DEVELOPMENT AND APPLICATION OF A UNIFIED BALANCING APPROACH WITH MULTIPLE CONSTRAINTS

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The development of a unified balancing approach with multiple constraints offers the engineer a powerful and flexible tool, broad in scope and not restricted to special rotor configurations. This method imposes no restrictions on the use of modal trail weight sets and/or modal influence coefficients, which have been proven effective for balancing high-speed rotors in previous "unified" approaches. This approach overcomes the limitations of earlier unifying efforts by permitting the application of orbit and/or weight constraints at any balancing speed not just at critical speeds. Modal trial weight sets for subsequent balancing can be predicted which will not disturb previously balanced speeds. An orbit constraint applied at an off-critical speed will only affect that individual orbit, but if applied at a critical speed, will affect all orbits and constrain the entire mode shape. In addition, correction weights may be constrained at any balance speed if, for example, a balance plane becomes unavailable (i.e. maximum weight removal limit reached) or if redundant planes exist. This method provides an analytic extension of the general influence coefficient methods and offers a least squares formulation that incorporates the constraints within the optimization procedure.

A special test facility is described which has been used to evaluate this approach to balancing. As

demonstrated, the analytic approach has been fully tested and found to be easily implemented. Test results compare favorably with the analytic methodology offered. No special problems have been encountered either in software implementation or obtaining excellent correlation from test rig experimental results.

INTRODUCTION

Two widely used and distinct linear approaches to balancing have been developed within the past 20 years. One is the modal approach, which has been widely accepted by industry and has been very effective in balancing a wide variety of high-speed rotating equipment [1].* The other approach consists of a group of balancing procedures, generally classified as influence coefficient methods, which has also attracted many supporters [2,3]. Recent publications have focused on the attractiveness of a combined or so-called "unified" approach, Darlow et al., [4,5]. The unified technique defines the use of modal trial weights which do not disturb previously balanced critical speeds (modes) and that are used to determine influence coefficients. The modal trial and correction weights are prescribed from experimentally determined influence coefficients, thus coupling the advantages of a modal technique with the influence coefficient approach.

These "unifying" efforts have encountered limitations in determining correction weights. The earlier approachs are, in effect, modal methods since they are predicated on the control of all orbits at a speed with a single modal weight set. Generally, such a procedure is not sufficient for balancing at off-critical speeds, such as balancing gas turbines at operating speed. To address this limitation, a final trim balance, using standard influence coefficient balancing, has been recommended [4]. It is apparent that these unifying approaches use a form of modal balancing to traverse critical speeds and the standard influence coefficient approach for trim balancing. This procedure is limited when general coupled modes are present or when synchronous orbit control is required at both the critical speed and at off-critical speeds. A truly unified approach would offer a single, integrated solution form for both steps and allow the engineer to evaluate the

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*Numbers in brackets refer to references.

trade-offs between balancing at the critical speeds and at off-critical speeds [6]. If modal constraints are to be imposed, the general flexibility of the influence coefficient balancing method should be extended to include such constraints.

This work describes the development of a general analytic approach to constrained balancing that is consistent with past influence coefficient methods (weighted least squares, point speed, etc.). The approach uses Lagrange multipliers [7] to impose orbit and/or weight constraints; these constraints are combined with the least squares minimization process to provide a set of coupled equations that result in a single solution form for determining correction weights. Proper selection of constraints results in the capability to: 1) balance higher speeds without disturbing previously balanced modes, thru the use of modal trial weight sets, 2) balance off-critical speeds, and 3) balance decoupled modes by use of a single balance plane. Furthermore, if no constraints are imposed, this solution form reduces to the general weighted least squares influence coefficient method.

This paper includes test data generated from a balancing rig operated at Mechanical Technology Incorporated. The test facility was used to examine the use of the general constrained balancing procedure and application of modal trial weight ratios. The data are compared to results obtained by using the general weighted least squares influence coefficient method. As the rig was designed to traverse up to two critical speeds, the algorithm for modal balancing, as well as off-mode correction, may be fully examined.

