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Gary T. Chapman and Murray Tobak

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OBSERVATIONS, THEORETICAL IDEAS, AND MODELING OF TURBULENT FLOWS  
--PAST, PRESENT, AND FUTURE--

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INTRODUCTION

Turbulence (or chaos) is one of the oldest and most difficult open problems in physics. Although the subject of this review is turbulence in the field of fluid dynamics, the problem of turbulence pervades many other fields; e.g., cosmology, the structure of the universe. At one time or another, it has occupied the minds of many of the great physicists, particularly in the early part of this century. The problem is so difficult that it has even defied the formulation of a consistent and rigorous definition. In this paper, we shall review the history of the subject and point to recent developments, as well as postulate future directions. To avoid merely enumerating a succession of isolated research events and accomplishments, the review will be presented in a context of the interactions between observations, theoretical ideas, and the modeling of turbulent flows. The context needs further elaboration, but first a few comments are in order regarding a basic premise of the review.

The basic premise is that turbulence can be understood within the framework of the continuum assumption of fluid dynamics. Accepting the premise implies accepting that the Navier-Stokes equations are a complete mathematical description of fluid flows, and hence, capable of describing turbulent flows. There are some experimental facts that might shed doubt on the validity of the assumption. For example, small amounts of long-chain polymers in water have a significant effect on turbulent properties, even though the polymers are dispersed and have dimensions significantly smaller than the dissipation scales of turbulence. Additional recent developments also may raise questions regarding the continuum assumption. Although it needs constant re-examination, the continuum assumption has formed the basis for the study of turbulence over its entire formal history. With precautions and reservations duly noted, the assumption will be accepted as the basis for this review as well. Acceptance of the Navier-Stokes equations also may raise criticism on mathematical grounds, inasmuch as existence has not been proven for solutions of the three-dimensional initial-value problem. Although no examples are known, the possibility cannot be ruled out that solutions become singular, especially at Reynolds numbers representative of fully-developed turbulence. This would imply that additional principles need to be introduced to ensure a complete theory. We shall proceed as

if this is not the case. Lanford [1] has compiled an excellent list of the presuppositions entailed by adoption of the Navier-Stokes equations as the framework for understanding turbulence.

To aid in the discussion, the context of interactions between observations, theoretical ideas, and modeling is illustrated in figure 1. First, let us define the terms: The term "observations" here includes not only empirical data from observations of the physical (real) world, but also empirical data from observations of computer simulations representing solutions of the full Navier-Stokes equations (hence the above-mentioned need for the continuum assumption) or other suitable simulations to be noted later. The term "theoretical ideas" is used here to denote the realm in which observations are transformed into (normally nonmathematical) idealizations or conceptualizations: e.g., the concept of a continuum or of an incompressible fluid. Conceptualizing or theorizing is vital to the study of turbulence, as it is to the study of any scientific discipline, but it represents both a positive and a negative aspect. On the positive side, it is essential that theoretical ideas be postulated, both to further the mathematical steps which follow, as well as to provide hypotheses against which to cast the observations. Theoretical ideas literally provide "a way of seeing." It is this aspect that also may be negative, for a way of seeing may color or bias our observations. These two aspects of the realm of theoretical ideas will surface as major points in the discussions to follow. Finally, the term "modeling" is used to denote the realm in which theoretical ideas are placed within a formal system by means of mathematics.

Information flows back and forth between each of the three realms. For example, observations typically lead to a theoretical idea which provides a basis for both new observations and a mathematical model. The mathematical model can be tested against observations and also provides implications against which to test the theoretical idea. Observations used to test the model can also lead to changes in the model. Once the study of a discipline has begun, it is difficult to tell in which of the three realms it originated. It is clear, however, that the role of theoretical ideas is a very powerful one in that it shapes our approach to the other realms in the sense of dictating "a way of seeing." (On this important point, see also Liepmann's essay [2], to which our own views are much indebted.) In this review, we shall try to show how the "way of seeing" has influenced the study of turbulence, first, by examining the subject in an historical context, and second, by examining recent developments. Finally, some future developments will be postulated.

## HISTORICAL PERSPECTIVE

Observations of turbulence are as old as recorded history. The Bible, for example, contains several references to turbulence or chaos. Leonardo da Vinci was

intrigued by turbulence, as his sketch reproduced in figure 2 (circa 1500) [3] indicates. But, the modern scientific study of turbulence dates from the late 1800s with the work of Osborne Reynolds. In reviewing the subject from that date to the present, one is struck by the appearance of three distinct movements, each of which (despite some overlap), can be characterized by a definite point of view with a reasonably well-defined beginning. The earliest of these, which has a strong non-deterministic flavor, will be referred to as the statistical movement; the next, which is predominantly observational, will be referred to as the structural movement; and the most recent will be called the deterministic movement. The three movements, with a few key events noted, are sketched in figure 3.

### Statistical Movement

As noted above, Osborne Reynolds' observations of transition in pipe flow in 1883 [4] mark the beginning of the scientific study of turbulence. Illustrative results of a repetition of Reynolds' experiments [5] are shown in figure 4. Reynolds' observations led him to decompose the velocity field into a mean flow plus a perturbation. Considering the perturbation flow either too complicated or incomprehensible, he time-averaged the Navier-Stokes equations on the basis of the decomposition and arrived at what are today called the Reynolds-averaged Navier-Stokes equations [6]. The averages of products of the perturbation terms appear in the mean-flow equations as (what are now called) Reynolds stresses. Reynolds' view that the perturbations (and hence, the observations of the flow's chaotic structure), were unpredictable or incomprehensible in detail was not merely the result of observations, but was a view that would find support in the generally accepted world-view of the time in which turbulence or chaos was considered synonymous with disorder (the term used in Webster's dictionary) or unpredictability (nondeterministic) or incomprehensibility. This viewpoint, which is formalized in the statistical theory of random perturbations, was bolstered by the success of the statistical mechanics approach to the kinetic theory of gases (e.g., Jean's book on the topic [7]). The statistical point of view gained even further support by the great success achieved in theoretical physics on the introduction of quantum mechanics.

The statistical view of turbulence (a theoretical idea) had two principal effects: First, it encouraged focusing further observations of the flow on means and various averages. Second, because the Reynolds-stress terms consisted of averages of products of perturbation quantities, it suggested that the extension required to complete the modeling had to involve the next higher moment, requiring in turn an even higher moment, and so forth, leading, of course, to the celebrated problem of closure. The closure problem has proven so formidable in modeling that, in the face of practical exigencies, it has been more or less set aside in favor of

phenomenological modeling. The latter has had considerable success in the practical realm, but has had essentially no impact in the realm of theoretical ideas.

The emphasis on observations of means and various averages led to significant development in instrumentation, particularly with regard to hot-wire anemometry. Development of the laser in the 1960s made possible the introduction of laser-doppler velocimetry (LDV), heralded as a great new tool in the study of turbulence. But nothing fundamentally new in the way of theoretical ideas has yet resulted from its introduction. Our contention is that the principal impediment is the statistical viewpoint itself; the use of the tool was prescribed too narrowly in accordance with its perceptions.

There is an extensive body of literature on the mathematical study of turbulence within the statistical theoretical framework. Included is not only the work of Reynolds, but that of Taylor, von Karman, Heisenberg, Kolmogoroff, Loitsianskii, Kraichnan, to mention only a few. The book by Hinze [8] contains an excellent summary of this work. Summaries of more recent work can be found in Monin and Yaglom [9] and in the contribution to this conference by Pouquet [10]. Kolmogoroff's "five-thirds law" [11], formulated for the statistical regime consisting of "locally isotropic turbulence," is an example of a particularly successful result of theoretical analysis within the statistical framework. It is successful in part because it ignores the regime of scales where most of the energy resides in most turbulent flows, and where the nature of the source of the turbulence is still evident. Even in the regime to which it ostensibly applies, the "five-thirds law" has been criticized on the basis that the turbulence is not truly isotropic, but intermittent. Hence, the law needs to be corrected for the nonspace-filling nature of turbulence [12]. Its blindness to these structural facts is precisely the disability of the statistical theoretical idea.

In our view, the principal shortcoming of the statistical approach is that the introduction of the statistical idea (predicated on a nondeterministic theoretical basis) at such an early stage of the study inhibits the interactions which otherwise would occur between observations, theoretical ideas, and modeling. The consequence is a paucity of imagery or structure about which to conceptualize. Our argument is not that statistical or averaging methods should have no role in the study of turbulence, but only that their introduction at the beginning of the study tends to stifle the flow of information and prevent conceptualization. Historically, this situation, coupled with the difficulties arising from the closure problem, encouraged the introduction of a line of research having an even more detrimental effect on theoretical ideas: phenomenological modeling.

Boussinesq's replacement of the molecular viscosity coefficient in the Navier-Stokes equations by a turbulent viscosity coefficient (eddy viscosity) in 1877 [13] and Prandtl's subsequent modeling of that term by means of the mixing-length idea in 1925 [14] had the effect of discouraging further theoretical ideas. First, Boussinesq's introduction of the eddy-viscosity concept entailed an inextricable

confounding of flow properties and material properties. Second, introduction of the mixing-length idea through analogy with ideas from the kinetic theory of gases inadvertently gave the impression that a powerful and consistent statistical approach had been established. The widespread adoption of these ideas in modeling the equations governing turbulent mean flows broke the link with the framework of the original Navier-Stokes equations, so that ideas from the latter framework concerning, e.g., new structures arising out of instabilities, became more or less irrelevant to turbulence studies. It was in this sense that adoption of the eddy-viscosity and mixing-length ideas effectively discouraged further theoretical ideas. Despite their great success in practice, the eddy-viscosity and mixing-length ideas, in fact, have serious shortcomings. The eddy-viscosity idea, for example, raises difficult conceptual problems which are, in effect, artificial, being the consequence of the nonphysical confounding of flow and material properties. Additionally, the conditions justifying the mixing-length idea rest on the assumptions of kinetic theory (small units traveling relatively long distances between interactions), and these conditions are not completely fulfilled by the properties of turbulent eddies. A more thorough review of phenomenological models and their application can be found in [15], and in papers at this conference by Launder [16] and W. C. Reynolds [17].

In the 1940s and 1950s, observations increasingly pointed to the structural content of turbulence. There was increasing evidence of intermittency, for example, and the realization that vorticity tended to concentrate in localized intervals of space and time (e.g., Batchelor and Townsend [18]). But the power of the statistical viewpoint--the ruling theoretical idea of the period--was sufficient to shape even the perception of the evidence of structure. An example is Landau's influential description of the origins of turbulence [19]. The study of laminar instability of flow past a flat plate had led to the discovery of the Tollmien-Schlichting waves and their measurement by Schubauer and Skramstad [20]. Although their occurrence was a deterministic, perfectly predictable event, in 1944 Landau postulated that turbulence was the result of an indefinitely large sequence of such events, and hence, in effect, unpredictable in detail.

