TWO SIMPLIFIED PROCEDURES FOR PREDICTING CYCLIC MATERIAL RESPONSE FROM A STRAIN HISTORY

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Simplified inelastic analysis procedures were developed at NASA Lewis and Pratt & Whitney Aircraft for predicting the stress-strain response at the critical location of a thermomechanically cycled structure. These procedures are intended primarily for use as economical structural analysis tools in the early design stages of aircraft engine hot section components where nonlinear finite-element analyses would be prohibitively expensive. Both simplified methods use as input the total strain history calculated from a linear elastic analysis. The elastic results are modified to approximate the characteristics of the inelastic cycle by incremental solution techniques. A von Mises yield criterion is used to determine the onset of active plasticity. The fundamental assumption of these methods is that the inelastic strain is local and constrained from redistribution by the surrounding elastic material.

In the procedure developed by PWA, a power law creep expression is combined with the elastic response to define the stress change over an increment of time. A Taylor series expansion is used to calculate the stress at the end of an increment from the stress at the beginning and the appropriate derivatives.

In the NASA Lewis procedure, a computer program (ANSYMP) was developed to predict the stress history at the critical location using the total strain-temperature history as input. Material cyclic stress-strain and creep properties and appropriate constitutive models are coded into user subroutines in the program. Plastic strains are computed iteratively for each increment of loading. Creep effects can be calculated on the basis of stress relaxation at constant strain, creep at constant stress or a combination of stress relaxation and creep accumulation.

The two simplified procedures were exercised on a combustor liner louver lip cycle and two thermomechanical fatigue test cycles. Both methods were able to capture the overall shapes of the hysteresis loops and to predict the stress levels to a degree of accuracy sufficient for most life prediction models.

INTRODUCTION

Requirements for better performance and fuel economy in aircraft gas turbine engines have adversely affected the durability of the hot section components. Structures such as combustor liners, turbine blades and vanes, and support structures experience severe gas temperature environments and operating conditions which often result in significant cyclic plastic and creep strains. Structural analysis under these conditions becomes a formidable undertaking. Improvements in the durability of hot section structures depend on accurate definition of the cyclic stress-strain response at the critical fatigue location.

Finite-element computer programs have been used with increasing frequency for the structural analysis of gas turbine engine components. In lower temperature components, design stress levels are maintained below the material yield strength. Linear finite-element analysis is sufficient for this type of component and the elastic solutions can be obtained rapidly and economically over the range of loading conditions constituting the flight cycle. However, for structures in the hot part of the engine, local stresses can exceed the material yield strength and can induce time-dependent creep deformation. Accurate prediction of the local inelastic stress-strain response under these conditions requires nonlinear structural analysis.

An example of a nonlinear structural analysis of a hot section component is described in Reference 1 for a half-scale louver-type combustor liner. This liner specimen was constructed in an identical configuration with current combustor liners in service and was tested in an induction heated experimental rig. The three-dimensional finite element model of a segment of the half-scale combustor liner (Figure 1) was subjected to the thermal loading cycle shown in Figure 2 (a). Both linear elastic and nonlinear stress-strain solutions for the critical fatigue location at the end of the louver lip are shown in Figures 2 (b) and (c) with letter designations corresponding to the time points shown in Figure 2(a). The linear thermoelastic analysis was conducted at a number of specific points throughout the thermal loading cycle. The results show compressive stresses beyond the material yield strength and a closed cyclic reponse. An incremental nonlinear analysis predicts a more realistic stress-strain response that includes cyclic plasticity and strain ratchetting. These analyses were performed using the MARC nonlinear finite-element code (Ref. 2). This and similar general purpose nonlinear codes use uncoupled classical time-independent plasticity and time-dependent creep models and sophisticated computational algorithms. With these features, a nonlinear finite element analysis represents a labor intensive, time-consuming and costly effort which is generally incompatible with the iterative nature of the design process.

This paper presents two simplified procedures for more economically estimating the local hysteretic response produced by cyclic thermal loading by using as input the total strain history calculated from an elastic analysis. The two procedures were exercised on two thermomechanical fatigue test cycles and a "faithful cycle" simulation of the strain-temperature history at the combustor liner louver lip. Both methods were able to capture the overall shapes of the hysteresis loops and to predict the stress levels to a degree of accuracy sufficient for most life prediction models. Development of these simplified analyses should reduce the need for nonlinear finite-element analysis in the early design stages for hot section components.

SYMBOLS

A, n = temperature-dependent constants in creep power law, equation (8)
E = modulus of elasticity
E_p = strain hardening slope of stress-strain curve
K, m = temperature-dependent constants in stress-strain equation
t = time
T = temperature
w = work hardening slope at maximum plastic strain
$\Delta\epsilon$ = strain increment
$\Delta\sigma$ = yield stress shift due to load reversal
Δt = time increment
ϵ = strain
$\dot{\epsilon}$ = strain rate
equation (1)
σ = stress
Ω = backstress
$\dot{\sigma}$ = $E\dot{\epsilon} - EA|\sigma|^{n-1}\sigma$
$\ddot{\sigma}$ = $-nEA|\sigma|^{n-1}\dot{\sigma}$
$\dddot{\sigma}$ = $-n(n-1)EA|\sigma|^{n-3}\sigma\dot{\sigma}^2 - nEA\sigma^{n-1}\ddot{\sigma}$
ν = Poisson's ratio

subscripts:

c = creep
e = elastic
i = beginning of increment
i+1 = end of increment
max = maximum value at start of unloading
p = plastic
t = total
y = yield

ANALYTICAL PROCEDURE

Within the gas turbine engine, certain hot section components experience a degree of inelastic or nonlinear stress-strain response. These components (combustor liners, turbine vanes, cases and other support structures) are subjected to cyclic thermomechanical loading where the predominant stress is produced by temperature differentials within the structure. In general, the resulting inelastic response is localized to the area of maximum temperature gradient while the remainder of the structure remains elastic. The stiffer elastic material acts as a constraining body and controls the cyclic strain range experienced by the inelastic region. A comparison of predicted results from nonlinear and linear elastic finite-element analyses in References 1,3 and 4 have shown that the cyclic total strain range and total strain history at the location of maximum inelastic response can be estimated from the linear thermoelastic analyses. Therefore, these simplified procedures assume, that the local mechanical strain and temperature histories at a location in the structure are known from previous linear analysis. Basic material (Hastelloy X) properties, including yield stresses and creep parameters developed for these analyses, are shown in Table I.

Simplified Procedure 1

This procedure for predicting local stress-strain response was developed at Pratt & Whitney Aircraft under contract to NASA Lewis Research Center and is fully documented in Reference 5. Development of the procedure assumes that the strain-temperature histories produced by a loading cycle are known from previous analysis. An incremental description of the histories together with the procedure described below is used to calculate the resulting stress history. Each increment in strain is assumed to be composed of either time-independent plasticity or time-dependent elastic and creep response.

$$\Delta\epsilon = \Delta\epsilon_p \tag{1a}$$

or

$$\Delta\epsilon = \Delta\epsilon_e + \Delta\epsilon_c \tag{1b}$$

Since the solution strategy is based on the prediction of stress increments, equations (1) are rewritten as

$$\Delta\sigma = \Delta\sigma_p \tag{2a}$$

or

$$\Delta\sigma = \Delta\sigma_e + \Delta\sigma_c \tag{2b}$$

During a loading cycle, the onset of plastic action is determined by the conventional yield surface concept taken from the classical time-independent plasticity model. The yield surface is assumed to be temperature dependent and isotropic with no strain hardening (fixed size and equal in tension and compression). Justification for this definition of the yield surface is based on two observations: (1) that at higher temperatures, Hastelloy X displays little cycle hardening and (2) that the variable temperature experienced in a thermomechanical cycle, which is the primary application of this procedure, reduces the amount of cyclic hardening developed at the lower temperatures.