ANALYTIC DEVELOPMENT

Consistent with past influence coefficient approaches to balancing, a linear relationship is assumed. Defining the orbit response vector as η, the influence coefficient matrix as [A], and T as the weight vector, this basic relationship is given by:

$$\eta = \eta_o + [A]\, T \tag{1}$$

where η_o represents the synchronous response orbits prior to balancing. The response vector, influence coefficient matrix, and weight vector are represented as complex quantitites.

Orbit constraints are specified by the orbit constraint vector D_o, and the orbit constraint matrix $[G_o]$, as:

$$[G_o]\eta = D_o \tag{2}$$

Similarly, balance weight constraints are specified by the weight constraint vector D_w, and the weight constraint matrix $[G_w]$ as:

$$[G_w]\, T = D_w \tag{3}$$

By defining a weighted summation of the residual orbits which includes the Lagrange balance weight and orbit constraints, the following relationship holds:

$$S = \eta^T\, [W]\, \eta + ([G_o]\eta - D_o)^T \lambda_o + ([G_w]T - D_w)^T \lambda_w \;{**} \tag{4}$$

Where $[W]$ is the least squares weighting matrix, and the vectors λ_o and λ_w are the Lagrange multipliers for orbit constraints and weight constraints respectively. The solution to the constrainted balancing problem requires minimization of Equation (4) with respect to λ_o, λ_w and T. Thus, by substituting Equation (1) into Equation (4), and taking the appropriate partial derivatives, the following Equations result:

$$[\bar{A}]^T[W]\eta + ([G_o][A])^T \lambda_o + [G_w]^T \lambda_w = 0 \tag{5}$$

**Superscript T is used to indicate matrix transpose and bar is used to indicate complex conjugate.

$$[G_o]\eta - D_o = 0 \tag{6}$$

$$[G_w]T - D_w = 0 \tag{7}$$

Further substitution of Equation (1) into Equations (5) and (6) provide the following relationships:

$$([\bar{A}]^T[W][A])T + (G_o[A])^T \lambda_o + [G_w]^T\lambda_w = -[\bar{A}]^T[W]\eta_o \tag{8}$$

$$[G_o]\,[A]T = D_o - [G_o]\eta_o \tag{9}$$

Equations (7), (8) and (9) are a set of simultaneous linear equations in λ_o, λ_w, and T. The solution of these equations for T satisfies the orbit constraints specified in Equation (2), the weight constraints specified in Equation (3) and minimizes the weighted response error term specified in Equation (4). Therefore, correction weights can be determined which satisfy the imposed constraints and minimize the residual response in the least squares sense. This solution form offers the flexibility of specifying orbit and/or weight constraints. Furthermore, if orbit constraints are imposed which require that the correction weight not disturb previously balanced speeds, the predicted correction weights are modal sets.

Modal trial weight ratios can be calculated using a formulation similar to that shown in equations [1] - [6]. That is, a modal weight constraint is applied and orbits are minimized. In this case the constraint matrix in equation [3] is replaced by a modal constraint matrix $[G_M]$ The resulting unweighted equations analogous to [7] and [8] are:

$$([\bar{A}]^T\,[A])T + [G_M]^T\,\lambda_M = 0 \tag{10}$$

$$[G_M]T = D_M \tag{11}$$

With this solution methodology, the user has great latitude in specifying the constraints as seen in the flow chart shown in Figure 1.

DESCRIPTION OF TEST FACILITY

An experimental rig was used to verify the previously discussed analysis. This rig is shown schematically in Figure 2. It consists of a shaft with overall length 660.0 mm (26 in.) supported on grease packed ball bearings. There are four disks on the shaft - two between the bearings and one outboard of each bearing. The disks between the bearings are made of aluminum and consist of a hub and rim with overall outside diameter 101.6 mm (4.00 in.). The overhung disks are made of steel and are 76.2 mm (3.00 in.) thick and have an outside diameter of 101.6 mm (4.00 in.). Each disk has 36 equally spaced circumferential holes for insertion of trial or correction weights. The rig is driven through a shaft mounted pulley and drive belt. The belt is driven with a universal motor and variable AC power supply. The maximum speed with this drive system is 10,000 rpm.

