TEMPERATURE FIELD AROUND A SPHERICAL, CYLINDRICAL, AND NEEDLE-SHAPED CRYSTAL, GROWING IN A PRE-COOLED MELT

G. P. Ivantsov

Translation of "Temperaturnuye Pole Vokrug Sharoobraznogo Tsilindricheskogo i igloobraznogo Kristalla Rastushchego v Pereokhlazhdennom Rasplave." Akademiia Nauk SSR, Doklady, Vol. 58, 1947, pp. 567-569.

(NASA-TM-77889) TEMPERATURE FIELD AROUND A SPHERICAL, CYLINDRICAL, AND NEEDLE-SHAPED CRYSTAL, GROWING IN A PRE-COOLED MELT (National Aeronautics and Space Administration) 6 p HC A02/MF A01 CSCL 20B G3/76

N85-32011

Unclas 21808

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON D.C. 20546 July 1985

STANDARD TITLE PAGE

1. Report No. NASA TM-77889	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle TEMPERATURE FIELD AROUND A SPHERICAL, CYLINDRICAL, AND NEEDLE-SHAPED CRYSTAL, GROWING IN A PRE-COOLED MELT		5. Report Date JULY, 1985
		6. Performing Organization Code
7. Author(s) G. P. Ivantsov		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address SCITRAN Box 5456 Santa Barbara, CA 93108		11. Contract or Grant No. NASw- 4004
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Temperaturnuye Pole Vokrug Sharoobraznogo Tsilindricheskogo i igloobraznogo Kristalla Rastushchego v Pereokhlazhdennom Rasplave." Akademiia Nauk SSR, Doklady, Vol. 58, 1947, pages 567-569.

16. Abstract

An examination is made of the growth of a single crystal in a pre-cooled melt. The conditions under which this occurs are described. It is found that the movement of the crystallization front is proportional to the root of time. The problem is solved based on the considerations presented.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified and Unlimited

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 5	22. Price

TEMPERATURE FIELD AROUND A SPHERICAL, CYLINDRICAL, AND NEEDLE-SHAPED CRYSTAL, GROWING IN A PRE-COOLED MELT

G. P. Ivantsov /*567

An examination is made of the growth of a single crystal in a pre-cooled melt under the following conditions.

The solid phase has $t=t_k$ with respect to the entire mass. The temperature field of the liquid phase, which is assumed to be fixed, obeys the Fourier equation:

$$\frac{\partial t}{\partial \tau} = a\nabla^2 t. \quad (1)$$

At the boundary of the phase interface

$$t = t_k, \quad (2)$$

$$-\lambda\left(\frac{\partial t}{\partial n}\right)_{t_k} = q\gamma\left(\frac{dn}{d\tau}\right)_{t_k}, \quad (3)$$

where λ is the melt heat conductivity coefficient; q--latent heat of crystalization; γ--specific weight which is identical for both phases.

The melt temperature at infinity is

$$t = t_0 \quad (t_k > t_0). \quad (4)$$

Certain solutions of this problem were obtained as follows:

Let us transform condition (3), replacing $\frac{dn}{d\tau}$ by $-\frac{\partial t}{\partial \tau}\Big/\frac{\partial t}{\partial n}$ and expanding the expression for the gradient:

$$\left[\left(\frac{\partial t}{\partial x}\right)^2+\left(\frac{\partial t}{\partial y}\right)^2+\left(\frac{\partial t}{\partial z}\right)^2\right]_{t_k} = \frac{q\gamma}{\lambda}\left(\frac{\partial t}{\partial \tau}\right)_{t_k}. \quad (3')$$

---

*Numbers in margin indicate pagination of foreign text.

In condition (3') let us introduce the arbitrary temperature function f(t) and we shall regard the expression obtained as the differential equation

$$\left(\frac{\partial t}{\partial x}\right)^2+\left(\frac{\partial t}{\partial y}\right)^2+\left(\frac{\partial t}{\partial z}\right)^2=f(t)\frac{\partial t}{\partial \tau}. \qquad (3'')$$

Formulation of the problem is: To find all surfaces of the crystalization front and the temperature fields corresponding to them in the liquid phase, which satisfy the differential equations (1) and (3") and the boundary conditions (2), (4) and (3'). /568

The order of solution of the problem is: To find the integrals of the non-linear equation of the first order (3") which includes the arbitrary functions. The form of the latter is determined using equation (1), and the arbitrary constants, by means of (2), (4) and (3').