The example illustrates how a ruling theoretical idea may shape our approach to the realms of observations and modeling in the sense of dictating "a way of seeing." Our severest criticism of the statistical movement is that it has resulted in a structureless theory having little power of conceptualization.

#### Structural Movement

This movement is dominated by observations. Early experimentalists studying turbulence noticed that their observations were not entirely in keeping with the purely nondeterministic statistical viewpoint. Observations of Tollmien-Schlichting

waves as evidence of initial instability in a transitional flow [20] have already been mentioned, as have been the observations of Batchelor and Townsend [18] regarding nonuniformities of vorticity in homogeneous isotropic turbulent flows. From the late 1950s to the present, a virtual flood of observations has been published concerning the structures that occur in turbulent flows. The following are outstanding examples: (1) The turbulent spot, first reported by Emmons [21]. Figure 5, taken from the work of Cantwell, Coles, and Dimotakis [22], is illustrative of this structure in turbulence. More recent detailed measurements have brought out additional structural features [22]. (2) Structures in wall-bounded shear flows, first observed by Kline and Runstadler [23]. Figure 6 illustrates their appearance [24]. (3) Structures in turbulent free shear layers, investigated extensively by Roshko [25] and others (cf. fig. 7 from [5] and [26]). Cantwell [27] has published an excellent review of these experimentally observed turbulent structures.

The structural movement has also included observations of computer simulations of turbulent flows. Examples are the results of Rogallo [28] for homogeneous isotropic turbulence, and those of Moin and Kim [29] for turbulent channel flow. A simulated hydrogen-bubble observation of the latter flow [30] is shown in figure 6, where it is compared with the already noted experimental observation of Kim, et al., [24]. The use of computer simulations has a deterministic character inasmuch as, strictly speaking, the computations are based on deterministic equations. However, many of the early analysts of these simulations adopted the same classical statistical methodology as that employed by the experimentalists. That is, for the most part, they measured only means and various averages. Some experimentalists were actively trying to measure and characterize the structures they were observing by developing a measurement methodology that reflected both the presence of coherent structures and their apparently random occurrence. These included conditional sampling techniques such as the method developed by Blackwelder and Kaplan [31], and the proper orthogonal decomposition method developed by Lumley [32,33]. The methods differ in the degree of subjective bias imposed by the experimenter, with the latter method having essentially none. Computer simulations of turbulent flows also have been analyzed by the same methods (e.g., [33] and [34]). Although they admit the presence of coherent structures, all of these methods contain the implicit assumption that the occurrence of structures is governed by random (and, thus probably incomprehensible) events. Full realization of the possibility of a deterministic and comprehensible chaotic flow behavior does not yet seem to have occurred within the structural movement.

The principal contribution of the structural movement has been the recognition of the presence and importance of structures in turbulence. Dryden [35] had recognized the possibilities in an early review (cf. Roshko's attribution [25]) as the following quotation makes clear: "It is necessary to separate the random process from nonrandom processes. It is not yet fully clear what the random elements are in turbulent flow." Despite its promise, the structural movement to date has had two



major shortcomings. The first is that no new theoretical ideas have emerged that could be translated into formal mathematical models. The prevailing viewpoint remains one of unpredictability or indeterminism, in which the coherent structures are conceived as having been sprinkled about randomly in time and space. In addition, the structures that have been observed cannot be fitted easily into the statistical models currently in use. Landahl [36] (cf. also his contribution to this collection [37]) is one of the few theoreticians who have attempted to incorporate structures within a compatible statistical model. Bushnell (cf. [38], this collection) has been instrumental in finding ways to use structural observations practically to improve aircraft performance. The second shortcoming is the Achilles' heel of the structural movement: the jungle of observational detail, lacking the ordering hand of theory. The difficulty stems from the unassimilated mixture of random and deterministic elements in the generally accepted "way of seeing" the coherent structures. While the structures are assumed to be randomly distributed in time and space, each occurrence is assumed governed by a locally deterministic cause (e.g., a local instability). A consistent, overall theory has been lacking which has the possibility of assimilating the random and deterministic elements into a single viewpoint allowing, e.g., deterministic chaos.

In summary, the structural movement has demonstrated the presence and importance of structures in turbulence, but so far, has not resulted in new theoretical ideas having the power to abet modeling. The principal criticism is precisely the inverse of that of the statistical movement: in place of theory without structure, the result to date has been structure without theory.

### Deterministic Movement

This is the most recent movement, although its origins date from the pioneering essays of Poincaré, the principal one bearing the title "On the Curves Defined by Differential Equations" [39]. The particular way of seeing inspired by Poincaré, strongly geometric as expressed in the language of topology, became known as the qualitative theory of differential equations. The field was advanced by the research of Andronov and his colleagues [40] who introduced the useful notions of "topological structure" and "structural stability." From the same line stems bifurcation theory, showing how structures may change with changes in conditions. The comprehensive review of Sattinger [41] demonstrates the extraordinary range of scientific disciplines in which bifurcation theory now plays a role. Applications to hydrodynamics are exemplified by the works of Joseph [42] and Benjamin [43]. Taken together, "nonlinear dynamical systems" is perhaps the best descriptive title for this body of theory. Its origins and some of its philosophical implications are traced in a recent interesting essay by M. W. Hirsch [44].

For the purposes of this review, dedicated to turbulence studies, the deterministic movement will be dated by the work of Lorenz in 1963 [45] and that of Ruelle and Takens in 1971 [46]. Lorenz discovered, via numerical computation, that a simple dynamical model of a fluid system yielded flow properties having bounded aperiodic behavior in time of a form apparently so chaotic, yet deterministic, that (in current terminology) it is said to indicate presence of a "strange attractor" (see fig. 8). Sparrow's book [47] contains a thorough study of the Lorenz equations (see also a thoughtful review of the book by Guckenheimer [48]). Unaware of Lorenz's results (but expert in the theory of dynamical systems), Ruelle and Takens independently proposed strange-attractor behavior as a model for turbulence, and argued via mathematical analysis that the turbulent state would be reached after the fluid system had undergone a finite and small number of bifurcations. A more accessible and updated account of the Ruelle-Takens thesis, fittingly first presented at a symposium honoring the mathematical heritage of Poincaré, recently has been published by Ruelle [49]. An example of a fluid system typifying the sequence proposed by Ruelle and Takens is the Taylor-Couette flow shown in figure 9 [50]. Within the same period (1967) Mandelbrot [51] posed the idea of non-space-filling curves (which he had named fractals) as a model for explaining intermittency in turbulence. Also during this period ideas were proposed for handling problems involving rapid transitions in the important scales of structures, directed particularly towards application to phase transitions in solids (cf., e.g., Wilson and Fisher [52]). This methodology goes by the name of "renormalization group theory"; its relevance to turbulence studies has been noted by several authors (e.g., Siggia [53]).

The above account notes the principal elements of the deterministic movement; they will be discussed in greater detail in the next section. Here, their historical significance will be touched on briefly to complete our historical perspective. First, the work on strange attractors has shown that chaotic behavior can occur in even simple deterministic systems (as few as three nonlinear ordinary differential equations). Second, deterministic chaotic behavior can occur after just a few bifurcations of a dynamical system. (Other routes to deterministic chaotic behavior involving, e.g., period-doubling, are possible as well, cf. [54].) Third, the ideas underlying bifurcation theory, strange attractors, fractals, and renormalization group theory provide a rich body of imagery, containing a considerable potential for conceptualization of turbulence structures. Taken together, the first and second points offer the possibility that chaos or turbulence in fluid dynamics can be understood as a state of a simple deterministic system. The third point suggests a basis for the construction of models.

Viewed from the framework of observations, theoretical ideas, and modeling already described, the impact of the deterministic movement can be summarized as follows: It has provided a basis for new observations, as exemplified by the work of Swinney and his colleagues on Taylor-Couette flow (fig. 9 and [50]). It has provided an impetus for mathematicians to return to the Navier-Stokes equations

themselves as a basis for modeling turbulent flows (e.g., Temam, et al., [55]) as well as simpler systems that exhibit chaotic behavior. Finally, the extensive studies of the Lorenz equations have shown the synergistic power of the computer when computational studies are carried out in conjunction with general mathematical theory. This is the eloquent point of Guckenheimer's review [48] of Sparrow's book [47]. The shortcoming of the movement (if what follows can be called a shortcoming for so new a movement) is that the effort to date has focused on simple systems; and, in fluid dynamics, principally on the mechanisms of transition. The application of current ideas to fully-developed turbulent flows has not yet been seriously undertaken, nor is it clearly evident yet that the understanding which has been gained can be converted into successful models for turbulent flows of practical interest. On the latter point, and to offset potential criticism from researchers focusing perhaps too exclusively on the ultimate goal of predictive power, we can do no better than cite the following from Hirsch's essay [44]: "The end result of a successful mathematical model may be an accurate method of prediction. Or it may be something quite different but not necessarily less valuable: a new insight . . ."

## RECENT DEVELOPMENTS

Our account of the deterministic movement took note of four recent developments which need further elaboration. These are bifurcation theory, strange attractors, fractals, and renormalization group theory. As indicated earlier, bifurcation theory and strange attractors emerged as part of the study of dynamical systems that originated with Poincaré. The study of fractals has roots in a number of areas, e.g., the study of Brownian motion and the meteorological studies of Richardson. Mandelbrot's book "The Fractal Geometry of Nature" [56] contains an excellent discussion of these origins. Finally, the study of phase transitions in condensed-phase matter was a principal source of inspiration for renormalization group theory. Each of these developments can be considered only briefly here; there are entire papers devoted to them elsewhere in this collection (cf. the contributions of Spiegel [57], Mandelbrot [58], and McComb [59]). Although they are often treated separately, and the perspective each brings to the subject is important, it is becoming clear that they cohere, and together offer the possibility of reflecting the greater part of many of the complex and chaotic processes in nature.

### Bifurcation Theory

Generally speaking, bifurcation theory is the study of equilibrium solutions of nonlinear evolution equations and how they change with changes in the parameters of

the problem. In fluid-dynamic applications, we are interested in equilibrium solutions of evolution equations of the form

$$\vec{U}_t = H(\vec{U}, \lambda), \quad (1)$$

where  $\vec{U}$  is the velocity vector and  $\lambda$  is a parameter (e.g., Reynolds number, angle of attack, Mach number). An equilibrium solution is taken to mean the solution to which  $\vec{U}(t)$  evolves after the transient effects associated with the initial values have died away. Equilibrium solutions may be time-invariant, time-periodic, quasi-periodic, or chaotic depending on conditions.

