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CONTAINING A CRACK

Renji Tang and F. Erdogan

LEHIGH UNIVERSITY
Bethlehem, Pennsylvania

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# A CLAMPED RECTANGULAR PLATE CONTAINING A CRACK

by

Renji Tang* and F. Erdogan

Department of Mechanical Engineering and Mechanics
Lehigh University, Bethlehem, PA 18015

## Abstract

In this paper the general problem of a rectangular plate clamped along two parallel sides and containing a crack parallel to the clamps is considered. The problem is formulated in terms of a system of singular integral equations and the asymptotic behavior of the stress state near the corners is investigated. Numerical examples are considered for a clamped plate without a crack and with a centrally located crack, and the stress intensity factors and the stresses along the clamps are calculated.

## 1. Introduction

A long strip containing a central or an edge crack and a rectangular block with an edge crack are two of the more widely studied geometries in fracture mechanics. The former generally simulates a single-edge-notched or a center-notched specimen and the latter the compact tension specimen. These specimens are often used for fatigue crack growth and fracture characterization of engineering materials. Typical studies using a variety of methods such as finite elements, Wiener-Hopf, weight function, Laurent series, conformal mapping, boundary collocation, and integral equations may be found in references [1]-[13]. In all these and similar studies the external loads are usually assumed to be such that the boundary conditions may be prescribed in terms of tractions only. However, in some cases the loads are applied to the specimen through "grips". In these problems it would be more appropriate to prescribe the boundary conditions along the grips in terms of displacements rather than tractions. The problem of a

*Permanent address: Associate Professor, Department of Mathematics and Mechanics, Lanzhou University, China

rectangular strip without a crack loaded through two rigid grips was considered in [14], where it was shown that the contact stresses along the grips are singular at the corners. In this paper the problem of a strip containing a crack is considered by using a technique which is somewhat different than that of [14] and which may be used to solve the clamped plate problem for an arbitrarily oriented crack.

## 2. Formulation of the Problem

We will first formulate the problem described in Fig. 1 where the clamped strip contains three cracks as shown. The "external load" in the problem is the relative clamp displacement $u_0$. One may express the stress and displacement components in the strip as follows:

$$\sigma_{ij}(x,y) = \sigma_{sij}(x,y) + \sum_{k=1}^{6} \int_{a_k}^{b_k} G_{ijk}(x,y,s_k,t) f_k(t)dt \ ,$$

$$(i,j=x,y \ ; \ k=1,\ldots,6) \ , \tag{1}$$

$$u_i(x,y) = u_{si}(x,y) + \sum_{k=1}^{6} \int_{a_k}^{b_k} V_{ik}(x,y,s_k,t) f_k(t)dt \ ,$$

$$(i=(x,y) \ , \ k=1,\ldots,6) \tag{2}$$

where $\sigma_{sij}$ and $u_{si}$ are associated with an infinite strip without any cracks and the functions $G_{ijk}$ and $V_{ik}$ represent the stresses and displacements in an infinite plate at the point $(x,y)$ due to dislocations $f_k$ distributed along the cuts $(a_k<t<b_k)$. In terms of the crack surface displacements the functions $f_k$ are defined as follows (Fig. 1):

$$f_i(x) = \frac{\partial}{\partial x}[u_y(x,s_i+0) - u_y(x,s_i-0)] \ , \ (i=1,2)$$

$$f_3(y) = \frac{\partial}{\partial y}[u_x(s_3+0,y) - u_x(s_3-0,y)] \ ,$$

$$f_j(x) = \frac{\partial}{\partial x}[u_x(x,s_j+0) - u_x(x,s_j-0)] \ , \ (j=4,5)$$

$$f_6(y) = \frac{\partial}{\partial y}[u_y(s_6+0,y) - u_y(s_6-0,y)] \ , \tag{3a-d}$$

where

$$s_{i+3}=s_i,\ a_{i+3}=a_i,\ b_{i+3}=b_i,\ a_i<s_i<b_i,\ (i=1,2,3) \tag{4}$$

and the Green's functions $G_{ijk}$ and $V_{ij}$ are given in Appendix A.

The solution which corresponds to an infinite strip may be expressed as follows:

$$u_{sx}(x,y) = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\{[A_1+(\frac{\kappa}{\alpha}+x)A_2]e^{-\alpha x}+[-A_3+(\frac{\kappa}{\alpha}-x)A_4]e^{\alpha x}\ e^{-i\alpha y}d\alpha,$$

$$u_{sy}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}[(A_1+A_2x)e^{-\alpha x}+(A_3+A_4x)e^{\alpha x}]e^{-i\alpha y}d\alpha\ , \tag{5a,b}$$

$$\sigma_{sxx}(x,y) = \frac{\mu}{\pi i}\int_{-\infty}^{\infty}\{-[\alpha(A_1+A_2x)+\frac{1+\kappa}{2}A_2]e^{-\alpha x}-[\alpha(A_3+A_4x)-\frac{1+\kappa}{2}A_4]e^{\alpha x}\}e^{-i\alpha x}d\alpha,$$

$$\sigma_{syy}(x,y) = \frac{\mu}{\pi i}\int_{-\infty}^{\infty}\{[-\frac{3-\kappa}{2}A_2+\alpha(A_1+A_2x)]e^{-\alpha x}+[\frac{3-\kappa}{2}A_4+\alpha(A_3+A_4x)]e^{\alpha x}\}e^{-i\alpha x}d\alpha,$$

$$\sigma_{sxy}(x,y) = \frac{\mu}{\pi}\int_{-\infty}^{\infty}\{-[\alpha(A_1+A_2x)+\frac{\kappa-1}{2}A_2]e^{-\alpha x}+[\alpha(A_3+A_4x)-\frac{\kappa-1}{2}A_4]e^{\alpha x}\}e^{-i\alpha x}d\alpha.$$

(6a-c)

The unknown functions $f_1,\ldots,f_6$ and $A_1(\alpha),\ldots,A_4(\alpha)$ are determined from the boundary conditions prescribed on the crack surfaces and the plate boundaries $x=0$ and $y=H$. In the actual problem the crack surfaces are free from tractions and along the boundaries of the plate we have (Fig. 1)

$$u_x(0,y)=0,\ u_y(0,y)=0,\ u_x(H,y)=u_0,\ u_y(H,y)=0,\ -\infty<y<\infty\ . \tag{7}$$

The superposition technique may be used to solve the problem by first obtaining the stress state in a clamped plate without any cracks under the boundary conditions (7). The equal and opposite of the stresses found in this solution may then be used as crack surface tractions in the perturbation problem. The ten unknowns $f_1,\ldots,f_6$, $A_1\ldots A_4$ would then have to be determined from

$$u_x(0,y)=0,\ u_y(0,y)=0,\ u_x(H,y)=0,\ u_y(H,y)=0,\ -\infty<y<\infty\ , \tag{8}$$

$$\sigma_{yy}(x,y_j) = p_j(s) = -\frac{3-\kappa}{\kappa-1}\frac{\mu u_0}{H}\ ,\ (y_j=s_j, s=x, a_j<s<b_j, j=1,2)\ ,$$

$$\sigma_{xx}(x_3,y) = p_3(s) = -\frac{\kappa+1}{\kappa-1}\frac{\mu u_0}{H}\ ,\ (x_3=s_3, s=y, a_3<s<b_3)\ ,$$

$$\sigma_{xy}(x,y_j) = p_j(s) = 0,\ (y_j=s_{j-3}, s=x, a_{j-3}<s<b_{j-3}, j=4,5)\ ,$$

$$\sigma_{xy}(x_3,y) = p_6(s) = 0\ ,\ (s=y, a_3<s<b_3)\ . \tag{9}$$

By substituting from equations (2), (5) and (A13)-(A20) (of Appendix A) into (8), by inverting the Fourier integrals and by evaluating the related infinite integrals, the functions $A_1,\dots,A_4$ may be expressed in terms of integrals of the following form:

$$A_i(\alpha) = \sum_{j=1}^{6}\int_{a_j}^{b_j} B_{ij}(\alpha,t)f_j(t)dt\ ,\ (i=1,\dots,4)\ , \tag{10}$$

where the functions $B_{ij}$ depend also on the constants $s_1$, $s_2$, $s_3$, $H$, and $\kappa$. From (10) and (6) the stress components $\sigma_{sij}$, $(i,j=x,y)$ are then obtained as follows:

$$\sigma_{sij}(x,y) = \sum_{k=1}^{6}\int_{a_k}^{b_k} C_{ijk}(x,y,s_k,t)f_k(t)dt\ ,\ (i,j=x,y)\ , \tag{11}$$

where the kernels $C_{ijk}$ are given in Appendix B.