The stress increment associated with time-independent plastic action is then calculated as

$$\sigma_{i+1} - \sigma_i = \Delta\sigma_p = \sigma_{y,i+1} - \sigma_{y,i} \qquad \text{for } \sigma_i = \sigma_{y,i} \text{ and } T_{i+1} \geq T_i \tag{3}$$

or

$$\Delta\sigma_{i+1} - \sigma_i = \Delta\sigma_p = (E_{p,i+1} + E_{p,i})\Delta\epsilon/2 \qquad \text{for } \sigma_i = \sigma_{y,i} \text{ and } T_{i+1} < T_i \tag{4}$$

For all other loading conditions the strain, or stress, is assumed to be time dependent elastic and creep response.

$$\Delta\epsilon_t = \Delta\epsilon_e + \Delta\epsilon_c \tag{5}$$

However, rather than consider a separate uncoupled creep model, an integrated or viscoplastic approach is used. This approach was pursued based on the observation that the cyclic material response is not purely elastic in either the loading or unloading parts of the response curves. Instead the transient response represents a simultaneous elastic and creep action. Development of the model is as follows:

$$\dot{\epsilon}_t = \dot{\epsilon}_e + \dot{\epsilon}_c \tag{6}$$

where $$\dot{\epsilon}_e = \dot{\sigma}/E \tag{7}$$

with Young's modulus, E, assumed to be temperature dependent but constant over any increment of loading. For the creep rate term, use is made of the short time monotonic creep model developed in Reference 1. This expression is

$$\dot{\epsilon}_c = (\sigma/A)^n \tag{8}$$

Thus, equation can be written as

$$\dot{\epsilon}_t = \sigma/E + Ao^n \tag{9}$$

or $$\dot{\sigma} = E\dot{\epsilon}_t - EA\sigma^n \tag{10}$$

An incremental solution of this nonlinear equation was developed using a Taylor series expansion.

$$\sigma_{i+1} = \sigma_i + \dot{\sigma}_i \Delta t + \ddot{\sigma}_i \Delta t^2/2! + \dddot{\sigma}_i \Delta t^3/3! + \ldots \tag{11}$$

For cyclic analysis the stresses used in equation (11) are effective values modified by a backstress, i.e:

$$\sigma^* = \sigma - \Omega$$
(12)

In a physical sense, the backstress is an internal stress generated by plastic deformation that changes the reference point for measurement of global stress. The function used to estimate the value of the backstress in these calculations is:

$$\Omega = \sigma_{max} - \sigma_i \qquad \text{for } \sigma_{max} \leq 2\sigma_y \tag{13}$$

or $$\Omega = \sigma_y \qquad \text{for } \sigma_{max} > 2\sigma_y \tag{14}$$

This is equivalent to considering a series of temperature-dependent circular yield surfaces pinned at the maximum tensile stress in the cycle and the back stresses as the centers of the yield circles. The limiting condition occurs when the diameter of the yield circle equals the maximum stress; a smaller (hotter) yield circle would predict reverse plasticity at the same sign of stress as the maximum stress which would be positive in this case. This is generally considered not possible and therefore the back stress is limited to the value of the yield stress for these temperatures.

Simplified Procedure 2

This fully automated procedure was developed at NASA Lewis Research Center for calculating the stress-strain history at the critical fatigue location of a structure subjected to cyclic thermomechanical loading. It has been implemented in a computer program (ANSYMP) which is documented in Reference 6. The procedure has been exercised on a wide variety of problems including multiaxial loading, nonisothermal conditions, different materials and constitutive models, and dwell times at various points in the cycles. Comparisons of the results of the simplified analyses for these problems with MARC inelastic solutions are reported in Reference 6. The basic assumption is that the total strain ranges calculated from linear elastic and nonlinear analyses are approximately equal. Another assumption of the method is that the effective stress-equivalent total strain hysteresis loops constructed from an elastic-plastic analysis will be parallel to the elastic hysteresis loop. The validity of these assumptions is demonstrated in References 1,3 and 4.

The procedure was set up to calulate the material cyclic response using the total strain obtained from an elastic analysis or strain measurements. Classical plasticity methods are used to characterize the yield surface by a yield condition to describe yielding under multiaxial stress states and by a hardening model to establish the location of the yield surface during cycling. This procedure was set up to accommodate itself to any yield criterion or hardening model. The only requirements are that the elastic input data, whether calculated or measured, be in a form consistent with the yield criterion and that the appropriate material properties be used in conjunction with the hardening model.

Most nonlinear computer programs use the von Mises yield criterion and incremental plasticity theory. Implicit in the von Mises yield criterion is the conversion of the total strain from a uniaxial stress-strain curve to modified equivalent total strain. The modified elastic equivalent total strain corresponds to the uniaxial total elastic strain multiplied by $2(1+v)/3$. This relationship must be taken into account for multiaxial problems in applying strain results from elastic finite-element programs or strain measurements as input for the simplified procedure.

In this study, all of the analyses were performed with the Hastelloy stress-strain properties and combined isotropic-kinematic hardening model used for the MARC nonlinear analyses in Reference 1. Creep computations were conducted with the same monotonic creep properties developed in Reference 1. Creep characteristics of the material were incorporated into the program with the creep model expressed by equation (8) and the constants given in Table I.

The yield stress shift ($\Delta\sigma_y$) due to load reversal under kinematic hardening is

$$\Delta\sigma_y=2(\sigma_y-w(\epsilon_{p,max})) \qquad (15)$$

where σ_y represents the current, not the initial, yield stress.

The procedure permits any of three creep options to be selected; (1) stress relaxation at constant strain, (2) cumulative creep at constant stress, and (3) a combination of (1) and (2). Option 1 was used for the simplified analyses conducted for this study since the problems involved strain controlled tests.

The elastic input data are subdivided into a sufficient number of increments to define the stress-strain cycle. To simulate the effects of time-dependent plasticity, dwell times for creep analysis were specified for all increments. The input data for the analyses were supplied by Pratt & Whitney Aircraft. The increments are analyzed sequentially to obtain the cumulative plastic and creep strains and to track the yield surface. An iterative procedure is used to calculate the yield stresses for increments undergoing plastic straining. First, an estimated plastic strain is assumed for calculating an initial yield stress from the stress-strain properties and the simulated hardening model. Second, a new plastic strain is calculated as

$$\epsilon_p = \epsilon_t - \epsilon_c - \sigma_y / E \qquad (16)$$

The yield stress is then recalculated using the new plastic strain. This iterative procedure is repeated until the new and previous plastic strains agree within a tolerance of 1 percent.

A FORTRAN IV computer program (ANSYMP) was created to automatically implement the simplified analytical procedure. The program consists of the main executive routine, ANSYMP, and four subroutines, ELAS, YIELD, CREEP, and SHIFT. The incremental elastic data and temperatures are read into subroutine ELAS. Material stress-strain properties as a function of temperature and a simulated hardening model are incorporated in subroutine YIELD and the creep characteristics are incorporated in subroutine CREEP. Subroutine SHIFT is required to update the temperature effects on the yield stress shift. SHIFT also serves the function of deciding the future direction of the yield surface under nonisothermal conditions by determining the relation of future to past thermal loading.

