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Fatigue Criterion to System Design, Life and Reliability

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FATIGUE CRITERION TO SYSTEM DESIGN, LIFE AND RELIABILITY

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SUMMARY

A generalized methodology to structural life prediction, design, and reliability based upon a fatigue criterion is advanced. The life prediction methodology is based in part on work of W. Weibull and G. Lundberg and A. Palmgren. The approach incorporates the computed life of elemental stress volumes of a complex machine element to predict system life. The results of coupon fatigue testing can be incorporated into the analysis allowing for life prediction and component or structural renewal rates with reasonable statistical certainty.

INTRODUCTION

Design of machine elements is based for the most part on yield stresses and fatigue limiting stresses. In addition to the components material properties, proper consideration must be given to the effects of notches, surface condition, component size, residual stress, temperature, duty cycle and environmental factors such as corrosive or chemical exposure. For most machine elements, individuals and organizations usually develop design methodology based upon engineering fundamentals found in most machine design texts and factors based upon their corporate experience and test data. As a result, it is not too unusual for different organizations or individuals, starting with the same or similar design requirements to reach dissimilar conclusions or designs while seemingly applying the same fundamental engineering principals to the problem.

Setting aside the subjective creative aspects of design, there appears to be nonuniformity of data from which numbers and design factors are selected as well as differences in the computer codes and boundary conditions used in the design process. To compound these difficulties, fatigue data which are used to establish fatigue limits are usually of a limited nature with the conditions under which the data were obtained not adequately defined or reported. Such items as temperature, humidity, number of specimens, specimen size and volume, heat treatment, hardness, surface finish and life distribution are not given.

The established fatigue limit for much of the reported data is a mean value. From a statistical viewpoint, the median value is equal to or less than the mean. This can be interpreted as meaning that before a fatigue limit is determined there is a probability that 50 percent or more of the specimens had failed. In other words, even at the fatigue limiting stress, life is finite. Of course, experienced design engineers have recognized this for years. They have added safety factors to their design procedure usually based on experience. While these procedures are generally adequate they can result in overdesigned, oversized, overweight and overcost structures.

A fundamental principle of good design is to recognize that any structure can fail. However, should it fail, it should be designed not to cause personal injury or secondary damage. In other words, if the structure fails, it fails in a safe or benign manner. Once a structure is designed to fail in a benign manner, it then can be designed for finite life whereby the overall size, weight, and cost can be reduced and still meet the reliability requirements of the application.

In view of the aforemention, it becomes the objective of the work reported herein to advance a generalized methodology for structural life prediction, design, and reliability based upon a fatigue criterion. The life prediction methodology is based in part on the work of W. Weibull (refs. 1 and 2) and G. Lundberg and A. Palmgren (refs. 3 to 6).

SYMBOLS

C dynamic load capacity

c stress-life exponent

e Weibull slope, $1/\alpha$

F probability of failure

f failure probability density function

h exponent

L life

L_A adjusted life at a 90-percent probability of survival

L_m mission life

L_{10} 10-percent life, life at which 90 percent of a population will survive

n load-life exponent

P applied load

$P(x)$ probability for occurrence of extent x

R cumulative renewal function

r renewal density function

S probability of survival

S_1 system probability of survival

l dummy variable of integration

t time or time function

V stressed volume

X time function or stress function

X_β characteristic strength

X_μ stress below which no specimens fail

z depth to critical maximum shearing stress

α shape parameter

β characteristic life

γ mean time to failure

η stress cycles to failure

μ location parameter or time below which no failure is expected to occur

σ_u fatigue limit

τ critical stress

STATISTICAL METHOD

In the late 1930's (circa 1937) W. Weibull in Sweden attempted to graphically linearize various types of statistical data distributions for small sample sizes. By trial and error, Weibull found that by having the $\ln\ln 1/S$ as the ordinate and $\ln X$ as the abscissa, where S is the probability of survival or statistical percent of samples surviving and X is a time function, most engineering data distributions will plot on a straight line (ref. 7). Hence, it became possible for small amounts of data to estimate a generalized population distribution for a population of infinite size. Having empirically determined this, Weibull developed a theoretical basis for what was to become known as the "Weibull distribution" or "Weibull plot" which was published in 1939 (ref. 1). Weibull (ref. 8) defines the distribution function as "an adequate expression for a large class of phenomena which have the property that the probability of nonoccurrence of an event is equal to the product of the elementary probabilities,"

$$S = \exp\left\{- [(X - \mu)/\beta]^{1/\alpha}\right\} \tag{1}$$

The function involves three parameters,

α the shape parameter
β the scale parameter
μ the location parameter

Where α the shape parameter is equal to 1. 0.5 and 0.28, the respective distributions approximated are exponential, Rayleigh, and normal (Gaussian). A typical Weibull plot for rollingelement bearing fatigue data is shown in figure 1. The slope of the line "e" which is called the "Weibull slope" is equal to $1/\alpha$. For most rolling-element bearing data e equals 1.1. The location parameter μ is a finite time under which there would be a zero probability of failure to occur. If failure can occur in one stress cycle then the location parameter μ would be assumed to be zero.

