of the measured and reconstructed time histories of airspeed (Fig. 6(a)), vertical windspeed (Fig. 6(b)), and total horizontal windspeed (Fig. (6c)). Estimates of these quantities are the most heavily affected by time variations in random atmospheric disturbances. Inertial type quantities such as position and groundspeed are less heavily affected. The results show no significant discrepancies between the measured and estimated values. Considering the fact that mountain wave activity was believed to exist, the vertical windspeeds, although large, are within believable limits. As expected, the filtered estimates exhibit higher frequencies than do the meteorological winds.

The results of the nonadaptive estimates are presented in Fig. 7, and similar comparisons are made. These comparisons show significant discrepancies. The estimates of airspeed and horizontal windspeed differ by more than 100 ft/sec. It is not believed that the meteorological estimates could have been this much in error. The vertical wind estimate reaches a peak value of nearly 60 ft/sec downdraft. Had these conditions actually occurred, it is doubtful that the reentering space shuttle could have cleared the lofty San Andreas

Mountains that border the STS-3 landing site. In Fig. 7 it is interesting to note that the behavior of the standard filter becomes more realistic toward the end of the time histories. These data correspond to data that were obtained just before landing and several minutes after the shuttle had dropped below the regions of heavy turbulence and mountain wave activity. Under these conditions the disturbances dropped to nearly zero and no adaptive estimation of the covariances of the disturbances were necessary.

CONCLUDING REMARKS

An adaptive algorithm that can be used to estimate the state-noise covariance for certain types of nonstationary processes has been developed. The algorithm, which was developed primarily for the purpose of air-data reconstruction, accounts for improper knowledge of the state-disturbance covariance matrix and, to some extent, accounts for unknown measurement biases. The algorithm is recursive and has the potential for real-time implementation. Because the adaptation criterion was formulated in a general sense, it has applications to fields other than air-data reconstruction.

An air-data reconstruction problem for the space shuttle is used to demonstrate the application of the algorithm. The algorithm exhibits superior performance as compared with a standard implementation of the discrete Kalman filter. The algorithm has potential for solving many types of nonstationary estimation problems for which the standard implementation of the Kalman filter is unsuited.

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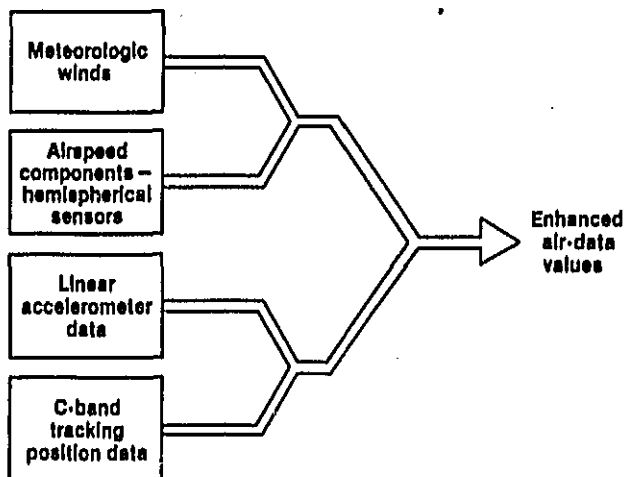
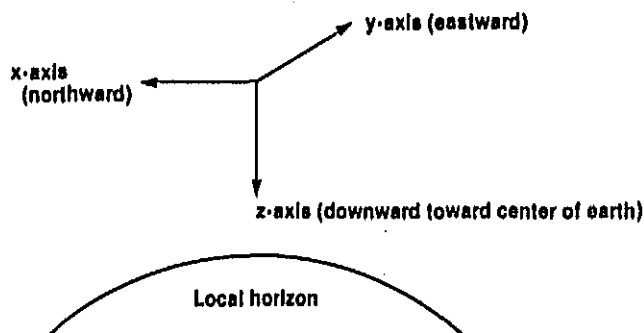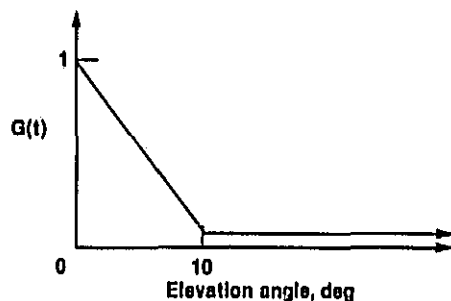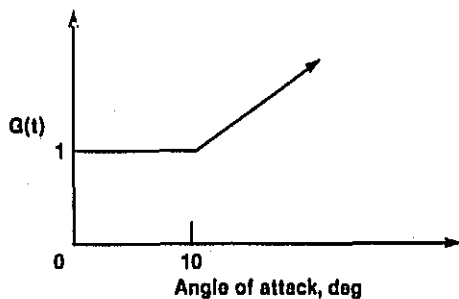Figure 1. Air-data estimation concept.