The ANSYMP program is available from the Computer Software Management Information Center (COSMIC), University of Georgia, Athens, Ga. 30602 under LEW 14011. A flow chart of the program and sample input and output data are presented in Reference 6.

The calculational scheme initially follows the effective stress-equivalent strain input data from subroutine ELAS until the occurrence of initial yielding. The stress-strain solution then proceeds along the yield surface as determined from the stress-strain properties in subroutine YIELD. At each increment during yielding the stress shift (difference between new yield stress and stress predicted from elastic analysis) from the original input data is calculated. Elastic load reversal is signaled when the input stress is less than the yield stress from the previous increment. During elastic unloading, the stresses are translated from the original elastic analysis solution by the amount of the calculated stress shift. Reverse yielding occurs when the stress reaches the reverse yield surface as determined from the hardening model incorporated in subroutine YIELD. Again, the solution follows the yield surface until another load reversal is indicated when the stress based on the shifted elastic solution is less than the yield stress. The elastic response during load reversal is obtained by translating the original elastic solution according to the new stress shift calculated during reversed yielding. The stress-strain response for subsequent cycles is computed by repeating this procedure of identifying load reversals, tracking reverse yield surfaces and translating the original elastic solution during elastic loading and unloading.

Creep computations are performed for increments involving dwell times using the creep characteristics incorporated in subroutine CREEP. Depending on the nature of the problem, the creep effects are determined on the basis of one of the three options provided in the subroutine.

PREDICTION OF THERMOMECHANICAL CYCLES

Two types of thermomechanical cycles were considered in the evaluation of the models; (1) a simple continuous thermomechanical cycle having the mechanical strain and temperature in phase (sinusoidal strain and temperature variation) resulting in a linear strain-temperature history and (2) and "a faithful cycle" that was representative of actual structural component response to thermal loading. Details of the faithful cycle are described in Reference 1 for the analysis of a gas turbine engine combustor liner. The thermomechanical tests were conducted on uniaxial tubular specimens which are also described in Reference 1. The hollow geometry, in combination with low frequency induction heating and internal air cooling, permitted testing with the prescribed mechanical strain and temperature histories. An axial extensometer attached to the internal ridges was used for strain control.

Linear Strain-Temperature Cycles

Two temperature histories were selected for the evaluation, 760°C to 982°C (1400°F to 1800°F) and 649°C to 982°C (1200°F to 1800°F). Each temperature cycle and a single mechanical strain cycle (approximately -0.001 to -0.0045 m/m) were imposed on a tubular test specimen with the resulting stress and strain response recorded from startup to stabilization. A description of the cycle parameters and loading sequence for the cycles is presented in Figure 3. Each test started at a constant 982°C (point A). The specimen was then compressed to a mechanical strain of -0.0045 (point B). A sinusoidal variation in temperature from 982°C to a minimum value of either 760°C or 649°C was then generated on the specimen in phase with a sinusoidal variation in mechanical strain from about -0.0045 to -0.0100. The period for both the temperature and strain histories was 1 minute. Cross plotting of these quantities resulted in the linear paths B-C and B-D in Figure 3. Predictions of the stress-strain responses for the 760°C to 982°C and 649°C to 982°C test cycles with the two simplified procedures are presented in Figures 4 and 5. Both methods essentially capture the overall shape, inelastic strain range and approximate stress levels of the experimental response curves. This would be considered as sufficient information for use as input to a life prediction model for a preliminary estimate of the base cyclic durability.

There is a tendency for the simplified analyses to overpredict the peak compressive stresses. This is especially true for the analytical cycles from the NASA procedure as shown in Figures 4 (b) and 5 (b). These discrepancies may be attributable to the use of monotonic stress-strain data for cyclic stress-strain problems.

Prediction of Faithful Cycle Response

Final evaluation of the models considered the combustor liner faithful cycle defined in Reference 1. The predicted strain-temperature response at the end of the louver lip differs from the previously defined conditions in that the strain and temperature are not continually in phase and the heatup and cooldown parts of the cycle are not identical. The nominal temperature-time history for this cycle is that shown for the louver lip in Figure 2 (a) and the approximate strain-time history imposed on the specimen was that obtained for the sixth cycle of the MARC nonlinear solution. Application of these loading spectrums on the thermomechanical specimen produced a stress-strain response considered to be representative of the local louver response. Prediction of the combustor liner "faithful cycle" response by the two simplified procedures is shown in Figure 6. Again the analytical results agreed reasonably well with the experimental data.

SUMMARY OF RESULTS

Two simplified analytical procedures are presented for predicting the local inelastic stress-strain response of a structure subjected to cyclic thermomechanical loading using as input the total strain history calculated from a linear elastic analysis. The first of these procedures was developed at Pratt & Whitney Aircraft under contract to NASA. The second procedure, which was developed at the NASA Lewis Research Center, is fully automated in a computer program (ANSYMP). These procedures were evaluated on their ability to predict the cyclic structural responses for three thermomechanically loaded test specimens. Both methods were able to simulate the overall shapes of the stress-strain hysteresis loops and to calculate the stress-strain histories to a degree of accuracy sufficient for most life prediction methods. The analyses were based on use of monotonic stress-strain material properties. It is likely that better agreement with experimental results would have been obtained if cyclic stress-strain data had been available. These simplified procedures provide economical structural analysis tools which can be applied in the preliminary design of hot section components of gas turbine engines where nonlinear finite-element programs would be prohibitively expensive and time-consuming to use.

REFERENCES

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3. McKnight, R. L.; Laflen J. H.; and Spamer, G. T.: Turbine Blade Tip Durability Analysis NASA CR-165268, 1981.
4. Kaufman A.: Evaluation of Inelastic Constitutive Models for Nonlinear Structural Analysis. NASA CP-2271, 1983.
5. Moreno, V.: Development of a Simplified Analytical Method for Representing Material Cyclic Response. NASA CR-168100, 1983.
6. Kaufman A.: Development of a Simplified Procedure for Cyclic Structural Analysis. NASA TP-1855, 1983.

TABLE I. - TEMPERATURE DEPENDENT PARAMETERS FOR SIMPLIFIED RESPONSE PROCEDURES

Temperature °C, (°F)	Young's Modulus, MPa (ksi)	Yield Stress, MPa (ksi)	Creep Parameters* A	Creep Parameters* n
427 (800)	170 03 (24.6 03)	314 (45.6)	-------	----
538 (1000)	170 03 (24.6 03)	314 (45.6)	-------	----
649 (1200)	161 03 (23.3 03)	303 (44.0)	-------	----
760 (1400)	152 03 (22.05 03)	252 (38.0)	592.54 (85.94)	4.15
815 (1500)	146 03 (21.15 03)	207 (30.0)	277.44 (40.24)	4.75
871 (1600)	137 03 (19.8 03)	103 (15.0)	188.05 (27.27)	5.25
927 (1700)	130 03 (18.85 03)	76 (11.0)	176.95 (25.66)	3.35
982 (1800)	123 03 (17.9 03)	48 (7.0)	91.53 (13.28)	3.15

*Equation (8).

