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A REVIEW OF PATH-INDEPENDENT INTEGRALS IN ELASTIC-PLASTIC FRACTURE MECHANICS

TASK IV INTERIM REPORT

Prepared by:
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Prepared for



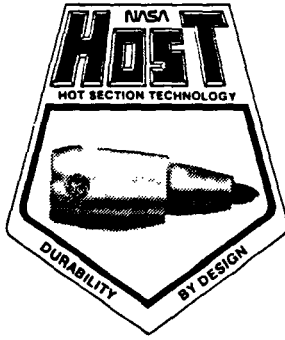
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1. INTRODUCTION

The J-integral has received much attention among researchers since it was introduced to fracture mechanics. As a result, a wide body of literature exists in this connection. The J-integral is a measure of severity of the deformation field at the crack tip and has proven to be a viable parameter for prediction of ductile crack initiation and growth under monotonic loading in the nonlinear regime. Because it can be determined from the far-field numerical solution, which is generally more accurate than the near-field solution, the J-integral has been quite popular in computational fracture mechanics. Unfortunately, however, the theoretical foundation upon which the J-integral is based does not permit it to be extended to more practical engineering problems. The path independence of the J-integral is valid only within the deformation theory of plasticity; hence, it cannot be used in the presence of substantially nonproportional loading and unloading after plastic deformation. The J-integral cannot be used in the presence of a temperature gradient and material inhomogeneity. Other factors such as body force and crack surface loading,^{*} although not as frequently encountered in applications as the above, are also excluded in the J-integral.

In recent years, there have been considerable efforts to modify or reformulate the path-independent (P-I) integral such that these limitations are removed. Accordingly, there emerged a number of new P-I integrals^{**} in the literature. Some of them are merely slight modifications of the original J-integral, but others are formulated on different theoretical bases. None of the new P-I integrals have yet been used enough to establish a consensus on the utility in fracture mechanics.

* The crack surface traction can be included in the J-integral simply by adding the crack surface path to the integration path.

** In the classical sense, the path-independent integral is a line integral performed along an open or closed curve. Here, the notion of "path independence" is somewhat different. As we will see, most path-independent integrals contain not only line integrals but also area integrals.

The purpose of this study is to review the new P-I integrals from the standpoint of theoretical basis of formulation. We will restrict our attention to rate-independent materials and will not consider the effects of inertia, body force,^{*} and large deformation. The P-I integrals in this category and to be reviewed in the text are the J-integral (Reference 1), Wilson and Yu's integral (Reference 2), Gurtin's integral (Reference 3), the J_θ -integral by Ainsworth et al. (Reference 4), the J^* -integral by Blackburn (Reference 5), the \hat{J} -integral by Kishimoto et al. (Reference 6), and the ΔT_p - and ΔT_p^* -integrals by Atluri et al. (Reference 7). These P-I integrals will be recast in consistent notation, and the similarities, differences, salient features, and limitations will be examined. Comments will also be made with regard to the physical meaning, the possibility of experimental measurement, and the ease of computation.

* The body force can be included easily in any P-I integral. Usually it generates an area integral term.

2. NOTATION

In most sources reviewed in this study, the index notation has been used. Accordingly, we will use the index notation for all the P-I integrals. The variables to be used in the text are described below. Unless stated otherwise, the definitions of variables are as given here. For the notation related to integration paths and areas, the reader is also referred to Figure 1.

a	Crack length
A	$\lim_{\varepsilon \rightarrow 0} (A_{\Gamma} - A_{\varepsilon})$
A_{Γ}	Area surrounded by Γ and the crack surface
A_{ε}	Area surrounded by Γ_{ε} and the crack surface
E	Young's modulus
J	The J-integral
J_G	Gurtin's P-I integral
J_W	Wilson and Yu's P-I integral
J_{θ}	Ainsworth's et al. P-I integral
\hat{J}	Blackburn's P-I integral
K_I	Mode I stress intensity factor
n_i	Outward unit normal vector
P	Potential energy
s	Arc length along the contour
S	Total external boundary
S_t	Part of S where traction is given
S_u	Part of S where displacement is given
T_i	Traction vector
u_i	Displacement vector
W	Strain energy density
x_i	The Cartesian coordinates
α	Thermal expansion coefficient
Γ	A counterclockwise path surrounding the crack tip, starting on the lower crack surface and ending on the upper crack surface
Γ_c	$\Gamma_c^+ + \Gamma_c^-$
Γ_c^+	Upper crack surface from Γ to Γ_{ε}
Γ_c^-	Lower crack surface from Γ_{ε} to Γ

Γ_ε	Γ with radius ε and with origin at the crack tip; $\varepsilon \rightarrow 0$ is implied without limit notation
δ_{ij}	Kronecker delta
Δ	Increment of the subsequent variable from time τ to $\tau + \Delta\tau$
ΔT	Atluri's P-I integral
ΔT_p	Atluri's et al. P-I integral
ΔT_p^*	Atluri's et al. P-I integral
ε_{ij}	Strain tensor
θ	Relative temperature
λ, μ	Lame's constants
ν	Poisson's ratio
σ_{ij}	Stress tensor
τ	Time
ϕ	Global thermodynamic potential
ψ	Helmholz free energy
Ω	Specific internal energy
$(\cdot)^e$	Elastic component of (\cdot) , such as $u_i^e, \varepsilon_{ij}^e, w^e$
$(\cdot)^p$	Plastic component of (\cdot)
$(\cdot)^\theta$	Thermal component of (\cdot) , such as ε_{ij}^θ