The integral equations for the unknown functions $f_1,\dots,f_6$ are obtained by substituting from (1) and (11) into the crack surface boundary conditions (9) and may be expressed as

$$\sum_{k=1}^{6} \int_{a_k}^{b_k} h_{nk}(s,t)f_k(t)dt = p_n(s) \ , \ (a_n<s<b_n, \ n=1,\dots,6; \ a_n=a_{n-3}, \ b_n=b_{n-3}, \ n=4,5,6) \tag{12}$$

where the kernels $h_{nk}$ $(n,k=1,\dots,6)$ are known in terms of $G_{ijk}$ and $C_{ijk}$ $(i,j=x,y; \ k=1,\dots,6)$ (see equations (1), (11), (9), Appendix A and Appendix B). For the crack problem shown in Fig. 1 (i.e., for $0<a_k<b_k<H$, $k=1,2$ and $s_2<a_3<b_3<s_1$) from appendices A and B it can be shown that the main diagonal elements of the kernels $h_{nk}$ which are contributed by $G_{ijk}$ are Cauchy kernels and consequently $h_{nk}$ can be expressed as follows:

$$h_{nk}(s,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{\delta_{nk}}{t-s} + k_{nk}(s,t) \ , \ (n,k=1,\dots,6) \tag{13}$$

where $\delta_{nk}=1$ for $n=k$, $\delta_{nk}=0$ for $n\neq k$, and $k_{nk}$, $(n,k=1,\dots,6)$ are known bounded functions in the closed intervals, $a_k\leq(t,s)\leq b_k$, $(k=1,\dots,6)$. If one wants to solve the problem shown in Fig. 1 for any crack geometry provided $0<a_k<b_k<H$, $k=1,2$ and $s_2<a_3<b_3<s_1$, one could let

$$f_k(t) = \frac{F_k(t)}{\sqrt{(t-a_k)(b_k-t)}} \ , \ (k=1,\dots,6) \tag{14}$$

and use a Gaussian integration formula to determine the unknown bounded functions $F_k(t)$, $(k=1,\dots,6)$[15]. From the definition of the unknown functions $f_k(t)$ as given by (3) and from Fig. 1 it is clear that the integral equations (12) must be solved under the following singlevaluedness conditions:

$$\int_{a_k}^{b_k} f_k(t)dt = 0 \quad , \quad (k=1,\dots,6) \ . \tag{15}$$

## 3. The Clamped Plate

From Fig. 1 it may be seen that the problem would reduce to that of a clamped plate if one lets $a_i=0$ and $b_i=H$, $(i=1,2)$. In this case technically the integral equations (12) and the singlevaluedness conditions (15) are still valid and the input functions $p_n(s)$ are still given by (9). However, the form of the solution is no longer given by (14). To determine the fundamental functions of the system of integral equations (12) the singular nature of the kernels $k_{nk}(s,t)$, $(n,k=1,\ldots,6)$ for $a_k=0$, $b_k=H$, $(k=1,2,4,5)$ must be investigated. This may be done through the asymptotic analysis of the infinite integrals giving the appropriate functions $C_{ijk}$ in Appendix B which are related to $k_{nk}$ (see, for example, [14] and [15]). First we observe that from (1), (11), (9), (12), (13) and Appendices A and B, the kernels $k_{nk}(s,t)$ may be expressed as follows:

$$k_{nk}(s,t) = k^G_{nk}(s,t) + k^C_{nk}(s,t) \ , \ (n,k=1,\ldots,6) \ , \tag{16}$$

$$k^C_{nk}(s,t) = \int_0^\infty K^C_{nk}(s,t,\alpha)d\alpha \ , \ (n,k=1,\ldots,6) \tag{17}$$

where $k^G_{nk}$ are the nonsingular parts of $G_{ijk}$ (see equation (1) and Appendix A) and $k^C_{nk}$ are the kernels contributed by $C_{ijk}$. If we now denote the asymptotic values of $K^C_{nk}$ for $\alpha\to\infty$ by $K^C_{nk\infty}(s,t,\alpha)$, the singular and Fredholm parts of the kernels $k^C_{nk}$ may be separated as follows:

$$k^C_{nk}(s,t) = k^S_{nk}(s,t) + k^F_{nk}(s,t) \ , \ (n,k=1,\ldots,6) \tag{18}$$

$$k^S_{nk}(s,t) = \int_0^\infty K^C_{nk\infty}(s,t,\alpha)d\alpha \ , \ (n,k=1,\ldots,6) \ , \tag{19}$$

$$k^F_{nk}(s,t) = \int_0^\infty [K^C_{nk}(s,t,\alpha)-K^C_{nk\infty}(s,t,\alpha)]d\alpha \ , \ (n_b k=1,\ldots,6) \tag{20}$$

where the kernels $k^F_{nk}(s,t)$ $(n,k=1,\ldots,6)$ are bounded in their respective closed domains. For $s_2<a_3<b_3<s_1$, after a relatively straightforward analysis it may be shown that

$$k^S_{nk}(s,t) = k^S_{nn}(s,t)\delta_{nk} \ , \ (n,k=1,\ldots,6) \ , \tag{21}$$

$$k_{33}^{s}(s,t) = 0 \quad , \quad k_{66}^{s}(s,t) = 0 \quad , \tag{22}$$

$$k_{nn}^{s}(s,t) = \frac{1}{2\kappa}\left[(3-\kappa^2)+12s\frac{d}{ds}+4s^2\frac{d^2}{ds^2}\right]\frac{1}{s+t}$$

$$-\frac{1}{2\kappa}\left[(3-\kappa^2)-12(H-s)\frac{d}{ds}+4(H-s)^2\frac{d^2}{ds^2}\right]\frac{1}{2H-s-t} ,$$

$$(n=1,2,4,5) \quad . \tag{23}$$

Because of (21) and (22) the dominant part of the integral equations (12) for n=3 and n=6 has only Cauchy type singularity and the unknown functions $f_3$ and $f_6$ have the form

$$f_j(t) = \frac{F_j(t)}{\sqrt{(t-a_3)(b_3-t)}} \ , \ (j=3,6) \quad . \tag{24}$$

Expressing now the remaining unknowns as

$$f_k(t) = \frac{F_k(t)}{(t-a_k)^{\beta}(b_k-t)^{\gamma}} \ , \ (k=1,2,4,5), \ 0<\mathrm{Re}(\beta,\gamma)<1 \ , \tag{25}$$

through a function-theoretic analysis it may be shown that (see, for example [15])

$$\beta = \gamma, \ 2\kappa\cos\pi\beta-(1+\kappa^2)+4(\beta-1)^2 = 0 \quad . \tag{26}$$

The characteristic equation (26) is identical to that found in [14] and in, for example, [16] for a quarter elastic plane fixed along one of its straight boundaries.