In certain cases it is possible to find the form of the arbitrary function and to solve the problem.

The complete integral of equation (3") obtained by the well known Lagrange-Charpy method has the form

$$\frac{1+C_1^2+C_2^2}{C_3}F(t)=x+C_1y+C_2z+C_3\tau+C_4, \qquad (5)$$

where $F(t)=\int\frac{dt}{f(t)}$; from this we have

$$t=\Phi(x+C_1y+C_2z+C_3\tau+C_4).$$

This form of the solution satisfies equation (1), but it is not of interest. Excluding all arbitrary constants from the complete integral (5), we obtain the particular integral

$$t=\Phi\left(\frac{x^2+y^2+z^2}{4a\tau}\right)=\Phi\left(\frac{r^2}{4a\tau}\right). \qquad (6)$$

Using equation (1), we determine the form of the arbitrary function Φ. It has the form

$$\Phi(u)=C'\left[\frac{e^{-u^2}}{u}-\sqrt{\pi}(1-\mathrm{erf}(u))\right]+C''=C'\Psi(u)+C'', \tag{7}$$

where

$$u=\frac{r}{2\sqrt{a\tau}}=\frac{R}{2\sqrt{a\tau}}\frac{r}{R}=u_0\frac{r}{R}. \tag{8}$$

Here R is the radius of the sphere which is the phase interface surface (r > R).

Considering the boundary conditions (2), (4) and (3') leads to the equation for determining u_0

$$\frac{c(t_k-t_0)}{q}=2\Psi(u_0)u_0^3e^{u_0^2} \tag{9}$$

(here c is the melt heat capacity), and it leads to the desired expression for the temperature field

$$\frac{t-t_0}{t_k-t_0}=\frac{\Psi(u)}{\Psi(u_0)}. \tag{10}$$

The temperature field around a cylindrical crystal obtained in a similar way from equation (6) for the planar case

$$\frac{t-t_0}{t_k-t_0}=\frac{\mathrm{Ei}(-u^2)}{\mathrm{Ei}(-u_0^2)}, \tag{11}$$

where u_0 is determined from the equation:

$$\frac{c(t_k-t_0)}{q}=-u_0^2e^{u_0^2}\mathrm{Ei}(-u_0^2). \tag{12}$$

We should note that in both cases the movement of the crystalization front is proportional to the root of time, which follows from expression (8) for u_0. The isothermal surfaces are spheres for the first case, and are cylinders for the second case. /569

Assigning the arbitrary relations between the arbitrary constants in the complete integral (5) and excluding the latter, we find the general integrals.

The following linear relationship is of interest out of the different types of relationships studied:

$$C_1=mC_2, \tag{13}$$

which leads to the following general integral, which describes the growth of a needle-shaped crystal:

$$t=\Phi(-z-m\tau+\sqrt{r^2+(z+m\tau)^2}). \qquad (14)$$

The isothermal surfaces in this case are confocal paraboloids of revolution, and for the planar case--confocal parabolic cylinders.

Determining the form of the arbitrary function Φ in solution (14) using equation (1) and the arbitrary constants using (2), (4) and (3'), we obtain the expression of the temperature field around the needle-shaped crystal growing in a pre-cooled melt:

$$\frac{t-t_0}{t_k-t_0}=\frac{\mathrm{Ei}\left(-\frac{wR}{2a}v\right)}{\mathrm{Ei}\left(-\frac{wR}{2a}\right)} \text{ where } v=\sqrt{\left(\frac{r}{R}\right)^2+\left(\frac{z-w\tau}{R}\right)^2}-\frac{z-w\tau}{R}. \qquad (15)$$

Here R = const is the radius of curvature of the paraboloid apex, which is the phase interface surface $\Phi\,(v=1)$, $w=\mathrm{const}$ is the growth rate of the needle in the direction of its axis.

The parameter $wR/2a$ is determined by the condition

$$\frac{c(t_k-t_0)}{q}=-\frac{wR}{2a}e^{wR/2a}\mathrm{Ei}\left(-\frac{wR}{2a}\right). \qquad (16)$$

Assigning the growth rate of the needle-shaped crystal w on the basis of these considerations, we consider that the problem is solved.