An undamped critical speed analysis of the rig indicated that there are two criticals below the maximum rig speed of 10000 rpm and are predicted to occur at 4648 and 6156 rpm. These two modes are flexible and the predicted mode shapes are shown in Figure 3. The rig is instrumented with eddy curent displacement probes at the disks in both horizintal and vertical directions. A fiber optic probe and signal conditioning circuit was used to acquire timing signal. Data from these probes was displayed on oscilloscopes. The synchronous vibration components (amplitude and phase with respect to the timing mark) were acquired digitally using a PDP 11/03 based data acquisition system.

Least Square Balancing

The unbalanced rotor was run to a speed where orbits began to grow rapidly to determine a baseline unbalanced condition. The resulting response is shown in Figure 4 for probe #1. Due to excessive vibration, it was not possible to exceed 4400 rpm. From this baseline condition, 4400 rpm was chosen as the balancing speed. Using the general least square influence coefficient method, the rotor was balanced as shown in Figure 4.

Constraint Balancing

Following the least squares balancing, a series of constrained balancing tests, as outlined in Table 1, were conducted.

Case No.	Number of Constraints	Condition
1	1	Weight Constraint at Plane #2 with 1.0 gm at 330° to simulate maximum weight removal limitation
2	2	Weight at Plane #2 with 0.9 gm at 15° Orbit at probe #1 with 0.0 mm
3	1	Modal Trial Weight Set Prediction. Plane #1 and #3 will be selected to have modal trial weight set installed.

Table 1 Constrained Balance Condition

As indicated in the following paragraphs, the constraints imposed were successfully matched by the test results.

Case 1: In Case #1, a single weight constraint was imposed. The correction weight for plane 2 was constrained to be 1.0 gm at 330°. The results for this balance run are shown for probe #1. (Figure 5). The response is slightly better than the least square balance response.

Case 2: Both weight and orbit constraints are imposed in this case. That is, the correction weight is specified at 0.9 gm at 15° in plane #2, while the orbit at probe #1 at 4400 rpm was constrained to be zero. The results for this run are shown in Figure 6. The response for probe 1 indeed approaches zero as specified.

Case 3: After balancing the rotor at 4400 rpm, trial weights were predicted for balancing at a higher speed which would not disturb the balanced state at 4400 rpm. Constraints were again used to specify that the modal trial weight set was to be installed in planes #1 and #3 only. The weight ratios were predicted as:

- Plane #1 1.89 at 358°
- Plane #2 0.00
- Plane #3 2.63 at 180°

The response was plotted as shown in Figure 7 after the following modal weigths were installed.

- Plane #1 0.70 gm. at 360°
- Plane #3 0.98 gm. at 180°

The result indicated that the balanced speed(4400 rpm) was not disturbed.

CONCLUSIONS

The analytic formulation of the constrained balance procedure as developed provides a balancing methodology that offers the flexibility of a unified approach yet removes restrictions imposed by earlier unified approaches. In particular, it offers the capability to impose constraints at critical and off-critical speeds. Since this method uses a single, consistent solution form, it allows for the evaluation of trade-offs between these balance conditions. Furthermore, it is totally compatible with influence coefficient methods and, in fact, reduces to the general weighted least squares solution when constraints are eliminated.

No limitations for determining the influence coefficients are imposed. Accordingly, using "modal" trial weight sets and/or redefining influence coefficients to be consistent with modal sets is possible. Therefore, one may use the full capabilities of modal weight sets for determing influence coefficients and directly integrate these results with the constrained balancing algorithm.

The test data has verified that this method offers a viable approach to balancing high-speed rotors; it also offers a great deal of flexibility to engineers when confronted with difficulties common to balancing rotating machinery. For example, orbits can be specified at any speed, to allow for smooth machine operation at the design operating speeds, control of bearing loads, etc. Balance weights can also be specified to compensate for balance planes which cannot accommodate more correction weight addition or removal. Furthermore, combinations of these types of constraints can be imposed simultaneously.

