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SOLAR ARRAY FLIGHT EXPERIMENT

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Science and Engineering Directorate

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16. ABSTRACT

This is a closed form solution for the longitudinal oscillation of the Solar Array Flight Experiment (SAFE) blanket for all phases of deployment. The frequency response shows that the blanket frequency increases shortly before full deployment. That fact causes a coupling between the mast and the blanket frequency but, because of the relatively high speed of deployment, a buildup of resonance is unlikely.

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BACKGROUND AND PURPOSE

The purpose of this study was to predict the longitudinal frequencies of the solar blanket for all states of deployment. Furthermore, a study was done to detect a possible crosscoupling between bending and longitudinal frequencies. Finally, the study attempted to predict potential problems caused by the station keeping maneuver. For better understanding of this problem, Figures 1 and 2 were included to show the construction of the SAFE hardware.

DESCRIPTION OF THE SAFE STRUCTURE

The SAFE structure consists of a deployable mast and 84 solar panels that are hinged sequentially on torsional springs, forming a deployable blanket. One end of the blanket is attached to the header of the mast; the other end is connected to the base of the mast.

In this study, an assumption was made that all boundaries around the blanket were rigid. At all times, the solar panels were guided by wires to deploy simultaneously with the mast.

The motion of the blanket is restricted to the xy-plane, consequently, a sail-type bulging is excluded (Fig. 3).

The first section consists of 58 panels or an equivalent of a system with 28 degrees-of-freedom. Deployed, it represents 70 percent of the total length.

For computation, the above system is restricted to six panels only (Fig. 4).

During the deployment each panel rotates and translates simultaneously. For computation, the rotational points chosen are as the center of mass of each panel. The motion of each panel is fully described by the angles θ_i'. Consequently, the angles become the "generalized coordinates." The geometry of the system requires that the angles are:

$$\theta_1' = \theta_2' \quad , \quad \theta_3' = \theta_4' \quad , \quad \theta_5' = \theta_6' \quad . \tag{1}$$

Only two points (P_1 and P_2) on the six-panel blanket of Figure 4 are free to move anywhere within the boundaries. This narrows down the degrees-of-freedom to

$$N = \frac{n}{2} - 1 \quad , \tag{2}$$

N = degrees-of-freedom

$n/2$ = panel pair .

THE MATH MODEL

All variables are expressed in natural coordinates θ_i', and the constants are:

$n = 84$ total number of panels

$l = 15$ in. width of the panel

$k = 0.795$ in. lb/rad torsional stiffness of the hinge

$m = 0.0106$ lb sec^2/in. mass of a single panel

$I = m/3 \cdot (l/2)^2 = 0.1992$ in. lb sec^2 inertia mass of a single panel

$\omega_o{}^2 = k/I = 3.992$ rad/sec^2 = a chosen abbreviation, called "the reference frequency"

$\omega_o \approx 2$ rad/sec

kinetic energy

$$T = \frac{m}{2} \sum_{i=1}^{n} v_i^2 + \frac{I}{2} \sum_{i=1}^{n} \dot{\theta}_i^2 \tag{3}$$

NOTE: $\dot{\theta}_i{}^1 = \theta_i$ [see equation (6)].

Potential energy without gravitational contribution.

$$V = \frac{k}{2} (\theta_1{}')^2 + \frac{k}{2} \sum_{i=1}^{k} (\theta_{i-1}{}' + \theta_i{}')^2 + \frac{k}{2} (\theta_n{}')^2 \tag{4}$$

Lagrange's equations:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}_i} \right) + \frac{\partial V}{\partial \theta_i} = 0 \quad . \tag{5}$$

The equations of motion are obtained by substituting equations (3) and (4) into equation (5). To keep the derivation simple, a math model with 2 degrees-of-freedom is used. Later, the 2 degrees-of-freedom matrices are extrapolated to represent 28 degrees-of-freedom. This technique is applicable only if the matrices are symmetric. Because of Lagrange's equation property, the mass and stiffness matrices are symmetrical.