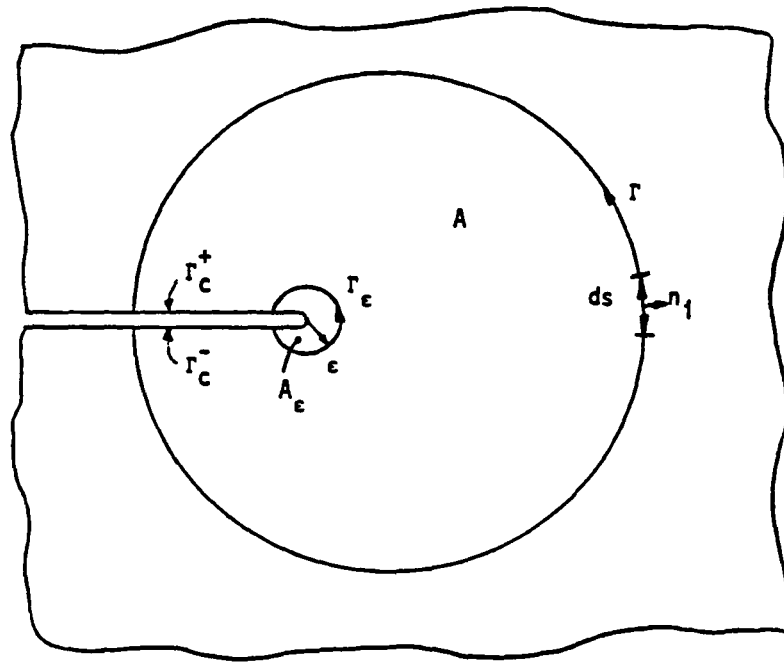


Figure 1. Integration Paths and Areas.

3. THE J-INTEGRAL

The introduction of the J-integral to fracture mechanics is due to Rice (Reference 1), although some related earlier studies can be found in Eshelby (Reference 8), Sanders (Reference 9), and Cherepanov (Reference 10). Later researchers extended the J-integral to large elastic deformation (References 11-13). In the following, we will review the concept of the J-integral.

Let us consider a cracked body as shown in Figure 1. We assume that the crack is straight and oriented in the x_1 -direction. We will also assume that the body force is negligible and the crack surface is traction-free. Then, the following integral denoted by J is independent of the path:

$$J = \int_{\Gamma} (n_1 W - t_i u_{i,1}) ds \quad (1)$$

where the strain energy density function W is given by

$$W = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad (2)$$

In view that W is a uniquely defined function of the current strain, the J-integral is considered to be valid for nonlinear elastic materials.

The path independence of J is easily proved by use of the equilibrium equation

$$\sigma_{ij,j} = 0 \quad (3)$$

and the divergence theorem

$$\int_C n_i f ds = \int_A f_{,i} dA \quad (4)$$

where f is a piecewise continuously differentiable function, C is a closed contour, and A is the area surrounded by C. The homogeneity of the material, at least in the x_1 direction, is also assumed to prove the path independence of J. In this connection, we note the work by Smelser and Gurtin (Reference 14) who showed that the J-integral can be extended to bimaterials without change, provided the bond line is straight and parallel to the crack.

The J-integral is a measure of the severity of crack tip deformation when Γ is taken sufficiently close to the crack tip. For linearly elastic materials, J can be related to the stress intensity factor by direct substitution of the singular solution (Reference 15). For instance, the Mode I solution yields

$$J = \frac{\kappa}{E} K_I^2 \quad (5)$$

where $\kappa = 1$ for plane stress problems, and

$$\kappa = 1 - \nu^2 \text{ for plane strain problems}$$

Equation 5 has been widely used to determine K_I from J computed from far-field data.

The J-integral is also valid for elastic-plastic materials within the deformation theory of plasticity. For power-law hardening materials, the crack tip field is characterized by the HRR (Hutchinson, Rice, and Rosengren) singularity (References 16 and 17). The J-integral is again related to the strength of the singular field by

$$\sigma_{ij} = \sigma_0 \left(\frac{EJ}{\sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(\theta, n) \quad (6)$$

and

$$\varepsilon_{ij} = \frac{\sigma_0}{E} \left(\frac{EJ}{\sigma_0^2 I_n r} \right)^{\frac{n}{n+1}} \tilde{\varepsilon}_{ij}(\theta, n) \quad (7)$$

where σ_0 is the yield stress, n is the hardening exponent, r and θ are the polar coordinates with the origin at the crack tip, I_n is a constant that is a function of n (References 16 and 17), and $\tilde{\sigma}_{ij}$ and $\tilde{\varepsilon}_{ij}$ are dimensionless functions of θ and n . A study by Kumar, et al. (Reference 18) revealed that J is a viable parameter for prediction of ductile crack extension under monotonic loading.

The J-integral computed from the numerical results of finite-element analyses, based on incremental theory of plasticity, is essentially path independent provided that the degree of nonproportionality in loading is not severe. It is also worth noting that McMeeking's work (Reference 19) shows the path dependence of J in the very vicinity of the crack tip where the loading is significantly nonproportional. One of the limitations on the utility of the J-integral is that it cannot be used in circumstances where unloading takes place after plastic deformation; this is significant since most crack-propagation problems involve unloading at either global or local level. Some researchers (References 20 and 21) correlated crack growth data to ΔJ defined operationally using the approximation formula for J in References 22 and 23. This type of approach, despite a lack of theoretical rigor, may be worthwhile insofar as no other parameters are available for crack-growth prediction in the elastic-plastic regime.