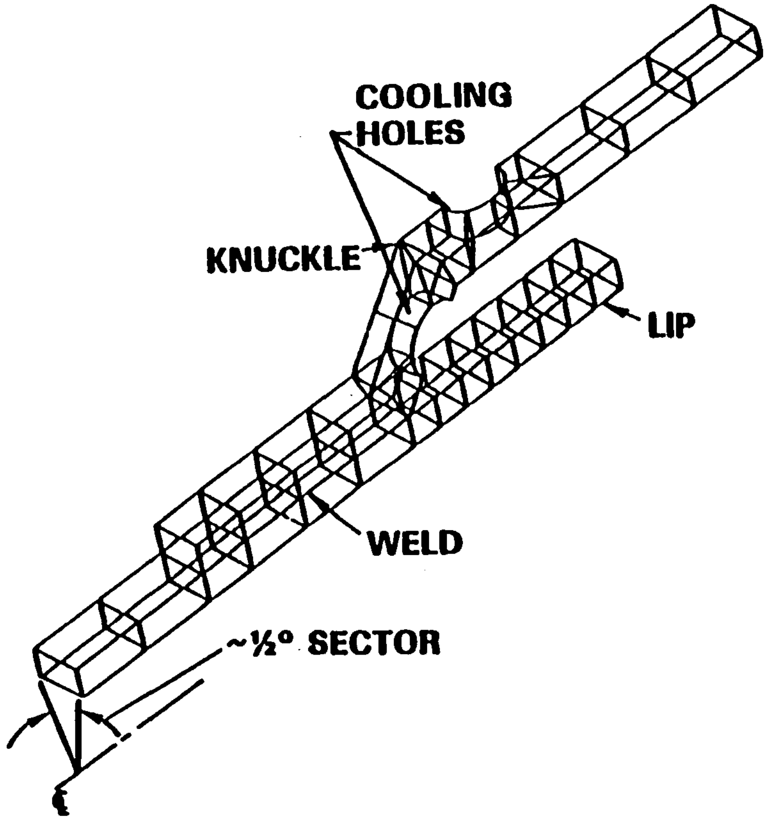
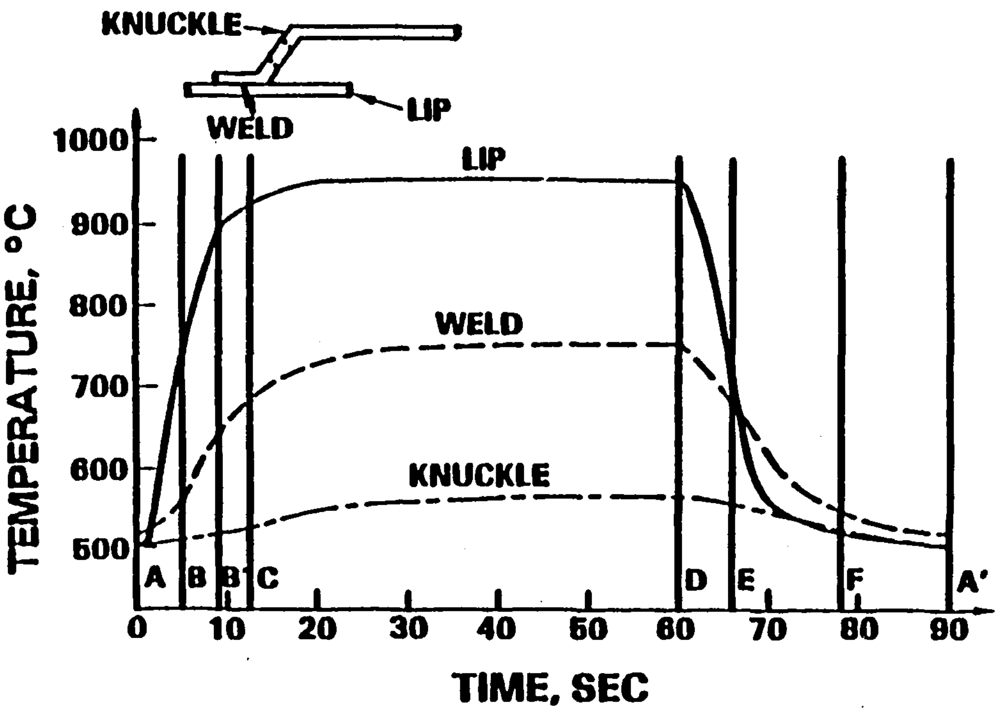
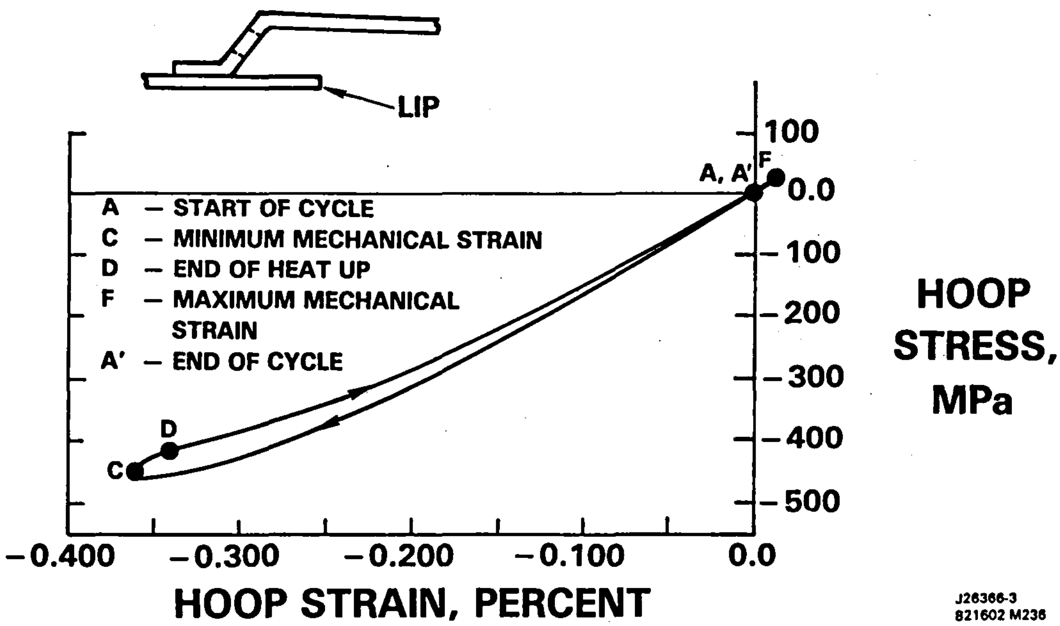


Figure 1. - Combustor liner finite element model.