The derivation is cumbersome but straight-forward. Using Figure 4, the center of mass of each panel can be described with trigonometric functions using a common origin. By differentiating the displacements of each panel with respect to time, the linear velocity (v_i) is established, and the rotational velocity needed in equation (3) is a time derivative of the angles between the panels $\dot{\theta}_i$. In order to keep the differential equations solvable, the assumption was made that the angles between the panels consist of a constant part and a time-varying part. This assumption implies that the panels oscillate around the equilibrium point θ_o.

The above assumption is defined as:

$$\theta_i{}^1(t) = \theta_o + \theta_i(t) \tag{6}$$

where

$\theta_i{}^1$ = total angle between the panels

θ_o = angle of deployment or the equilibrium position

θ_i = oscillatory angle .

Using Figure 4, the displacement of the center of mass for the first panel is:

$$x_1 = \left(\frac{p}{2}\right) \sin \theta_1{}^1 = \left(\frac{p}{2}\right) \sin (\theta_o + \theta_1) \tag{7}$$

where the time derivative is:

$$\dot{x}_1 = \left(\frac{p}{2}\right) \dot{\theta}_1 \cos (\theta_o + \theta_1)$$

for

$$\lim_{\theta_1 \to o} \dot{x}_1 = \left(\frac{p}{2}\right) \dot{\theta}_1 \cos \theta_o \quad \text{corollary} \quad \lim_{\theta_1 \to 0} \dot{y}_1 = -\left(\frac{p}{2}\right) \dot{\theta}_1 \sin \theta_o \quad .$$

Consequently, the total linear velocity of the first panel's CM is:

$$v_1{}^2 = \dot{x}_1{}^2 + \dot{y}_1{}^2 = \left(\frac{p}{2}\right)^2 \left[(\dot{\theta}_1{}^2 \cos^2 \theta_o + \dot{\theta}_1{}^2 \sin^2 \theta_o)\right]$$

For simplicity, $s = \sin^2 \theta_o$, $c = \cos^2 \theta_o$. The rest of the velocities can be easily seen from Figure 4.

$$\left.\begin{aligned} v_1{}^2 &= \left(\frac{p}{2}\right)^2 (\dot{\theta}_1{}^2\, c + \dot{\theta}_1{}^2\, s) \\ v_2{}^2 &= \left(\frac{p}{2}\right)^2 [(3\dot{\theta}_1)^2\, c + \dot{\theta}_1{}^2\, s] \end{aligned}\right\} \text{1st panel pair}$$

$$\left.\begin{aligned} v_3{}^2 &= \left(\frac{p}{2}\right)^2 [(4\dot{\theta}_1 + \dot{\theta}_3)^2\, c + \dot{\theta}_3{}^2\, s] \\ v_4{}^2 &= \left(\frac{p}{2}\right)^2 [(4\dot{\theta}_1 + 3\dot{\theta}_3)^2\, c + \dot{\theta}_3{}^2\, s] \end{aligned}\right\} \text{2nd panel pair} \qquad (8)$$

$$\left.\begin{aligned} v_5{}^2 &= \left(\frac{p}{2}\right)^2 [(4\dot{\theta}_1 + 4\dot{\theta}_3 + \dot{\theta}_5)^2\, c + (\dot{\theta}_5{}^2\, s] \\ v_6{}^2 &= \left(\frac{p}{2}\right)^2 [(4\dot{\theta}_1 + 4\dot{\theta}_3 + 3\dot{\theta}_5)^2\, c + \dot{\theta}_5{}^2\, s] \end{aligned}\right\} \text{3rd panel pair}$$