Rice (Reference 24) has shown that the J-integral can be interpreted as the rate of potential energy decrease for two bodies differing in crack length by an infinitesimal amount, namely

$$J = - dP/da \quad (8)$$

where

$$P = \int_V W(\epsilon_{ij}) dV - \int_{S_t} t_i u_i dA \quad (9)$$

is the potential energy, V is the volume of the body, and S_t is the boundary where traction is prescribed. In view of Equation 8, J is often referred to as the "crack driving force" or "energy release rate." This interpretation is, of course, valid within the framework of nonlinear elasticity. The relation of Equation 8 makes possible the experimental measurement of J as discussed by Dowling and Begley (Reference 25) and Rice, et al. (Reference 26).

4. WILSON AND YU'S INTEGRAL

Wilson and Yu (Reference 2) modified the J-integral to include thermal strain. Their integral (for convenience, we will use J_W) is given by

$$J_W = \int_{\Gamma} (n_1 W - t_i u_{i,1}) ds - \alpha(3\lambda + 2\mu) \int_A [\frac{1}{2}(\theta \varepsilon_{ii})_{,1} - \varepsilon_{ii} \theta_{,1}] dA \quad (10)$$

where

$$W = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} \quad (11)$$

and ε_{ij} includes the thermal strain. The stress component σ_{ij} is related to ε_{ij} by

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} - \alpha(3\lambda + 2\mu) \delta_{ij} \theta \quad (12)$$

Notice that the modification was made only for the case of homogeneous, isotropic, linearly elastic materials. Alternative forms of J_W were also suggested in Reference 2 using different definitions of W :

$$J_W = \int_{\Gamma} (n_1 W^* - t_i u_{i,1}) ds + \alpha(3\lambda + 2\mu) \int_A \varepsilon_{ii} \theta_{,1} dA \quad (13)$$

and

$$J_W = \int_{\Gamma} (n_1 \bar{W} - t_i u_{i,1}) ds - \alpha(3\lambda + 2\mu) \int_A \theta \varepsilon_{ii,1} dA \quad (14)$$

where

$$W^* = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} - \alpha(3\lambda + 2\mu) \theta \varepsilon_{ii} \quad (15)$$

and

$$\bar{W} = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} + \alpha(3\lambda + 2\mu) \theta \varepsilon_{ii} \quad (16)$$

The path Γ must be replaced by $\Gamma + \Gamma_c$ if the crack surface is not free of traction. The near-field expression for J_W can be written as

$$J_W = \int_{\Gamma_\varepsilon} (n_1 W - t_i u_{i,1}) ds \quad (17)$$

Using the singular solutions (References 15 and 27), J_W can be related to the stress intensity factors for a bounded crack-tip temperature.

It is noted that material inhomogeneity can be included in J_W in a manner similar to Atluri's (Reference 7); that is, the additional term in $\partial W/\partial x_1$ due to material inhomogeneity can be expressed in terms of stress, strain, temperature, and the derivatives of these parameters. The resulting equation is, however, identical to the J^* -integral (Equation 31) to be discussed later.

The singularity of the temperature field at the crack tip, if not bounded, must be weaker than the square-root singularity in order for the area integral to exist. The possibility of an unbounded temperature at the crack tip was discussed in Nguyen (Reference 28) and Bui (Reference 29). Notice also that for a bounded θ the integrand of the area integral in Equations 10 and 14 behaves like $r^{-3/2}$ near the crack tip while the integrand in Equation 13 behaves like $r^{-1/2}$. In this sense, Equation 13 would be somewhat better for the computational purpose.

Wilson and Yu's integral was reconsidered by McCartney (Reference 30) with a thermodynamic background. He showed that

$$J_W = \int_{\Gamma} (n_1 \rho \Psi - t_i u_{i,1}) ds + \int_A \rho \eta \theta_{,1} dA \quad (18)$$

where ρ is the density of the material, Ψ is the helmoltz free energy, η is the entropy, and θ is the absolute temperature. For the adiabatic process ($\eta = \text{constant}$), Equation 18 reduces to

$$J_W = \int_{\Gamma} (n_1 \rho \Omega - t_i u_{i,1}) ds \quad (19)$$

where Ω is the specific internal energy. It is worth noting that Equation 18 can be written as

$$J_W = \int_{\Gamma} (n_1 \rho \Omega - t_i u_{i,1}) ds - \int_A \rho \theta \eta_{,1} dA \quad (20)$$

Here, $\Psi = \Omega - \theta \eta$ has been used. Then, replacing Γ with $\Gamma + \Gamma_c$ and using the divergence theorem, the near-field expression is obtained as

$$J_W = \int_{\Gamma_{\varepsilon}} (n_1 \rho \Omega - t_i u_{i,1}) ds \quad (21)$$

It is of interest to notice the work of Nguyen (Reference 28) and Germain, et al. (Reference 31) in which they showed that Equation 12 can be expressed as

$$J_W = - \partial \phi / \partial a \quad (22)$$

where ϕ is a global thermodynamic potential with arguments such as strain, temperature, crack length, and other internal parameters. Equation 22 is equivalent to Equation 8 for J and, as such, carries a significant meaning from the standpoint of experimental measurement. Notice also that Equation 2 is equivalent to Equation 17 if Ω is identical to W at the crack tip. This condition is satisfied if the entropy at the crack tip is maintained constant. For a stationary crack, this implies a finite crack tip temperature.

