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Simultaneous Earth Observations From Two Satellites

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SIMULTANEOUS EARTH OBSERVATIONS FROM TWO SATELLITES

1 MOTIVATION

Inferences of characteristics of the earth's surface are made based upon remote observations of reflected (visible) and emitted (infrared) radiance from the surface It is well known that the radiance measured from a scene on the earth's surface depends upon the orientation of the scene relative to the direction of the sun and line of sight of the observer There is an advantage if two satellites in different orbits are used to make these observations at the same time At times when their subsatellite points are collocated, cross calibration can be performed, and at other times observations can be made to exploit the angular dependencies of the observations The question arises as to how often collocated, contemporaneous observations are possible using two satellites This work answers that question First, a simple closed form analytical expression is derived in Section 2 Secondly, a numerical, statistical solution is derived in Section 3 The two are compared with five examples in Section 4, and conclusions are given in Section 5

2 ANALYTICAL SOLUTION

Consider two orbits with inclinations i_1 and i_2 respectively If $i_2 > i_1$ the maximum angle between the planes of these two orbits is $i_1 + i_2$, the minimum angle is $i_2 - i_1$, and the average angle (for a year) is i_2

An encounter is defined to occur whenever the distance between the subsatellite points of 1 and 2 is less than D_{MAX} The probability that the ground track of satellite 2 will lie within encounter range of the ground track of satellite 1 is

$$P_2 = 4\, D_{MAX}/(2\, \pi\, R \sin i_2)$$

$$R = \text{Radius of the Earth} = 6378\ 165 \text{ kilometers}$$

This does not guarantee an encounter, since satellite 1 might not be at the right place in its orbit when satellite 2's subpoint is close to the ground track of satellite 1 Once the subsatellite point of 2 is within D_{MAX} of the ground track of 1, the probability that the subsatellite point of 1 will also be there is

$$P_1 = D_{MAX}/(2\, \pi\, R)$$

The probability that the subsatellite points of 1 and 2 will be located within D_{MAX} of each other is the product of probabilities

$$P = P_1\, P_2$$

$$\text{or} \quad P = D_{MAX}^2/(\pi\, R)^2 \sin i_2$$

The number of seconds in a year is

$$TY = (365 \text{ days/year})\ (24 \text{ hours/day})\ (3600 \text{ seconds/hour})$$

$$TY = 3\ 1536 \times 10^7 \text{seconds/year}$$

The total time per year during which the two satellites will be in a state of encounter with each other is

$$ET = P\ TY$$

$$ET = 3\ 156 \times 10^7 D_{MAX}{}^2/(\pi\, R)^2 \sin i_2 \qquad \text{(seconds/year)}$$

The speed of the subpoint of a satellite is given by

$$v = R\,(\mu^{1/2})/(R + h)^{3/2} \text{ (km/sec)}$$

$$h = \text{Satellite height (km)}$$

$$\text{where } \mu = GM = 3\ 98603\text{x}10^5 \text{ (km}^3 \text{ sec}^{-2}\text{)}$$

and G is the universal gravitational constant and M is the mass of the earth The yearly average relative speed of the subsatellite points 1 and 2 is

$$v_{rel} = (v_1^2 + v_2^2 - 2\, v_1\, v_2 \cos i_2)^{1/2}$$

The duration of an average encounter is

$$T_D = D_{MAX}/v_{rel}$$

The number of encounters per year is

$$NUM = ET/T_D$$

$$NUM = 3\ 15\text{x}10^7\, D_{MAX}^2\, (v_1^2 + v_2^2 - 2\, v_1\, v_2 \cos i_2)^{1/2}/(\pi R)^2\, D_{MAX} \sin i_2$$

$$NUM = \ 078\, D_{MAX}\, (v_1^2 + v_2^2 - 2\, v_1\, v_2 \cos i_2)^{1/2}/\sin i_2 \text{ (encounters/year)}$$

Some sample results are given in Section 4

3 NUMERICAL, STATISTICAL SOLUTION

The cartesian coordinates of the subsatellite point of the k^{th} ($k=1$ or 2) satellite relative to inertial space are given by (Reference 1)

$$x_k = R\,(\cos \Omega_k \cos \eta_k + \sin \Omega_k \cos i_k \sin \eta_k)$$

$$y_k = R\,(\sin \Omega_k \cos \eta_k + \cos \Omega_k \cos i_k \sin \eta_k)$$

$$z_k = R\,(\sin i_k \sin \eta_k)$$

where i_k = the inclination of the k^{th} satellite's orbit

Ω_k = the longitude of the ascending node of the k^{th} satellite's orbit

$\Omega_k = \Omega_{ok} + \dot{\Omega}_k\, t$

t = time

Ω_k = time derivative of Ω_k

$\dot{\Omega}_k = (-3/2)\, J_2\, R^2\, \mu^{1/2} \cos i_k\, (R + H)^{-7/2}$

J_2 = coefficient of the second zonal harmonic of the earth's potential = 00108263 (non-dimensional)