For fatigue data, if η stress cycles to failure is substituted for X in equation (1) and e for $1/\alpha$, and where $F = 1 - S$, equation (1) can be written

$$F = 1 - \exp\left\{-\left(\frac{\eta}{\beta}\right)^{e}\right\} \tag{2}$$

The characteristic life β is the 63.2 percent failure life of the population distribution.

Weibull stated that the probability of survival S could be expressed as

$$\ln \frac{1}{S} \sim \tau^{c}\eta^{e}V \tag{3}$$

where V is the volume representation of stress concentration referred to herein as stressed volume, τ is the critical shearing stress and c is an exponent denoting a stress-life relation where

$$\eta \sim \tau^{-c} \tag{4}$$

The values of c and e can be determined experimentally.

The effect of stressed volume can be illustrated wherein two specimens of stress volumes V_1 and V_2 respectively are subjected to equal stress τ. If η_1 is determined at a probability of survival S_1, then the probability of survival S_2 for V_2 for the life is given by the following expression

$$S_2 = S_1^{V_2/V_1} \tag{5}$$

where $V_2 > V_1$.

For the same probability of survival, the specimens with the larger stressed volume will have the lower life. This principle was applied to successfully normalize rolling-element fatigue data by Carter (ref. 9) and Zaretsky (ref. 10). This principle was applied by Grisaffe (ref. 11) in a Weibull analysis of shear bond strength of plasma-sprayed alumina coatings on stainless steel (fig. 2). Grisaffe showed that the calculated mean bond strength decreased with increasing test area in accordance with equation (5) (fig. 3). Further, the stress at which no specimens failed could be determined by restating equation (1) as follows:

$$F_x = 1 - \exp\left[-\left(\frac{X - X_\mu}{X_\beta}\right)^e\right] \qquad (6)$$

where the parameters are now defined as

Fx statistical percent of specimens which when tested at one set of conditions, fail at given stress or lower

X stress

X_μ stress below which no specimens failed

X_β characteristic strength

e Weibull slope

LUNDBERG - PALMGREN THEORY

Lundberg and Palmgren were contemporaries of Weibull. A problem existed in the rolling-element bearing industry to establish the lives of these bearings short of extensive testing. Taking the Weibull approach a step further, Lundberg and Palmgren (ref. 3) took equation (3) and added another element, the depth to the critical maximum shearing stress, z where

$$\ln \frac{1}{S} \sim \frac{\tau^c n^e V}{z^h} \qquad (7)$$

The rationale for including the depth z was that the initiation of a fatigue crack occurred at the depth z and that the distance the crack needs to travel to the surface until a spall (pit) occurs is equivalent to a critical crack length. In other words the greater the distance the maximum shearing stress below the surface the longer it takes for a fatigue crack to propagate to the surface, the longer the fatigue life. Lundberg and Palmgren took the rolling bearing geometry, kinematic, stress theory of Hertz (ref. 12) and Thomas and Hoersch (ref. 13) and incorporated then into equation (7) to obtain a series of equations which relate the rolling-element fatigue life of a bearing to the applied load wherein the resultant life would be in millions of inner race revolutions. Knowing the speed of the bearing the life in hours can be determined. Lundberg and Palmgren further refined their approach to bearing life prediction to include a fictitious load designated C, the dynamic load capacity. The dynamic load is the theoretical load which when applied to the bearing would result in a life of one million inner-race revolutions and where

$$C = P \sqrt[n]{L_{10}} \qquad (8)$$

P applied load

L_{10} 10-percent life, life at a 90-percent probability of survival, inner-race revolutions

n load-life exponent usually taken as 3.

Equation (8) can be written:

$$L_{10} = \left(\frac{C}{P}\right)^n \tag{9}$$

Hence, the predicted life of a bearing can be determined by knowing C and P.

The Lundberg-Palmgren theory is now an international standard used by every manufacturer of bearings around the world (ref. 14). The theory has been applied to other machine elements and mechanical transmission systems (refs. 15 to 28). Table 1 is a list of applications of the Lundberg-Palmgren theory to life prediction.