5. GURTIN'S INTEGRAL

Gurtin (Reference 3) proved the following conservation law for the thermoelastic-field-satisfying Equations 3 and 12 and the steady-state temperature condition, $\theta_{,ii} = 0$:

$$\int_S \left[n_i W - t_k u_{k,i} - \frac{\alpha^2(3\lambda+2\mu)^2}{2(\lambda+\mu)} \theta^2 n_i + \frac{\alpha\mu(3\lambda+2\mu)}{(\lambda+\mu)} \left(\theta \frac{\partial u_i}{\partial n} - u_i \frac{\partial \theta}{\partial n} \right) \right] dA = 0 \quad (23)$$

where S is the boundary of the volume, $\partial/\partial n = n_j \partial/\partial x_j$, and W is the strain energy density defined by

$$W = \mu \varepsilon_{ij} \varepsilon_{ij} + \frac{\lambda}{2} (\varepsilon_{kk})^2 \quad (24)$$

Equation 23 is valid in two or three dimensions. In particular, for the former with traction-free crack surface,

$$J_G = \int_\Gamma \left[n_1 W - t_k u_{k,1} - \frac{\alpha^2(3\lambda+2\mu)^2}{2(\lambda+\mu)} \theta^2 n_1 + \frac{\alpha\mu(3\lambda+2\mu)}{(\lambda+\mu)} \left(\theta \frac{\partial u_1}{\partial n} - u_1 \frac{\partial \theta}{\partial n} \right) \right] ds \quad (25)$$

is path independent provided that $\theta u_{1,2}$ and $u_1 \theta_{,2}$ are continuous across the crack. This condition is a severe restriction from the application standpoint. For instance, the continuity of the two quantities cannot be satisfied for Mode I crack problems. It would be more practical to change the integration path from Γ to $\Gamma + \Gamma_c$, thereby eliminating the restrictions. With this modification and with the assumption that the singularity of the temperature at the crack tip is weaker than $r^{-1/2}$, the near-field expression for J_G can be written as

$$J_G = \int_{\Gamma_\varepsilon} (n_1 W - t_1 u_{i,1}) ds \quad (26)$$

This equation and Equation 17 yield the same results, although different definitions of W were used.

Equation 25 clearly has a computational advantage over J_W because it does not contain area integrals, and it is possible to determine J_G experimentally

by measuring the variables in the integrand along the boundary. This type of measurement was performed by Read and McHenry (Reference 32) to determine J for a single-edge notch specimen. Notice, however, that the J_W -integral does not require the steady-state temperature condition, but J_G does.

6. THE J_θ -INTEGRAL BY AINSWORTH, ET AL.

The J_θ -integral introduced by Ainsworth, et al. (Reference 4) is given by

$$J_\theta = \int_\Gamma (n_1 W - t_i u_{i,1}) ds + \int_A \sigma_{ij} \varepsilon_{ij,1}^\theta dA \quad (27)$$

where ε_{ij}^θ is the thermal strain,

$$W(\varepsilon'_{ij}) = \int^{\varepsilon'_{ij}} \sigma_{ij} d\varepsilon'_{ij} \quad (28)$$

and

$$\varepsilon'_{ij} = \varepsilon_{ij} - \varepsilon_{ij}^\theta$$

Notice that ε'_{ij} includes the elastic and plastic strain due to mechanical loads. For elastic deformation it can be easily shown that J_θ is identical to J_W . The J-integral is valid only within the deformation theory of plasticity, and it cannot be used with unloading after plastic deformation and material inhomogeneity. Ainsworth, et al. (Reference 4) asserts that J_θ is related to the potential energy by

$$J_\theta = \lim_{\Delta a \rightarrow 0} \frac{1}{\Delta a} [\int_\Gamma n_j \sigma_{ij} \Delta u_i ds - \int_A \sigma_{ij} \Delta \varepsilon_{ij} dA] \quad (29)$$

where Δ denotes the increment as the crack length changes from a to $a + \Delta a$. In fact, they started from Equation 29 and arrived at Equation 27 in their derivation. It is noted, however, that J_θ given by Equation A7 in Reference 4 is not identical to Equation 27 unless the integration of $n_1 W$ along Γ_ε vanishes. Notice also that their Equation A7 is the same as the \hat{J} -integral given by Equation 36.

7. THE J*-INTEGRAL BY BLACKBURN

Blackburn (Reference 5) proposed the J*-integral defined by

$$J^* = \int_{\Gamma_c} (\frac{1}{2} \sigma_{ij} u_{i,j} dx_2 - t_i u_{i,1} ds) \quad (30)$$

or alternatively

$$J^* = \int_{\Gamma+\Gamma_c} (\frac{1}{2} \sigma_{ij} u_{i,j} dx_2 - t_i u_{i,1} ds) + \int_A (\frac{1}{2} \sigma_{ij} u_{i,j,1} - \frac{1}{2} \sigma_{ij,1} u_{i,j}) da \quad (31)$$