Figure 2. Topodetic axis coordinate definitions.

(a) Error weighting for radar-derived position data.

(b) Error weighting for hemispherical sensor-derived airspeed data.

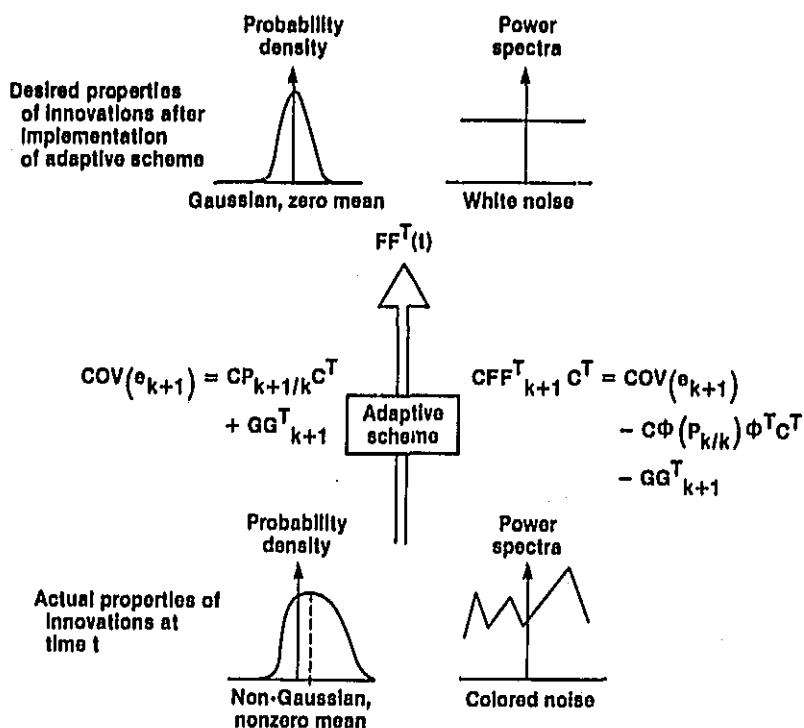Figure 3. Measurement error weighting schedules.

$$COV\left(e_{k+1}\right) = CP_{k+1/k}C^T + GG^T_{k+1}$$

$$CFF^T_{k+1} C^T = COV\left(e_{k+1}\right) - C\Phi\left(P_{k/k}\right)\Phi^T C^T - GG^T_{k+1}$$