SYSTEM LIFE AND RELIABILITY

The life and relibability of a system is based upon the lives and reliabilities of all of its components. Having determined the life of the individual components using Lunberg-Palmgren, the probability of survival of the entire system is as follows

$$S_T = S_1 \cdot S_2 \cdot S_3 \ldots S_n \tag{10}$$

The system life-reliability equation can be written as follows

$$\ln \frac{1}{S_T} = \ln \frac{1}{0.9} \left[\left(\frac{L}{L_1}\right)^{e_1} + \left(\frac{L}{L_2}\right)^{e_2} + \left(\frac{L}{L_3}\right)^{e_3} \ldots + \left(\frac{L}{L_\beta}\right)^{e_n}\right] \tag{11}$$

where L_1, L_2, L_3 . . . L_n are the lives of each component of the system at a 90 percent probability of survival. The system life L can be determined at each system probability of survival S_T. For a 90-percent probability of survival

$$1 = \left(\frac{L}{L_1}\right)^{e_1} + \left(\frac{L}{L_2}\right)^{e_2} + \left(\frac{L}{L_3}\right)^{e_3} \ldots + \left(\frac{L}{L_n}\right)^{e_n} \tag{12}$$

MISSION LIFE

A system does not usually operate at one constant load in actual service. Miner's Rule is used to sum fatigue damage of a mission profile consisting of loads and time-at-loads. For a given probability of survival, the mission life for the system L_m is

$$L_m = \left(\frac{t_a}{L_a} + \frac{t_b}{L_b} + \frac{t_c}{L_c} + \frac{t_d}{t_d}\right)^{-1} \tag{13}$$

where t_a, t_b, t_c and t_d are the fraction of the total time at loads P_a, P_b, P_c and P_d, respectively, and L_a, L_b, L_c and L_d are the system lives at a given probability of survival at loads P_a, P_b, P_c and P_d, respectively. Weibull plot or distribution can be constructed using this method by determining the mission life at varying probabilities of survival. This method was used by Lewicki to determine the fatigue life of a turboprop reduction gearbox (ref. 28). The analysis was compared to actual field data. The results of this comparison are shown in figures 4 and 5.

COMPONENT REPLACEMENT RATE

The Weibull analysis is a valuable tool for predicting the life of components or systems. The analysis would describe the failure rate in field service only if all components were put into service at the same time and if failed components were not replaced. But, a certain number of components must be kept in operation, failure rates with replacements is of interest. As an example, if there are 10 000 bearings in the field, the user or manufacturer must know how many spare bearings will be needed over a given period in order to keep 10 000 bearings in operation at all times. Over the interval of service operation it may take 15 000 replacements to keep all 10 000 bearings running. The Weibull analysis can never exceed 100 percent, but field failures can and often do exceed 100 percent (ref. 19).

The cumulative probability of failure for the first time, assuming constant service conditions, is a function of the length of time that the bearing has been running. Failure is the complimentary function to survival according to the following relation:

$$F(t) = 1 - S(t) \tag{14}$$

The probability density function for failure is

$$f(t) \equiv \frac{dF}{dt} \tag{15}$$

The instantaneous probability of failure for the first time in any time interval from t to $t + \Delta t$ is then given by

$$P(\text{first failure in } \Delta t \text{ interval}) = f(t)\Delta t \tag{16}$$

By using renewal theory (refs. 29 and 30), the probability of having to replace a bearing in the field is written as follows:

$$P(\text{making a replacement}) = r(t)\Delta t \tag{17}$$

where $r(t)$ is called the renewal density. The renewal density is calculated from the following summation (ref. 29):

$$r(t) = \sum_{k=1}^{X} f_k(t) \tag{18}$$

where $f_k(t)$ is the k-fold convolution of f with itself and is computed by the following recurrence relation (ref. 29):

$$f_{k+1}(t) = \int_0^t f_k(T) f_1(t - T) dT \qquad (f_1 \equiv f) \tag{19}$$

The expression $f_k(t)\Delta t$ gives the probability of a k^{th} failure occurring in the time interval t to $t + \Delta t$. Since, when a failure occurs, it can be for any value of k, it follows that the renewal density function should be defined as the sum over all the f_k's.

The total number of replacements made during the first t units of time is obtain by integrating the renewal density function as follows:

$$R(t) = \int_0^t r(T) dT \tag{20}$$

For a group of bearings that has been operating for some time with failures occurring and replacements being made, it is also important to know the MTBF (mean time expected between failures). According to the renewal density theorem (ref. 30),

$$\lim_{t \to \infty} r(t) = \frac{1}{\int_0^\infty t f(t) dt} \equiv \frac{1}{\gamma} \tag{21}$$

Therefore, the mean time between failures is calculated as

$$\text{MTBF} = \frac{\gamma}{\text{total number of bearings}} \tag{22}$$

Reference 19 reports the use of a computer program to evaluate equation (18) for rolling-element bearings. A Weibull slope of 1.1 was assumed. Figure 6 shows plotted results, which give the renewal density function for a case of where the failed bearings are removed from service and replaced with new or restored bearings. For comparison, the probability density for failures with no replacement (Weibull density function) is plotted also. The area under the curves represents the probability of failure.