Changes in equilibrium solutions can occur at two levels. The first occurs as a result of instability in equation (1). As the parameter  $\lambda$  is varied, a critical value  $\lambda_c$  can be reached beyond which the original solution becomes unstable. New solutions, called bifurcating solutions, appear, some of which may be stable, and some unstable to small perturbations. By stable and unstable we mean the following: If a small perturbation of the solution decays to zero as  $t \rightarrow \infty$ , the solution is said to be asymptotically stable; if the perturbation grows, the solution is said to be asymptotically unstable. Stable branches of bifurcating solutions can be either local or global. A bifurcation solution is said to be local if it can be mapped onto the original solution without cutting the solution space; if it cannot, the bifurcation solution is said to be global. In addition, the bifurcation can be supercritical or subcritical, as illustrated in figure 10. In a supercritical bifurcation (shown by the pitchfork bifurcation), there is at least one branch of stable bifurcating solutions that is continuous with the original solution at the bifurcation point  $\lambda_c$ . Thus, for a small change in  $\lambda$  across  $\lambda_c$ , there is a stable bifurcating solution that is  $O(\Delta)$  close to the original solution such that as  $\lambda - \lambda_c \rightarrow 0$ ,  $\Delta \rightarrow 0$ . This is not the case for a subcritical bifurcation shown on the right of figure 10. Here, for a small change in  $\lambda$  across  $\lambda_c$ , there is no branch of stable bifurcating solutions that is continuous with the original branch. This type of bifurcation normally leads to hysteresis behavior because the critical point for the upper branch in the case shown does not occur at the same value of  $\lambda_c$  as it does for the lower branch. The symmetrical bifurcation curves shown in figure 10 often result from idealized problems. In practice, there is less enforced symmetry, or there is a boundary condition, or a scale that was suppressed in the idealized problem. When these are brought into consideration, the idealized bifurcation diagram may undergo an unfolding. This is illustrated in figure 10 with the pitchfork. The idealized pitchfork has the following form (to leading order)

$$\psi^3 - \lambda\psi = \psi(\psi^2 - \lambda) = 0, \quad (2)$$

whereas the general (unfolded) bifurcation to this order has the form

$$\psi^3 + a(\lambda)\psi^2 + b(\lambda)\psi + c(\lambda) = 0. \quad (3)$$

For the case shown in figure 10,  $a = 0$ , and  $b$  represents the effect of a small imperfection. Bifurcation in the case of Taylor-Couette flow between rotating cylinders has this form, in which the  $c$  term is the result of including ends in the concentric cylinders, rather than treating the idealized problem in which the ends are at plus-and-minus infinity. The first stage of the (idealized) Taylor-Couette flow problem typifies a common type of bifurcation in which an original time-invariant equilibrium solution is replaced at the bifurcation point by another time-invariant equilibrium solution, in this case one describing the Taylor vortices. A second type of bifurcation is the "Hopf" bifurcation in which the original time-invariant equilibrium solution is replaced by a branch of stable equilibrium solutions which are time-periodic solutions. The Hopf-type of bifurcation is common in aerodynamics, for example, the Karman vortex street in the wake behind a circular cylinder for  $Re > Re_c \approx 50$ . A third type of bifurcation of great interest occurs when a quasi-periodic equilibrium solution is replaced by a bounded aperiodic solution having chaotic properties. Taylor-Couette flow for  $R/R_c = 23.5$  [50] illustrates this type of behavior. It is suggested that this behavior indicates the presence of a strange attractor--the subject to be treated in the next section.

The second level at which changes in equilibrium solutions occur focuses on the class of equilibrium solutions that is time-invariant. Here, we concentrate attention on the singular points in the equilibrium flows where  $\vec{U} = 0$ . With  $\vec{U}_t = 0$  in equation (1), we can recast equation (1) to directly describe particle trajectories or streamlines:

$$\vec{U} = \vec{X}_t = G(\vec{X}, \lambda), \quad (4)$$

where  $\vec{X}$  is the spatial coordinate of the fluid element. Here, as  $\lambda$  crosses  $\lambda_c$ , a singular point may bifurcate into multiple singular points, or a new pair of singular points may appear, or a pair disappear. However, bifurcation at this level need not imply nonuniqueness in the governing flow equations. Equilibrium solutions may remain stable and unique on either side of  $\lambda_c$ . The bifurcation of singular points in the flow will be referred to as structural bifurcation. All structural bifurcations are global in the mapping sense described earlier. Structural bifurcations in fluid flows are described with examples in [60].

The general topic of bifurcation theory has received considerable attention in the past few years with development of an extensive body of literature. Examples of this genre of work are given in [41,61,62].

### Strange attractors

The recognition that bifurcation to a bounded aperiodic solution can occur, indicating presence of a strange attractor, represents a significant step in the

study of turbulence. Strange attractors appear in forced dissipative systems and can occur with relatively small nonlinearities (cf. the Lorenz system in fig. 8). The following account is an attempt to give a more geometric sense to the term.

First, we need to introduce some additional terminology. By the state of a fluid-dynamic system, we shall mean a complete specification of the velocity field at an instant in time. The space of all states is the state space. The term orbit will refer to a solution of the differential equations determining points in state space (i.e., the Navier-Stokes equations), regarded as a curve in state space, and the solution flow will refer to the motion on the state space that advances each point along its respective orbit. The term equilibrium solution that was introduced earlier in connection with bifurcation theory, represents the long-term behavior of a solution flow after transients have died away. Time-invariant equilibrium solutions can be represented as fixed points in state space. Time-periodic equilibrium solutions can be represented as closed paths (i.e., circles) in state space. Equilibrium solutions having two incommensurable periods are representable on tori (called 2-tori) in state space. These orbits are called attractors if orbits starting sufficiently close to them converge to them. Convergence in this sense is equivalent to the notion of asymptotic stability introduced earlier.

Whereas the Landau theory of turbulence [19] supposed that tori of increasing dimension ( $n$ -tori) would succeed each other in an indefinite sequence of supercritical bifurcations, Ruelle and Takens [46] argued that beyond a 2-torus, a "strange attractor" would appear on the next bifurcation. Lanford [1] has presented a useful qualitative description of how a model strange attractor might succeed a 2-torus. His description is reproduced in figure 11. The solution flow shown in part (a) is primarily in the direction of the arrows around the torus with relatively small transverse motion. Plotted against time, a solution would appear as noisily periodic. Parts (b), (c), and (d) show successive intersections of a set of solution curves that undergo squeezing and stretching (b), rotation (c), and folding (d) in one circuit of the torus. Four iterations of this "return mapping" produce the complex layered structure shown in (e). As Lanford has noted, the separate phases (a)-(d) would occur simultaneously in real examples. In fact, the outcome depends only on the nature of the mapping which takes a point in the cross-section A into the next place where the solution curve through that point recrosses A. This mapping is called the return mapping or the Poincaré mapping. One assumes that the sequence of return mappings has a limit, and the limit is representative of the strange attractor. It must have the following characteristics: First, solutions "phase-mix" (cf. Joseph [42]). Unlike periodic or quasi-periodic functions, the strange attractor has an autocorrelation function which decays rapidly in time. This is a property shared by all observations of turbulent flows. Second, the strange attractor has a sensitive dependence on initial conditions. Solutions which start out even infinitesimally close together must eventually depart from each other. In Lanford's description of a model strange attractor (fig. 11), it is the

stretching property that ensures the eventual departure of adjacent solutions. Finally, each member of the set of solutions comprising the strange attractor occupies zero volume in state space. It is this characteristic that forces the strange attractor to have noninteger dimensionality, or, in Mandelbrot's terms [63], fractal measure. In Lanford's model attractor, the folding property (d) gives rise to a multilayered structure that does not occupy any volume, and it is this property that accounts for the attractor's noninteger dimensionality. It is also this property that manifests itself in terms of intermittency.

For a more comprehensive study of the various connections between strange attractors and turbulence, the reader is referred to Lanford's review articles [1,64]. Ott's review [65] of strange attractors in a dynamical systems framework has an excellent section on the connection with fractal dimensionality. The connection as a possible property of solutions of the incompressible Navier-Stokes equations was apparently first aired by Foias and Temam [66], and has been recently sharpened [67,68]. The formulation of appropriate measures and dimensional descriptions of strange attractors is a subject of intense current interest [69-71].

## Fractals

The fractal idea as a description of turbulence precedes that of the dimensionality of the strange attractor and, in the geometric form put forward by Mandelbrot [56], has considerable conceptual power. Hence, it will be in that context that fractals and fractal dimensionality will be briefly described. A fractal curve is a curve that is everywhere continuous but nowhere differentiable. An example of a fractal curve is the Brownian motion of a particle. Lest one choose to disregard the fractal idea too quickly, one should recall that the Navier-Stokes equations can be considered an ensemble average over a set of Brownian-motion curves. To illustrate these curves and their properties, two examples have been selected. These are shown in figure 12. The simplest example is the Koch curve. This curve is constructed by the following recursive procedure: take a line one unit long and divide it into three equal segments. Remove the center segment and replace it with two equal segments to form a hat (see fig. 12). This process is repeated recursively on each of the new segments that are formed at successive steps. Now the length of the resulting curve increases without limit as the number of repetitions ( $n$ ) increases without limit, but the curve does not fill up any space. Only a line with apparent texture results. Another way to think of this is to take three of these Koch curves and form an equilateral triangle. Now note that we have a finite area enclosed (an island) by a perimeter (coastline) that is infinitely large. This point is well described in Mandelbrot's book [56] in the chapter entitled "How long is the Coast of Britain?" The following question arises: Is there a way to form a relationship

between the line length and the unit of size at any point in the iteration? There is, and it is as follows: Let

$$L = s^\mu \quad (5)$$

where  $L$  is the length of the line and  $s$  is the length of the element used to construct  $L$ . Hence, for the Koch curve we have

$$\left(\frac{4}{3}\right)^n = \left[\left(\frac{1}{3}\right)^n\right]^\mu$$

or

$$\mu = 1 - \frac{\ln 4}{\ln 3}$$

Now  $\mu$  can be interpreted as the difference between the Euler dimension ( $D_E$ ) of the element ( $s$ ), which in this case is 1, and the dimension of the line  $L$ , which is called the fractal dimension ( $D_f$ ). Hence,

$$\mu = D_E - D_f$$

or, for the Koch curve,  $D_f = \ln 4 / \ln 3$  which is about 1.28. Note that if  $D_f = D_E$ , the length of the line does not depend on the size of the unit of construction, which is what one expects for smooth curves.