## 4. The Stress Intensity Factors

In the crack problem shown in Fig. 1, once the functions $f_1,\ldots,f_6$ giving the crack surface displacement derivatives or the bounded functions $F_1,\ldots,F_6$ defined in (14) are determined, the Modes I and II stress intensity factors at the crack tips may be obtained from the following standard relations:

$$k_1(a_i) = \frac{2\mu}{1+\kappa} \lim_{t\to a_i} \sqrt{2(t-a_i)}\, f_i(t) = \frac{2\mu}{1+\kappa} \frac{F_i(a_i)}{\sqrt{(b_i-a_i)/2}} , \quad (i=1,2,3)$$

$$k_2(a_j) = \frac{2\mu}{1+\kappa} \lim_{t\to a_j} \sqrt{2(t-a_j)}\, f_j(t) = \frac{2\mu}{1+\kappa} \frac{F_j(a_j)}{\sqrt{(b_j-a_j)/2}} ,$$

$$(a_j=a_{j-3},\ b_j=b_{j-3},\ j=4,5,6) ,$$

$$k_1(b_i) = -\frac{2\mu}{1+\kappa} \lim_{t\to b_i} \sqrt{2(b_i-t)}\, f_i(t) = -\frac{2\mu}{1+\kappa} \frac{F_i(b_i)}{\sqrt{(b_i-a_i)/2}} , \quad (i=1,2,3),$$

$$k_2(b_j) = -\frac{2\mu}{1+\kappa} \lim_{t\to b_j} \sqrt{2(b_j-t)}\, f_j(t) = -\frac{2\mu}{1+\kappa} \frac{F_j(b_j)}{\sqrt{(b_j-a_j)/2}} ,$$

$$(a_j=a_{j-3},\ b_j=b_{j-3},\ j=4,5,6) . \qquad (27a\text{-}d)$$

These relations are based on the definition of the stress intensity factors in terms of normal and shear cleavage stresses at the crack tips of the form (Fig. 1)

$$k_1(a_1) = \lim_{x\to a_1} \sqrt{2(a_1-x)}\, \sigma_{yy}(x,s_1) , \ \ldots$$

$$k_2(a_1) = \lim_{x\to a_1} \sqrt{2(a_1-x)}\, \sigma_{xy}(x,s_1) , \ \ldots \qquad (28a,b)$$

In the "clamped plate" problem it was shown that at the corners $y=s_i$, $x=0$, $x=H$, $(i=1,2)$ the displacement derivatives have a singularity of the order $r^{-\beta}$, where $r$ is a small distance (in $x$ direction) from the corners (see eqs. (25) and (26)). This implies that at the corners the stresses may also have a singularity of the order $r^{-\beta}$, $r$ now being the radial distance from the corner. In practical applications one may be particularly interested in the behavior of interface stresses $\sigma_{xx}$ and $\sigma_{xy}$ along the clamps. We first observe that the stress state at any point in the medium is given in terms of the density functions $f_1,\ldots,f_6$ and the kernels $G_{ijk}$

and $C_{ijk}$ (see (1) and (11)). Next, we note that the singular behavior of the stresses is determined by the dominant parts of the related kernels only and these dominant kernels can be separated through a relatively simple asymptotic analysis of $G_{ijk}$ and $C_{ijk}$ given in the Appendices A and B. Also, the singular behaviors of the stresses in the medium at all four corners have the same form. Thus, after following a procedure similar to that described by the equations (16) through (23), for a small distance r from the corner x=0, $y=s_1$ the stresses along the clamp x=0 may be expressed as

$$\sigma_{xx}(0,s_1-r) \cong \frac{2\mu}{\pi(1+\kappa)}\left[\int_0^H K_{11}^s(y,t)f_1(t)dt + \int_0^H K_{14}^s(y,t)f_4(t)dt\right] ,$$

$$\sigma_{xy}(0,s_1-r) \cong \frac{2\mu}{\pi(1+\kappa)}\left[\int_0^H K_{41}^s(y,t)f_1(t)dt + \int_0^H K_{44}^s(y,t)f_4(t)dt\right] ,$$

$$(y=s_1-r) , \qquad (29a,b)$$

where

$$K_{11}^s(s_1-r,t) = \frac{t(t^2-r^2)}{(t^2+r^2)^2} + \frac{1}{2\kappa}\left[\frac{(\kappa^2-1)t}{t^2+r^2} + \frac{2t(t^2-r^2)}{(t^2+r^2)^2}\right] ,$$

$$K_{14}^s(s_1-r,t) = -\frac{r(r^2+3t^2)}{(t^2+r^2)^2} - \frac{1}{2\kappa}\left[\frac{(1+\kappa^2)r}{t^2+r^2} + \frac{4t^2r}{(t^2+r^2)^2}\right] ,$$

$$K_{41}^s(s_1-r,t) = -\frac{r(r^2-t^2)}{(t^2+r^2)^2} - \frac{1}{2\kappa}\left[\frac{(1+\kappa^2)r}{t^2+r^2} - \frac{4t^2r}{(t^2+r^2)^2}\right] ,$$

$$K_{44}^s(s_1-r,t) = \frac{t(t^2-r^2)}{(t^2+r^2)^2} + \frac{1}{2\kappa}\left[\frac{(1-\kappa^2)t}{t^2+r^2} + \frac{2t(t^2-r^2)}{(t^2+r^2)^2}\right] . \qquad (30a\text{-}d)$$

The kernels given in (30) are "singular" in the sense that they become unbounded as t and r approach the end point (t=0, r=0) simultaneously and have the order $t^{-1}$ or $r^{-1}$. To evaluate the stresses given in (29) we first note that by substituting $f_1(t)$ and $f_4(t)$ from (25), the leading terms of the typical integrals in (29) may be expressed as follows (see, for example [15]):

$$\int_0^H \frac{tf_1(t)}{t^2+r^2}\,dt \cong \frac{\pi F_1(0)}{2H^\beta \sin\frac{\pi\beta}{2}}\,\frac{1}{r^\beta}\,,$$

$$r\,\frac{d}{dr}\int_0^H \frac{tf_1(t)}{t^2+r^2}\,dt \cong -\,\frac{\pi\beta F_1(0)}{2H^\beta \sin\frac{\pi\beta}{2}}\,\frac{1}{r^\beta}\,,$$

$$\int_0^H \frac{rf_4(t)}{t^2+r^2}\,dt \cong \frac{\pi F_4(0)}{2H^\beta \cos\frac{\pi\beta}{2}}\,\frac{1}{r^\beta}\,,$$

$$r^2\,\frac{d}{dr}\int_0^H \frac{f_4(t)}{t^2+r^2}\,dt = -\,\frac{\pi(\beta+1)F_4(0)}{2H^\beta \cos\frac{\pi\beta}{2}}\,\frac{1}{r^\beta}\,. \tag{31a-d}$$

Thus, it is seen that the leading terms in the expressions of the interface stresses $\sigma_{xx}$ and $\sigma_{xy}$ around the corner $x=0$, $y=s_1$ have a singularity of the form $r^{-\beta}$. Similar to (28), defining now the "stress intensity factors" by

$$k_1(0) = \lim_{r\to 0} \sqrt{2}\, r^\beta \sigma_{xx}(0,s_1-r)\,,$$

$$k_2(0) = \lim_{r\to 0} \sqrt{2}\, r^\beta \sigma_{xy}(0,s_1-r)\,, \tag{32a,b}$$

and substituting from (30) and (31) into (29), we obtain

$$k_1(0) = \frac{\sqrt{2}\,\mu}{\kappa H^\beta \sin\pi\beta}\left[(\kappa+1-2\beta)\cos\frac{\pi\beta}{2}\,F_1(0)-(\kappa+3-2\beta)\sin\frac{\pi\beta}{2}\,F_4(0)\right],$$

$$k_2(0) = -\,\frac{\sqrt{2}\,\mu}{\kappa H^\beta \sin\pi\beta}\left[(\kappa-1+2\beta)\sin\frac{\pi\beta}{2}\,F_1(0)+(\kappa-3+2\beta)\cos\frac{\pi\beta}{2}\,F_4(0)\right]. \tag{33a,b}$$

The stresses elsewhere along the interfaces $x=0$ and $x=H$, of course, may be evaluated from eqs. (1) and (11) by using the appropriate kernels from Appendices A and B. The stress intensity factors in the other corners may be obtained by following a procedure similar to that described in (29)-(33).