The functions plotted in figure 6 were numerically integrated and are shown plotted in figure 7. These are the cumulative functions for renewal or failure. The cumulative renewal functions indicate 100 percent replacement bearings will have been needed by the time 8 L_{10} intervals have elapsed. By comparison, at the time of 8 L_{10} the Weibull distribution shows only 65 percent of the original bearings will have failed. The difference in total failures would be due to replacement bearings failing.

The mean time between failure (MTBF) from equation 22 can be obtained from figure 6. The MTBF is the inverse of the probability density for failure. As an example a probability density for failure of 0.12 would give a MTBF value of 8.3 L_{10}.

FAGIGUE LIFE MODELING

The Weibull and Lundberg-Palmgren analyses have been primarily applied to high cycle fatigue with the material subjected to a Hertzian stress field. However, as indicated in reference 11, the Weibull analysis can be applied to other types of durability problems. What is important is that a material element only knows the state of stress it is subjected to and not whether it is in a bearing, gear, shaft, compressor or turbine. The crack propagation time in high cycle fatigue is generally a small fraction of the total time to failure. In low cycle fatigue, the crack propagation time is generally a significant fraction of the time to failure. In either case, the end result is total fracture of the component making it no longer useful for its intended purpose. If the failure distribution is within standard statistical ranges, then it can be represented by the Weibull analysis. Hence, the analysis should be blind to high or low cycle fatigue. This was recognized by Ioannides and Harris (ref. 31). Using Weibull and Lundberg-Palmgren, they introduced a "fatigue limiting stress" and integrated the computed life of elemental stress volumes to predict life. This approach leads to a method of applying Lundberg-Palmgren life prediction techniques to other components besides those subject to Hertzian loading. It also allows an investigator to use the results of coupon testing to predict the life of complex shaped components subjected to nonHertzian cyclic stressing. Ioannides and Harris (ref. 31) applied their analysis successfully to rotating beam fatigue (fig. 8), flat beams in reversed bending (fig. 9) and beams in reverse torsion. Based upon their approach or a modification thereof, design procedures for structures subjected to fatigue loading can be formulated which allows for finite life determination in the initial design stage with reasonable statistical certainity.

DESIGN OF COMPLEX STRUCTURES

In recent years, finite element stress analysis of complex structures subjected to thermal and mechanical loading has reach a high degree of sophistication and reliability. Now computers perform calculations in mere seconds and minutes for problems which only a short time ago would take hours or be impossible to perform. An example of a turbine blade analyzed using finite element analysis is shown in figure 10. Each element of material has associated with it a stress. For design purposes, this stress is within certain established stress or strain limits. The ultimate life of the component is usually based upon empirical calculations or extrapolation from field experience. The results are at best highly speculative. By subjecting the component to expensive product improvement programs (PIP) and by "make and break" techniques, component lives over a products life time can usually be extended to the useful life of the product.

The key to cost effective design is to be able to maximumize component life and minimize cost at the completion of the final design stage without the need for extensive testing or field experience. It is proposed that this objective can be accomplished as follows:

(1) Having determined the stress of each elemental volume of a component using finite-element analysis, it is possible to establish a life of the elemental stressed volume using either equation (3) or (7).

(2) Using equation (12) it is then possible to establish the life of the entire component.

(3) Carrying the process a step further, applying the principle of equation (8) a "dynamic" capacity of the component can be determined.

(4) Once the dynamic capacity is determined, the mission life of a component or system can be established using equation (13).

(5) Using renewal theory (eqs. (18) and (22)), the number of replacements required over a systems useful life and the mean time between failures (MTBF) can be determined.

By design iteration, design life can be maximized and cost minimized. Further, by knowing failure rates, production guidelines can be set prior to a product entiring the market place.

In the area of bearing technology, vast technological improvements occurred between initial publication of Lundberg-Palmgren (1947) and contemporary development. These technological improvements were factored into the analysis by the use of life adjustment factors (ref. 32) wherein the adjusted life L_A at a 90-percent probability of failure is

$$L_A = (D)(E)(F)(G)(H)\left(\frac{C}{P}\right)^n \tag{23}$$

where the life adjustment factors are

D Materials Factor
E Processing Factor
F Lubrication Factor
G Speed Effect
H Misalignment Factor

A similar approach in the design of power-transmitting shafts was taken by Loewenthal (ref. 33). The Loewenthal approach is based on a fatigue limiting stress based on combined torsion, bending, and axial loading. The fatigue limiting stress is modified by ten factors which in principle are in part incorporated in the Weibull and Lundberg-Palmgren analyses. Those factors outside the analysis can be based upon experimental coupon fatigue data.

Most machine elements are subjected to their maximum design load only for a very small faction of their mission life cycle. Hence, by designing to a life criterion and a mean cubic load rather than a fatigue limiting stress, it is possible to reduce the size of the machine element without a reduction in system reliability.