η_k = true anomaly of the k^{th} satellite

$\eta_k = \eta_{ok} + \dot{\eta}_k\, t$

$\dot{\eta}_k$ = time derivative of η_k

$\eta_k = \mu^{1/2}\,(R + h)^{-3/2}$

The distance between the two subsatellite points at any time t is given by

$$D = ((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z^2 - z_1)^2)^{1/2}$$

By computing D at sufficiently small increments of time throughout the year the times are found for which D is less than D_{MAX}, which by definition are encounter times The mean and root mean square values of the number of encounters are computed from 36 different computer runs for which the initial conditions of orbit 1 (i_1, h_1, Ω_{01}, η_{01}) and orbit 2 (i_2, h_2, η_{02}) are held fixed for each of the 36 computer runs, while Ω_{02} is zero for the first run and is incremented by 10 degrees for each successive run Each of the 36 computer runs simulates 1 year of flight for the two satellites

4 COMPARISON BETWEEN ANALYTICAL AND NUMERICAL SOLUTIONS

Table I shows a comparison between the analytical and numerical results for satellite 1 in a Shuttle type orbit and satellite 2 in a sun synchronous orbit The second comparison is for satellites in low inclination orbits, the third is for an equatorial satellite and a polar orbiter, the fourth comparison is for a shuttle type orbit and a geosynchronous orbit, the last comparison in the table has a sun synchronous orbit for satellite 1 and a geosynchronous orbit for satellite 2 The table shows that the analytical and mean numerical solutions of the number of encounters per year are within 12 percent of each other over the whole range of orbits and encounter diameters (D_{MAX} = 20, 50, and 100 km) The table also presents the RMS over the 36 solutions for each satellite pair

5 CONCLUSIONS

There are pathological cases where the analytical solution fails (e g , two satellites in the same orbit, but displaced in true anomaly such that they never encounter each other, or two satellites in the same orbit, with one posigrade and the other retrograde will have encounters exactly twice per orbit) Aside from these and perhaps a few other pathological cases the analytical solution works very well For all the cases analyzed in Table 1, the two satellites come within 20 km of each other about 10 to 15 times per year and within 100 km of each other about 50 to 100 times per year Encounters are sufficiently frequent that these opportunities may be exploited for cross calibration between on-board sensors or to augment the look angle range of special scenes There are some practical questions which need to be answered before proceeding with a two-satellite approach It might not be practical or desirable to conduct experiments at the encounter sites On the average the encounters will be split evenly between day and night Detailed trajectory analyses would be required for specific ground truth experiments

Reference

1 Danby, J M A , 1962 Fundamentals of Celestial Mechanics, Macmillan Company

TABLE 1 — NUMBER OF TIMES PER YEAR FOR WHICH TWO SATELLITES HAVE SUBPOINTS WHICH ARE SIMULTANEOUSLY WITHIN D_{MAX} OF EACH OTHER

Satellite 1		Satellite 2		Analytical Expression	D_{MAX} = 20 km Solution			D_{MAX} = 50 km Solution			D_{MAX} = 100 km Solution		
i_1	h_1	i_2	h_2		Analytical	Numerical		Analytical	Numerical		Analytical	Numerical	
						Mean	RMS		MEAN	RMS		MEAN	RMS
(°)	(km)	(°)	(km)	(Encounters per yr)	(Encounters per yr)			(Encounters per yr)			(Encounters per yr)		
28 0	500	98 21	705 0	NUM = 829 D_{MAX}	16 6	15 7	3 65	41 4	41 36	5 5	82 9	84 08	2 5
10 0	500	20 0	705 0	NUM = 555 D_{MAX}	11 1	10 0	3 69	27 8	26 9	3 76	55 5	54 7	2 59
0 0	300	90 0	1000 0	NUM = 765 D_{MAX}	15 3	14 1	737	38 2	36 9	1 16	76 5	74 1	1 48
0 0	35863	28 0	500 0	NUM = 1 11 D_{MAX}	22 2	21 1	743	55 5	54 1	2 37	111 0	109 4	2 23
0 0	35863	98 21	705 0	NUM = 542 D_{MAX}	10 8	9 67	745	27 1	25 6	1 25	54 2	52 5	1 76

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16 Abstract

Simultaneous co-located observations from two different orbits lead to several advantages (i.e., cross calibration of sensors and a wider range of solar-zenith and sensor look angles). The question was asked "How many times per year (on the average) do the sub-satellite points of two satellites simultaneously come within D kilometers of each other?". For the Space Station (altitude: 500 km, inclination: 28°) and a sun synchronous satellite (altitude 705 km, inclination 98.21°) the answers are 16, 41 and 82 times per year for encounter distances D of 20, 50, and 100 km respectively. The relationship between encounters per year and distance D is linear. The answers were obtained in two ways:

(1) a closed form statistical approach which led to a simple algebraic expression, and (2) a "Monte Carlo" type computer solution. The largest difference between the two solutions was less than 12%.

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