## 5. The Results

The first example considered is the problem of a clamped plate without a crack in order to compare the results with that given in [14] where a different technique was used to solve the problem. In [14] the measure of the external loads was the resultant force rather than the displacement $u_0$ used in this paper. Hence, the easiest direct comparison may be made for the ratio of the stress intensity factors, $k_2(0)/k_1(0)$ (see (32) for the definitions). For the length to width ratio $H/2s_1 = 2$ and $\kappa=3-4\nu$ this comparison is shown in Table 1.

Table 1. Comparison of $k_2(0)/k_1(0)$ obtained in this paper (second row) and in [14] (third row), $H/2s_1=2$.

| $\nu$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| $-k_2/k_1$ | 0 | 0.118610 | 0.212352 | 0.302276 | 0.396751 | 0.504213 |
| $(-k_2/k_1)$ (from [14]) | 0 | 0.118614 | 0.212355 | 0.302280 | 0.396751 | 0.504209 |

For various relative plate dimensions and for $\kappa=1.8$, the normalized stress intensity factors defined by (32) are shown in Table 2. In this case (26) gives the power of stress singularity as $\beta=0.28882$.

Table 2. Normalized stress intensity factors at the corner $x=0$, $y=s_1$ of a rectangular clamped plate for $\kappa=1.8$, plate length=H, plate width=$2s_1$ (see insert in Fig. 3)

| $H/s_1$ | $k_1(0)/(\frac{\mu u_0 s_1^\beta}{s_1})$ | $k_2(0)/(\frac{\mu u_0 s_1^\beta}{s_1})$ | $-k_2(0)/k_1(0)$ |
|---|---|---|---|
| 1 | 2.030255 | -0.613697 | 0.302276 |
| 2 | 1.024510 | -0.309685 | 0.302276 |
| 4 | 0.478944 | -0.144773 | 0.302276 |
| 6 | 0.318621 | -0.096312 | 0.302276 |
| 8 | 0.240840 | -0.072800 | 0.302276 |
| 10 | 0.194190 | -0.058699 | 0.302276 |

Figures 2 and 3 show the normalized stresses $\sigma_{xx}(0,y)$ and $\sigma_{xy}(0,y)$ along the clamp for the case of plane strain and for $\kappa=1.8$. The corresponding Mode II stress intensity factor is shown in Fig. 4.

The second example considered is a clamped plate containing a central crack (see the insert in Fig. 5). The normalized Mode I stress intensity factor calculated at the crack tips $y=\mp c$, $x=H/2$ is shown in Table 3 for $\kappa=1.8$ and $\kappa=2.2$. For comparison the results obtained in [6] by using the technique of conformal mapping for $\kappa=2.2$ are also included in the table. The agreement seems to be fairly good for small cracks. However, for larger cracks there is some discrepency. For $\kappa=2.2$ and $H/2s_1=1$ the normal stress along the clamp is shown in Fig. 5.

Table 3. Crack tip stress intensity factor $k_1(a_3)$ in clamped plate containing a symmetrically located central crack, $H/2s_1=1$, $c=(b_3-a_3)/2$ (Fig. 5).

| $\frac{2c}{H}$ | $k_1(a_3)/(\frac{\mu u_0}{H}\sqrt{c})$ | | |
|---|---|---|---|
| | $\kappa=1.8$ | $\kappa=2.2$ | $\kappa=2.2$ (Ref. 6) |
| 0.1 | 3.15135 | 2.60029 | 2.60000 |
| 0.2 | 3.00550 | 2.49602 | 2.50000 |
| 0.4 | 2.78474 | 2.17958 | 2.17391 |
| 0.5 | 2.34322 | 2.00829 | 2.01724 |
| 0.667 | 1.98095 | 1.73227 | 1.78663 |
| 0.8 | 1.71592 | 1.52883 | 1.64815 |

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APPENDIX A

The Green's Functions $G_{ijk}$ which appear in equations (1) and (2).

$$G_{xxk}(x,y,y_k,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{(t-x)[(t-x)^2-(y-y_k)^2]}{[(t-x)^2+(y-y_k)^2]^2}, \quad (k=1,2;\ y_k=s_k), \tag{A1}$$

$$G_{yyk}(x,y,y_k,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{(t-x)[3(y-y_k)^2+(t-x)^2]}{[(t-x)^2+(y-y_k)^2]^2}, \quad (k=1,2;\ y_k=s_k), \tag{A2}$$

$$G_{xyk}(x,y,y_k,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{(y-y_k)[(y-y_k)^2-(t-x)^2]}{[(t-x)^2+(y-y_k)^2]^2}, \quad (k=1,2;\ y_k=s_k), \tag{A3}$$

$$G_{xx3}(x,y,x_3,t) = -\frac{2\mu}{\pi(1+\kappa)} \frac{(y-t)[3(x-x_3)^2+(t-y)^2]}{[(t-y)^2+(x-x_3)^2]^2}, \quad (x_3=s_3), \tag{A4}$$

$$G_{yy3}(x,y,x_3,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{(y-t)[(t-y)^2-(x-x_3)^2]}{[(t-y)^2+(x-x_3)^2]^2}, \quad (x_3=s_3), \tag{A5}$$

$$G_{xy3}(x,y,x_3,t) = -\frac{2\mu}{\pi(1+\kappa)} \frac{(x-x_3)[(x-x_3)^2-(t-y)^2]}{[(t-y)^2+(x-x_3)^2]^2}, \quad (x_3-s_3), \tag{A6}$$

$$G_{xxj}(x,y,y_j,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{(y-y_j)[(y-y_j)^2+3(t-x)^2]}{[(t-x)^2+(y-y_j)^2]^2}, \quad (j=4,5;\ y_j=s_{j-3}), \tag{A7}$$

$$G_{yyj}(x,y,y_j,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{(y-y_j)[(y-y_j)^2-(t-x)^2]}{[(t-x)^2+(y-y_j)^2]^2} , \quad (j=4,5;\ y_j=s_{j-3}) , \tag{A8}$$

$$G_{xyj}(x,y,y_j,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{(t-x)[(t-x)^2-(y-y_j)^2]}{[(t-x)^2+(y-y_j)^2]^2} , \quad (j=4,5;\ y_j=s_{j-3}) , \tag{A9}$$

$$G_{xx6}(x,y,x_6,t) = -\frac{2\mu}{\pi(1+\kappa)} \frac{(x-x_6)[(x-x_6)^2-(t-y)^2]}{[(t-y)^2+(x-x_6)^2]^2} , \quad (x_6=s_3, \tag{A10}$$

$$G_{yy6}(x,y,x_6,t) = \frac{2\mu}{\pi(1+\kappa)} \frac{(x-x_6)[(x-x_6)^2+3(t-y)^2]}{[(t-y)^2+(x-x_6)^2]^2} , \quad (x_6=s_3) , \tag{A11}$$