A better example, and one more closely related to turbulence, is the second example in figure 12, namely, that of a surface. One may think of this surface as a surface of vorticity. It is distorted in a recursive manner as follows: Divide the unit square into nine small squares; now, remove the four corner squares and the center square and replace them by building a small square box over the open squares (center box down for convenience). With each of the 29 sides of the new figure, repeat the process. The surface becomes more and more distorted with each step, and the actual surface area increases without limit as the number of iterations ( $n$ ) increases. Hence, the surface in two dimensions becomes more and more distorted, but never fills up space in three dimensions. In a manner similar to that used for the Koch curve, the fractal dimension is found to be  $\ln 29 / \ln 9 + 1$  or about 2.54. Now, this is a rather simplistic model for a sheet of vorticity that has been distorted into a parcel of turbulence because of instabilities. Even though the model is simplistic, it is true that a hot wire passing the distorted sheet would exhibit intermittency. The observations that high-Reynolds-number turbulent flows exhibit intermittency go back to a paper by Batchelor and Townsend [18]. These authors noted that in high-Reynolds-number homogeneous turbulent flows, the vorticity was not distributed uniformly but was concentrated on sheets or other localized regions of space. Attempts have been made to derive a fractal dimension for this



turbulence based on higher-order statistical information. Values between 2.0 and 3.0 have been derived. However, the reduction of information on the higher-order statistics to a fractal dimension requires specification of a topological form of the turbulence, and this step has led to considerable disagreement. Mandelbrot [56] suggests that reasonable topologies bound the value of the fractal dimension between 2.5 and 2.7. In a recent attempt to establish a basis for these values, Chorin [72] calculated the distortion of a vortex tube using the Euler equations. That calculation showed that the vorticity contracted (in an  $L_2$  norm sense) to a fractal dimension of about 2.5, in reasonable agreement with the lower bound postulated by Mandelbrot.

### Renormalization group theory

As noted earlier, the concept of the renormalization group has its roots in the study of condensed phase matter, in particular, certain crystal problems. The concept behind it is easiest to understand in that context. An example is shown at the top of figure 13. Here, a portion of a square lattice of molecules (dots) is shown on the top left. The interactions of four of the molecules are computed, so that they can be replaced by a supermolecule (X) which has the combined property of the four. Then, four of the supermolecules treated together form a still larger supermolecule and so on. One seeks scaling properties of the system that become invariant with repetitions of the process, so that only a small number of reclusterings may be required. A more detailed description of this process can be found in the paper by Wilson and Kogut [73]. The idea has been used by Feigenbaum et al. [74] in a novel way that calls attention to the universal scaling numbers for period-doubling. The idea has been used by Siggia [53] as follows: A fluid-dynamics problem with turbulence is being solved computationally by the use of finite-difference methods. In order to conserve computational time, a coarse grid is preferred. In that case, a subgrid turbulence model is required to handle the dissipation that occurs below the resolution of the grid. At the bottom of figure 13, a way to develop this subgrid turbulence model is sketched by means of a scale description. A problem is set up where the large-scale structures are forced. A fine grid is used so that dissipation by the subgrid scales is not important. A slightly coarser grid is then used with a dissipation term chosen so as to keep the midscales unchanged. The problem is repeated with a still coarser grid and dissipation term that, again, keeps the midscales unchanged. Several repetitions of the process should suffice to reveal the existence of a dissipation-scaling relation such as  $d \sim s^\gamma$ , where  $d$  is the dissipation,  $s$  is the grid size, and  $\gamma$  an exponent to be determined. This is a simple numerical application of a renormalization group idea. Another example is presented by McComb ([59], in this collection). Additional attempts should be forthcoming.

## FUTURE DIRECTIONS

In our view, future directions for the study of turbulence will reflect the recent developments of the deterministic movement, together with statistical elements and structural observations that are consistent with the deterministic approach. In a previous paper [60], the authors suggested a framework for studying nonlinear problems in flight dynamics. That same framework is proposed here as being suited to the deterministic approach to the study of turbulence. The framework has four premises involving the elements structure, change, chaos, and scale:

- (1) All flows have structure.
- (2) Structures change in systematic ways with changes in parameters.
- (3) Some changes lead to a special class of structures, chaos.
- (4) Structures have various scales.

The premises allow the following interpretation of observations: Flow structures are interdependent. Changes in structures occur in discrete ways at definite and repeatable values of parameters. Chaos in fluid systems can occur after a finite number of bifurcations (discrete changes). Chaos is deterministic and can be represented by a strange attractor of finite dimensionality. Finally, the various scales of turbulence interact (a restatement of the observation that the structures are interdependent).

Taken together with a corresponding set of mathematical ideas, the four premises form a strong theoretical framework (a way of seeing) for the understanding, and (potentially) the modeling of turbulent flows. The premises of the theoretical framework are:

- (1) Structures are describable in topological terms.
- (2) Changes in structure are describable by bifurcation theory.
- (3) Chaos is describable by the theory of strange attractors and fractals.
- (4) Scales are describable by group theory ideas.

Where this body of theory will lead in the modeling of turbulent flows is not yet completely obvious. However, at least the following seems likely: (1) Some form of averaging of the Navier-Stokes equations probably still will be required. Whatever its form, the averaging will be carried out such that: (a) it allows the representation of at least the major structural and subcritical bifurcations that occur both in the outer flow (away from boundary layers) and within the turbulence itself, and (b) it incorporates chaotic information. (2) The chaotic portion of the problem probably will be modeled by a finite-dimensional strange attractor along the lines of current developments in dynamical systems theory. Progress here hinges on the formulation of appropriate measures of the strange attractor which will allow a rational finite-dimensional representation of its essential nature. The representation will be driven by the mean flow and, in turn, supply information to the mean-flow equations to be used in forming the Reynolds stresses. This is where renormalization-group theory ideas will be required to resolve only the essential

scales of the problem. Topological ideas will be used to replace the formerly ubiquitous "turbulent eddies" (and the currently, perhaps equally nonspecific, "small-scale chaotic structures") by a concise and descriptive grammar of structural forms.

The ideas which have been discussed perhaps can be illustrated more clearly by examples. Two flows have been selected to illustrate some of the ways in which flow properties change with changes in the governing parameters. Our purpose here is twofold: First, to demonstrate the extent to which descriptions based on the language of the proposed theoretical framework are compatible with observations; and second, to demonstrate some of the principal structural features of flows which we believe should be capturable by a mathematical model constructed according to the prescription above.

The first example is that of the two-dimensional cylinder immersed in an incompressible crossflow that is uniform and steady far upstream (fig. 14). The main parameter is Reynolds number ( $Re$ ). We examine a sequence of flows as  $Re$  is increased from very low values ( $Re < 7$ ) to values of the order of  $10^6$ . Sketch (a) in figure 14 depicts the regime resulting from a Hopf-type bifurcation occurring at  $Re \approx 50$ , in which the previously steady flow is replaced by a time-periodic flow (cf. [75]). As  $Re$  is increased by increments, at least two additional bifurcations occur, leaving the wake with a quasi-periodic large-scale structure and with chaotic small-scale structures superposed on the free-shear layers (sketch (b)). The small-scale structures work their way forward in the wake as  $Re$  increases, until they are in the vicinity of the separation points on the cylinder. At a definite and repeatable value of  $Re$  ( $Re \approx 3.5 \times 10^5$  [76,77]) an antisymmetric interaction occurs having opposite effects on the separation points, which leaves the mean flow asymmetric and disrupts the previously quasi-periodic form of the wake flow (sketch (c)). As has been documented in [77], this event has all of the characteristics of a subcritical bifurcation, including hysteresis, inasmuch as the return to a symmetric mean flow with reduction in  $Re$  from higher values occurs at a lower critical value of  $Re$  than had the onset of asymmetry. With a small further increase in  $Re$ , a second interaction restores the symmetric character of the mean flow (sketch (d)). This event also involves the hysteresis with reduction in  $Re$  from above, characteristic of a subcritical bifurcation. Finally, at sufficiently high values of  $Re$  (sketch (e)) the chaotic structures have moved forward of the separation points, rendering the boundary-layer flow on the cylinder turbulent. A periodic structure reasserts itself in the wake, now with a new fundamental period reflecting the scale of the chaotic structures characteristic of both the boundary layer and the free-shear layers. The example confirms first that descriptions based on the language of the theoretical framework are in fact compatible with observations. Starting with the onset of flow separation at  $Re \approx 7$  (describable as a structural bifurcation, not shown in fig. 14), the onset of each of the flows sketched in figure 14 can be described as either a subcritical bifurcation or a

structural bifurcation of the mean flow, occurring at a definite and repeatable critical value of  $Re$ . Second, with regard to computations, the example suggests that a mathematical model capable of capturing the sequence of important bifurcations will have to acknowledge the existence of small-scale chaotic structures and how they may interact with large-scale periodic structures.

The second example is that of a two-dimensional airfoil immersed at zero angle of attack in a compressible flow that is uniform and steady far upstream (fig. 15). The main parameter here is free-stream Mach number ( $M_\infty$ ); we examine a sequence of flows as  $M_\infty$  is increased in the transonic speed range, with Reynolds number ( $Re$ ) held fixed. The value of  $Re$  is supposed sufficiently high to maintain a turbulent attached boundary layer over the major part of the airfoil at the lowest value of  $M_\infty$ . In sketch (a) of figure 15,  $M_\infty$  is low enough so that no shock waves develop. As  $M_\infty$  increases, shock waves develop (sketch (b)) but the boundary-layer flow remains attached. An additional increment in  $M_\infty$  strengthens the shock waves and the boundary-layer flow separates behind the shocks but remains essentially steady except for the presence of small-scale chaotic structures. This is a structural bifurcation of the mean flow (sketch (c)). A further incremental change in  $M_\infty$  results in a large-scale periodic fluctuation, originating in the separated-flow regime and akin to vortex-shedding. It is describable as a Hopf-type bifurcation, and observations [78] have shown it to be subcritical (sketch (d)). Finally, with another increment in  $M_\infty$ , the large-scale periodic fluctuation vanishes and the flow returns to an essentially steady separated structure (sketch (e)). This event is also describable as a subcritical bifurcation, involving hysteresis.