NOTE: Because of equation (1), only odd or only even subscripts are needed. If one substitutes equation (8) into equation (3), the kinetic energy is:

$$T = \frac{m}{2}\left(\frac{p}{2}\right)^2 \left[\begin{array}{ll} \dot{\theta}_1{}^2\, C & + \dot{\theta}_1{}^2\, s \\ +(3\dot{\theta}_1)^2\, c & + \dot{\theta}_1{}^2\, s \\ +(4\dot{\theta}_1 + \dot{\theta}_3)^2\, c & + \dot{\theta}_3{}^2\, s \\ +(4\dot{\theta}_1 + 3\dot{\theta}_3)^2\, c & + \dot{\theta}_3{}^2\, s \\ +(4\dot{\theta}_1 + 4\dot{\theta}_3 + \dot{\theta}_5)^2\, c & + \dot{\theta}_5{}^2\, s \\ + (4\dot{\theta}_1 + 4\dot{\theta}_3 + 3\dot{\theta}_5)^2\, c & + \dot{\theta}_5{}^2\, s \end{array}\right] + \frac{I}{2}(\dot{\theta}_1{}^2 + \dot{\theta}_3{}^2 + \dot{\theta}_5{}^2)\cdot 2 \qquad (9)$$

Because of the rigid boundaries, the end requirements are:

$$\sum_{i=1}^{n} \theta_i^1 = \sum_{i=1}^{n} (\theta_o + \theta_i) = n\,\theta_o + \sum_{i=1}^{n} \theta_i = \text{constant}$$

or

$$\sum_{i=1}^{n} \theta_i = \text{constant} - n\theta_o = 0$$

in this case

$$\theta_1 + \theta_2 + \ldots + \theta_6 = 0 \quad ,$$

since

$$\theta_1 = \theta_2 \quad , \quad \theta_3 = \theta_4$$

or

$$\theta_1 + \theta_3 + \theta_5 = 0 \quad \rightarrow \quad \theta_5 = -\theta_1 - \theta_3 \quad , \quad \dot{\theta}_5 = -\dot{\theta}_1 - \dot{\theta}_3 \tag{10}$$

The mass moment of each panel rotated around CM_{PANEL} and the inertia of each panel is:

$$I = \frac{m}{3} \cdot \left(\frac{1}{2}\right)^2 \quad . \tag{11}$$

Using the above restrictions, the following substitutions were made:

$$T = \frac{3}{2} I \begin{bmatrix} \dot{\theta}_1^2\, c & + \dot{\theta}_1^2\, s \\ +(3\dot{\theta}_1)^2\, c & + \dot{\theta}_1^2\, s \\ +(4\dot{\theta}_1 + \dot{\theta}_3)^2\, c & + \dot{\theta}_3^2\, s \\ +(4\dot{\theta}_1 + 3\dot{\theta}_3)^2\, c & + \dot{\theta}_3^2\, s \\ +(4\dot{\theta}_1 + 4\dot{\theta}_3 - \dot{\theta}_1 - \dot{\theta}_3)^2\, c & + (\dot{\theta}_1^3 + \dot{\theta}_3)^2\, s \\ +(4\dot{\theta}_1 + 4\dot{\theta}_3 - 3\dot{\theta} - 3\dot{\theta}_3)^2\, c & + (\dot{\theta}_1 + \dot{\theta}_3)^2\, s \end{bmatrix} + I[\dot{\theta}_1^2 + \dot{\theta}_3^2 + (\dot{\theta}_1 + \dot{\theta}_3)^2] \tag{12}$$

Using the chain rule on equation (12) and differentiating the kinetic energy partially in respect to $\dot{\theta}_i$ and later with respect to time, the equation of motion becomes:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) = I\left\{3(48\ddot{\theta}_1 + 24\ddot{\theta}_3)\cos^2\theta_o + 16\ddot{\theta}_1 + 8\ddot{\theta}_3\right\}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_3}\right) = I\left\{3(24\ddot{\theta}_1 + 16\ddot{\theta}_3)\cos^2\theta_o + 8\ddot{\theta}_1 + 16\ddot{\theta}_3\right\}$$