$$G_{xy6}(x,y,x_6,t) = -\frac{2\mu}{\pi(1+\kappa)} \frac{(y-t)[(t-y)^2-(x-x_6)^2]}{[(t-y)^2+(x-x_6)^2]^2} , \quad (x_6=s_3) , \tag{A12}$$

$$V_{xk}(x,y,y_k,t) = -\frac{1}{\pi(1+\kappa)} \{\frac{\kappa-1}{4} \log[(x-t)^2+(y-y_k)^2] - \frac{(x-t)^2}{(x-t)^2+(y-y_k)^2}\},$$

$$(k=1,2;\ y_k=s_k) , \tag{A13}$$

$$V_{yk}(x,y,y_k,t) = \frac{-1}{\pi(1+\kappa)} \{\frac{\kappa+1}{2} \text{Arctan} \frac{y-y_k}{x-t} - \frac{(x-t)(y-y_k)}{(x-t)^2+(y-y_k)^2}\} ,$$

$$(k=1,2;\ y_k=s_k) , \tag{A14}$$

$$V_{x3}(x,y,x_3,t) = -\frac{1}{\pi(1+\kappa)} \{\frac{\kappa+1}{2} \text{Arctan} \frac{x-x_3}{t-y} - \frac{(t-y)(x-x_3)}{(t-y)^2+(x-x_3)^2}\},$$

$$(x_3=s_3) , \tag{A15}$$

$$V_{y3}(x,y,x_3,t) = \frac{1}{\pi(1+\kappa)} \{\frac{\kappa-1}{4} \log[(t-y)^2+(x-x_3)^2] - \frac{(t-y)^2}{(t-y)^2+(x-x_3)^2}\},$$

$$(x_3=s_3) \ , \qquad \text{(A16)}$$

$$V_{xj}(x,y,y_j,t) = -\frac{1}{\pi(1+\kappa)} \{\frac{1+\kappa}{2} \text{Arctan} \frac{y-y_j}{x-t} + \frac{(x-t)(y-y_j)}{(x-t)^2+(y-y_j)^2}\} \ ,$$

$$(j-4,5;\ y_j=s_{j-3}), \qquad \text{(A17)}$$

$$V_{yj}(x,y,y_j,t) = \frac{1}{\pi(1+\kappa)} \{\frac{\kappa-1}{4} \log[(x-t)^2+(y-y_j)^2] + \frac{(x-t)^2}{(x-t)^2+(y-y_j)^2}\},$$

$$(j=4,5;\ y_j=s_{j-3}) \ , \qquad \text{(A18)}$$

$$V_{x6}(x,y,x_6,t) = -\frac{1}{\pi(1+\kappa)} \{\frac{\kappa-1}{4} \log[(t-y)^2+(x-x_6)^2]$$

$$+ \frac{(t-y)^2}{(t-y)^2+(x-x_6)^2}\} \ , \quad (x_6=s_3) \ , \qquad \text{(A19)}$$

$$V_{y6}(x,y,x_6,t) = \frac{1}{\pi(1+\kappa)} \{\frac{\kappa+1}{2} \text{Arctan} \frac{x-x_6}{t-y} + \frac{(t-y)(x-x_6)}{(t-y)^2+(x-x_6)^2}\},$$

$$(x_6=s_3) \ . \qquad \text{(A20)}$$

## APPENDIX B

The kernels $C_{ijk}$ which appear in equation (11)

$$C_{xxk} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{xxk}(x,t,\alpha) - m_{xxk}(H-x,H-t,\alpha)]\cos\alpha(y-y_k)d\alpha, \quad (k=1,2;\ y_k=s_k) \ , \tag{B1}$$

$$C_{xx3} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{xx3}(x,x_3,\alpha) + m_{xx3}(H-x,H-x_3,\alpha)]\sin\alpha(t-y)d\alpha, \quad (x_3-s_3) \ , \tag{B2}$$

$$C_{xxj} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{xxj}(x,t,\alpha) + m_{xxj}(H-x,H-t,\alpha)]\sin\alpha(y-y_j)d\alpha \ , \quad (j=4,5;\ y_j=s_{j-3}) \tag{B3}$$

$$C_{xx6} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{xx6}(x,x_3,\alpha) - m_{xx6}(H-x,H-x_3,\alpha)]\cos\alpha(t-y)d\alpha, \quad (x_3=s_3) \ , \tag{B4}$$

$$C_{yyk} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{yyk}(x,t,\alpha) - m_{yyk}(H-x,H-t,\alpha)]\cos\alpha(y-y_k)d\alpha, \quad (k=1,2;\ y_k=s_k) \ , \tag{B5}$$

$$C_{yy3} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{yy3}(x,x_3,\alpha) + m_{yy3}(H-x,H-x_3,\alpha)]\sin\alpha(t-y)d\alpha \ , \quad (x_3=s_3) \ , \tag{B6}$$

$$C_{yyj} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{yyj}(x,t,\alpha) + m_{yyj}(H-x,H-t,\alpha)]\sin\alpha(y-y_j)d\alpha \ , \quad (j=4,5;\ y_j=s_{j-3}) \ , \tag{B7}$$

$$C_{yy6} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{yy6}(x,x_3,\alpha)-m_{yy6}(H-x,H-x_3,\alpha)]\cos\alpha(t-y)d\alpha \ ,$$

$$(x_3=s_3) \ , \qquad (B8)$$

$$C_{xyk} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{xyk}(x,t,\alpha)+m_{xyk}(H-x,H-t,\alpha)]\sin\alpha(y-y_k)d\alpha \ ,$$

$$(k=1,2;\ y_k=s_k) \ , \qquad (B9)$$

$$C_{xy3} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{xy3}(x,x_3,\alpha)-m_{xy3}(H-x,H-x_3,\alpha)]\cos\alpha(t-y)d\alpha \ ,$$

$$(x_3=s_3) \ , \qquad (B10)$$

$$C_{xyj} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{xyj}(x,t,\alpha)-m_{xyj}(H-x,H-t,\alpha)]\cos\alpha(y-y_j)d\alpha \ ,$$

$$(j=4,5;\ y_j=s_{j-3}) \ , \qquad (B11)$$

$$C_{xy6} = \frac{\mu}{\pi(1+\kappa)} \int_0^\infty [m_{xy6}(x,x_3,\alpha)+m_{xy6}(H-x,H-x_3,\alpha)]\sin\alpha(t-y)d\alpha \ ,$$

$$(x_3=s_3) \ , \qquad (B12)$$

$$m_{xxk}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(-1+\kappa^2-2\alpha x+2\alpha t+4\alpha^2 tx)+4\kappa\alpha H(\alpha x+\alpha t-\alpha H)]e^{-\alpha(t+x)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(\alpha x-\alpha t)+2\alpha H(1-\kappa^2-2\alpha H+2\alpha t+2\alpha x-4\alpha^2 xH+4\alpha^2 tx)]e^{\alpha(t-x)}\},$$

$$(k=1,2) \ , \qquad (B13)$$

$$m_{xx3}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(-1-\kappa^2-2\alpha x-2\alpha x_3-4\alpha^2xx_3)+4\kappa\alpha H(-1-\alpha x+\alpha H-\alpha x_3)]e^{-\alpha(x+x_3)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(1+\alpha x-\alpha x_3)+2\alpha H(-1-\kappa^2-2\alpha x-2\alpha H+2\alpha x_3-4\alpha^2xH+4\alpha^2xx_3)]e^{-\alpha(x-x_3)}\},$$

(B14)

$$m_{xxj}=-D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(-1-\kappa^2-2\alpha x-2\alpha t-4\alpha^2tx)+4\kappa\alpha H(-1-\alpha x-\alpha t+\alpha H)]e^{-\alpha(t+x)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(1+\alpha x-\alpha t)+2\alpha H(-1-\kappa^2-2\alpha H+2\alpha t-2\alpha x-4\alpha^2xH+4\alpha^2tx)]e^{\alpha(t-x)}\},$$