CONCLUSION

A generalized methodology to structural life prediction, design, and reliability based upon a fatigue criterion is advanced. The life prediction methodology is based in part on work of W. Weibull and G. Lundberg and A. Palmgren. The approach incorporates the computed life of elemental stress volumes of a complex machine element to predict system life. The results of coupon fatigue testing can be incorporated into the analysis allowing for life prediction and component or structural renewal rates with reasonable statistical certainty.

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TABLE 1. - LUNDBERG-PALMGREN FATIGUE LIFE ANALYSIS
[Based upon 1939 Weibull theory.]

Analysis	Year published
Bearings	1947
Spur gears	1975
Helical gears	1976
Restored and refurbished bearings	1977
Load-life relation	1978
Toroidal drive (traction)	1976
Optimization of multiroller traction drive	1980
Simplified life analysis for traction drives	1980
Nasvytis drive	1981
Spiral bevel gears	1982
Planetary assemblies	1982
Transmission assemblies	1983
Rotary beam, torsion x tension-compression	1984

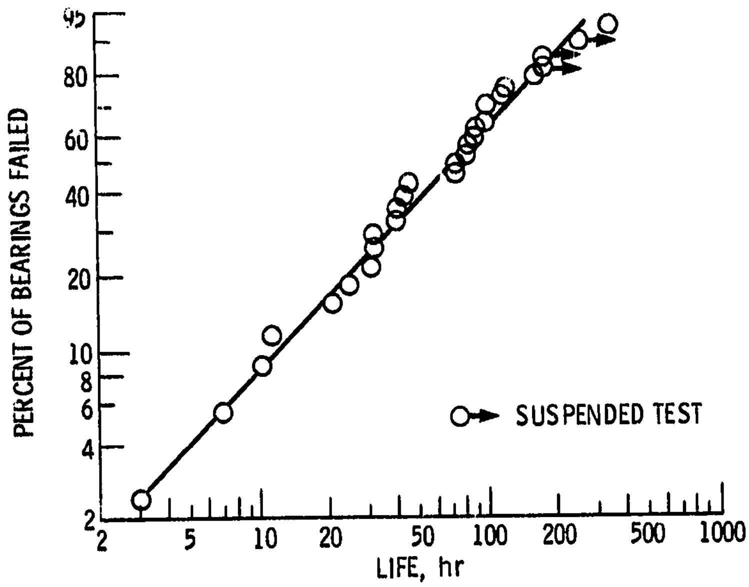


Figure 1. - Weibull distribution of bearing fatigue life for 140-mm bore-size angular-contact ball bearing. MIL-L-7808 lubricant; thrust load, 9500 lbs; speed, 10 000 rpm; temperature, 121 °C (250 °F).

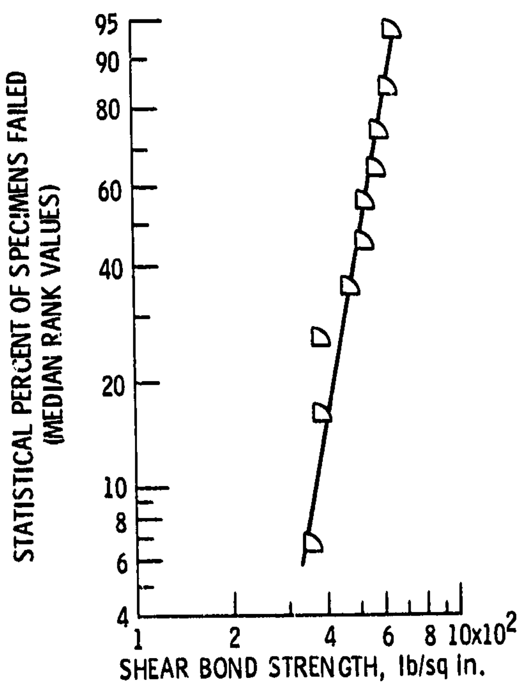


Figure 2. - Weibull distribution for bond-shear-strength for coating of spray powder on stainless steel (ref. 11).