It can be shown that the second term in Equation 31 vanishes in the elastic case; thus, J* is identical to J. In the thermoelastic case, it is identical to J_W, J_G, and J_θ in view of the near-field expression of these P-I integrals. Furthermore, the J*-integral given by Equation 28 is path independent even in the presence of material inhomogeneity since only the equilibrium and the divergence theorem were used to obtain Equation 31 from Equation 30. For power-law hardening materials, direct substitution of the HRR field furnishes the relation between J* and J:

$$J^* = g(n)J \quad (32)$$

where n is the hardening coefficient, and g(n) can be computed once the angular variation of field variables is known (see Appendix). For general elastic-plastic materials with smooth stress-strain curves, the integrands in Equation 31 are continuous everywhere in the body if the material undergoes either loading or unloading in the global sense. For loading conditions that give rise to loading and unloading zones in the body at the same time and for materials with kinks in the stress-strain curves (such as bilinear material), the continuity of the stresses and the derivatives of displacement must be guaranteed across the loading-unloading boundary or the elastic-plastic boundary for the path independence of J*. The continuity of the stresses can be proven from the equilibrium of a thin slice of the body containing the boundary and from the continuity of the effective stress. It is also believed that the strain will be continuous across the boundary (excluding the elastic/

perfectly plastic material) considering that the stress continuity will ensure the continuity of the elastic strain and the plastic strain will change continuously across the boundary (but the plastic strain rate is discontinuous). Continuity of strain will then guarantee the continuity of the displacement gradient; hence, use of the divergence theorem (Equation 4), and consequently the path independence of J^* , is also justified under these circumstances. Unfortunately, no numerical results for J^* are found for problems of this sort.

The major difference of J^* from other P-I integrals is that the strain energy density function does not appear in the integral. Instead, an explicit expression $\frac{1}{2}\sigma_{ij}u_{i,j}$ was used. This type of approach will expand the scope of the applicability of the integral, but it makes the physical meaning obscure. In fact, the physical meaning of J^* in the plastic regime is not known and may be difficult to ascertain. The determination of J^* through experiment is not likely unless J^* is related to potential in a manner analogous to Equation 8. Notice that Equation 30 requires accurate measurement of near-field variables, and Equation 31 has an area integral that can be determined only by measuring the field data everywhere in A. Neither of these is expected to be feasible. Other P-I integrals containing area integrals have the same problem.

The computation of Equation 31 requires caution in view that the integrand behaves like r^{-2} in the vicinity of the crack tip. The second term can be written near the crack tip as follows:

$$\int_0^r \int_{-\pi}^{\pi} [r^{-2} f(\theta) + \text{less singular terms}] r d\theta dr \quad (33)$$

Thus, for J^* to exist (this is guaranteed by Equation 27), we must have

$$\int_{-\pi}^{\pi} f(\theta) d\theta = 0 \quad (34)$$

This can be readily verified for linearly elastic materials, as noted by Atluri (Reference 33). For other types of materials, the angular distribution of the field variables is not known explicitly.

Blackburn, et al. (Reference 34) presented some numerical results of J^* for specimens subjected to loading, unloading, and temperature gradient.

8. THE \hat{J} -INTEGRAL BY KISHIMOTO, ET AL.

The \hat{J} -integral proposed by Kishimoto, et al. (Reference 6) is given by

$$\hat{J} = -\int_{\Gamma_{\text{end}}} t_i u_{i,1} ds \quad (35)$$

where Γ_{end} is the path surrounding the so-called "fracture process zone" in which the continuum mechanics fails to work effectively. In the process to arrive at Equation 35, the equilibrium equation and the divergence theorem were used. Also, it was assumed that the displacement field on Γ_{end} is maintained constant in the crack propagation, namely $\partial u_i / \partial a = 0$ on Γ_{end} . With this assumption, \hat{J} represents the rate of work done on the fracture-process zone by the surrounding medium. Then they convert Equation 35 into the following by assuming that Γ_{end} shrinks and vanishes:

$$\hat{J} = -\int_{\Gamma+\Gamma_c} t_i u_{i,1} ds + \int_A \sigma_{ij} \varepsilon_{ij,1} dA \quad (36)$$

They further assumed that

$$\int_{\Gamma_\varepsilon} W^e ds = 0 \quad (37)$$

and obtained

$$\hat{J} = \int_{\Gamma+\Gamma_c} (n_1 W^e - t_i u_{i,1}) ds + \int_A \sigma_{ij} \varepsilon_{ij,1}^* dA \quad (38)$$

where

$$\varepsilon_{ij}^* = \varepsilon_{ij} - \varepsilon_{ij}^e = \varepsilon_{ij}^p + \varepsilon_{ij}^\theta \quad (39)$$

and

$$W^e = \int_0^{\epsilon_{ij}^e} \sigma_{ij} d\epsilon_{ij}^e \quad (40)$$

For linear elastic problems, Equation 37 is not satisfied unless a non-singular crack-tip field is enforced in the analysis. With the singular elastic field, Equation 38 is equivalent to

$$\hat{J} = \int_{\Gamma_\epsilon} (n_1 W^e - t_i u_{i,1}) ds \quad (41)$$

which is the usual J-integral. Thus, \hat{J} computed from Equation 38 is not identical to \hat{J} computed from Equation 36. For thermoelastic problems it can be easily shown that \hat{J} given by Equation 38 is identical to J_θ , J_w , J_G , and J^* provided that the material is homogeneous. It is also noted that the physical meaning they claim is valid if there is a finite, rigid, fracture-process zone at the crack tip - independent of the crack size. It is unclear, however, whether or not the singular crack tip field is recovered when Γ_{end} shrinks and vanishes and, if it is recovered, whether or not the physical meaning remains intact.