$(j=4,5)$ (B15)

$$m_{xx6}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(1-\kappa^2+2\alpha x-2\alpha x_3-4\alpha^2xx_3)+4\kappa\alpha H(-\alpha x-\alpha x_3+\alpha H)]e^{-\alpha(x+x_3)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(-\alpha x+\alpha x_3)+2\alpha H(-1+\kappa^2-2\alpha x+2\alpha H-2\alpha x_3+4\alpha^2xH-4\alpha^2xx_3)]e^{-\alpha(x-x_3)}\},$$

(B16)

$$m_{yyk}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(-3-\kappa^2+6\alpha t+2\alpha x-4\alpha^2xt)+4\kappa\alpha H(2-\alpha x-\alpha t+\alpha H)]e^{-\alpha(t+x)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(-\alpha x+\alpha t+2)-2\alpha H(-\kappa^2-3+2\alpha x-6\alpha t+6\alpha H+4\alpha^2xt-4\alpha^2xH)]e^{\alpha(t-x)}\},$$

$(k=1,2)$ , (B17)

$$m_{yy3}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(-3+\kappa^2+2\alpha x-6\alpha x_3+4\alpha^2xx_3)+4\kappa\alpha H(-1+\alpha x-\alpha H+\alpha x_3)]e^{-\alpha(x+x_3)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(1-\alpha x+\alpha x_3)+2\alpha H(-3+\kappa^2+2\alpha x+6\alpha x_3-6\alpha H+4\alpha^2Hx-4\alpha^2xx_3)]e^{-\alpha(x-x_3)}\}$$

(B18)

$$m_{yyj}=-D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(-3+\kappa^2-6\alpha t+2\alpha x+4\alpha^2xt)+4\kappa\alpha H(-1+\alpha x+\alpha t-\alpha H)]e^{-\alpha(t+x)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(1-\alpha x+\alpha t)+2\alpha H(-3+\kappa^2+2\alpha x+6\alpha t-6\alpha H-4\alpha^2xt+4\alpha^2xH)]e^{\alpha(t-x)}\},$$

$$(j=4,5), \qquad \text{(B19)}$$

$$m_{yy6}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(\kappa^2+3-2\alpha x-6\alpha x_3+4\alpha^2xx_3)+4\kappa\alpha H(-2+\alpha x+\alpha x_3-\alpha H)]e^{-\alpha(x+x_3)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(-2+\alpha x-\alpha x_3)+2\alpha H(-3-\kappa^2+2\alpha x-6\alpha x_3+6\alpha H-4\alpha^2xH$$

$$+4\alpha^2xx_3)]e^{-\alpha(x-x_3)}\}, \qquad \text{(B20)}$$

$$m_{xyk}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(1+\kappa^2-2\alpha x-2\alpha t+4\alpha^2tx)+4\kappa\alpha H(-1+\alpha x+\alpha t-\alpha H)]e^{-\alpha(t+x)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(-1+\alpha x-\alpha t)+2\alpha H(-1-\kappa^2+2\alpha H-2\alpha t+2\alpha x-4\alpha^2xH+4\alpha^2tx)]e^{\alpha(t-x)}\},$$

$$(k=1,2), \qquad \text{(B21)}$$

$$m_{xy3}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(1-\kappa^2-2\alpha x+2\alpha x_3-4\alpha^2x_3x)+4\kappa\alpha H(-\alpha x+\alpha H-\alpha x_3)]e^{-\alpha(x+x_3)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(\alpha x-\alpha x_3)+2\alpha H(1-\kappa^2-2\alpha x-2\alpha x_3+2\alpha H+4\alpha^2x_3x-4\alpha^2Hx)]e^{-\alpha(x-x_3)}\},$$

$$\text{(B22)}$$

$$m_{xyj}=D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(-\kappa^2+1-2\alpha x+2\alpha t-4\alpha^2tx)+4\kappa\alpha H(-\alpha x-\alpha t+\alpha H)]e^{-\alpha(t+x)}$$

$$+[2\kappa^2(e^{-2\alpha H}-1)(\alpha x-\alpha t)+2\alpha H(1-\kappa^2+2\alpha H-2\alpha t-2\alpha x-4\alpha^2xH+4\alpha^2tx)]e^{\alpha(t-x)}\},$$

$$(j=1,2), \qquad \text{(B23)}$$

$$m_{xy6}=-D^{-1}(\alpha)\{[\kappa(e^{2\alpha H}-1)(-1-\kappa^2+2\alpha x+2\alpha x_3-4\alpha^2 xx_3)+4\kappa\alpha H(1-\alpha x-\alpha x_3+\alpha H)]e^{-\alpha(x+x_3)}$$

$$+[2\kappa^2(e^{-2\alpha x}-1)(1-\alpha x+\alpha x_3)+2\alpha H(1+\kappa^2-2\alpha H+2\alpha x_3-2\alpha x+4\alpha^2 xH-4\alpha^2 xx_3)]e^{-\alpha(x-x_3)}\},$$

(B24)

$$D(\alpha) = \kappa^2[e^{2\alpha H}+e^{-2\alpha H}-2]-4\alpha^2 H^2 \ . \qquad \text{(B25)}$$

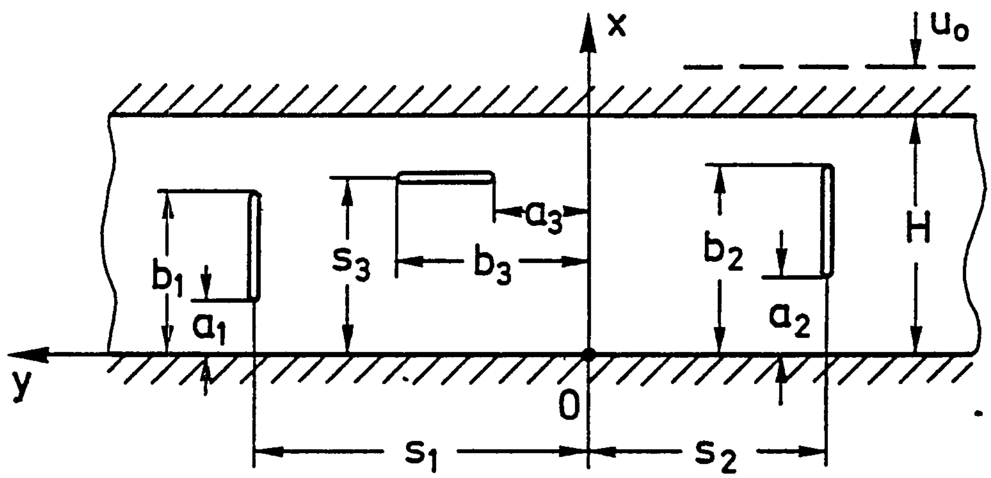


Fig. 1 The basic crack geometry considered.