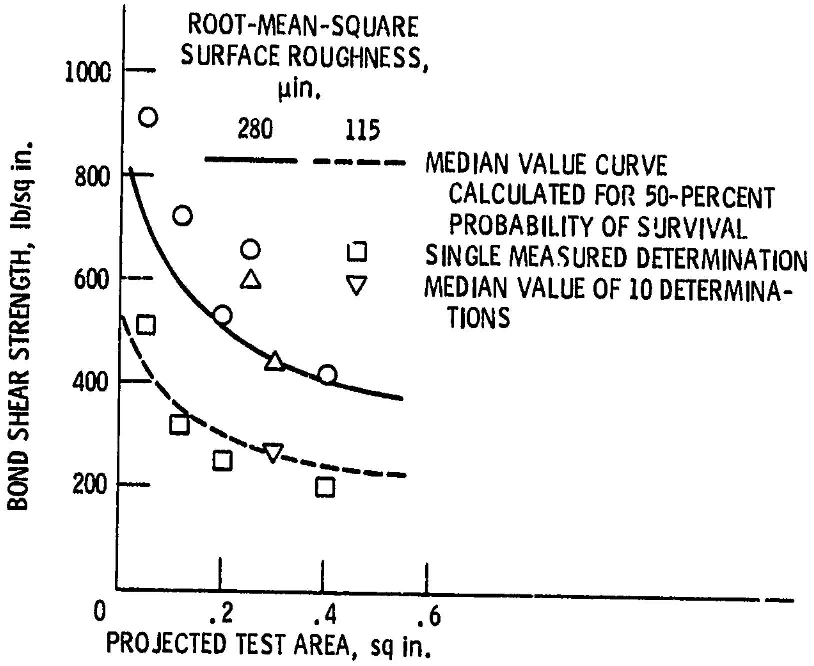


Figure 3. - Effect of projected test area on measured and calculated bond shear strength of coatings at two surface roughnesses (ref. 11).

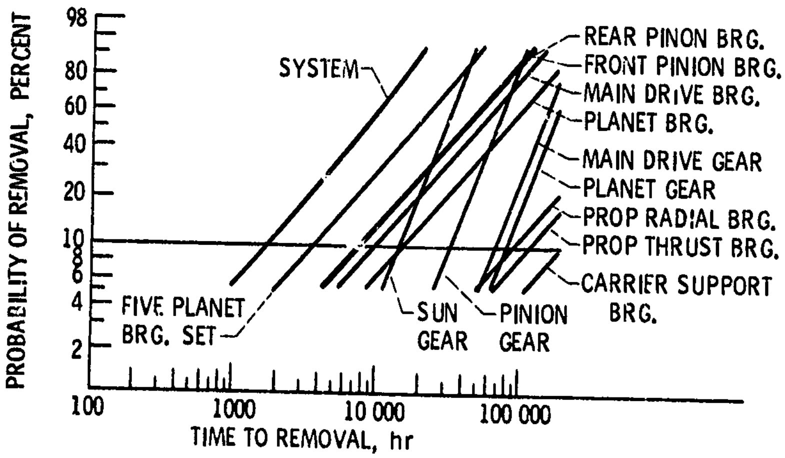


Figure 4. - Analytically predicted system and component fatigue mission lives. Combined material and material processing life adjustment factors used for bearings. Gear material is baseline material-no factors needed. No lubrication life adjustment factors used (ref. 28).

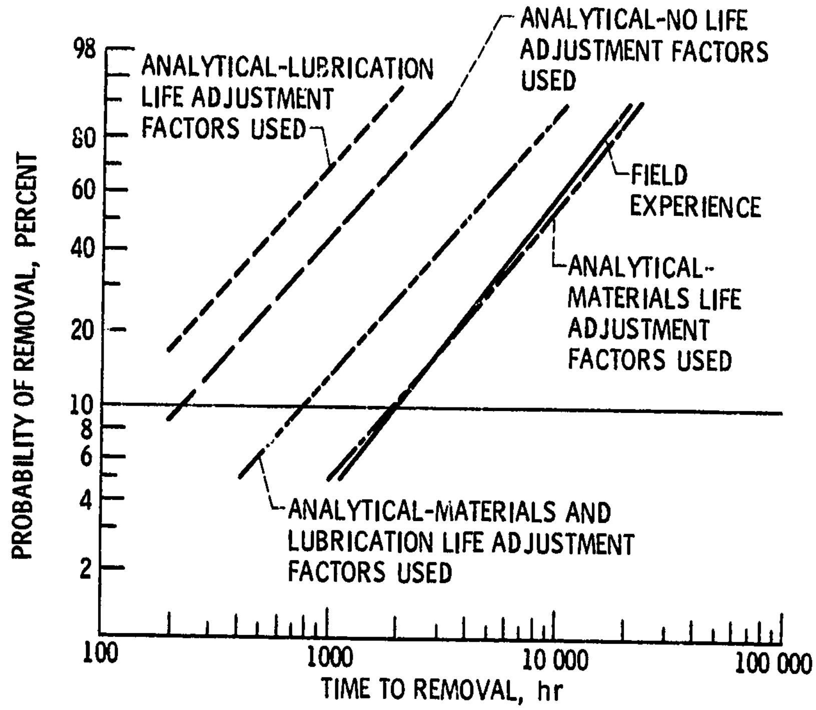


Figure 5. - Turboprop reduction gearbox fatigue lives (field experience) compared with analytically predicted system mission lives. (Material factor implies combined material and material processing factor.) (Ref. 28.)