Using Figure 4 and equation (4) the potential energy is:

$$V = \frac{k}{2}(\theta_1{}')^2 + \frac{k}{2}(\theta_1{}' + \theta_2{}')^2 + \frac{k}{2}(\theta_2{}' + \theta_3{}')^2 + \frac{k}{2}(\theta_3{}' + \theta_4{}')^2 + \frac{k}{2}(\theta_4{}' + \theta_5{}') + \frac{k}{2}(\theta_5{}' + \theta_6{}')^2$$
$$+ \frac{k}{2}(\theta_6\ )^2$$

because of equations (1), (6) and (10), V reduces to:

$$V = \frac{k}{2}(\theta_o + \theta_1)^2 + \frac{k}{2}(2\theta_o + 2\theta_1)^2 + \frac{k}{2}(2\theta_o + \theta_1 + \theta_3)^2 + \frac{k}{2}(2\theta_o + 2\theta_3) + \frac{k}{2}(2\theta_o - \theta_1)^2$$
$$+ \frac{k}{2}(2\theta_o - 2\theta_1 - 2\theta_3)^2 + \frac{k}{2}(\theta_o - \theta_1 - \theta_3)$$

according to equation (5) the partial potential energy is:

$$\frac{\partial V}{\partial \theta_1} = k\ (0 + 12\theta_1 + 6\theta_3)$$

$$\frac{\partial V}{\partial \theta_3} = k\ (\theta_o + 6\theta_1 + 10\theta_3) \quad .$$

If one collects all the terms, the final equations of motion are:

$$I\left\{3(48\ddot{\theta}_1 + 24\ddot{\theta}_3)\cos^2\theta_o + 16\ddot{\theta}_1 + 8\ddot{\theta}_3\right\} + k(0 + 12\theta_1 + 6\theta_3) = 0$$

$$I\left\{3(24\ddot{\theta}_1 + 16\ddot{\theta}_3)\cos^2\theta_o + 8\ddot{\theta}_1 + 16\ddot{\theta}_3\right\} + k(\theta_o + 6\theta_1 + 10\theta_3) = 0 \quad . \qquad (13)$$

Abbreviated for convenience, in short:

$$\omega_o^2 = \frac{k}{I} \quad . \tag{14}$$

If equation (14) is substituted in equation (13), the equations of motion are:

$$\begin{bmatrix} 16 & 8 \\ 8 & 10 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_3 \end{Bmatrix} + 3\cos^2\theta_o \begin{bmatrix} 48 & 24 \\ 24 & 16 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_3 \end{Bmatrix} + \omega_o^2 \begin{bmatrix} 12 & 6 \\ 6 & 10 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_3 \end{Bmatrix} = -\omega_o^2 \begin{Bmatrix} \cdot \\ \theta_o \end{Bmatrix} \tag{15}$$

Using symmetry, the equation of motion for a system with 5 degrees of freedom is:

$$[M] \left\{\ddot{\theta}_i\right\} + \omega_o^2 \, [K] \left\{\theta_i\right\} = -\omega_o^2 \left\{\theta_o\right\}$$

The equation is explained as follows:

$$[M] = \begin{bmatrix} 16 & & & & \\ 8 & & \text{symmetrical} & & \\ & & & & \\ & & & & \\ 8 & & & 8 & 16 \end{bmatrix} + 3\cos^2\theta_o \begin{bmatrix} 16*9 & & & & \\ & & \text{symmetrical} & & \\ 8*15 & 16*7 & & & \\ 8*11 & 8*11 & 16*5 & & \\ 8*7 & 8*7 & 8*7 & 16*3 & \\ 8*3 & 8*3 & 8*3 & 8*3 & 16*3 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 12 & & & & \\ & & \text{symmetrical} & & \\ 7 & 12 & & & \\ 6 & 7 & 12 & & \\ 6 & 6 & 7 & 12 & \\ 5 & 5 & 5 & 6 & 10 \end{bmatrix}$$

[M] = mass matrix consists of two parts, static and dynamic.

[K] = stiffness matrix

* = multiplication sign.