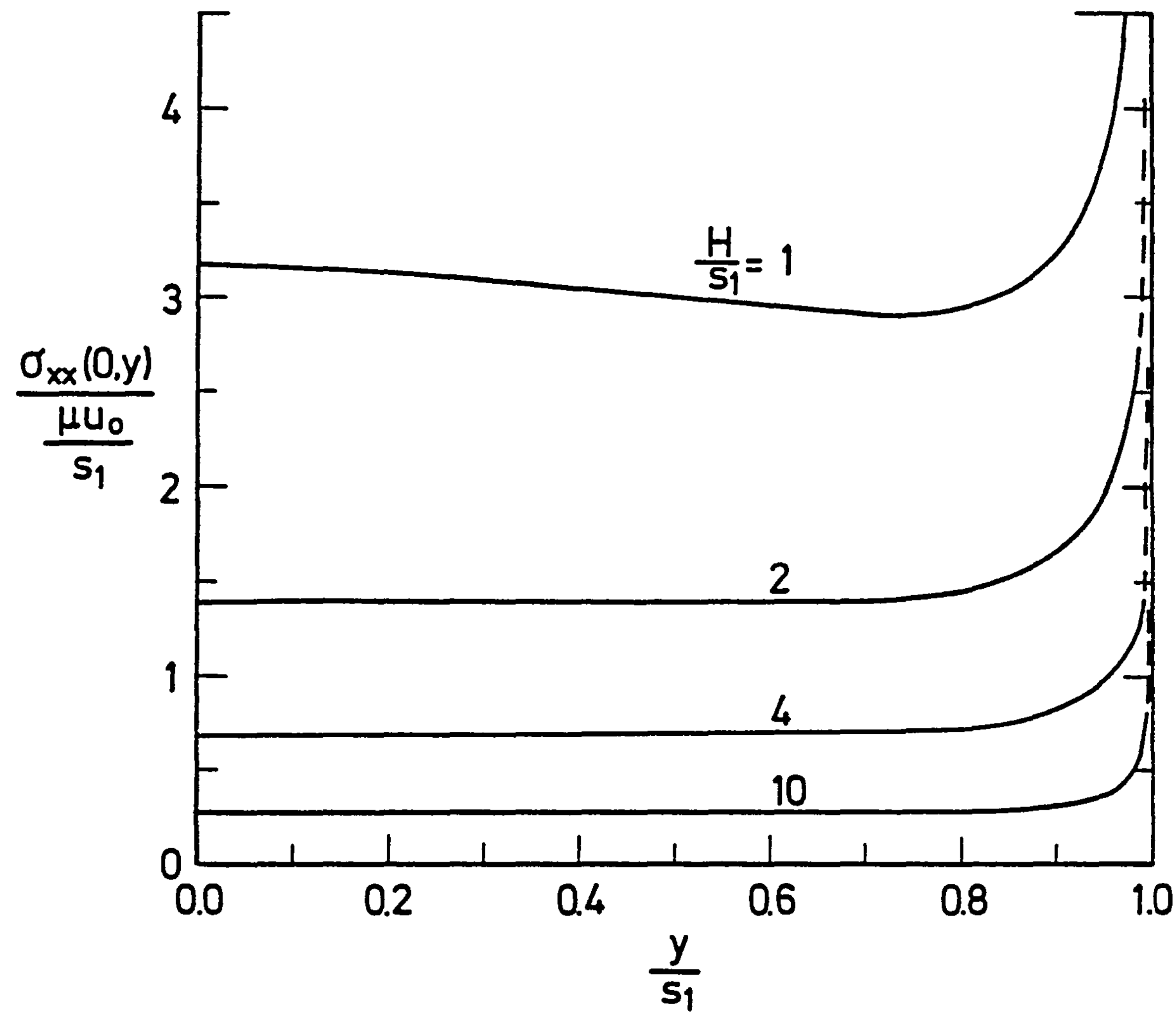


Fig. 2 Normal stress along the clamped boundary in a plate without a crack ($\kappa$=1.8).

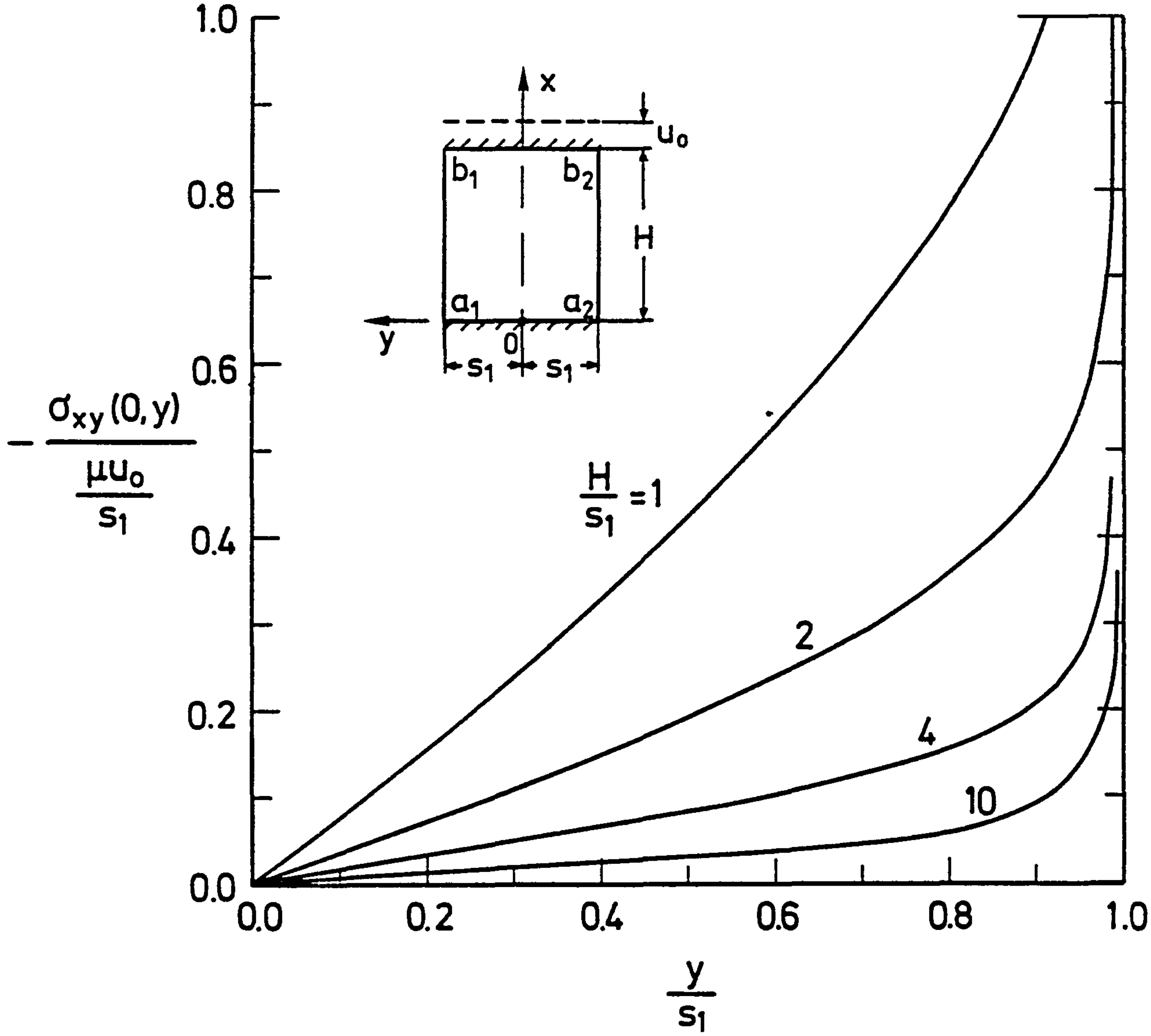


Fig. 3 Shear stress along the clamped boundary in a plate without a crack ($\kappa$=1.8).