Figure 4. Use of innovations vector to adaptively compute the state disturbance covariance matrix.
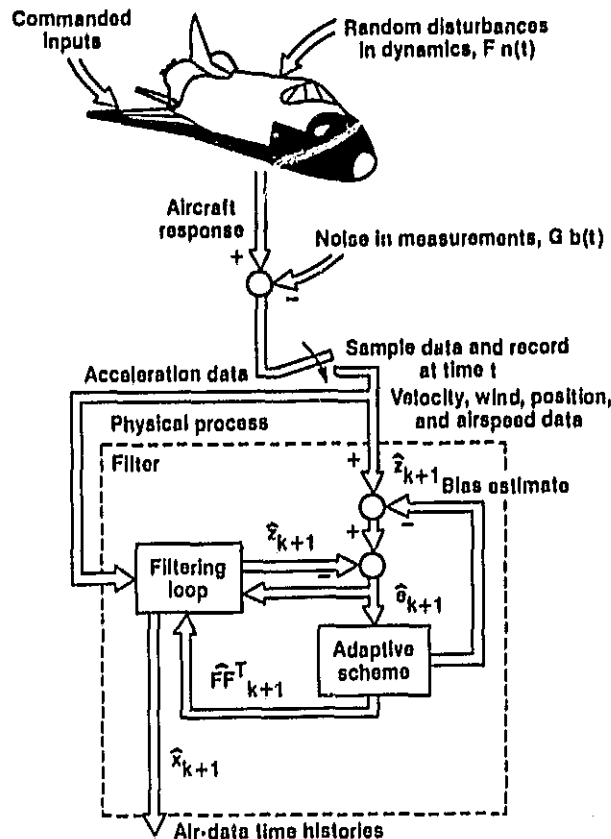
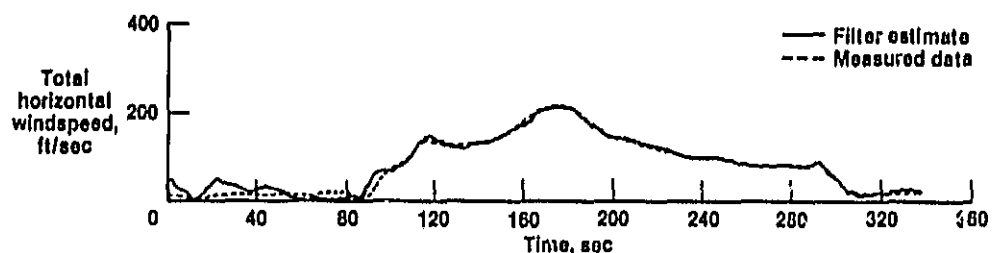Figure 5. *Adaptive air-data reconstruction algorithm.*
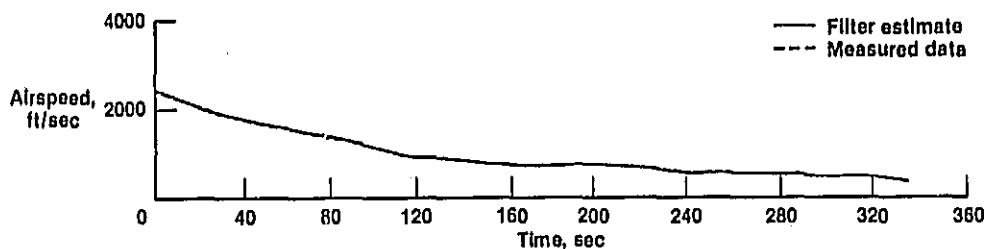
(a) *Airspeed.*