Application of modal trial weight sets will not affect the lower speeds that have already been balanced. This practice is especially useful when rotors are very sensitive to imbalance. In fact, the application of modal trial weight set can reduce the number of trial weight runs since the set will not disturb those balanced lower modes.

ACKNOWLEDGEMENTS

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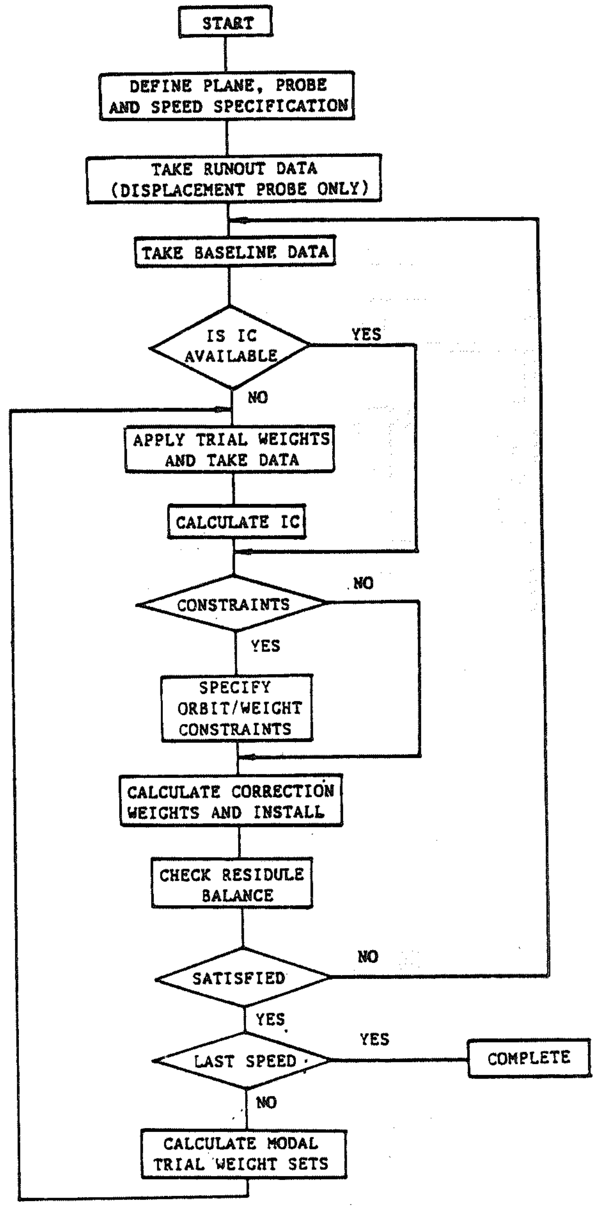