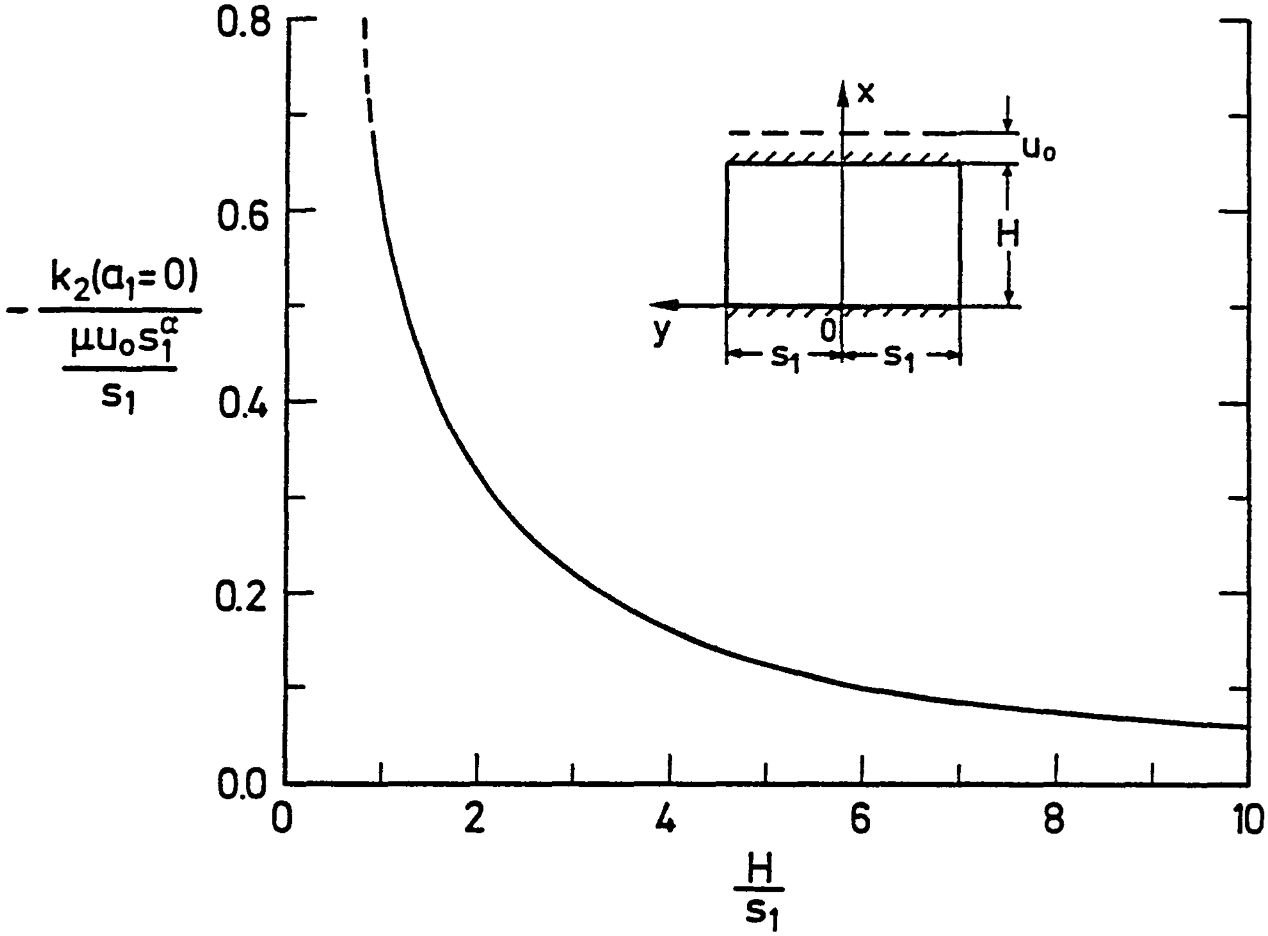


Fig. 4 Normalized Mode II stress intensity factor at the corners of a clamped rectangular plate without a crack ($\kappa$=1.8).

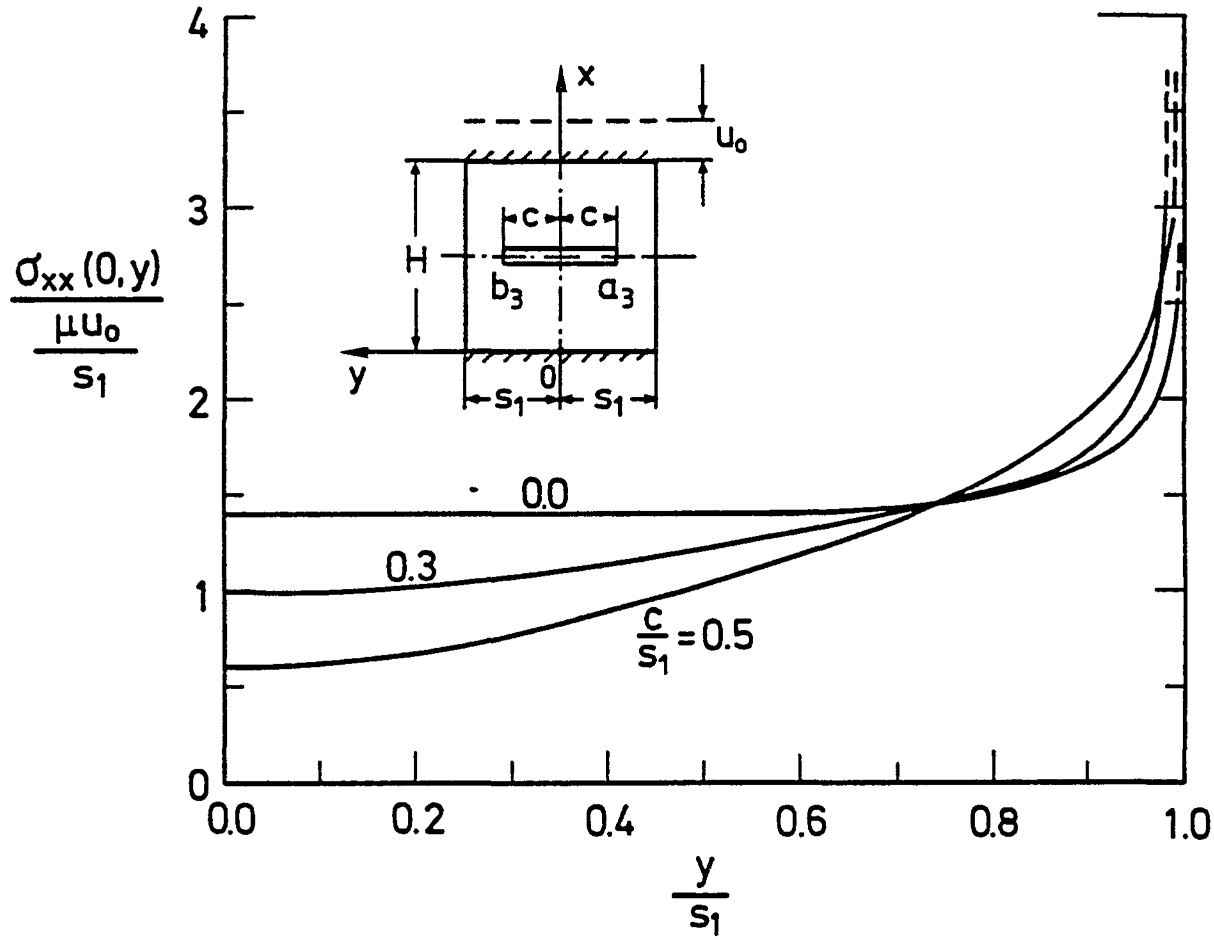


Fig. 5 Normal stress along the clamped boundary in a plate with a central crack ($H/2s_1=1$), $\kappa=2.2$.

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16 Abstract

In this paper the general problem of a rectangular plate clamped along two parallel sides and containing a crack parallel to the clamps is considered. The problem is formulated in terms of a system of singular integral equations and the asymptotic behavior of the stress state near the corners is investigated. Numerical examples are considered for a clamped plate without a crack and with a centrally located crack, and the stress intensity factors and the stresses along the clamps are calculated.

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