For elastic-plastic problems, the crack tip field is dominated by plastic deformation, as verified by Hutchinson (Reference 16) for power-law hardening materials. The hypothesis of Equation 37 is, therefore, acceptable for a vanishingly small fracture-process zone, and Equation 38 would be practically identical to Equation 36. It is also mentioned that \hat{J} , given by Equation 36, can be related to J for linear elastic and power-law hardening materials by

$$\hat{J} = \begin{cases} J(2\nu^2 + \nu + 3)/4 & \text{elastic materials, plane stress} \\ J(3 - 2\nu)/4(1 - \nu) & \text{elastic materials, plane strain} \\ J h(n) & \text{power-law hardening materials} \end{cases} \quad (42)$$

where $h(n)$ is a function of the hardening coefficient only (see Appendix).

The \hat{J} -integral given by Equation 36 can be used for elastic-plastic materials subjected to loading and unloading. Aoki, et al. (Reference 35)

demonstrated the path independence of \hat{J} for a specimen subjected to global loading and then unloading. For problems undergoing local unloading, no numerical results are available; however, it is expected that \hat{J} will again be path independent in view that the integrand of the line integral will be continuous across the loading/unloading boundary as described in the discussion of the J^* integral. Equation 36 can also be used with a temperature gradient and material inhomogeneity since no constitutive relations were assumed in the derivation.

The computation of the area integral in \hat{J} is similar to that of the J^* integral. The experimental measurement of \hat{J} would be as difficult as that of J^* .

9. THE ΔT -INTEGRALS BY ATLURI, ET AL.

Atluri (Reference 36) proposed a path-independent integral, given in the incremental form, which takes into account finite strain, body force, inertia, and crack surface traction. Temperature gradient and material inhomogeneity were not included in the formulation. The small-deformation version of the ΔT -integral is given by

$$\begin{aligned}\Delta T &= \int_{\Gamma+\Gamma_c} [n_1 \Delta W - n_j (\sigma_{jk} + \Delta \sigma_{jk}) \Delta u_{k,1}] ds - \int_A \sigma_{jk,1} \Delta u_{j,k} dA \\ &= R + \int_{\Gamma_\epsilon} [n_1 \Delta W - n_j (\sigma_{ij} + \Delta \sigma_{ij}) \Delta u_{i,1}] ds\end{aligned}\quad (43)$$

where

$$\Delta W = (\sigma_{ij} + \frac{1}{2} \Delta \sigma_{ij}) \Delta u_{i,j} \quad (44)$$

and R is the contribution of the discontinuities in the material response along the loading/unloading boundary. The incremental stress $\Delta \sigma_{ij}$ is related to the incremental strain by the Prandtl-Reuss equation:

$$\Delta \sigma_{ij} = 2\mu \Delta \epsilon_{ij} + \lambda \delta_{ij} \Delta \epsilon_{kk} - \frac{12\alpha\mu^2 \Delta \epsilon_{kl} \sigma'_{kl} \sigma'_{ij}}{\sigma'_{mn} \sigma'_{mn} (6\nu + 2\partial F / \partial W^P)} \quad (45)$$

where

$$\sigma'_{ij} = \sigma_{ij} - 1/3 \delta_{ij} \sigma_{kk} \quad (46)$$

$$F = \frac{3}{2} \sigma'_{ij} \sigma'_{ij}{}^2 - F_0(W^P) = 0 \quad (47)$$

and

$$W^P = \int \epsilon_{ij}^P \sigma_{ij} d\epsilon_{ij}^P \quad (48)$$

The ΔT -integral defined by Equation 43 includes the measure of the severity of conditions near the crack tip and also the effect of the transition from loading to unloading zones in A. The ΔT -integral was proven to be path independent for paths in either the entirely loading zone or entirely unloading zone. Atluri (Reference 36) also showed, for the case of nonsingular crack tip field, that the ΔT -integral implies the rate of change in potential energy (per unit crack length) in the time interval τ to $\tau + \Delta\tau$ of two bodies that are identical in shape and load history but have an infinitesimal difference in crack length.

Atluri, Nishioka, and Nakagaki (Reference 7) later presented two modified versions of the ΔT -integral. In the new integrals, denoted by ΔT_p and ΔT_p^* , the contribution of the elastic-plastic boundary was expressed in terms of field variables. The ΔT_p^* and ΔT_p integrals are given by:

$$\begin{aligned}\Delta T_p^* &= \int_{\Gamma_e} [n_i \Delta W - (t_i + \Delta t_i) \Delta u_{i,1} - \Delta t_i u_{i,1}] ds \\ &= \int_{\Gamma + \Gamma_c} [n_i \Delta W - (t_i + \Delta t_i) \Delta u_{i,1} - \Delta t_i u_{i,1}] ds \\ &\quad + \int_A [\Delta \sigma_{ij} (\epsilon_{ij,1} + \frac{1}{2} \Delta \epsilon_{ij,1}) - \Delta \epsilon_{ij} (\sigma_{ij,1} + \frac{1}{2} \Delta \sigma_{ij,1})] dA\end{aligned}\tag{49}$$

and

$$\begin{aligned}\Delta T_p &= \int_S [n_i \Delta W - (t_i + \Delta t_i) \Delta u_{i,1} - \Delta t_i u_{i,1}] ds \\ &= \int_{\Gamma + \Gamma_c} [n_i \Delta W - (t_i + \Delta t_i) \Delta u_{i,1} - \Delta t_i u_{i,1}] ds \\ &\quad + \int_{A_s - A_\Gamma} [(\sigma_{ij,1} + \frac{1}{2} \Delta \sigma_{ij,1}) \Delta \epsilon_{ij} - (\epsilon_{ij,1} + \frac{1}{2} \Delta \epsilon_{ij,1}) \Delta \sigma_{ij}] dA\end{aligned}\tag{50}$$