Extensive studies of the phenomenon have been carried out both experimentally [78] and computationally [79] for a biconvex airfoil. The computations, initiated by Levy [79], were based on the Reynolds-averaged Navier-Stokes equations with closure achieved by means of a phenomenological (mixing-length) turbulence model. Computational results were able to capture essential features of the bifurcations, including the critical values of  $M_\infty$  signalling both the onset and the termination of periodic fluctuations as well as the form and frequency of the fluctuations themselves. The computational results were not sufficiently accurate, however, to demonstrate the subcritical nature (i.e., the presence of hysteresis) of the bifurcations.

Levy's computational results are extremely encouraging in that they demonstrate that even rather simple modeling of the averaged Navier-Stokes equations may be sufficient to capture essential features of important bifurcations. Improvement of the modeling to enable capturing, e.g., the subcritical nature of bifurcations, would appear to hinge on the successful implementation of the second part of the prescription outlined earlier. In particular, as we have noted in the first example, the model must be capable of acknowledging more fully the effect of interactions between small-scale chaotic and large-scale periodic structures.

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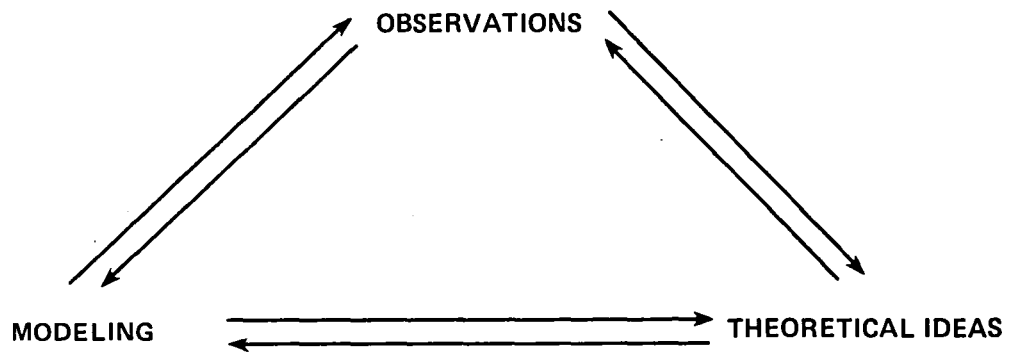


Fig. 1. Observations, theoretical ideas, modeling, and their interactions.



Fig. 2. Very early observations of turbulence--sketch by Leonardo da Vinci, circa 1500.

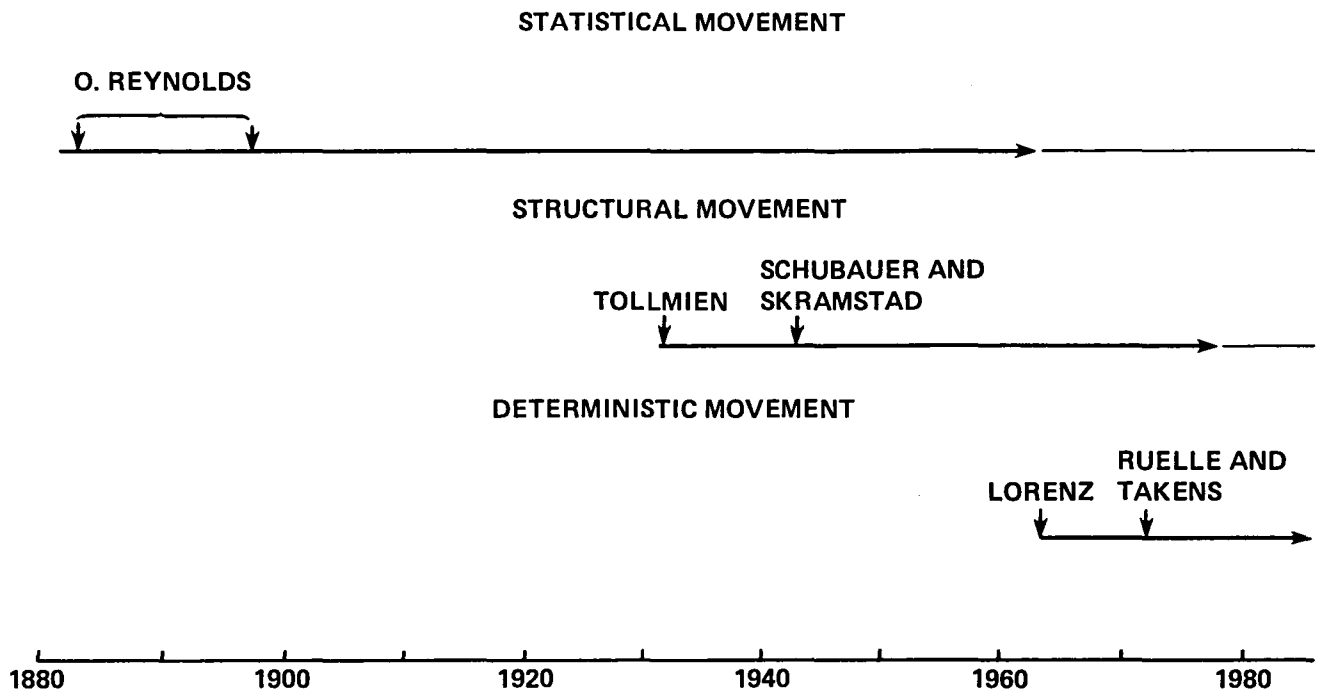


Fig. 3. Three movements in the history of turbulence research: statistical, structural, and deterministic.

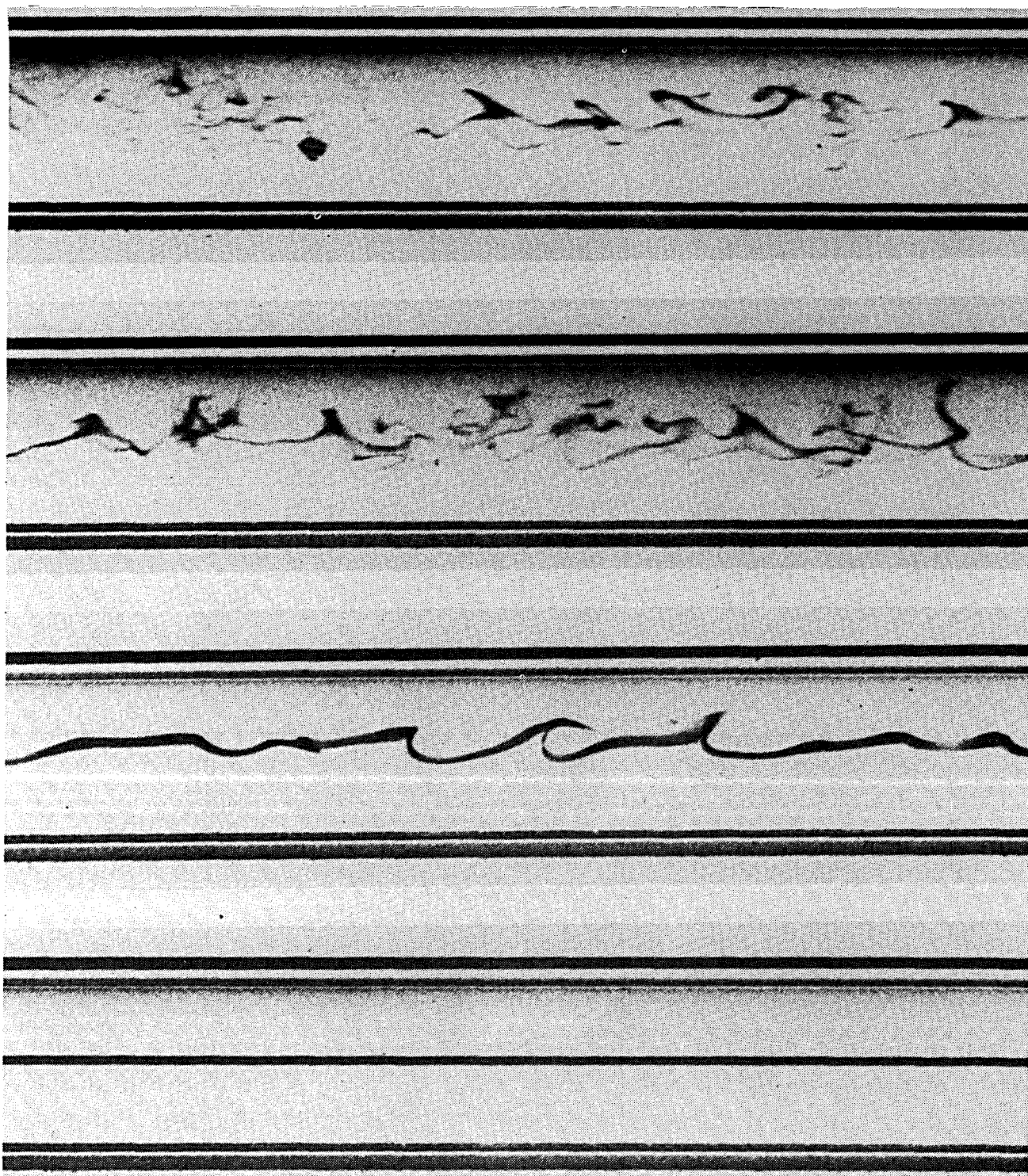


Fig. 4. First systematic study of turbulence--a repetition of O. Reynolds' 1883 dye experiments (from [5]).