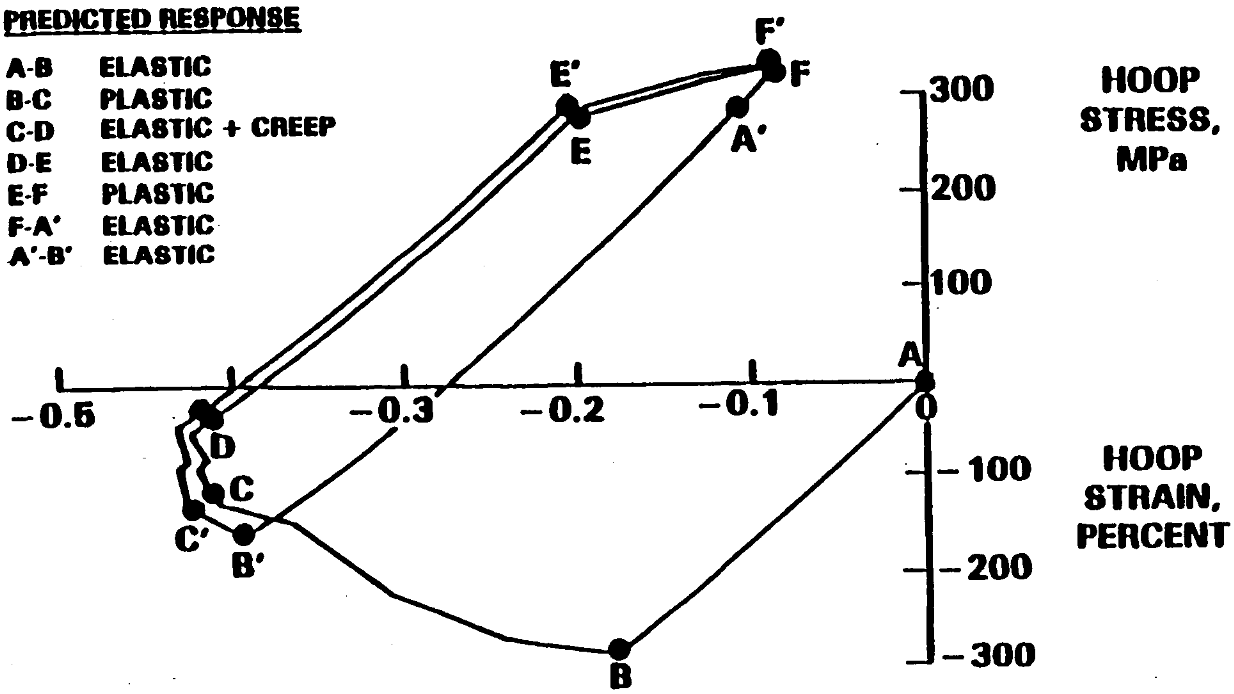


(a) - Louver temperature response.

Figure 2. - Combustor liner structural liner results.



(b) - Elastic analysis stress-strain response at louver lip.



(c) - Nonlinear analysis stress-strain response at louver lip

Figure 2. - Concluded.