DISCUSSION

Equation of motion (15) was treated as an eigenvalue problem. A small desk computer was used to compute the roots of the problem since equation (15) can be rewritten as:

$$[M] = \lambda_i [K]$$

or

$$\lambda_i = \omega_i^2 \quad .$$

In Figure 5 the natural frequency was drawn as a function of the deployment angle θ_o. As expected, the natural frequency of the blanket increases shortly before full deployment because the structure becomes stiffer. At full deflection, all the blanket modes (longitudinal) coincide at $f \approx 0.3$ Hz.

Using the finite element method, the dynamics of the whole system were simulated. For 70 percent deployment, the natural frequency of it is 0.06 Hz (Fig. 5, point R_2). To answer the question of a potential problem, due to a mutual excitation between blanket and mast, the frequency response curve (2) was drawn. Points R_1 and R_2 were given by simulation and R_3 was implicitly computed using the beam equation.

The curves (1) (Fig. 5) intersect with curve (2) at S_1, S_2, S_3, ... S_{28}. Those are points of mutual coupling that if given sufficient time, can build up to a problem – causing resonance. To investigate the time the mast is exposed to the sympathetic oscillation of the blanket, the dwell-time concept was introduced.

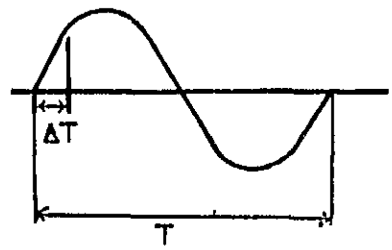
DWELL FACTOR T_{dw}

T_{dw} is a factor that defines the percent of the mast period, the mast and blanket are essentially at coincident frequencies. This factor is defined as the ratio, in percent, of the time a changing blanket natural frequency lies between the 1/2 power points of the mast first mode response, and is shown by the formula on Table 1. As shown in this table, the lower blanket modes ($i = 1, 2, 3$) coincide with the mast first natural frequency at S_1, S_2, and S_3 with frequencies $f_i = 0.070$, 0.082 and 0.092. Since the blanket natural frequency is only coincident with the mast natural frequency for less than 1/4 of a mast period, the response does not have sufficient time to grow to a significant level.

CONCLUSION

An obvious problem is not evident. A possible further study could determine the dynamic behavior under the station-keeping conditions.

TABLE 1. THE DWELL FACTOR FOR THE LOWEST THREE MODES

i	f_i (Hz)	S_i (Hz/sec)	T_{DW} (%)	Pictorial Representation of T_{DW}
1	0.070	0.00109	9	
2	0.082	0.00076	18	
3	0.092	0.00069	25	
.	.			
.	.			
.	.			
28	0.240	0.00000	∞	

S_i = rate of change of blanket mode frequency at the intersection with the mast first bending mode (Fig. 5).