In the ΔT_p -integral, S is the external boundary including the crack surface, A_Γ is the area enclosed by Γ , and A_s is the total area. The area integrals on the extreme right-hand sides of Equations 49 and 50 vanish when the loading

is proportional. Thus, inasmuch as the deformation theory of plasticity is applicable, the following identity holds:

$$\Delta T_p^* = \Delta T_p = \Delta J \quad (51)$$

where

$$\Delta J = \int_{\Gamma_\varepsilon} [n_1 \Delta W - (t_i + \Delta t_i) \Delta u_{i,1} - \Delta t_i u_{i,1}] ds \quad (52)$$

is considered the increment of the J-integral due to the increment of the external loading in the time interval τ to $\tau + \Delta\tau$.

Equation 49 shows that ΔT_p^* is a direct measure of severity of deformation at the crack tip, but it is not experimentally measurable. On the other hand, in view of Equation 50, ΔT_p is not related to the immediate crack-tip field unless the loading is proportional. Notice, however, ΔT_p can be determined by measuring field data along the external boundary. Nakagaki, et al. (Reference 37) presented some numerical data to demonstrate the path independence of ΔT_p^* and ΔT_p for a compact tension specimen that was plastically loaded, unloaded, and then reloaded. They also computed ΔJ at the peaks using Equation 52 and showed that it is no longer path independent after the specimen is unloaded.

A salient feature of the ΔT_p^* and ΔT_p -integrals is that they are based on the incremental theory of plasticity; this enables these P-I integrals to be used even in nonproportional loading and elastic unloading following plastic deformation. It can also be verified, by adding $-(2\mu + 3\lambda) \alpha \sigma_{ij} \Delta\theta$ to Equation 45 and following the procedure in Reference 7, that Equations 49 and 50 can be used without change for thermomechanical loading. Material inhomogeneity is also taken into account in Equations 49 and 50.

The computation of ΔT_p^* would be similar to other P-I integrals involving area integrals. The behavior of the integrand of the area integral of ΔT_p^* in the vicinity of the crack tip is similar to the area integral of Equation 31.

Atluri et al. (Reference 7) showed that ΔT_p is related to the rate of an incremental energy ($\Delta\Pi$) with respect to crack length if the loading is proportional. For nonproportional loading, the physical meaning of ΔT_p is not clear nor is that of ΔT_p^* .

10. ENGINEERING APPLICATIONS

In this report, several P-I integrals have been examined on a theoretical basis. To be useful for engineering applications, a P-I integral must satisfy each of three conditions.

First, the integral must be path independent for realistic loading and temperature conditions. The integral must be calculable without difficulties and must be reasonably path-independent; that is, path-independence must be achieved at least numerically, even though no theoretical justification exists, outside of the intense deformation zone at the crack tip.

Second, the integral must be determined (or closely approximated) from measurements made on laboratory test specimens at various stages of loading. If the integral can be expressed as the rate of change of a potential with crack length, it can in principle be determined from a load-displacement record. If the integral consists only of a line integral applicable over the specimen boundaries, it may be measured using strain gages and extensometers as in Reference 25. Area integrals would require a whole-field stress and strain analysis. While such may be possible, they are tedious, and results are not immediately available.

Third, the integral must correlate various types of crack-propagation behavior. That is, crack propagation must depend on the imposed value of the integral and be independent of crack size and specimen or component geometry. This condition is most likely to be satisfied if the integral is a measure of the crack-tip severity and if a sound physical meaning is associated with the P-I integral in relation to crack propagation.

The above three aspects are summarized in Table 1 in conjunction with the deformation and temperature conditions under which the integrals are path independent.

Table 1. Summary of P-I Integrals.

(1) P-I Integral	Measure of Crack Tip Severity	Physical Meaning (2)			Capability to Handle				Computation (Integrals Involved)	Experimental Measurement (5)
		Elastic	Thermo- elastic	Plastic	Prop. loading (plastic regime)	Nonprop. & Loading Unloading	Thermal Strain	Material Inhomogeneity		
J	Yes	$-\frac{\partial P}{\partial a}$	-	$-\frac{\partial P}{\partial a}$	Yes	No	No	No	Line	Yes
J _W	Yes	$-\frac{\partial P}{\partial a}$	$-\frac{\partial \phi}{\partial a}$ (3)	-	No	No	Yes	No	Line + Area	Yes
J _G	Yes	$-\frac{\partial P}{\partial a}$	$-\frac{\partial \phi}{\partial a}$	-	No	No	Yes	No	Line	Yes
J ₀	Yes	$-\frac{\partial P}{\partial a}$	$-\frac{\partial \phi}{\partial a}$	$-\frac{\partial P}{\partial a}$	Yes	No	Yes	No	Line + Area	Yes
J*	Yes	$-\frac{\partial P}{\partial a}$	$-\frac{\partial \phi}{\partial a}$	Unknown	Yes	Yes	Yes	Yes	Line + Area	No
\hat{J}	Yes	Rate of work done to crack tip by surrounding material (4)			Yes	Yes	Yes	Yes	Line + Area	No
ΔT_p^*	Yes	$-\frac{\Delta \Pi}{da}$ for prop. loading			Yes	Yes	Yes	Yes	Line + Area	No (6)
ΔT_p	No	Unknown for nonprop. loading			Yes	Yes	Yes	Yes	Line + Area	Yes

Note: (1) $J = J_W = J_G = J_0 = J^*$ for elastic and thermoelastic deformation of homogeneous materials,
 $J = \Sigma \Delta T_p^* = \Sigma \Delta T_p$ for monotonic proportional loading.