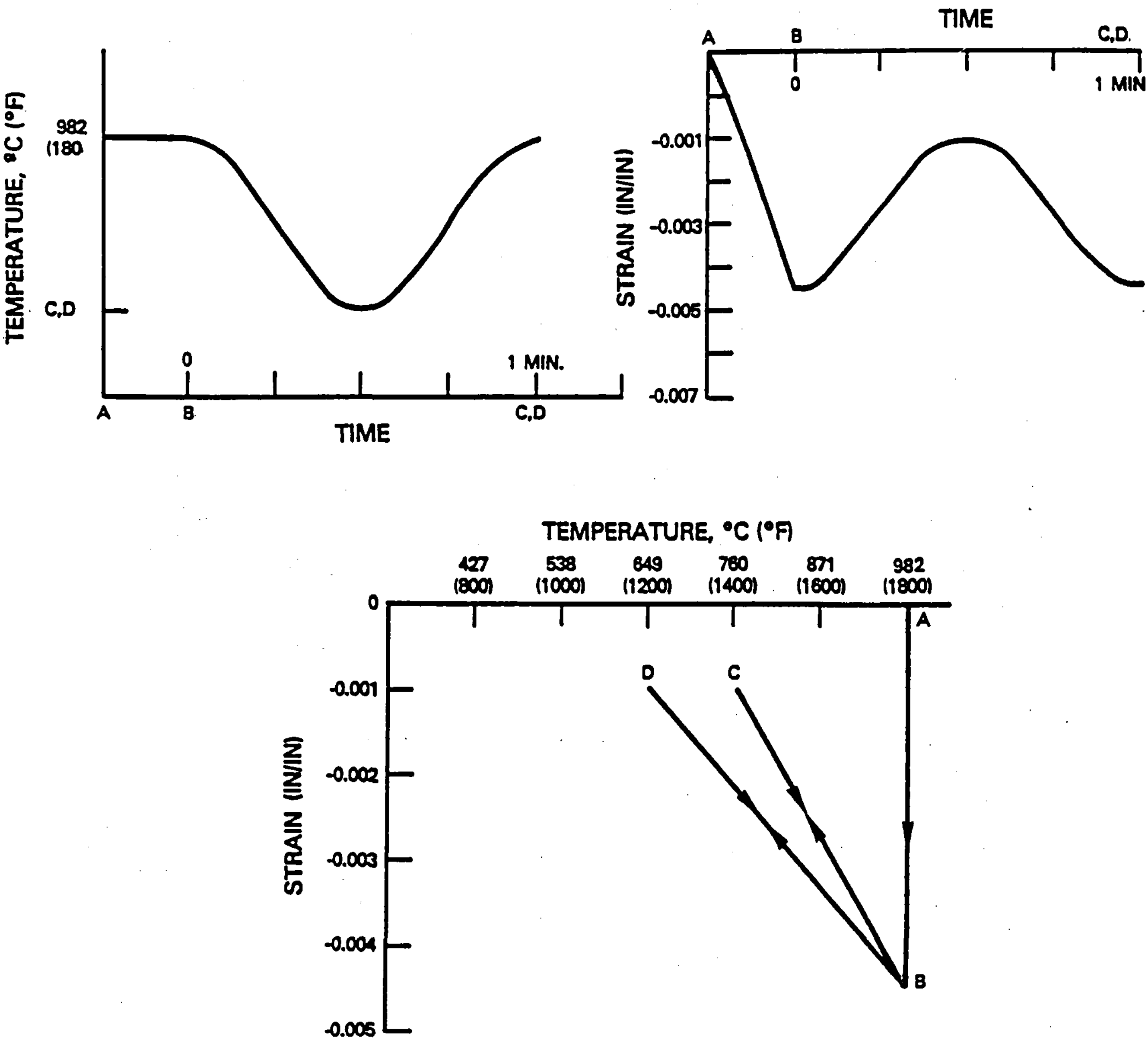
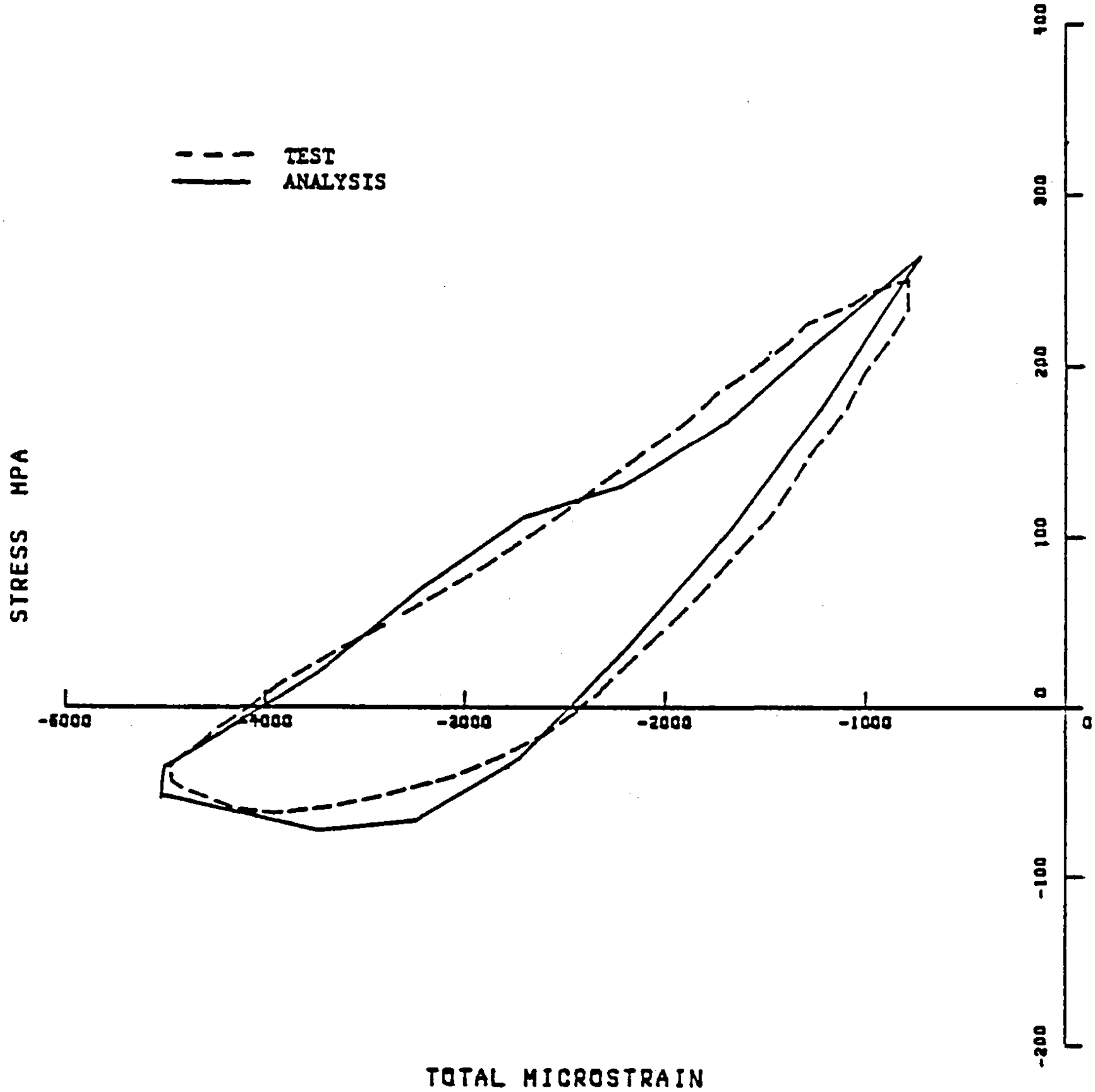
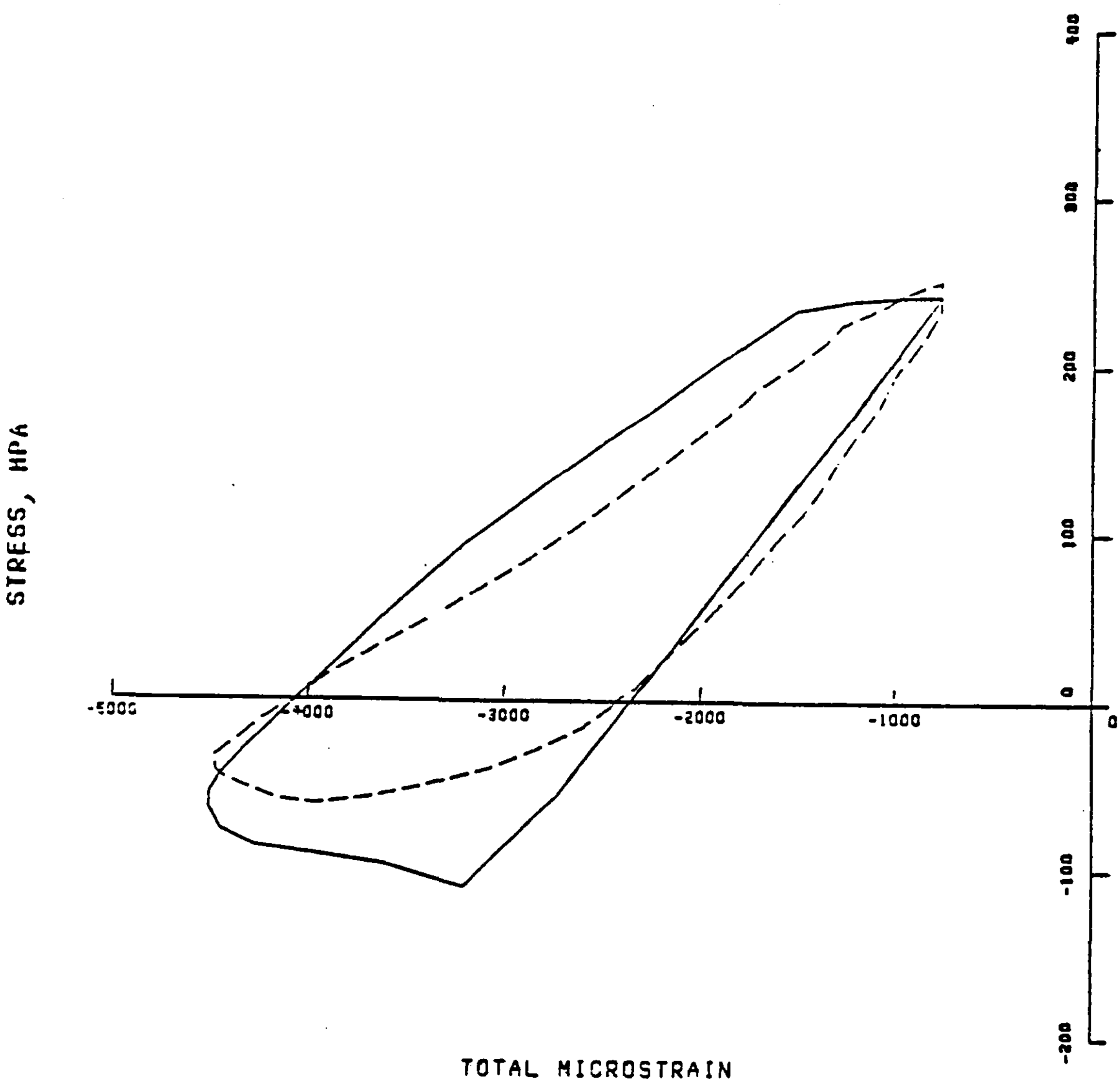


Figure 3. – Temperature and Strain Input Histories for Linear Thermomechanical Cycles.