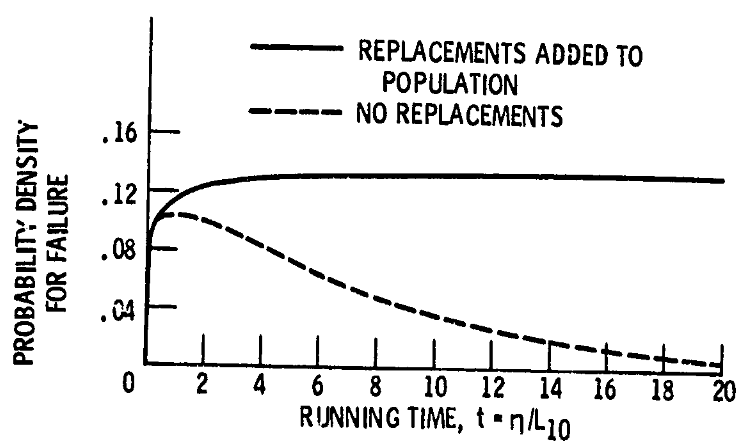


Figure 6. - Renewal density functions compared for rolling-element bearings. Also shown is probability density for failure of new bearing assuming no replacements. Area under curves represents probability of failure.

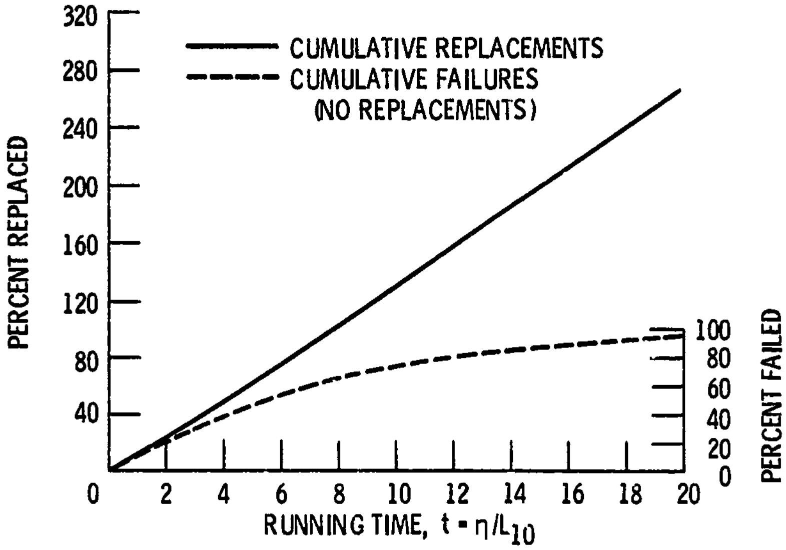
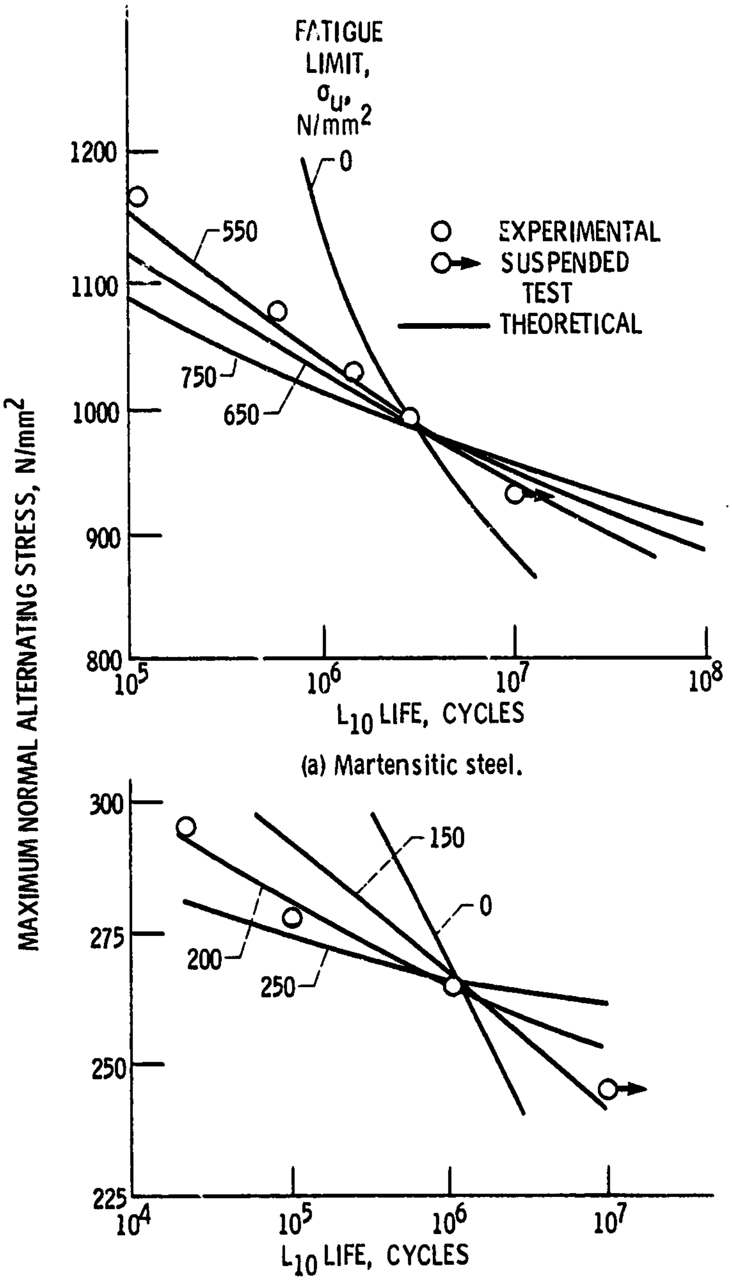