Figure 6. *Time history comparisons of STS-3 air data reconstructed using adaptive, nonstationary filtering algorithm.*
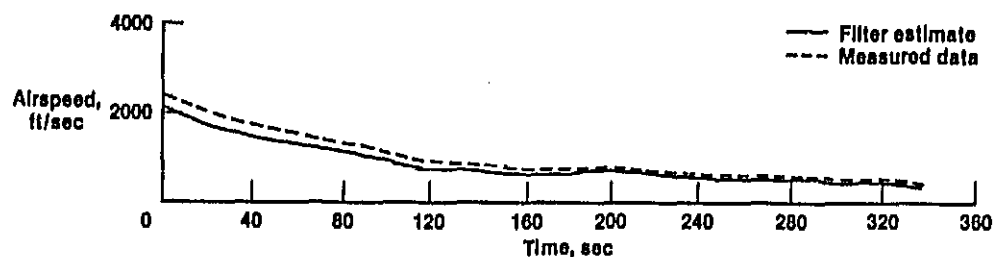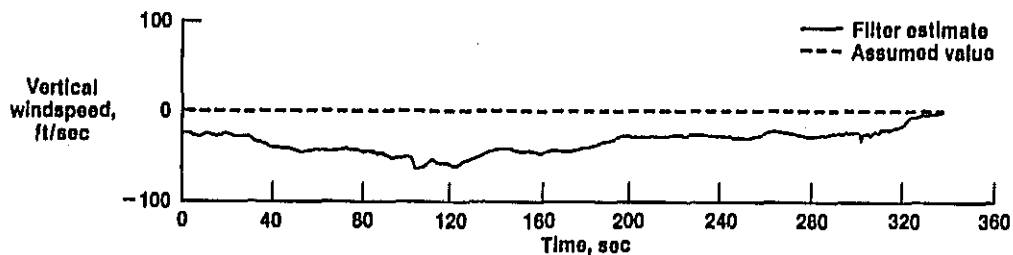
7
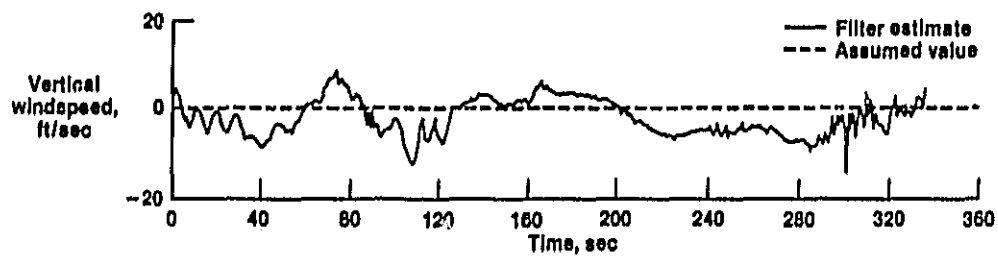
(b) Vertical wind.

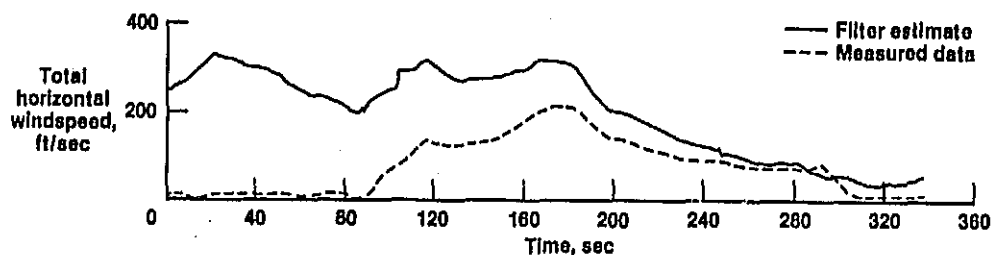(c) Total horizontal windspeed.

Figure 6. Concluded.

(a) Airspeed.

(b) Vertical windspeed.

(c) Total horizontal windspeed.

Figure 7. Time history comparisons of STS-3 air data reconstructed using standard implementation of discrete Kalman filter.

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16. Abstract

The dynamics model and data sources herein used to perform air-data reconstruction are discussed, and a brief discussion of the Kalman filter is included. The discussion indicates the need for adaptive determination of the noise statistics of the process. The filter innovations are presented as a means of developing the adaptive criterion, which is based on the estimation of the true mean and covariance of the filter innovations. A method for the numerical approximation of the mean and covariance of the filter innovations is presented.

The algorithm as developed is applied to air-data reconstruction for the space shuttle, and data obtained from the third landing are presented. To verify the performance of the adaptive algorithm, the reconstruction is also performed using a constant covariance Kalman filter. The results of the reconstructions are compared, and the adaptive algorithm exhibits better performance.

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