Fig. 1 Balancing Procedure Flow Chart

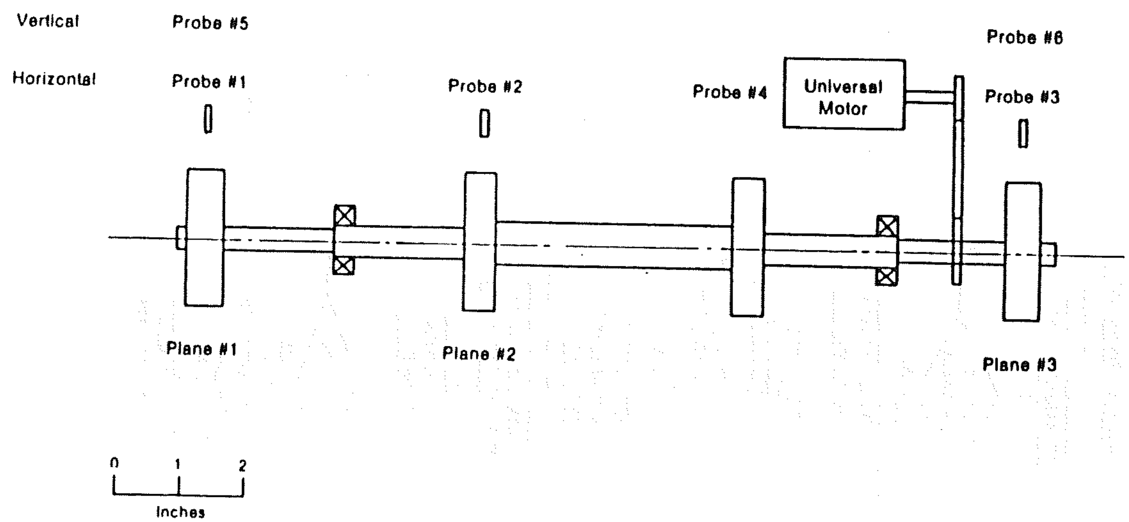

Fig. 2 Schematic of Balancing Rig

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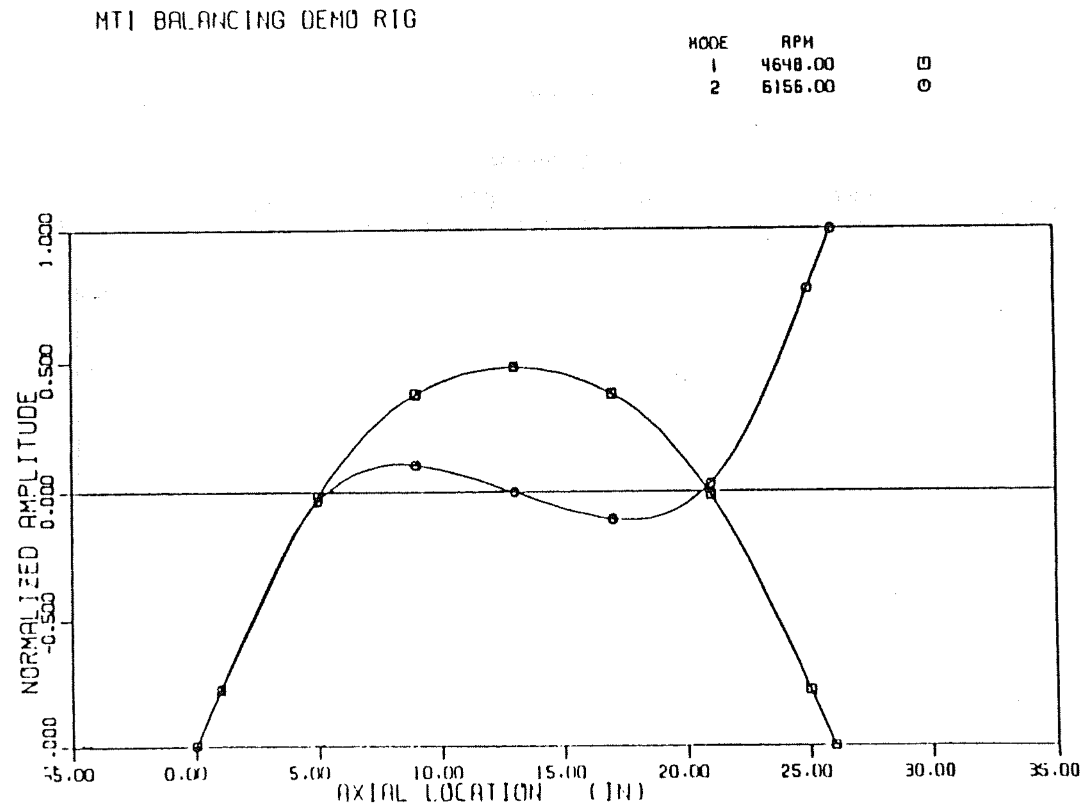