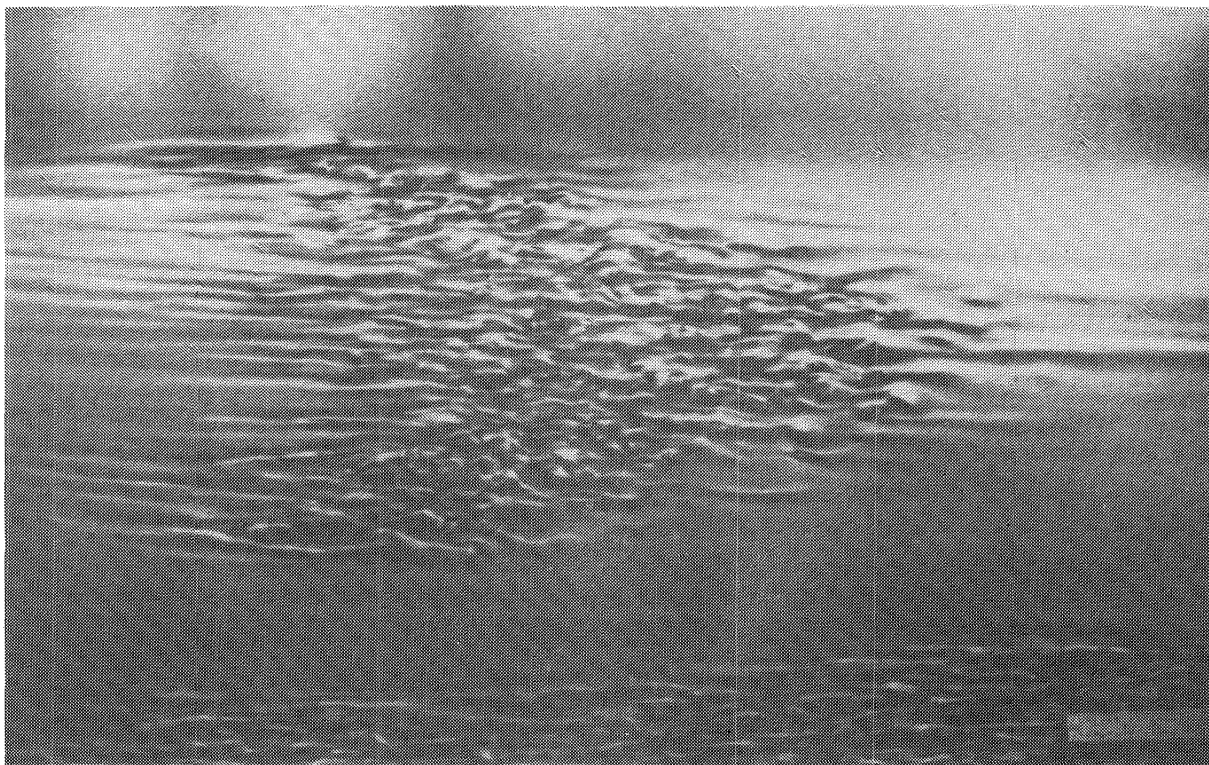


Fig. 5. Top and side views of Emmons turbulent spot (from [22]).

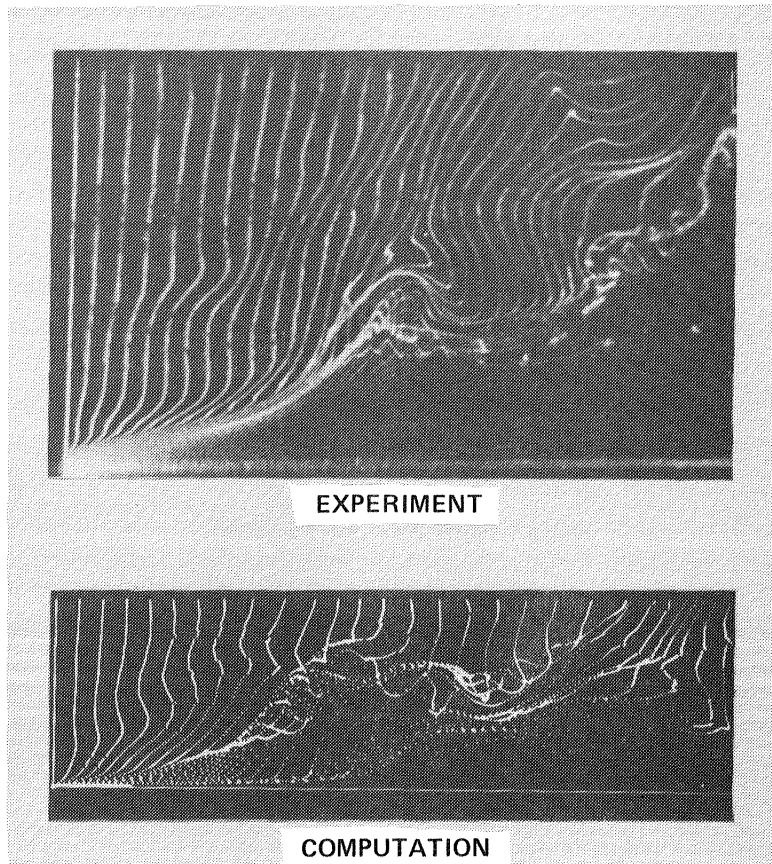


Fig. 6. Structures in turbulent wall shear flows (from [30]).



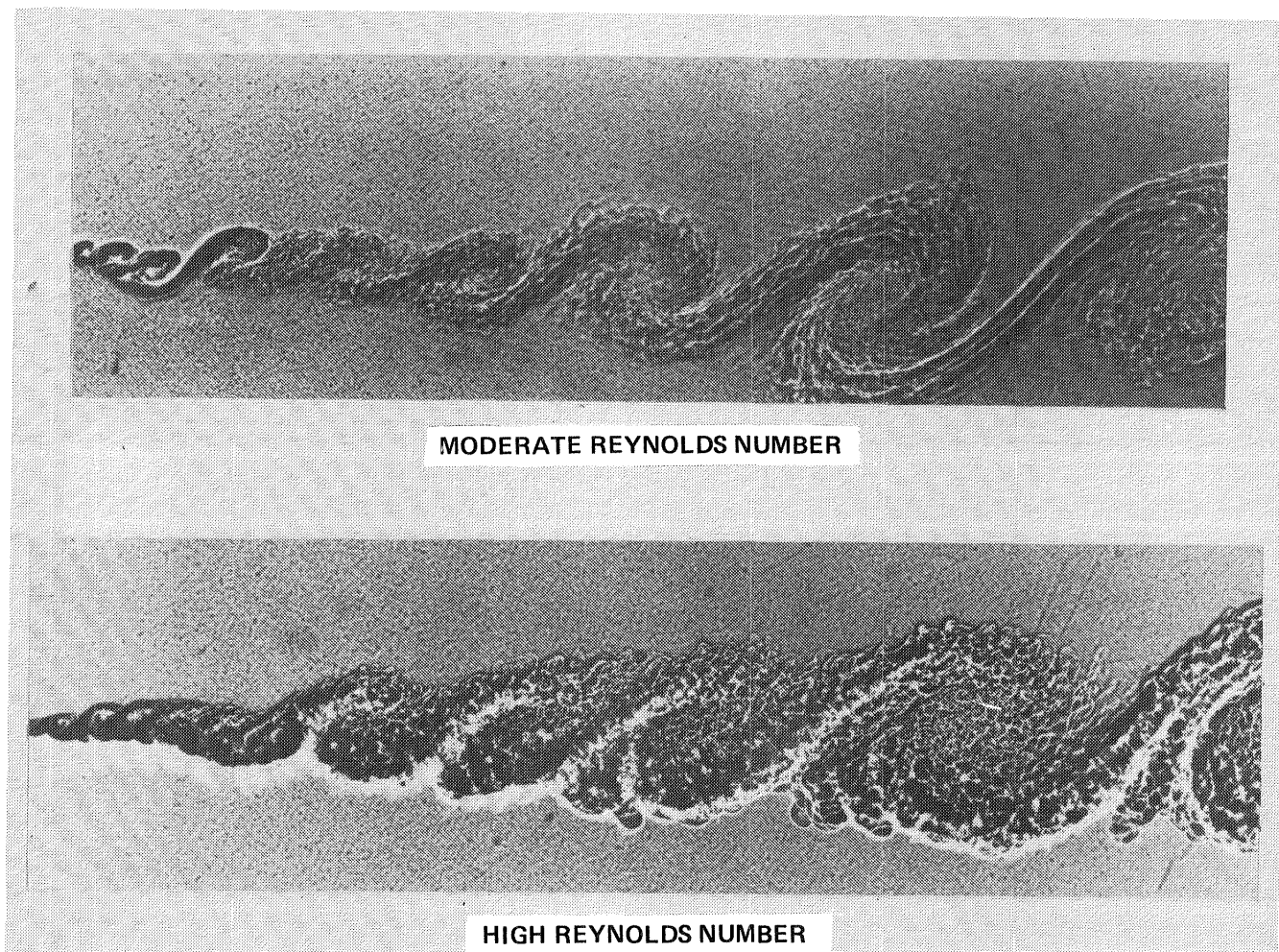


Fig. 7. Coherent structures in turbulent free shear layers (from [5]).

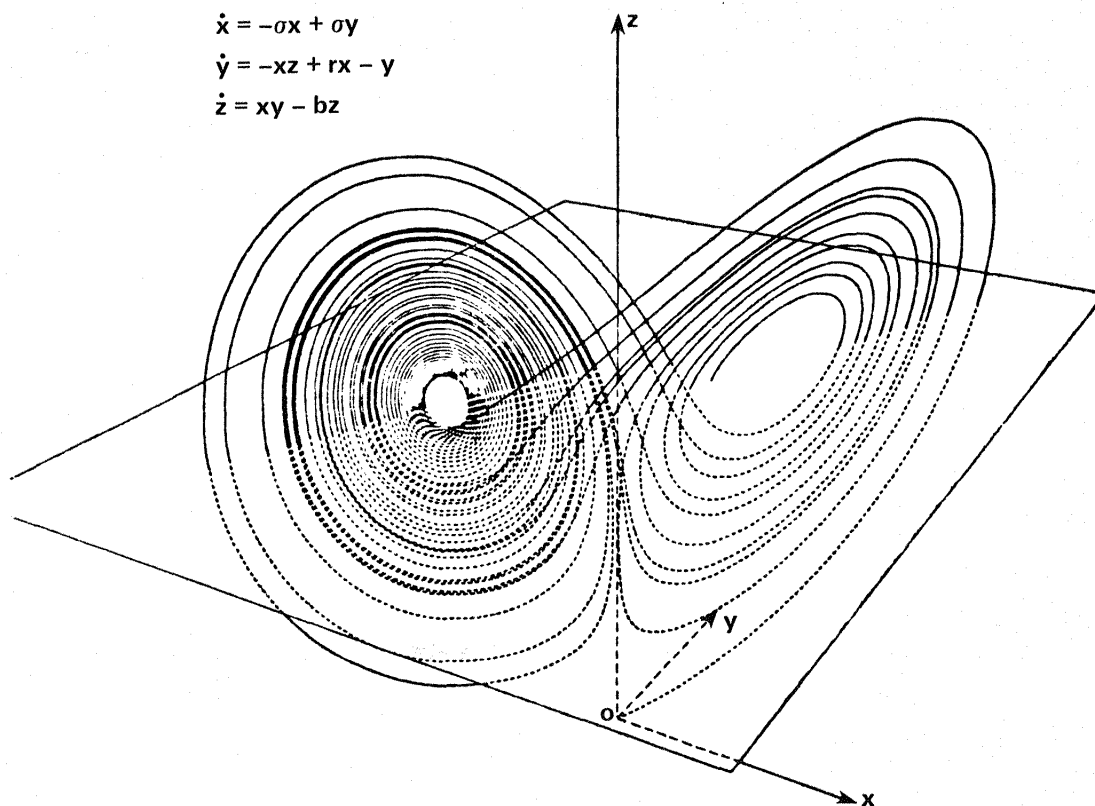


Fig. 8. The Lorenz attractor.