f_i = frequency at intersection

i = blanket modal number

$\zeta \approx 0.01$

$T = 1/f_i$

$\Delta T = 2\zeta\ f_i 2/S_i$

$T_{dw} = \Delta T/T * 100\%$

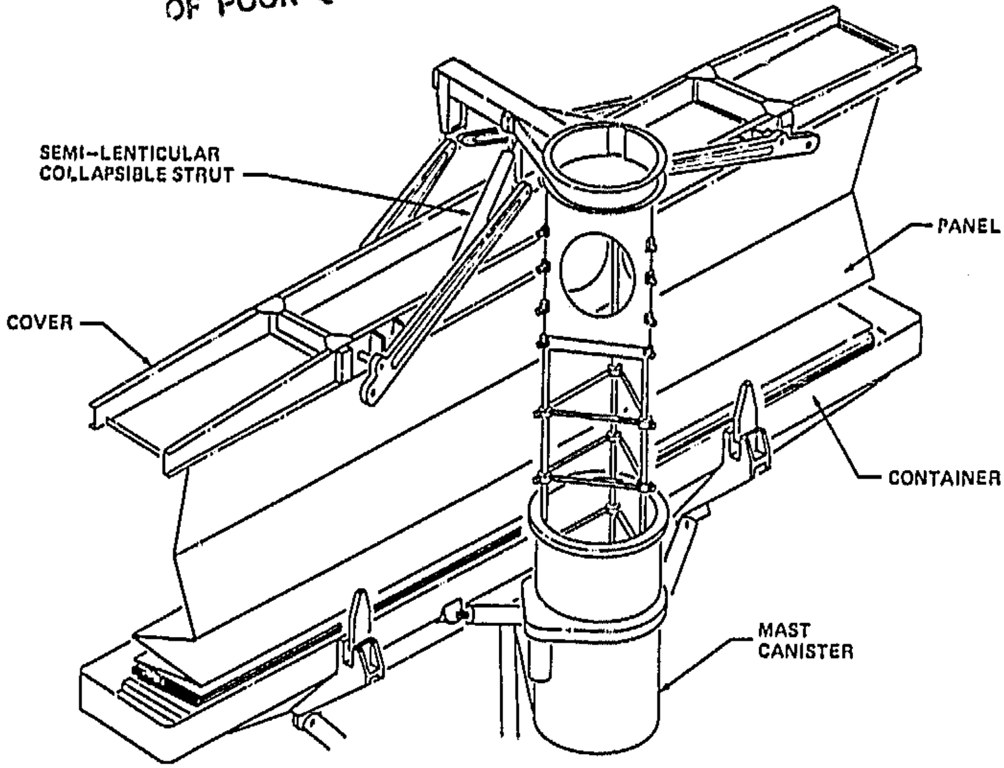


Figure 1. Summary view SAFE structure.

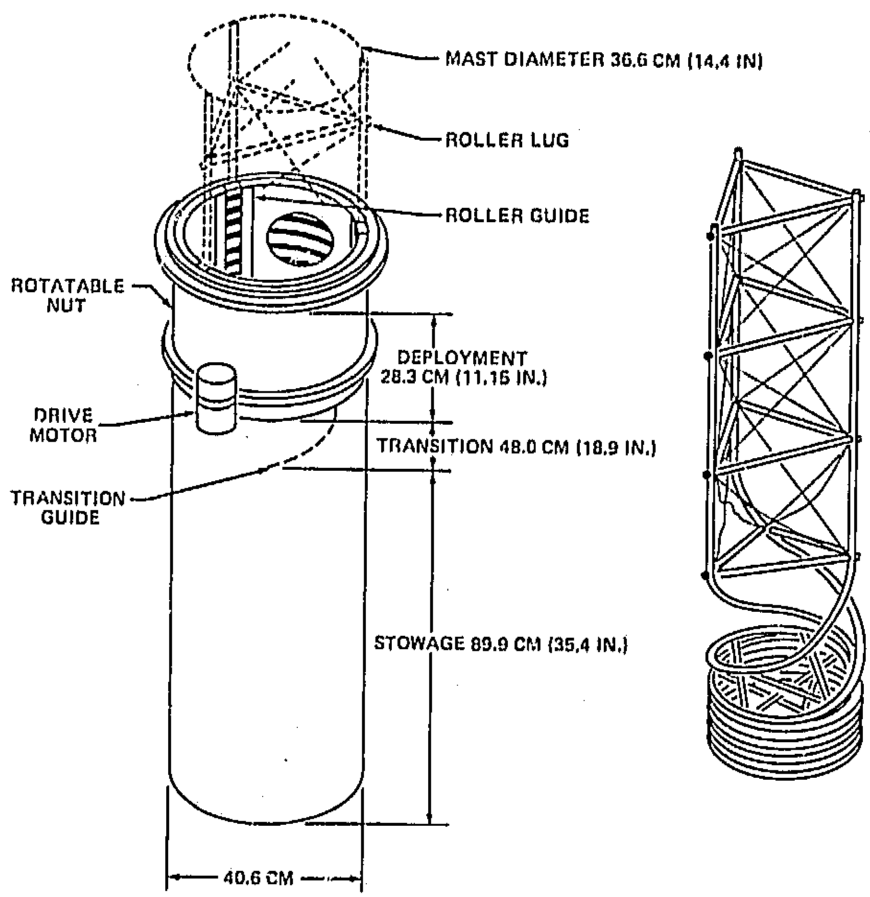


Figure 2. Solar array extension mast.