Fig. 3 Mode Shape Plot

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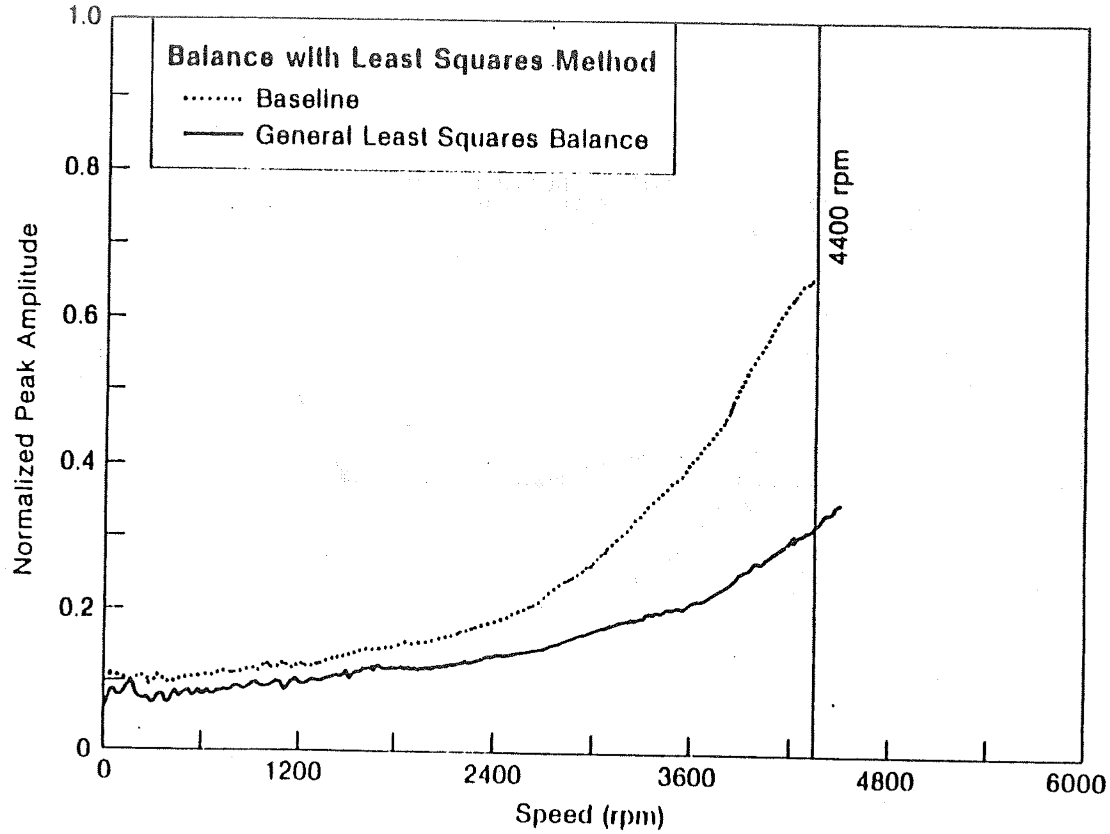

Fig. 4 Response vs. Speed (Balance with General Least Squares Method)