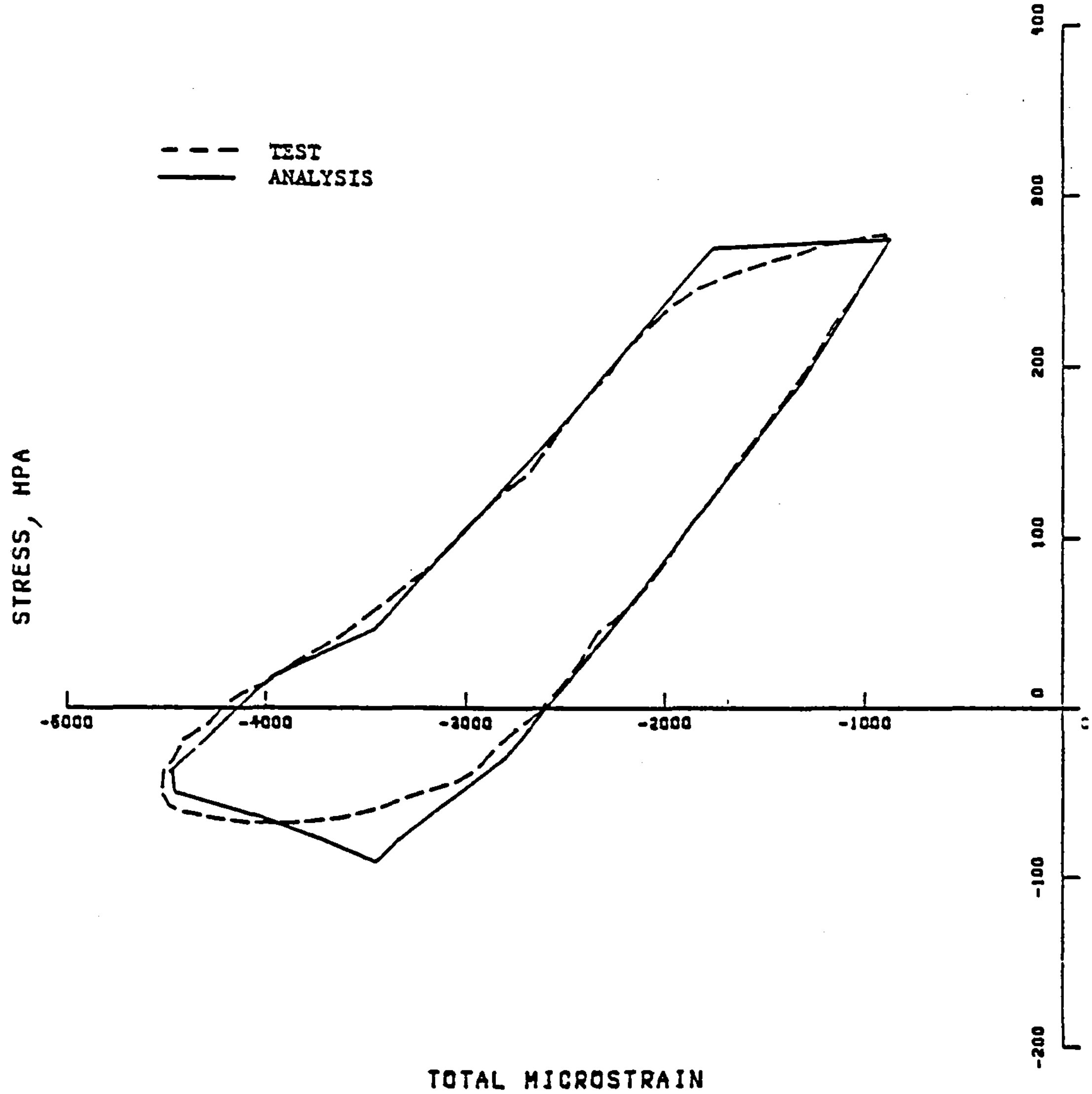
(a) Simplified Procedure 1

Figure 4. - Simulation of 760 to 982°C thermomechanical test cycle.



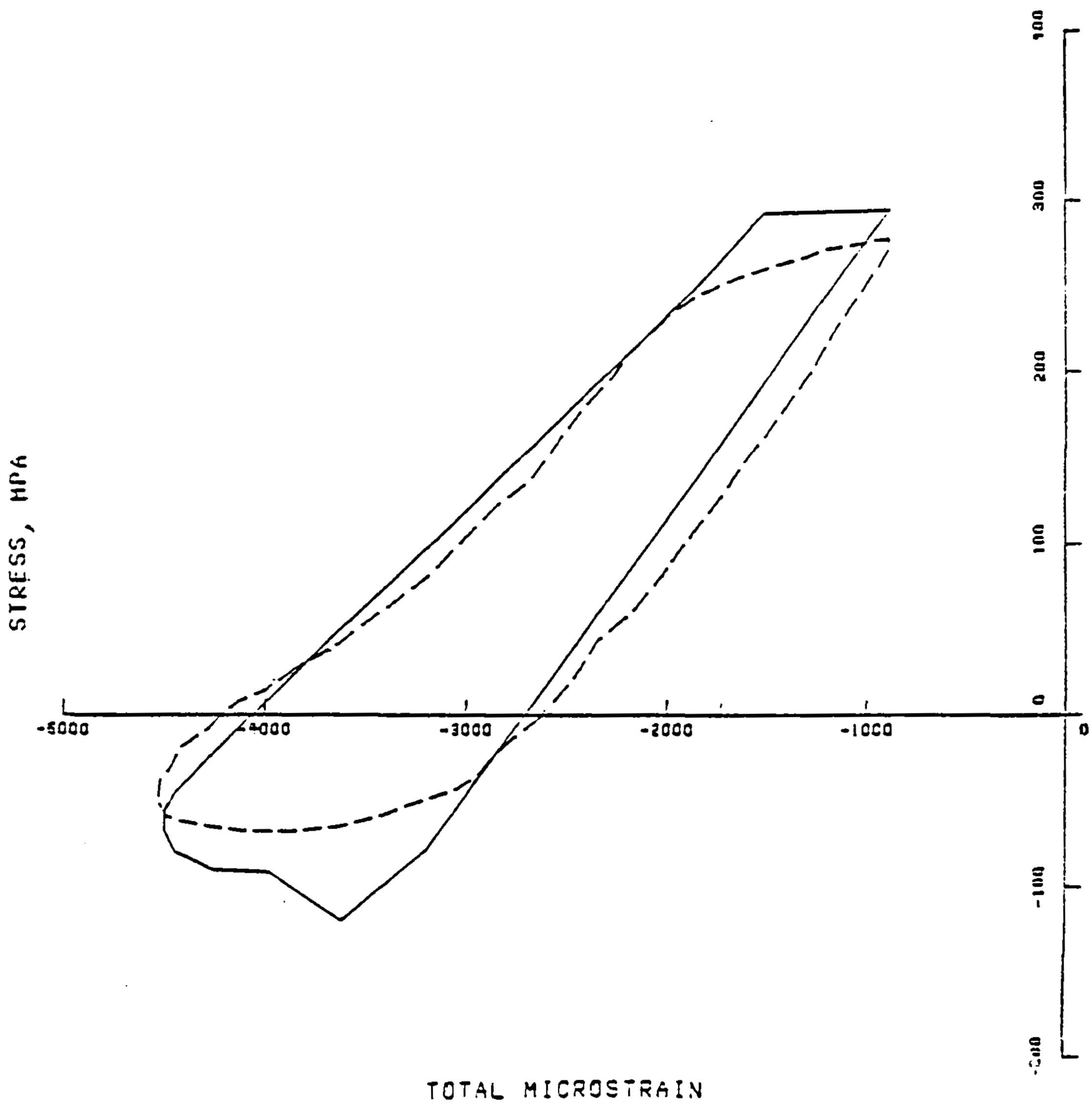
(b) Simplified Procedure 2

Figure 4. - Concluded.