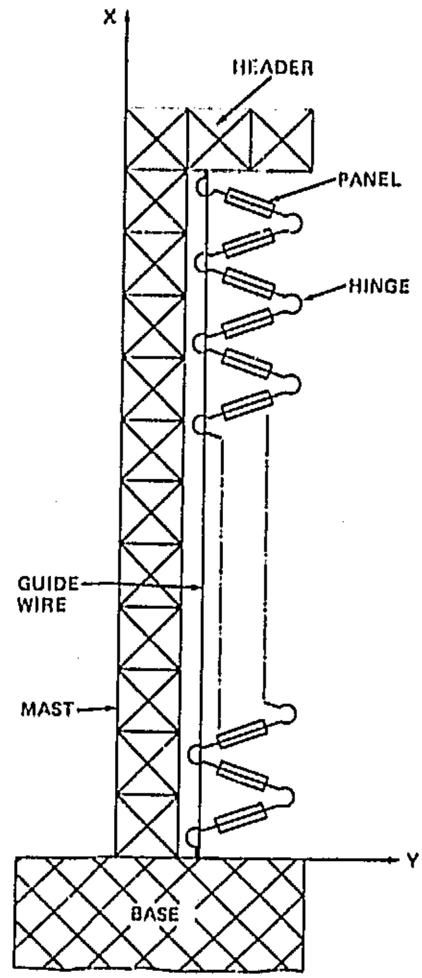


Figure 3. The overall view of the SAFE model.

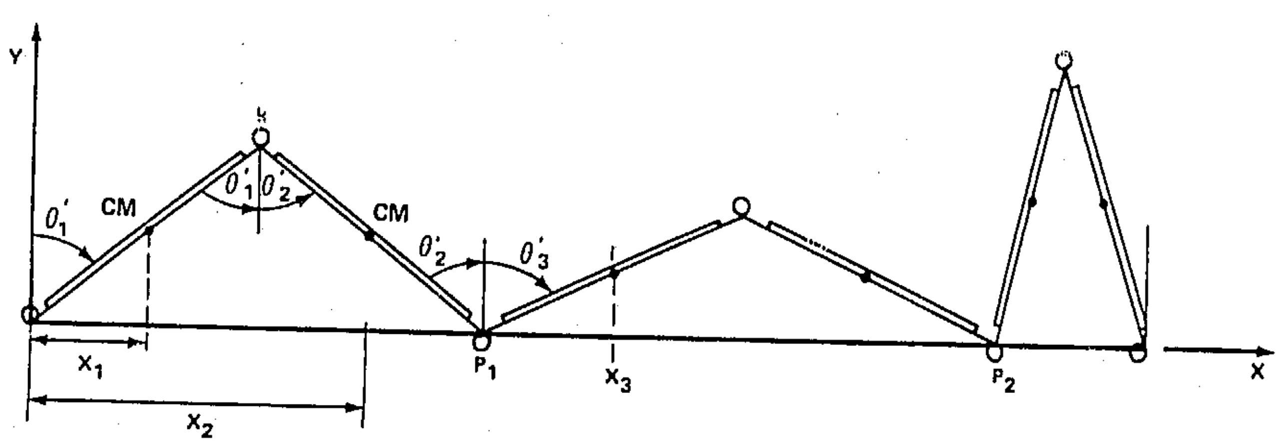


Figure 4. A closeup section of the blanket.