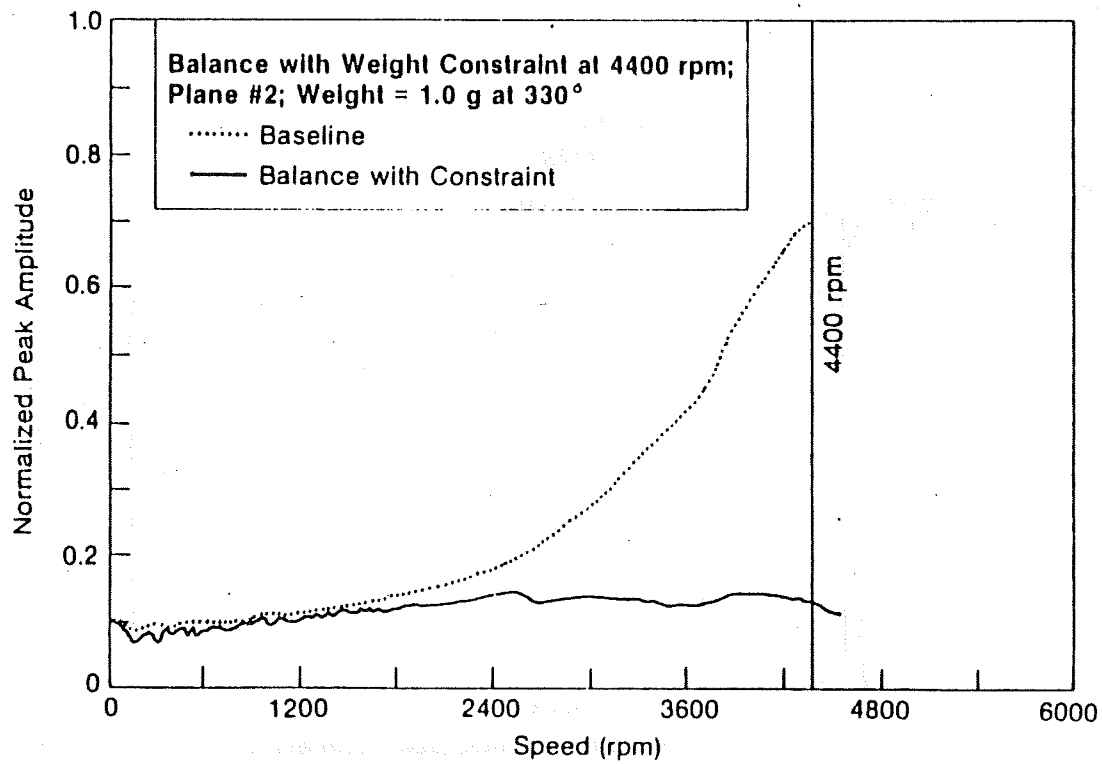

Fig. 5 Response vs. Speed (Balance with Weight Constraint)

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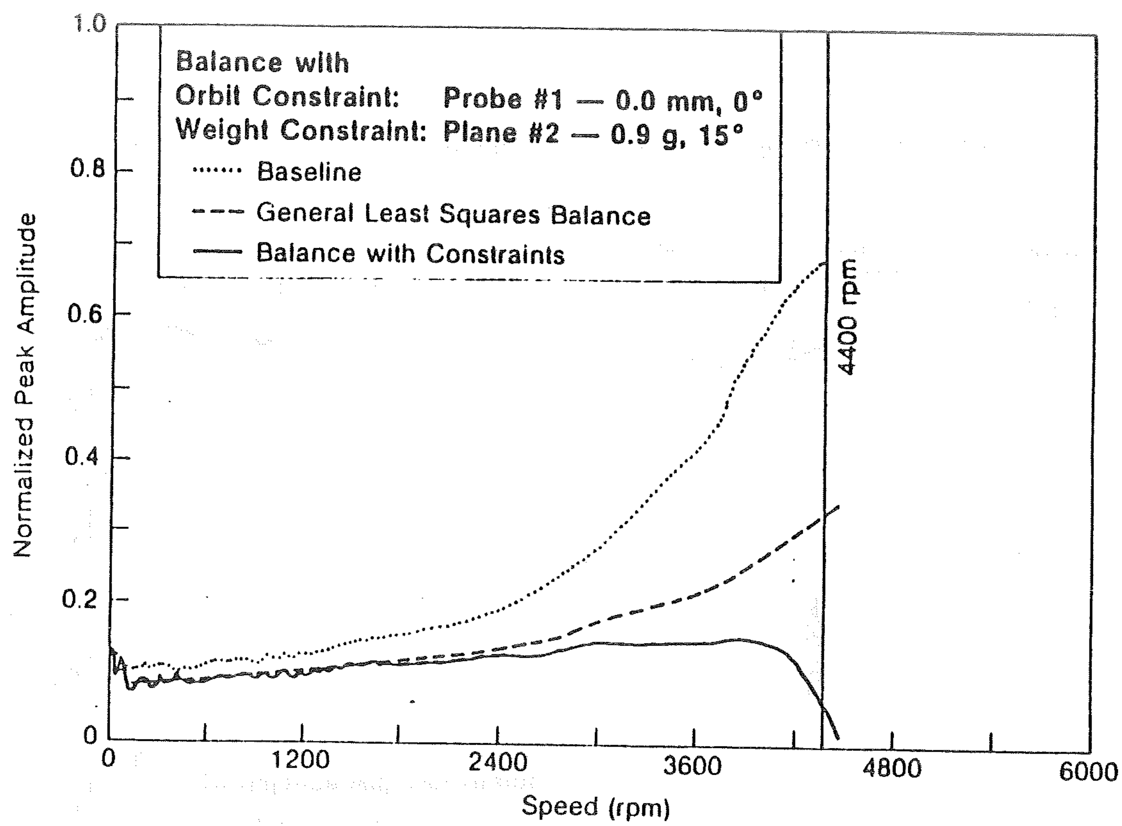