(2) P = Potential energy, ϕ = Global thermodynamic potential, $\Delta \Pi$ = Incremental energy

(3) Provided that the internal energy is identical to the strain energy at the crack tip

(4) With the assumption of a rigid fracture process zone at the crack tip independent of the crack size

(5) Yes if it can be expressed as the rate of a potential, or if it can be expressed as a line integral along the boundary

(6) Yes if the loading is proportional

11. CONCLUSIONS

We have reviewed a number of P-I integrals proposed for use in elastic-plastic fracture mechanics. The results are summarized in Table 1. It is seen that there exists as yet no P-I integral with well-established physical background and all the desirable features encountered in applications.

The J -integral and J_θ -integral may be used only for monotonic loading without substantial nonproportionality. For cyclic loading, utilization of these P-I integrals may be possible only by use of appropriate operational definitions of the range of these quantities.

The J_W and J_G integrals are usable only for thermoelastic problems with homogeneous material properties. These integrals may be useful for prediction of crack growth in a rather small temperature gradient field and under small-scale-yielding conditions.

The path independence of the J^* , \hat{J} , ΔT_p , and ΔT_p^* integrals is maintained for more general elastic/plastic problems including nonproportional loading, unloading, temperature gradients, and material inhomogeneities. This clearly is a salient feature upon application to crack-growth analyses under cyclic and thermomechanical loading; however, the physical meaning of these P-I integrals should be further investigated.

Finally, it is noted that no significant effort has yet been made to utilize these P-I integrals to consolidate crack-growth data under mechanical or thermomechanical load cycling. Experimental efforts along with further analytical studies are needed to evaluate the utility of these P-I integrals in fracture mechanics.

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APPENDIX

Substitution of the HRR singular solution into the near-field expression of the J^* integral furnishes

$$\int_{\Gamma_\varepsilon} (\frac{1}{2} \sigma_{ij} \varepsilon_{ij} n_1 - \sigma_{ij} n_j u_{i,1}) ds = \alpha K^{n+1} \varepsilon^{(n+1)(s-2)+1} I^* \quad (53)$$

where $s = (2n + 1)/(n + 1)$, ε is the radius of Γ_ε , K is a constant related to the boundary condition (Reference 16), and

$$I^* = \int_{-\pi}^{\pi} \left[\left(\frac{1}{2} \tilde{\sigma}^{n+1} \cos\theta - \sin\theta [\tilde{\sigma}_r (\tilde{u}_\theta - \partial \tilde{u}_r / \partial \theta) - \tilde{\sigma}_{r\theta} (\tilde{u}_r + \partial \tilde{u}_\theta / \partial \theta)] \right. \right. \\ \left. \left. + [n(s - 2) + 1] \cos\theta (\tilde{\sigma}_r \tilde{u}_r + \tilde{\sigma}_{r\theta} \tilde{u}_\theta) \right) \right] d\theta \quad (54)$$

Here $\tilde{\sigma}_e$, $\tilde{\sigma}_r$, $\tilde{\sigma}_\theta$, $\tilde{\sigma}_{r\theta}$, \tilde{u}_r , and \tilde{u}_θ are functions of θ only (Reference 16), associated with the angular distribution of the corresponding stress or displacement components in the vicinity of the crack tip. Notice that I^* is a function of n only. Equation 53 can be written as

$$J^* = g(n)J$$

where $g(n) = I^*/I$, and I is defined by Equation 24 of Reference 16. Similarly,

$$\hat{J} = h(n)J$$

where

$$h(n) = \hat{I}/I$$

and

$$\begin{aligned}
\hat{I} = & - \int_{-\pi}^{\pi} \left(\sin\theta [\tilde{\sigma}_r(u_\theta - \partial u_r / \partial \theta) - \tilde{\sigma}_{r\theta}(u_r + \partial \tilde{u}_\theta / \partial \theta)] \right. \\
& \left. + [n(s - 2) + 1] \cos\theta (\tilde{\sigma}_r \tilde{u}_r + \tilde{\sigma}_{r\theta} \tilde{u}_\theta) \right) d\theta
\end{aligned} \tag{55}$$

Again, \hat{I} is a function of n only.

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16 Abstract The objective is to review the path-independent (P-I) integrals in elastic-plastic fracture mechanics which have been proposed in recent years to overcome the limitations imposed on the J-integral. The P-I integrals considered herein are the J-integral by Rice, the thermo-elastic P-I integrals by Wilson and Yu and by Gurtin, the J*-integral by Blackburn, the J_0 -integral by Ainsworth et al., the \hat{J} -integral by Kishimoto et al., and the ΔT_p and ΔT_p^* integrals by Atluri et al. The theoretical foundation of these P-I integrals is examined with emphasis on whether or not path independence is maintained in the presence of nonproportional loading and unloading in the plastic regime, thermal gradients, and material inhomogeneities. The similarities, differences, salient features, and limitations of these P-I integrals are discussed. Comments are also made with regard to the physical meaning, the possibility of experimental measurement, and computational aspects.					
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