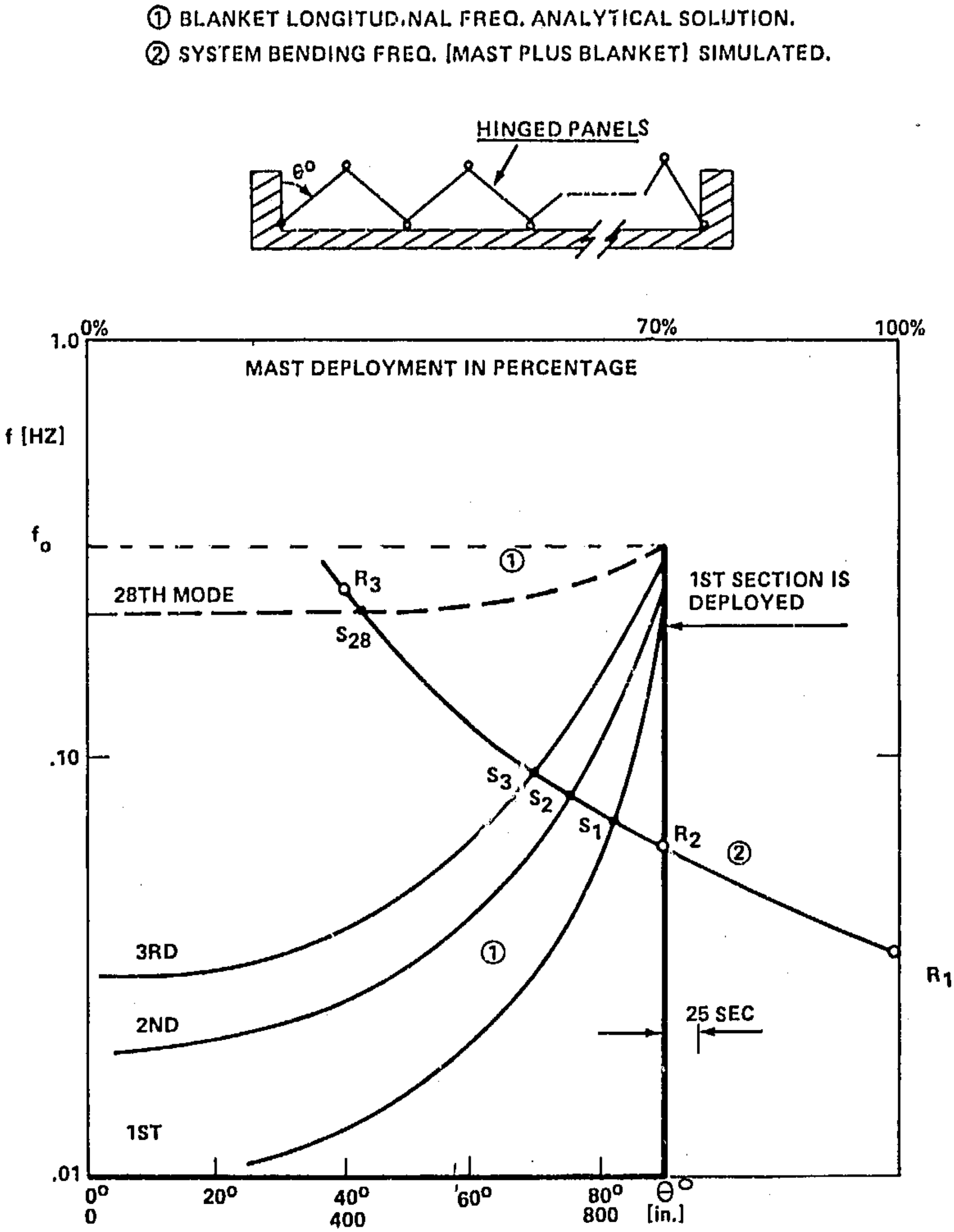


Figure 5. Blanket natural frequency versus deployment.

APPROVAL

SOLAR ARRAY FLIGHT EXPERIMENT

By Jerry Slaby

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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