Fig. 6 Response vs. Speed (Balance with Orbit and Weight Constraints)

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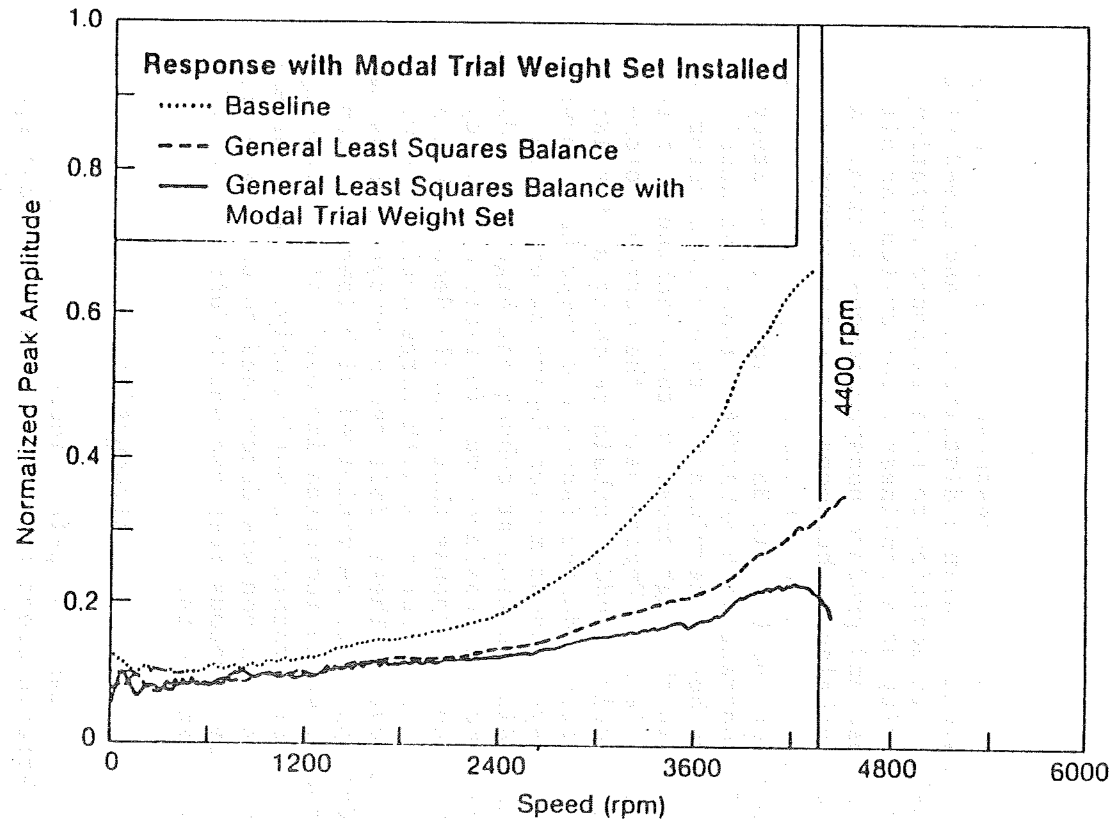

Fig. 7 Response vs. Speed (with Modal Trial Weight Set Installed)

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