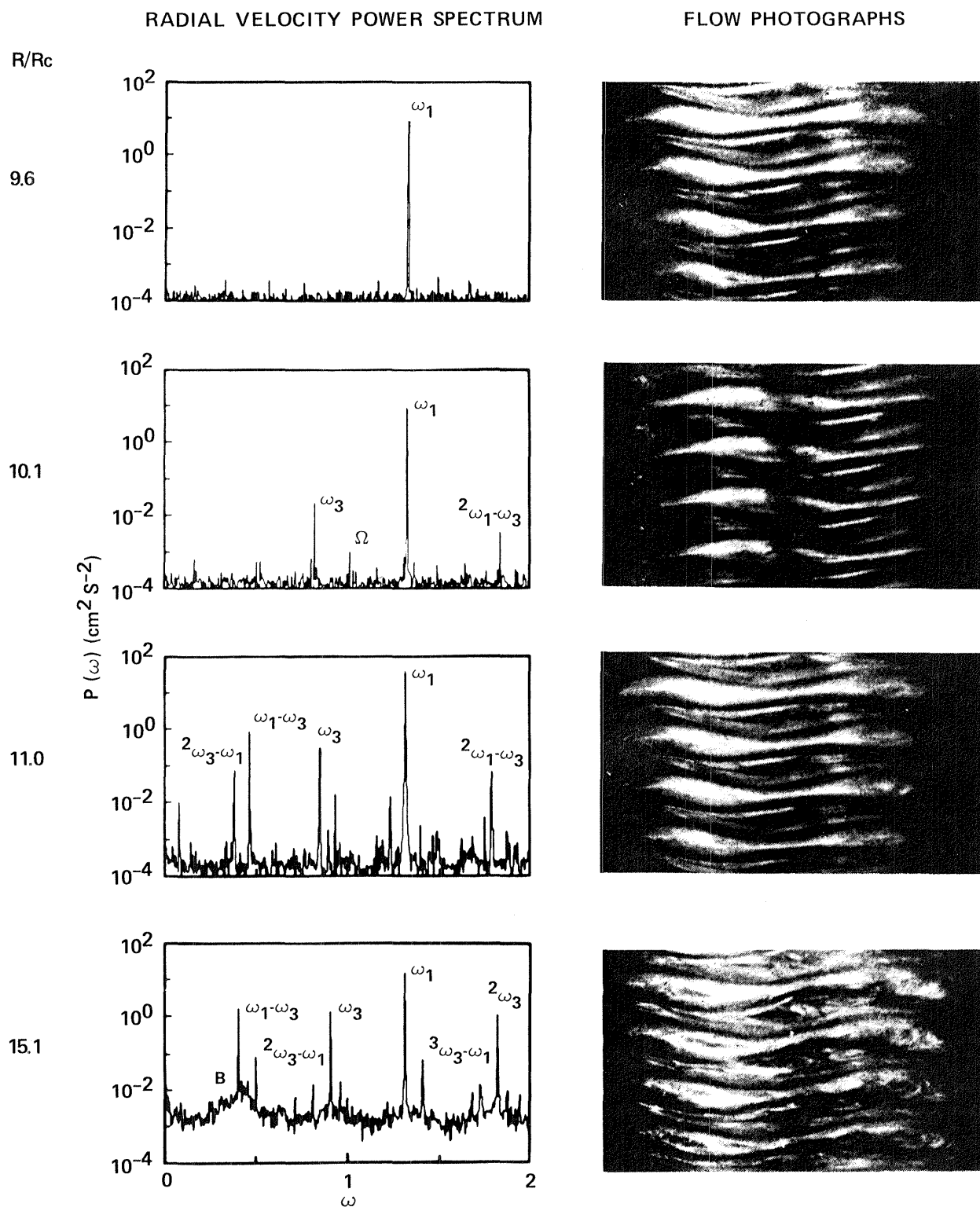


Fig. 9. Stages in Taylor-Couette flow with increasing Reynolds number  $R$ ;  $R_c$  = critical  $R$  for onset of Taylor instability (from [50]).

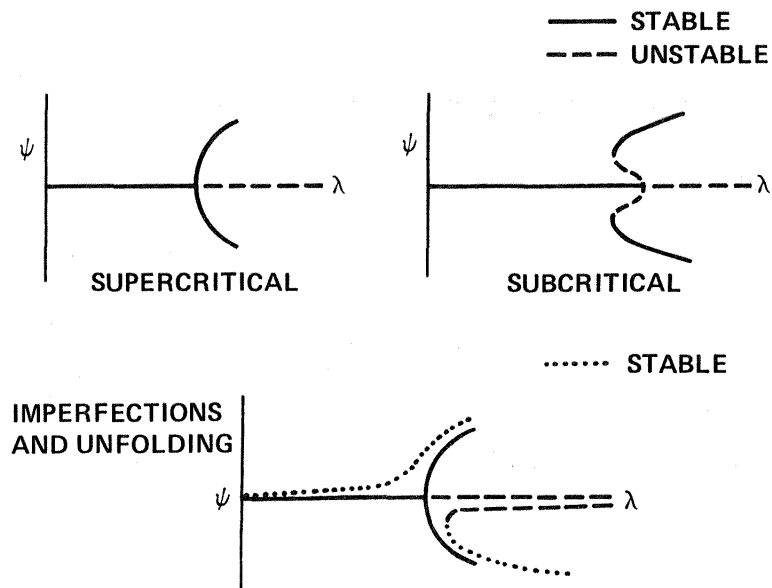


Fig. 10. Supercritical and subcritical bifurcations and the unfolding of a bifurcation curve due to imperfections.

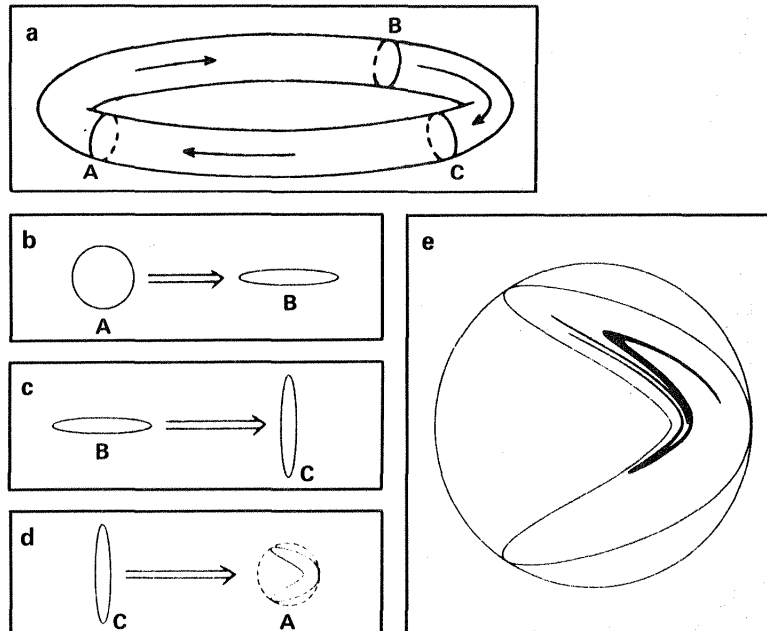


Fig. 11. Qualitative behavior of a model strange attractor.

KOCH CURVE:



$$L = 1$$



$$L = 4/3$$

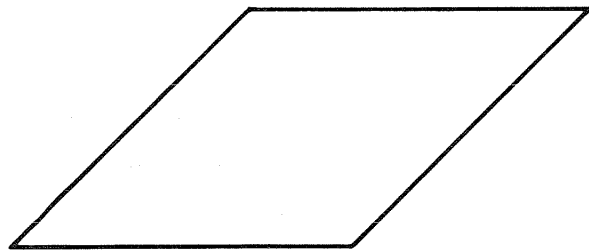


$$L = (4/3)^2$$

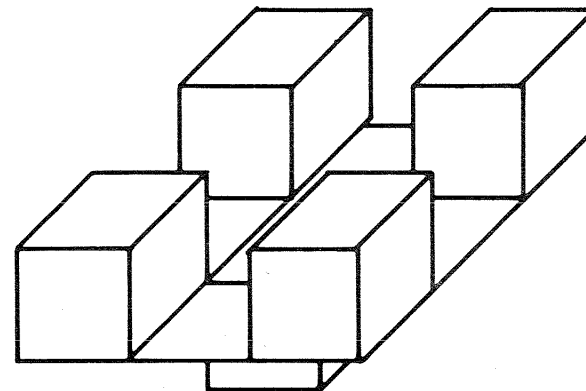


$$L = (4/3)^n$$

SURFACE:



$$A = 1$$



$$A = 29/3^2$$



$$A = (29/3^2)^n$$

Fig. 12. Examples of fractals.