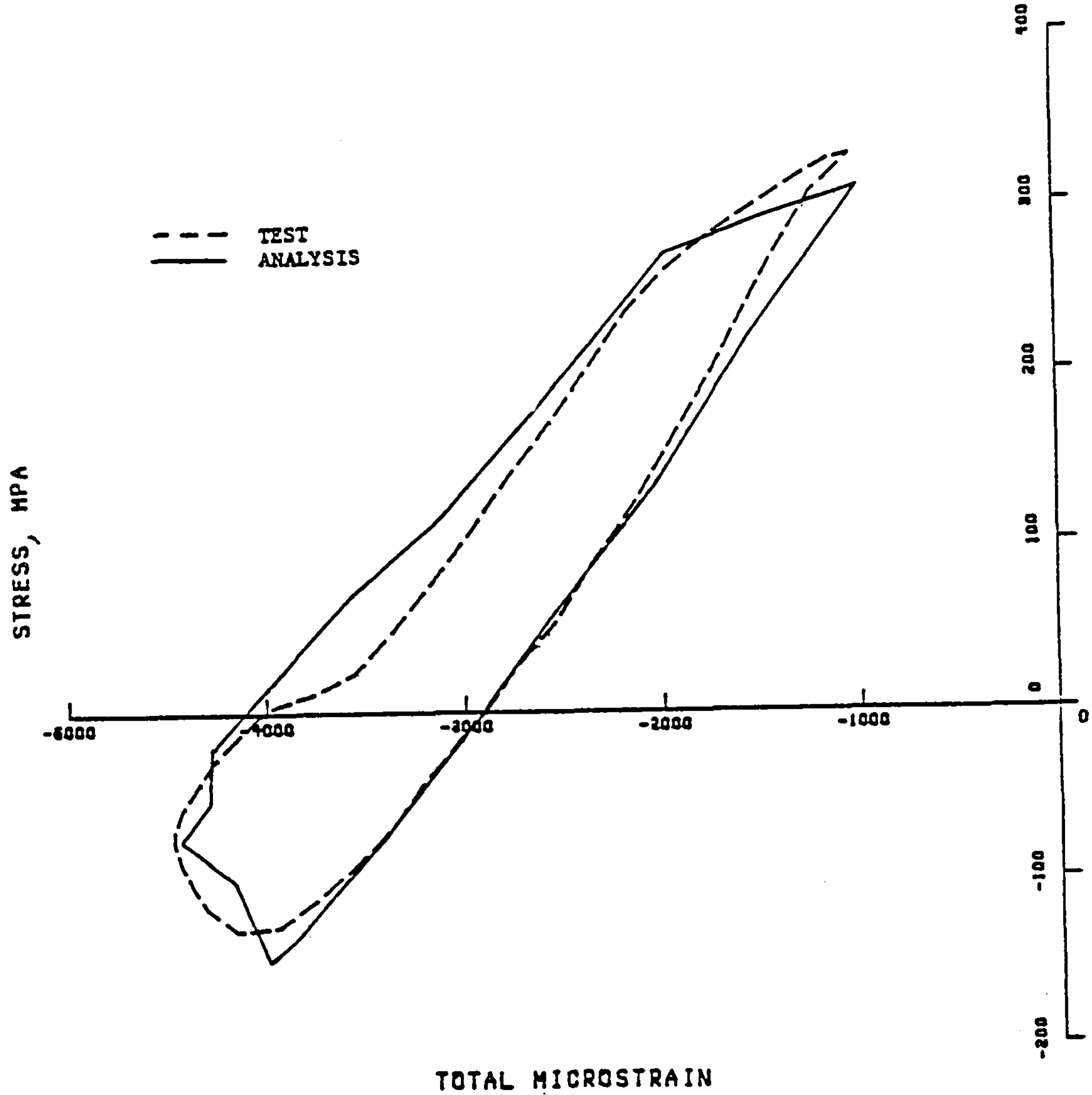
(a) Simplified Procedure 1

Figure 5. - Simulation of 649 to 982°C thermomechanical test cycle.



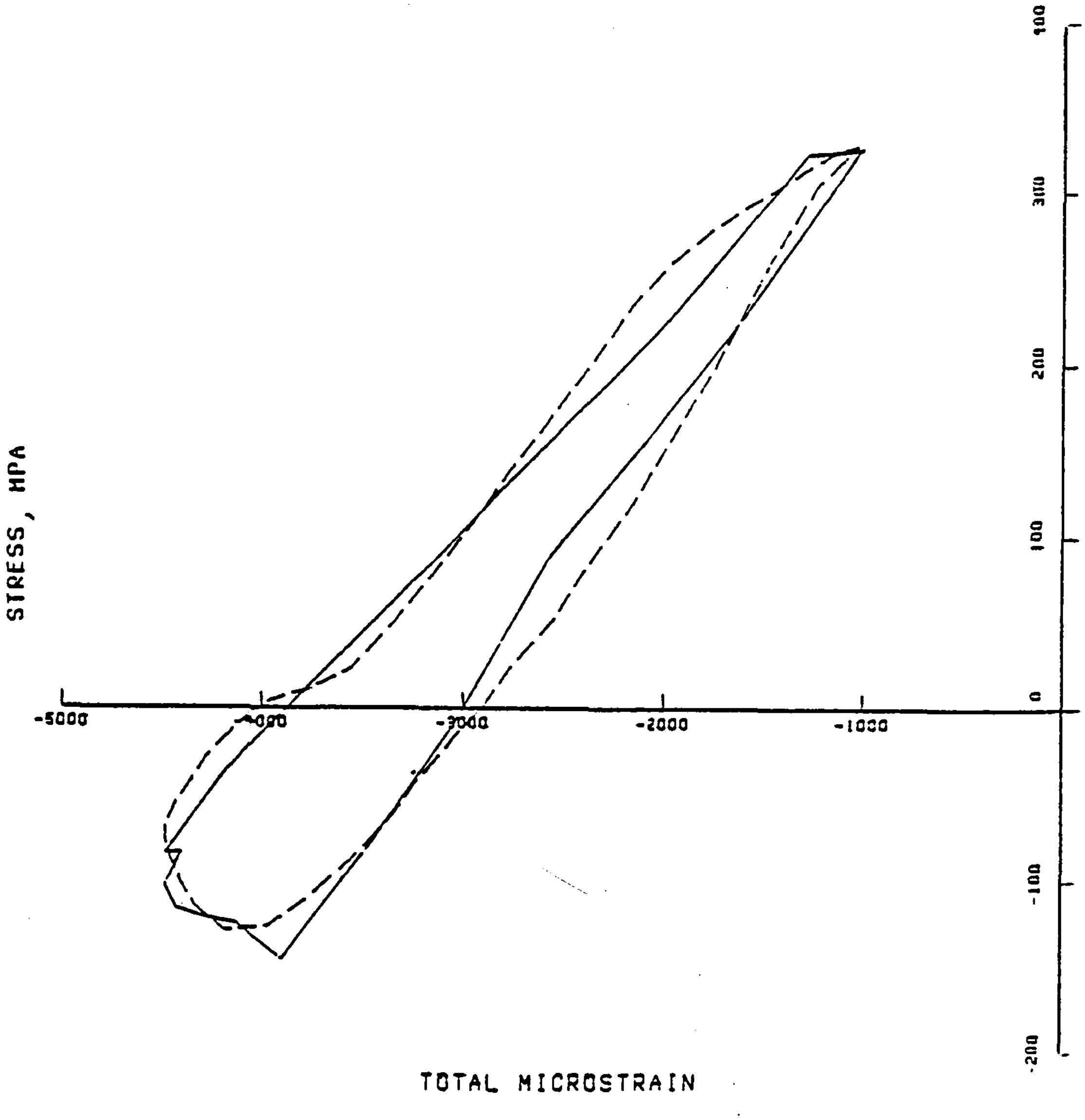
(b) Simplified Procedure 2

Figure 5. - Concluded.



(a) Simplified Procedure 1

Figure 6. - Simulation of combustor liner "faithful cycle".



(b) Simplified Procedure 2

Figure 6. - Concluded.