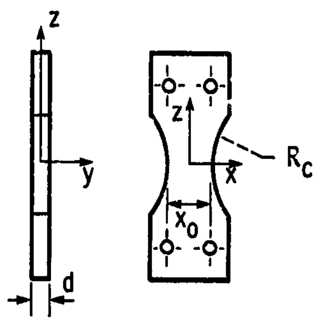
Figure 7. - Cumulative renewal and failure for rolling-element bearings.



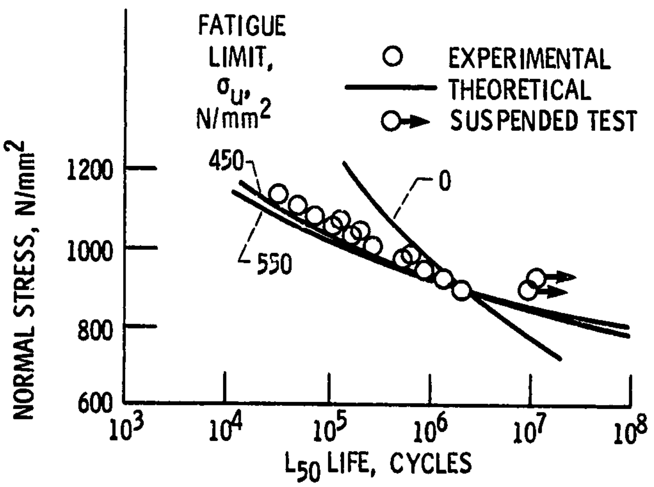
(a) Martensitic steel.

(b) Soft annealed steel.

Figure 8. - Comparison between experimental and predicted stress-life relation for AISI 52100 steel rotating beam specimens for calculated values of fatigue limit σ_u. (Ref. 31.)



(a) Flat beam configuration.



(b) Stress-life relationship for calculated values of fatigue limit σ_u.

Figure 9. - Comparison between experimental and predicted stress-life relation for flat beam specimens made from bearing steel and tested in reverse bending (ref. 31).

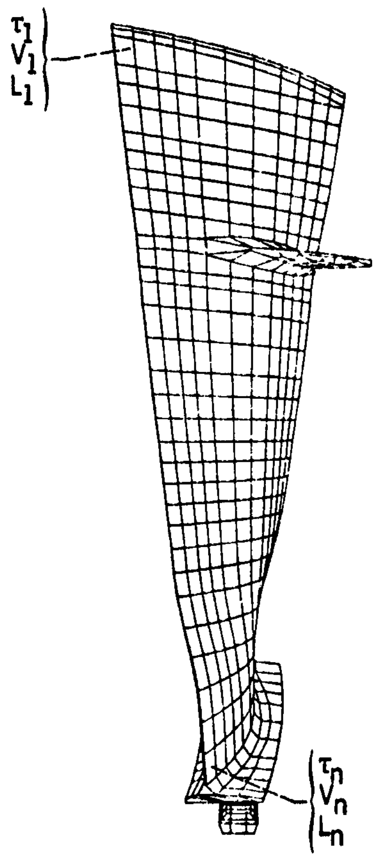


Figure 10. - Finite element analysis of turbine engine fan blade.

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15. Supplementary Notes

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16. Abstract

A generalized methodology to structural life prediction, design, and reliability based upon a fatigue criterion is advanced. The life prediction methodology is based in part on work of W. Weibull and G. Lundberg and A. Palmgren. The approach incorporates the computed life of elemental stress volumes of a complex machine element to predict system life. The results of coupon fatigue testing can be incorporated into the analysis allowing for life prediction and component or structural renewal rates with reasonable statistical certainty.

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