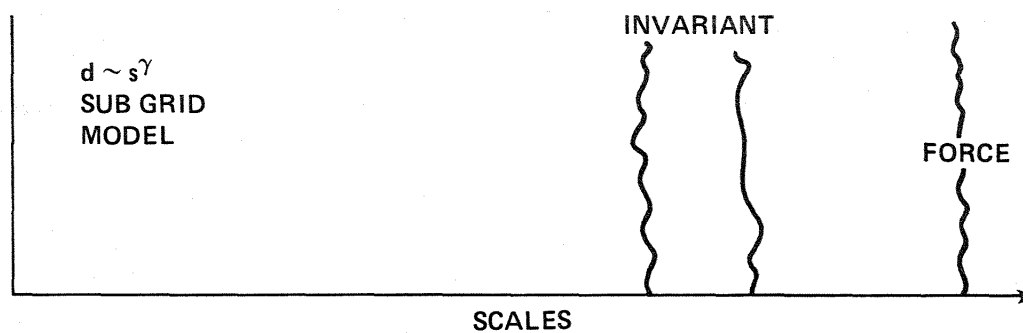
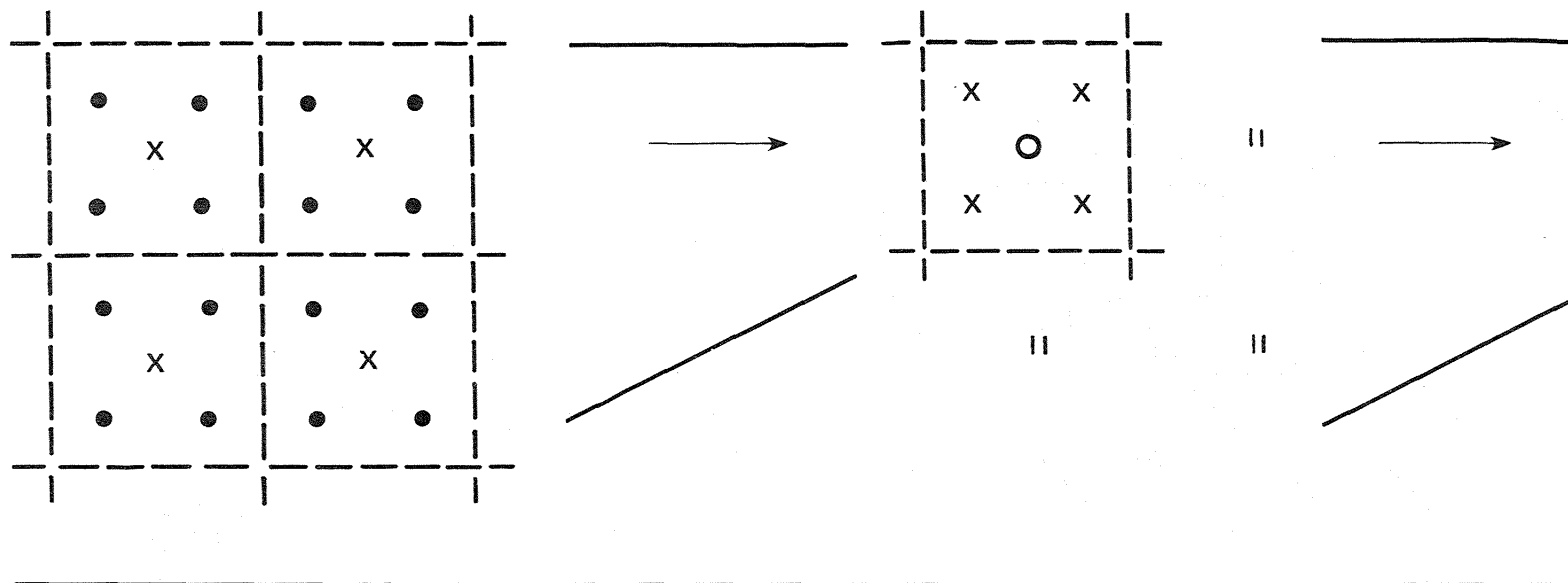


Fig. 13. Renormalization group ideas.

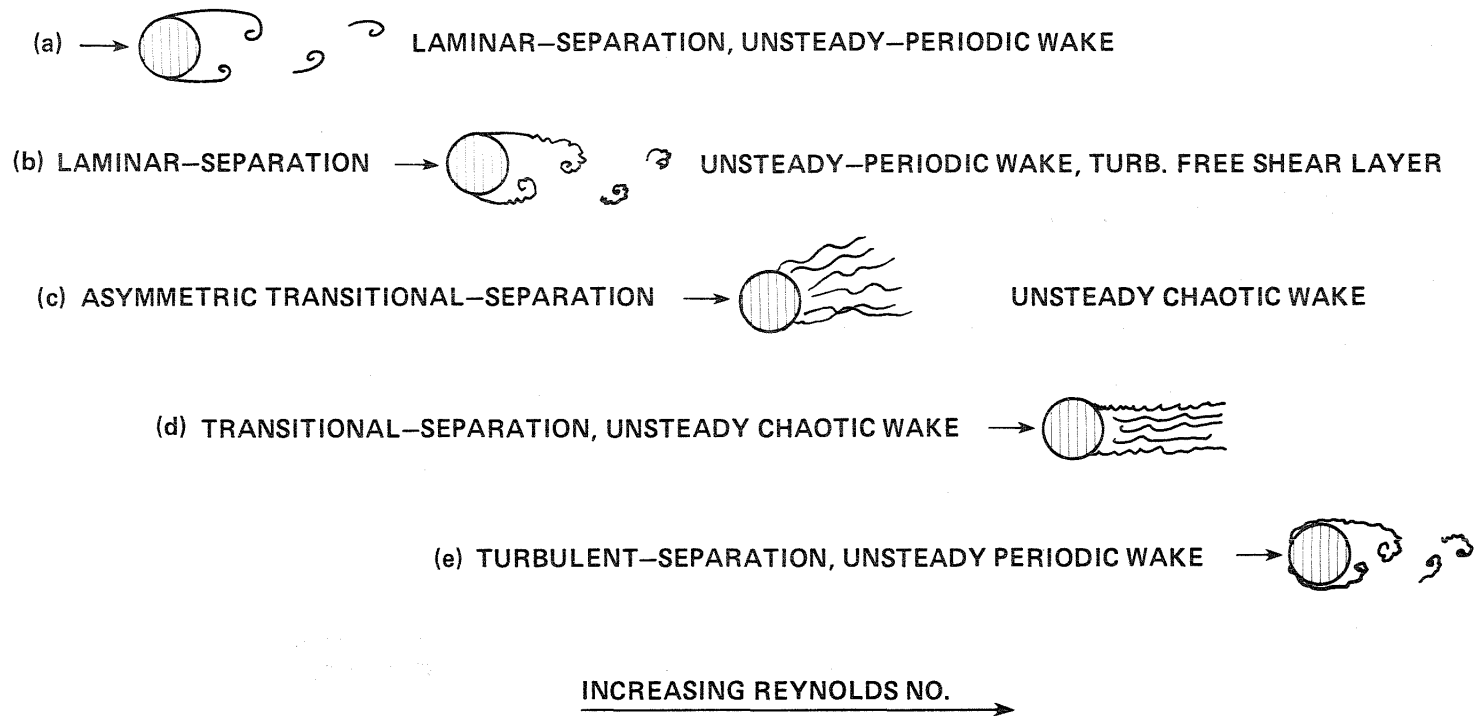


Fig. 14. Stages of incompressible flow past a circular cylinder with increasing Reynolds number.

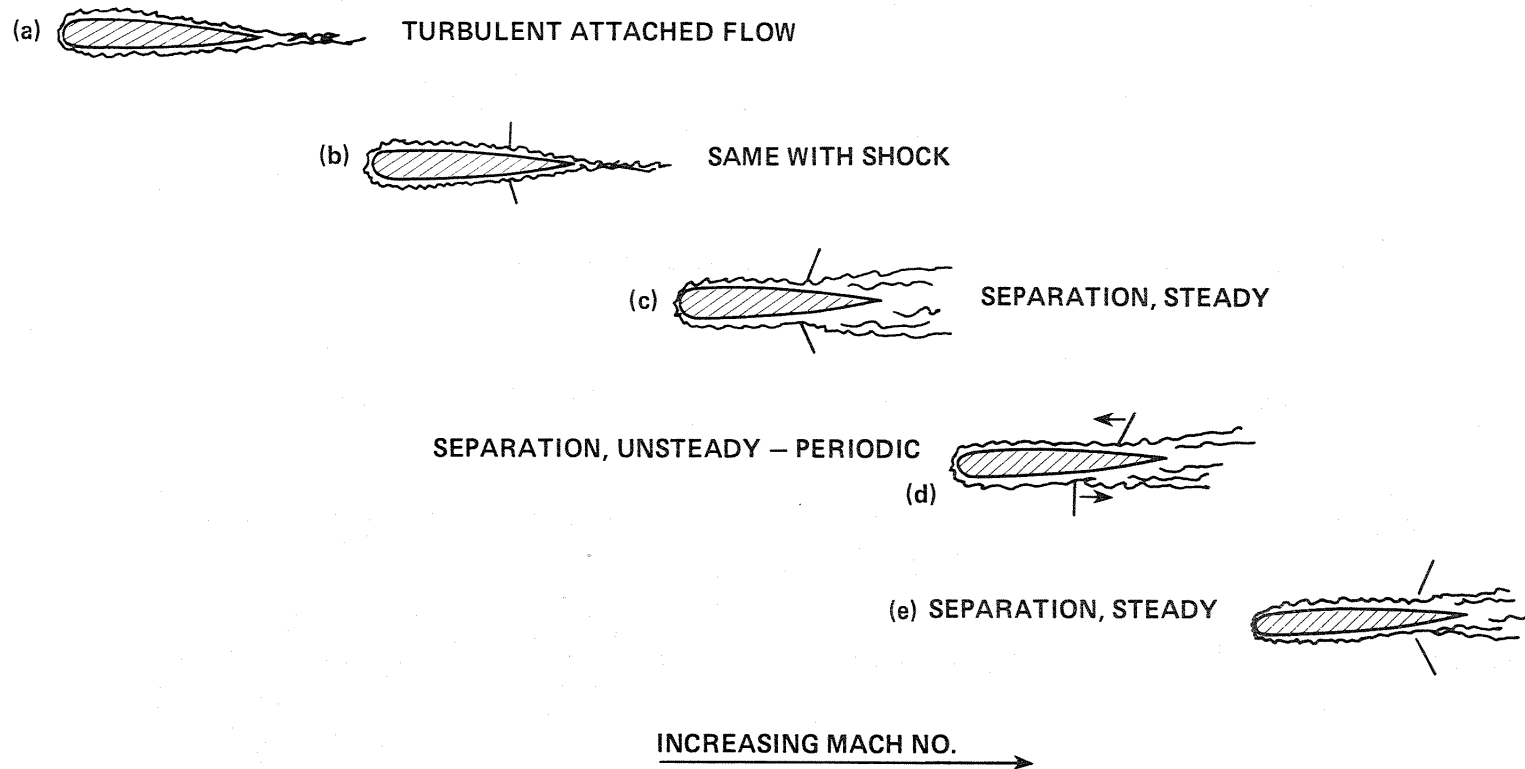


Fig. 15. Stages of transonic flow past a symmetric airfoil at zero angle of attack with increasing Mach number.



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16. Abstract  The study of turbulence is analyzed in a historical context featuring the interactions between observations, theoretical ideas, and modeling within three successive movements. These are identified as predominantly statistical, structural and deterministic. The statistical movement is criticized for its failure to deal with the structural elements observed in turbulent flows. The structural movement is criticized for its failure to embody observed structural elements within a formal theory. The deterministic movement is described as having the potential of overcoming these deficiencies by allowing structural elements to exhibit chaotic behavior that is nevertheless embodied within a theory. Four major ideas of this movement are described: bifurcation theory, strange attractors, fractals, and the renormalization group. A framework for the future study (and potentially modeling) of turbulent flows is proposed, based on the premises of the deterministic movement.					
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